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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

TOPOLOGICAL EFFECTS IN QUASI ONE-DIMENSIONAL AND TWO-DIMENSIONAL SUPERCONDUCTORS WITH SPIN-ORBIT INTERACTION

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The work was performed at the Institute of Physics of Azerbaijan National Academy of Sciences of laboratory of Condensed matter states and quantum phases

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COMMON CHARACTERISTIC OF THE DISSERTATION

The actuality of the topic. Nowadays, the discovery of the quantum Hall effect is considered as one of the greatest achievements contributing to the evolution of the physics. The reason for it that after its discovery at 1980, the effect gave rise to a new generation of matter known as topological phases, which at present have a number of effective applications capable to make revolution at science and technology. Of course, the development of topological insulators and topological superconductors first has made a revolution in theoretical research, and then it has confirmed itself via the advanced experimental applications. Topological insulators can be described as follows: they behave themselves as an insulator in the interior but at surface they contain conducting states. Surface states of the topological insulators looks like as surface states of the quantum Hall effect appeared at the motion of the electrons in the strong magnetic field.

The scientific direction which is known as "topological quantum states" of condensed matter were first encountered with the discovery of the integer and later fractional quantum Hall effects, and the entangled gapped quantum spin-liquid state of integer-spin "quantum spin chains" in the two-dimensional electron systems. The common and important feature of these discoveries was that they could not be explained by known fundamental laws of "condensed-matter physics". It was not at the time apparent that there could be any connection between these two physical phenomena, in the last 30-35 years we understand that their common feature is that they involve "topologically nontrivial" entangled states of matter that are fundamentally different from the previously-known "topologically trivial" states, and this lies at the heart of their unexpected properties.

Properties of topological insulator make these materials attractive for new technologies that require quantum-level manipulations. Chen Yong Laboratory at Purdue University in the US state of Indiana focuses on raising the quality of existing topological insulators, points to spintronics as a field likely to benefit from the material.

Spintronics is the exploring of the intrinsic spin of the electron, and its related magnetic moment, rather than electronic charge in physical processes. Today, a number of memory elements and devices based effects of giant magnetoresistance for computer technology and modern microelectronics basis spintronics were discovered.

Based on the existence of time-reversal and particle-hole invariants in the most of superconductor compounds led to the suggestion of the idea that these compounds can exhibit topological superconductor phase characteristics and then possibility of existence of such a topological superconductor phase was confirmed both theoretically and experimentally. We note that, surface states which occur in the topological superconductors are similar to Dirac cone being energy spectrum of relativistic particles. The quasiparticles are chiral in these states and are protected by time-reversal symmetry and particle-hole symmetry. An external excitation such as paramagnetic impurity, phonon can't destroy these states. The conductive surface states emerge under certain conditions in the topological superconductors, and quasiparticles localized as zero -energy states at the peak point of the Dirac cone, behave such as Majorana fermions. The conductive surface states emerge under certain conditions in the topological superconductors, and in localized zero-energy state quasiparticles at the peak point of the Dirac cone behave such as Majorana fermions. The protection of the Majorana quasiparticles against external excitement also contributes utilizing of Majorana fermions in quantum computing as a tool of information carriers. Majorana particle is its own antiparticle, it is emerged "half" of normal fermion and in the form zero-energy particle; massless, and spin of a particle is same to a normal fermion. Theoretical calculations were shown that Majorana fermions appear in the two-dimensional spinless p+ip wave superconductor model associated with magnetic vortices, as well as at the end of topological wire in one-dimensional spinless p-wave superconductors.

The considering that, it is easier to determine the location and movement of quasiparticle in one-dimensional systems, it has attracted great interest of physicists that particles can be used carriers of quantum information qubit as Majorana fermion in one-dimensional superconducting wires. To be carriers of quantum information Majorana fermions have to satisfy the following conditions:

- The formation of a qubit, i.e., |0> and |1> state. The |0> state means that there are no Majorana fermions at the given point, or two Majorana fermions are vanished being combined together, and the |1> state implies the presence of Majorana fermion at the given point;
- Transferring information by altering location Majorana fermions;
- Protecting information from external influences and reading information (measuring).

As we know, bits are used by information unit of classic computers, but quantum bits or qubits are utilized by quantum computers for the memory units. The qubits are consist of a two-state quantum system. Also, in the quantum theory it is allowed that two distinct quantum states can be applied to a single particle at the same time. When we consider conditions of superposition and chaos, this may come as a surprise to us. Because we have never encountered particles which are the different states at the same times in our daily lives. The superposition is that, the two distinct quantum states must be represented as a single particle at the same time in quantum physics. Heisenberg uncertainty principle and Schrodinger's cat paradox attempt to explain this state.

The purpose and objectives of the study. The main purpose of this work is an investigation into topological phase of the time-reversal invariant new class superconductors, studies of these superconductors' properties in the topological phase, an emergence condition of Majorana fermions. The second scientific direction in this work is an investigation of the topological phase in the quasi one-dimensional semiconductors in the presence of the charge-density wave. In order to solve the above mentioned problems one considers a quasi one-dimensional superconductor of equidistant parallel wires on a plane. It is shown that a time-reversal invariant topological phase appears when the intra-wire s-wave symmetric and inter-wire d-wave symmetric Cooper pairs set up in the structure.

The following problems have been given and resolved for reaching purpose:

1. Our aim is to investigate a time-reversal invariant topological phase which can be realized in periodic quasi one-dimensional superconductor structures on a plane in the absence of a magnetic field. s-wave intra-wire and d-wave inter-wire pairings take place in quasi one-dimensional systems and the sign of wave function of Cooper pairings will be changed by crossing the nodal point between two Fermi surfaces in the absence of a magnetic field. Thereby it should be theoretically shown that time-reversal invariant superconductor system may be transformed into the topological phase;
2. In according to location of correlation between electron in every intra-wire and nearest neighbor inter-wire in the structure consisting of a N-chain on a plane shows that characteristic of all system can be given by studying two topological superconductors chain. One of the goals of dissertation is to show mathematical basic of this logical result, i.e. possibility of expressing a Hamiltonian of N-chain with Hamiltonian of two chains;
3. Investigation of the emergence conditions of Majorana fermions in the topological phase of N-chain superconductor on a plane;
4. Investigation of the emergence of Majorana fermions' dynamics which they arise at the end of a wire and amplitude and period alter in the Josephson current if time-reversal invariant topological phase takes place in the quasi one-dimensional superconductors;
5. Arising of the topological phase in quasi one-dimensional wires on a plane in the presence of applied magnetic field, and in this case changing of the Majorana fermions properties has not been considered in the literature. This problem should be clarified in the dissertation, and studying how the change

energy spectrum symmetry of the Majorana quasiparticles when both Rashba and Dresselhaus spin-orbit interactions exist in the system;

6. The time-reversal invariant and particle-hole symmetry are preserved in one- and quasi one-dimensional system in the presence of Peierls-Frohlich instability when the system is tranformed into the phase with charge-density wave.This fact also indicates that the structure can be transmitted to the topological phase. In the dissertation have to study possibility of transmitting to the topological phase in one- and quasi one-dimensional systems with charge-denisty wave under influence of spin-orbit interactions and external magnetic fields, and show that under which conditions Majorana fermions emerge at the ends of a wire in the topological phase;
7. The emergence of Majorana fermions in the system can be proved by measuring the tunnel current. Therefore, one of the aim of the present work is to calculate the transient current through a tunnel junction made by semiconductors with charge-density wave;
8. One of the objectives of this dissertation is to demonstrate application of charge-density wave systems in quantum computer technology.

Research methods. The methods of computational mathematical, quantum mechanics, computer software packages and other computing technologies were used. Have been studied methods such as using the second quantization operators of the approximation of the Hamiltonian to find the energy spectrum by diagonalization of the Schrödinger equation and to utilize the "Mathematica" software package for technical problems in the study of equations.

The basic provisions for defense:

- ❖ The conditions for the formation of Majorana fermions in one- and quasi-one-dimensional charge-density wave anomaly systems have been investigated and prospects for using of Majorana fermions as quantum information carriers in quantum computers have been studied.
- ❖ The anomaly of zero potential of tunnel current have been studied in one- and quasi-one-dimensional charge-density wave systems. It has been proposed that zero-bias anomaly in tunnel current used for recording information in quantum computers.
- ❖ Time-reversal invariant new superconductor systems have been proposed, properties of the topological phase, conditions of formation of the Majorana-Kramers pairs, and a fractal Josephson system also have been studied in these systems.
- ❖ All changes in the topological phase of the new quasi-one-dimensional superconductor and in the properties of the zero energy Majorana modes have been explored in the case of distortion of the time-reversal invariance.

Scientific novelties of the dissertation:

1. Alternative new systems have been proposed for time-reversal invariant superconductors. These systems can be fabricated by filling a superconducting material into the cavities of dielectric matrices like zeolite and asbestos. It has shown that the equidistant wires on a plane should transform to a time-reversal invariant topological superconducting state belonging to DIII symmetry class provided that s-wave intra-wire and d-wave inter-wire Cooper pairings are formed.
2. The energy spectrum of these systems has been calculated, and possibilities of emerging Majorana-Kramers pairs at the ends of the wire have been studied.
3. In quasi-one dimensional system, the fractal Josephson current with 4π has been indicated to emerge as the result of tunneling Majorana-Kramers pairing at the end of the wire. In

addition Josephson current inside of the bulk will flow between nearest-neighborings wires. It is also possible to differ them from each other.

4. An external magnetic field destroys the time-reversal invariant. In this case changes in the energy spectrum and topological phase have been studied. It is shown that if the system contains both Rashba and Dresselhaus spin-orbit interactions, then Majorana energy dispersion becomes asymmetric even in the absence of magnetic field. In the presence of magnetic field, instead of the Majorana- Kramers pairs a single Majorana fermion emerges at every end of the wire, and the apllied magnetic field introduces a new asymmetry to the energy spectrum of the Majorana fermions. The alters in the energy spectrum have been numerically calculated under influence of the external magnetic field and the Rashba-Dresselhaus spin-orbit interactions.
5. The problem of turning into topological phase has been studied based on the mechanism of charge-density wave Peierls instability in one- and quasi-one-dimensional systems which differ from superconductor. It has been presented that charge-density wave systems turn into topological phase under the influence of the strong spin-orbit interaction and applied magnetic field, and in the topological phase, the Majorana fermions form at the ends of the wire.
6. In the tunnel experiment, on the one side, system was transmitted to the topological phase, when by taken a one-dimensional system with Peierls instability, and other side, tunnel effect has been explored between the two systems when by taken other metallic system. The occurrence of zero energy mod has been displayed inside gap by altering applied magnetic field.

Theoretical and practical significance of the research. One of the main result is a creation of Majorana fermion in topological superconductor wires, which can be used as a quantum computing unit qubit in quantum computers. Recently, idea of using Majorana fermions as a information carrier in the quantum computers has been suggested. The reason of this is a stability of Majorana fermion against external inelastic scattering factors such as temperature fluctuations, phonon, scattering electron-electron correlations and etc. as a result of system symmetry. Currently, the topological phase is created in superconducting crystals, where exist strong spin-orbit interactions, such as InAs, InSb etc., by transforming them to a superconducting phase by means of proximity effect. A conducting states are created in the center of the superconductor band gap by means of an external magnetic field, which is characterized by Majorana quasiparticles. As a result of this procedure, the topological phase in the superconductivity occurs at low temperatures resulting from the proximity effect. On the other hand, application of the magnetic field for transformation of s-wave superconductor to p-wave one and formation of the topological phase reduces the critical temperature of the superconductivity. The new method, suggested by us for emergence of Majorana fermions in a topological semiconductor in the presence of a charge-density wave, has several advantages. First, there are many semiconductor materials where exist Peierls instability with formation of the charge-density wave at higher critical temperatures, even at room temperature. Therefore, this mechanism is more suitable for application to the quantum computers. On the other hand, magnetic field affect very weak on the critical temperature of the charge-density wave phase.

There isn't a need magnetic field for realization of a time-reversal invariant topological superconductor state belonging to Z_2 symmetry class. This property of superconductors makes them even more relevant for application. We have shown that a quasi one dimensional superconductor, consisting of equidistant wires on a plane, can be transformed into the topological phase provided that s-wave symmetric Cooper pairs inside of wire and d-wave symmetric pairs between nearest-neighboring wires are formed. In this systems Majorana fermion emerges without applying magnetic field.

Approbation and application

The main results of the dissertation were discussed at the following international and national scientific conferences:

- "İnternational Conference on Ternary and Multinary Compounds, ICTMC-18" (Salzburg-2012),
- "1st International Scientific Conference of young scientists and specialists", (Baku-2014),
- "5th International Advances in Applied Physics and Materials Science Congress, APMAS" (Fethiye-2015).

The dissertation was also discussed at scientific seminars of the Institute of Physics of ANAS.

Printed works. The presented list of articles in the end of the dissertation to appropriate dissertation topic were published in the following journals:

Azerbaijan journal of Physics, 2012, 2015, 2019; The Reports of National Academy of Sciences of Azerbaijan, 2015; Acta Physica Polonica A, 2016; JETP Letters, 2016; Republic of Azerbaijan Intellectual Property Agency, Patent İ 2019 0002.

Name of the organization performed dissertation. The dissertation was performed at the Institute of Physics of Azerbaijan National Academy of Sciences of "Condensed matter states and quantum phases" laboratory.

Structure and volume of the dissertation work.

The dissertation is presented by introduction, four chapters, main results, 143-pages computer text. The list of references contain 185 titles. The content of the dissertation include 21 pictures and 1 table.

Summary of the dissertation

The dissertation consists of introduction, four chapters, the main results and a bibliographic list of used literature.

In the introduction of the dissertation description of the actuality of the topic, the purpose of the dissertation, the resolved problems for reaching purpose, the basic provisions for defense, scientific novelties and practical significancies of the obtained results are displayed.

In the first chapter was indicated brief overview of the basic theoretical considerations about the properties of the materials needed for further analysis of the theoretical informations obtained in the dissertation. This chapter was shown mechanizm formation of topological insulators and superconductors. A brief description of the invention of topological insulators with the quantum Hall effect in the 1980's, with the well-known experiment by von Klitzing et al. on two-dimensional electron gases was displayed. The information about the discovery of quantum Hall effect led to the birth of topological phases was entered. The quantum spin Hall insulators model given by Kane-Mele was also displayed. A similar classification was also applied to topological superconductors.

In the second chapter are described in detail concept of inducing Majorana modes by using Kitaev model of one-dimensional spinless p-wave superconductor. In the model by Kitaev realizes a one-dimensional topological superconducting wire with single Majorana modes on each end. The model is an example of a nontrivial one-dimensional system in the "D" class. As the "Kitaev wire" is one of the simplest system to realize Majorana fermions, its implementation was the subject of a number of theoretical proposals. The important ingredients are: a superconductor, spin-orbit coupling, and time-reversal breaking. In contrast to the Kitaev model, in 2010, Oreg Y., Rafael F., Open F., and Lutchyn R. M., Sau J. D., Das Sarma S. gave theory of the real model, how Majorana fermions form in one-dimensional systems. They have shown emergence of Majorana fermions in the real system. We note that in the model proposed by Oreg and et al. must be a strong spin-orbit interaction, a Zeeman magnetic field influencing the spin for formation of a topological phase in superconductors. This implies that in this model, energy spectrum is according to Dirac dispersion, and Majorana fermions formed at the point of zero-energy mode. In this model authors have indicated that zero-energy Majorana bound states are formed in various situations when semiconductor wires are situated in proximity to a conventional s-wave superconductor. This occurs when the external magnetic field, the superconducting gap, or the chemical potential vary along the wire. Next the experimental observation of the Majorana fermions by Mourik and others is investigated.

In the third chapter time-reversal invariant new superconductor systems are proposed, and in this system topological phase properties are studied. In 1970 quasi-one-dimensional superconductors have been fabricated artificially by Bogomolov group at the Institute named after A. F. Ioffe. Thus, they have received periodic structures by filling a superconducting material into the cavities of dielectric matrices like zeolite and asbestos crystals under high pressure up to 30 kbar. The regular set of cavities of 5–10Å diameter in zeolite and from 20–30Å up to 100–150Å diameter in asbestos form a periodic lattice of different geometry in one-, two-, and three-dimension. It have been demonstrated the critical temperatures of quasi-one-dimensional superconductors become higher than the critical temperatures of the bulk superconductors. High stress field around the filaments may guarantee higher value of Rashba spin-orbit interactions in the structures and confirms that they turn into the topological phase. This fact is also shown experimentally. We have investigated in our work possibility of the time-reversal invariant superconductivity in new quasi-one-dimensional systems. Here we have also studied probability of transforming superconductor which is protected by time-reversal symmetry into topological phase in the absence of the magnetic field. We have shown that s-wave intra-wire and d-wave inter-wire Cooper pairings can be formed in quasi-one-dimensional chain systems. The equidistant superconducting wires, aligned along x-axes in {x, y} plane, with s-wave intra- and d-wave inter-wire pairings, in the presence of spin-orbit interactions and arbitrary directed homogeneous magnetic field B are described by Hamiltonian

$$\widehat{H} = \sum_j \{\widehat{H}_{j,j} + \widehat{H}_{j,j+1} + \widehat{H}_{j+1,j}\}. \qquad (1)$$

where $\widehat{H}_{j,j+1} = \widehat{H}^{+}_{j,j+1}$ and

$$\widehat{H}_{j,j} = \sum_{\sigma,\acute{\sigma}} \int \frac{dk_x}{2\pi} \{\psi^{+}_{j,\sigma}(k_x)\xi_{k_x}\psi_{j,\sigma}(k_x) +$$
$$+\psi^{+}_{j,\sigma}[2sink_x\left(\alpha(\sigma_y)_{\sigma,\acute{\sigma}} + \beta(\sigma_x)_{\sigma,\acute{\sigma}}\right) +$$
$$+\omega_z\left((\sigma_z)_{\sigma,\acute{\sigma}}cos\theta + \beta(\sigma_x)_{\sigma,\acute{\sigma}}sin\theta cos\varphi + (\sigma_y)_{\sigma,\acute{\sigma}}sin\theta sin\varphi\right)]\psi_{j,\acute{\sigma}}$$
$$+\Delta_0\psi^{+}_{j,\uparrow}(k_x)\psi^{+}_{j,\downarrow}(-k_x) + \Delta_0^{*}\psi_{j,\downarrow}(-k_x)\psi_{j,\uparrow}(k_x)\},$$

$$\widehat{H}_{j,j+1} = \sum_{\sigma,\acute{\sigma}} \int \frac{dk_x}{2\pi} \{t_{\perp}\psi^{+}_{j,\sigma}(k_x)\psi_{j+1,\sigma}(k_x) +$$
$$+i\psi^{+}_{j,\sigma}\left(\bar{\alpha}(\sigma_x)_{\sigma,\acute{\sigma}} + \bar{\beta}(\sigma_y)_{\sigma,\acute{\sigma}}\right)\psi_{j+1,\acute{\sigma}} +$$
$$+\Delta_1\psi^{+}_{j,\uparrow}(k_x)\psi^{+}_{j+1,\downarrow}(-k_x) + \Delta_1^{*}\psi_{j,\downarrow}(-k_x)\psi_{j+1,\uparrow}(k_x)\}.$$

The changes in the energy spectrum symmetry of the Majorana quasiparticles are calculated:

$$\{E^2 + \xi_k^2 - S - \omega_z^2 - |\Delta(k)|^2\}^2 - 4\xi_k^2(S + \omega_z^2) - 4\Delta^2(k)\omega_z^2 -$$
$$-4(\omega_z sin\theta_0 H)^2 - 8\xi_k E\omega_z sin\theta_0 H = 0. \qquad (2)$$

Note that if $\theta_0 = 0$ and $\omega_z = 0$, the energy spectrum obtained from $det|E_0 - H_0| = 0$ - is as follows:

$$E_0 = s\sqrt{(\xi_k^2 \pm \epsilon_s)^2 + \Delta^2(k)},$$

where $s = \pm$ and $\epsilon_s = \sqrt{\left(k_x^2 + k_y^2\right)(\alpha^2 + \beta^2) - 4\alpha\beta k_x k_y}$. The condition $\Delta\left(k_y\right) = 0$ determines the nodal points of the order parameter. $\Delta\left(k_y\right)$ changes sign at $cosk_y = \mp\frac{\Delta_0}{2\Delta_1}$ if $|\Delta_0| < 2\Delta_1$ as moving along k_y from $k_y = 0$ to $k_y = \pi$. At the nodal points

$$E_{0N} = s\left[2t_{\parallel}cosk_x + \mu - t_{\perp}\frac{\Delta_0}{2\Delta_1} \pm \epsilon_{sN}(k_x, k_{yN})\right]$$

and

$$\epsilon_{sN}\left(k_x, k_{yN}\right) = 2\sqrt{(sin^2 k_x + \delta^2)(\alpha^2 + \beta^2) - 4\alpha\beta\delta sin k_x}$$

is the value of ϵ_s at the nodal point $k_{yN} = \arccos(-\frac{\Delta_0}{2\Delta_1})$. $\epsilon_{sN}\left(k_x, k_{yN}\right)$ varies between the maximal

$$\epsilon_{SN}^{max}\left(\pm\frac{\pi}{2}, k_{yN}, \alpha, \beta\right) = 2\sqrt{\alpha^2 + \beta^2}\sqrt{1 + \delta^2 \mp \frac{4\alpha\beta\delta}{\alpha^2 + \beta^2}}$$

and minimal

$$\epsilon_{SN}^{min}\left(k_{x0}, k_{yN}, \alpha, \beta\right) = 2\delta\frac{(\alpha + \beta)|\alpha - \beta|}{\sqrt{\alpha^2 + \beta^2}}$$

values. The order parameter switches sign as the nodal point is crossed. On the other hand, the spin-orbit interactions split the Fermi surfaces. The splitted Fermi surfaces around the nodal points lie in the energetic interval of $\epsilon_{sN}\left(k_x, k_{yN}\right)$ from each other. Non-trivial time-reversal invariant topological phase with $\nu = 1$ is realized when the maximal value of the kinetic energy term (the first three terms) is smaller than the minimal value of the spin-orbit interactions mediated splitting energy $\epsilon_{sN}\left(k_x, k_{yN}, \alpha, \beta\right)$, $\left|2t_{\parallel} + \mu - t_{\perp}\frac{\Delta_0}{\Delta_1}\right| < \epsilon_{SN}^{min}(k_{x0}, \alpha, \beta)$. The superconductor is fully gapped when $\left|2t_{\parallel} + \mu - t_{\perp}\frac{\Delta_0}{\Delta_1}\right| > \epsilon_{SN}^{max}(k_{x0}, \alpha, \beta)$.

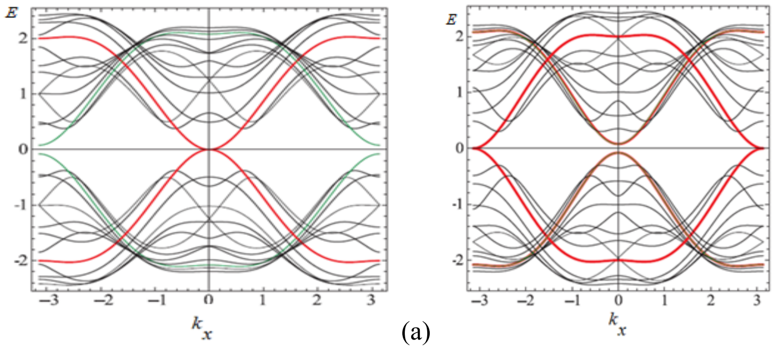

Figure 1: The energy spectrum of strongly anisotropic two-dimensional superconductor as a function of k_x for different values of $k_y = 0.0, \pm 0.5, \pm\frac{\pi}{3}, \pm 1.5, \pm\frac{2\pi}{3}, \pm 2.5, \pm 3.0$. In the both cases we choose $2t_{\parallel} = 1, |\Delta_0| = 0.5 \cdot 2t_{\parallel}$, $t_{\perp} = 0.2 \cdot 2t_{\parallel}$, $\alpha = 0.6 \cdot 2t_{\parallel}$, $\beta = 0$, $\mu = -0.061 \cdot 2t_{\parallel}$ and $0.061 \cdot 2t_{\parallel}$.

The band structure of the topological superconductor with zero energy surface states for this case is drawn in figures 1 (a) and (b). For $\Delta_0 < 0$, Majorana edge states appear at $k_x = 0$, which are shown in figure 1 (a) by red curves. The zero energy states move to the Brillouin zone boundaries, $k_x = \pm\pi$, for $\Delta_0 > 0$ (see figure. 1 (b)).

In addition, we have investigated to alter formation of in the topological phase of the new quasi-one-dimensional superconductor and properties of the zero-energy Majorana mode when the time-reversal invariant is destroyed by applying external magnetic field. The energy dispersion for the Bogoliubov–de Gennes quasi-particles in the presence of an external magnetic field is expressed

$$(E^2 - \xi_k^2 - \epsilon_s^2 - \omega_z^2 - |\Delta|^2)^2 - 8\xi_{k_x}\omega_z sin\theta \Phi_{\mathrm{k}}(\varphi)E -$$
$$-4\xi_k^2(\epsilon_s^2 + \omega_z^2) - 4\left(\omega_z sin\theta \Phi_{\mathrm{k}}(\varphi)\right)^2 - 4\omega_z^2|\Delta|^2 = 0 \quad (3)$$

Where $\Phi_{\mathrm{k}}(\alpha, \beta, \varphi) = sink_x(\alpha sin\varphi + \beta cos\varphi) - sink_y(\alpha cos\varphi + \beta sin\varphi)$ determines the azimuthal angle φ dependence of the energy. $\Phi_{\mathrm{k}}(\alpha, \beta, \varphi)$ emerges due to interference between the co-planar vector fields, such as the magnetic field and the spin-orbit interactions, and introduces a linear in E term in equation (3). This linear term destroys a symmetry of the energy dispersion, and it vanishes for a magnetic field, perpendicular $\theta = 0$) to the superconducting plane, yielding from equation (3) for the energy spectrum,

$$E^2 = \xi_k^2 + \epsilon_s^2 + \omega_z^2 + \left|\Delta(k_y)\right|^2 \pm 2\sqrt{\xi_k^2(\epsilon_s^2 + \omega_z^2) + \left|\Delta(k_y)\right|^2 \omega_z^2}$$

which hosts zero energy state

$$E(0) = s\left|\omega_z \pm \sqrt{\tilde{\mu}^2 + |\Delta(0)|^2}\right|$$

at the center of the Brillouin zone. This expression shows that for $\omega_z > \sqrt{\tilde{\mu}^2 + |\Delta|^2}$ the topological non-trivial phase is realized, where one Majorana bound-state resides at $k = 0$; while for $\omega_z < \sqrt{\tilde{\mu}^2 + |\Delta|^2}$ a topologically trivial gapped state takes place with one Majorana bound-state at the edges.

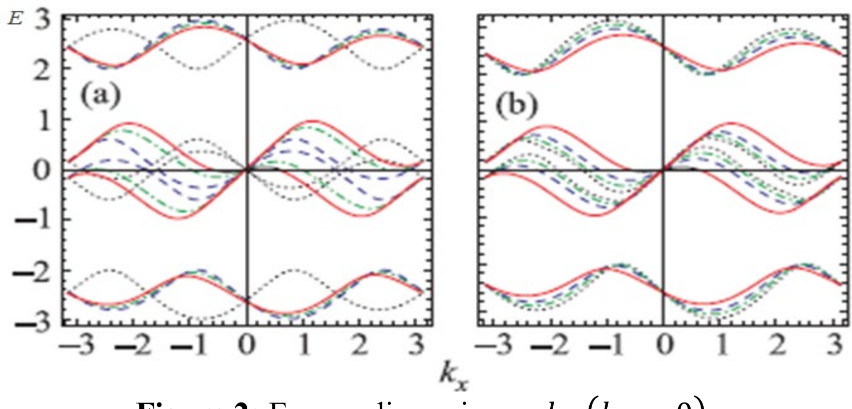

Figure 2: Energy dispersion vs k_x $(k_y = 0)$.

Dresselhaus spin-orbit interactions constant $\beta = 0$ and different values of the in-plane magnetic orientation: $\varphi = \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{\pi}{4}$ for dotted (black), dot-dashed (green), dashed (blue), and solid (red) curves, respectively in figure 2 (a); $\varphi = \frac{\pi}{4}$ and different values of $\beta = 0.4, 0.3, 0.2, 0.0$ for dotted (black), dot-dashed (green), dashed (blue), and solid (red) curves, respectively in figure 2 (b). The dimensionless parameters for both figures are chosen (in the unit of $2t_{\parallel}$) to be $2t_{\perp} = 0.1, \mu = 0, \alpha = 0.5, \epsilon_z = \sqrt{1.7}, |\Delta| = 0.7$.

In the fourth chapter we have shown that Majorana fermions are formed not only in superconductors, but also in a new system - charge-density wave quasi-one dimensional semiconductors. In this study, we have displayed theoretically that formation of a topological phase and emerging of Majorana fermions on the edge of one- and quasi-one dimensional system with Peierls instability in the charge-density wave state in the presence of spin-orbit interaction and external magnetic field in the system. The principle of this work is that a new non-superconductor system can also transform into topological phase and Majorana fermions can form in this system. These systems are one- and quasi-one-dimensional charge-density wave systems. The model considered here is essentially a one-dimensional Hubbard model with on-site Coulomb interactions in the presence of both Rashba and Dresselhaus spin-orbit interactions and a Zeeman magnetic field. Hamiltonian of the system is given by the following form:

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_{int} \tag{4}$$

where $\widehat{H}_0$ and $\widehat{H}_{int}$ are expressed noninteracting part of the Hamiltonian and correlation between electron, correspondingly in momentum space

$$\widehat{H}_0 = \sum_{0<k<\frac{G}{2}} \sum_{\sigma,\sigma'} \{\xi_k c^{+}_{k,\sigma} c_{k,\sigma'} \delta_{\sigma,\sigma'} + \omega_z c^{+}_{k,\sigma} (\sigma_x)_{\sigma,\sigma'} c_{k,\sigma'} + \\ +\alpha \sin(kd) c^{+}_{k,\sigma} (\sigma_z)_{\sigma,\sigma'} c_{k,\sigma'} + \\ +\beta \sin(kd) c^{+}_{k,\sigma} \left(\sigma_y\right)_{\sigma,\sigma'} c_{k,\sigma'} + (k \leftrightarrow k - \frac{G}{2})\},$$

$$\hat{H}_{int} =$$

$$= \frac{1}{2N} \sum_{0<q<G} \sum_{\sigma} \{ \sum_{\substack{-G/2<k<G/2-q \\ q-G/2<k'<G/2}} U(k,k',q)\, c^{+}_{k+q,\sigma} c_{k,\sigma} c^{+}_{k'-q,-\sigma} c_{k',-\sigma} +$$

$$+ \sum_{\substack{G/2-q<k<G/2 \\ -G/2<k'<q-G/2}} U(k,k',q) c^{+}_{k,\sigma} c_{k+q-G,\sigma} c^{+}_{k',-\sigma} c_{k'-q+G,-\sigma} \}.$$

The pole of the single particle Green's function $G^{-1}(E,k) = E - \hat{\mathcal{H}}$ determines the quasiparticle energy:

$$E^2_{CDW} = \xi_k^2 + \gamma^2 \sin^2 k + |\Delta|^2 + \omega_z^2 \pm \pm 2\sqrt{\xi_k^2 \gamma^2 \sin^2 k + \omega_z^2 |\Delta|^2 + \xi_k^2 \omega_z^2}. \quad (5)$$

The energy spectrum at the center of the Brillouin zone for the topological charge-density wave with gapped "bulk" states and zero energy end states can be written as

$$E^{(0)}_{CDW} = E(0) = \left| \omega_z - \sqrt{\mu_t^2 + |\Delta|^2} \right|$$

Figure 3: The energy spectrum is plotted

The energy spectrum is plotted according to equation (5) for fixed values of $t = 0.5, \tilde{\alpha} = 0.8$ in figure 3 and for the following values of the dimensionless parameters: (a) $\tilde{\Delta} = 0.7, \tilde{\omega}_z = \sqrt{1.3}, \tilde{\mu} = -0$, (b) $\tilde{\Delta} = 0.7, \tilde{\omega}_z = \sqrt{2.18}, \tilde{\mu} = -0.3$. A magnetic-field-dominated gap at the center of the band for $\omega_z^2 > \mu_t^2 + |\Delta|^2$ turns to the pairing-dominated one for $\omega_z^2 < \mu_t^2 + |\Delta|^2$. A quantum phase transition from a topological nontrivial to trivial phase occurs at $\omega_z^2 = \mu_t^2 + |\Delta|^2$.

Peierls instability systems have been studied extensively for a long time, and even in recent years, special systems have occured on the Si (111) and Si (555) surface in a zigzag arrangement chain. Quasi-one dimensional chains are fabricated artificially by nesting of gold and silver atoms along chains on these surfaces using a scanning tunneling microscope. In the systems charge-density wave is formed as a result of Peierls instability i.e. a period-doubling lattice, and the system transforms from metallic phase to dielectric.

In the tunnel experiment, tunnel junction of a one-dimensional system with Peierls instability and a metallic system are investigated. Deformation potential arises around the chains in these systems with Peierls instability, at the result, Rashba spin-orbit interaction is emerged. We have studied a theory of emergence of zero-energy Majorana mode in band gap by measuring the tunnel current with changing the external magnetic field or the spin-orbit interaction in the system.

The zero-energy Majorana state in the Peierls gap can be experimentally detected from the tunneling experiments, where the conductivity of the tunneling contact is expressed through the one-particle density of states $\rho(\epsilon, T)$:

$$\frac{\delta G(V,T)}{G^{(0)}} = \int_{-\infty}^{\infty} \frac{d\epsilon}{4T} \frac{\delta\rho(\epsilon)}{\rho^{(0)}} \left[\frac{1}{\cosh^2\frac{\epsilon - eV}{2T}} + \frac{1}{\cosh^2\frac{\epsilon + eV}{2T}} \right]. \qquad (6)$$

At $T = 0$ this expression is written

$$\frac{\delta G(\epsilon)}{G^{(0)}} = \frac{\left[\rho(\epsilon,0) - \rho^{(0)}\right]}{\rho^{(0)}} = \frac{\delta\rho(\epsilon)}{\rho^{(0)}},$$

where $\rho^{(0)}$ is the density of states of a pure system. The density of states is found from the conventional expression

$$\rho(\varepsilon) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sum\nolimits_n \delta\left(\epsilon - E_n(k)\right)$$

where $E_n(k)$ is the energy spectrum for $n = 1,2,3,4$ given by equation (5).

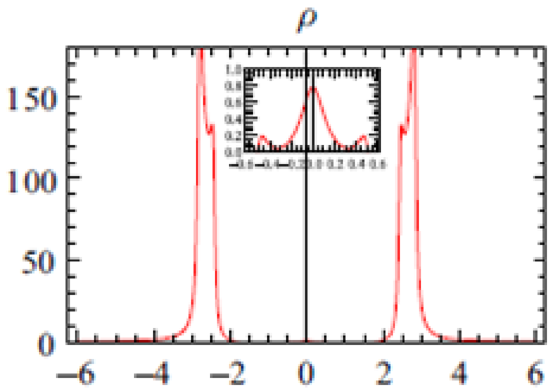

Figure 4: The relative change in the density of states, $\tilde{\alpha} = 0.3, \tilde{\Delta} = 1.0, \tilde{\omega}_z = \sqrt{2.0}$.

The evolution of the central peak in $\delta\rho(\epsilon)/\rho^{(0)}$ is depicted in Fig. 4, where the central peak emerges only for special values of the external parameters satisfying the critical condition $\omega_z^2 = \mu_t^2 + |\Delta|^2$.

The dissertation is completed with results and a list of literature.

MAIN RESULTS

1. A time-reversal invariant topological superconductivity can be realized in a quasi-one dimensional structure on a plane, which is fabricated by filling the superconducting materials into the periodic channel of dielectric matrices like zeolite and asbestos. It is shown that the equidistant wires on a plane should transform to a time-reversal invariant topological superconducting state belonging to DIII symmetry class provided that formation of intra-wire s-wave and inter-wire d-wave Cooper pairs.
2. All alters in the energy spectrum and topological phase of the equidistant superconducting wires are investigated in the case of distortion of time-reversal invariance by applying an in-plane magnetic field to the system. Zeeman magnetic field is shonw to introduce an asymmetry in the Majorana energy dispersion in the general case when both Rashba and Dresselhaus spin-orbit interactions exist in the system. Instead of Kramer-Majorana pair, which appears at the end of a wire in the absence of Zeeman magnetic field, only one Majorana fermion locates at each end in the presence of the magnetic field. The changes in the energy spectrum in the presence of magnetic field and Rashba-Dresselhaus spin orbit interactions are studied theoretically and numerically.
3. The characteristic of all system can be given by studying structure of the Hamiltonian with two wires on a plane.
4. It is demonstrated that, Majorana-Kramers pairs form at ends of the wires in these systems. Instead of Kramers pair, only one Majorana fermion which don't depend on spin arises at each end of the one-dimensional wire in the presence of the magnetic field. These fermions will be caused a 4π periodic Josephson current by flowing through the ends of the quasi-one-dimensional superconductors on a plane.
5. Majorana fermions, emerging at each end of wires of the time-reversal invariant topological quasi-one dimensional

superconductor, may tunnel from one wire to other wire and produces a fractional Josephson current of 4π period. Thereby the current with two periods appears in the system: apart from the normal Josephson current with 2π period in the "bulk", a fractional current with 4π period will appear due to tunnelling of Majorana-Kramer pairs on the end of the wires.

6. We have proposed a theory of formation of a topological phase and emerging of Majorana fermions on the edge of one- and quasi-one dimensional system with Peierls instability in the charge-density wave state in the presence of spin-orbit interaction and external magnetic field in the system.
7. We have displayed a protection of particle-hole symmetry at a certain temperatur range and existence of Majorana fermions in the quasi-one-dimensional system despite of transition from commensurate phase to incommensurate phase.
8. A tunneling current is calculated in the junction of metal and semiconductor with strong spin-orbit interaction and Zeeman magnetic field in the charge-density wave state. A tunneling current is demonstrated to flow through the junction due to emerging a zero-energy Majorana mode in the centre of the band gap when the system is transformed to a topological phase. According to the our calculations, there is no state in the band gap if the system is not in the topological phase. If a strong spin-orbit interaction exists in the system, the latter turns to the topological phase under external magnetic field, and Majorana fermions corresponding to zero modes emerge at the edge of the system, a peak arises around zero energy. The appearance of this peak also indicates the presence of the Majorana fermion. A speculation is provided on experimental objects where the theoretically calculated tunneling current may be observed.

List of published scientific papers in the dissertation topic

1. Məmmədova, S. O. Topoloji izolyator ifratkeçiricilərdə Mayorana fermionlarının kvant informasiya daşıyıcısı olan qubit kimi meydana çıxma mexanizmləri // Azerbaijan Journal of Physics, - Baku: - 2019. C. XXV, №3, - s. 47-49.
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