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ABSTRACT

of the dissertation for the degree of Doctor of Science

THREE AND FOUR FREQUENCY INTERACTIONS OF LASER RADIATIONS IN METAMATERIALS AND NONLINEAR OPTICAL CRYSTALS

Speciality: 2211.01 – Solid State Physics

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GENERAL CHARACTERISTIC OF THE WORK

Relevance of the topic and degree of elaboration. The study of the interactions of laser radiation with different environments is related to the wide use of the effects resulting from these effects in solving various application problems of laser physics.

Nonlinear optical interaction processes play an important role in the creation of strong coherent radiation sources, in determining the material parameters of the environment, in particular, the non-linear receptivity and refractive index, as well as in the non-linear conversion of frequencies from one region to another.

Nonlinear optical effects have also been observed in metamaterials with new perspectives for the development of photonics, in addition to non-linear crystals. The growth of interest in metamaterials is also related to the creation of super- and hyper-lenses with high efficiency on their basis, as well as frequency-tunable converters, miniature antennas and phase shifters. In order to study these new materials in detail, it is necessary to expand the theoretical understanding of frequency conversion processes in them.

One of the fields of application of nonlinear optical effects is optical fibers. The nonlinearity of optical effects in fibers is related to the fact that the refractive index depends on the intensity of the incident radiation and changes as a result of inelastic scattering of waves. Since these materials are more resistant to the interference of electromagnetic fields, the study of nonlinear processes occurring inside them is one of the prerequisites for the design of fiber communication systems.

The discovery of low-loss silicon fibers not only revolutionized the field of optical fiber communication, but also laid the foundation for nonlinear fiber optics as a new field of research.

From the point of view of parametric processes covering the electromagnetic spectrum, the issue of high efficiency of laser frequency converters remains relevant even today.

From this point of view, finding the optimal parameters of the issue to investigate the reasons that prevent the increase of efficiency and increase the efficiency emerges as an important theoretical issue.

Although many years of experimental and theoretical research on the study of frequency transformations have made certain contributions to this field, not all nonlinear processes in nonlinear optics can be quantitatively and qualitatively explained. Because a number of difficulties appear in the analysis of nonlinear processes for real environments and real laser beams. Therefore, various theoretical approximations are used in nonlinear optics to investigate such processes. The most widely used research method in the scientific literature is the constant field approximation (CFA). In this approximation, the complex amplitude of the pump wave is assumed to be constant, and the phase changes of the interacting waves are neglected. In this case, although the solution of the truncated system of equations describing the interaction of waves becomes easier, this approximation correctly describes only the initial stage of the interaction process, and therefore it loses information about some qualitatively important features of the nonlinear optical process. For a more visual description of optical processes in a nonlinear media, it is appropriate to consider phase changes and find the optimal parameters of the problem. Therefore, it is important to apply an approximation that can take into account the phase changes of the waves and give more accurate results. Both the constant amplitude approximation and the constant intensity approximation (CIA), which takes into account the reverse effect of the generated or amplified wave on the phase of the pump wave, were used in the research work. In this approximation, no restriction is placed on the phase of the fundamental wave, with the real part of the complex amplitude remaining constant.

From this point of view, the study of efficiency for finding the optimal parameters of specific problems in metamaterials, nonlinear optical crystals and optical fibers, taking into account phase changes of waves, determines the actuality of the topic.

Object and subject of research. The research objects are the crystals with the second and third order nonlinear susceptibilities, optical resonators, metamaterials and optical fibers. As a subject of research, the three frequency interaction in optical fibers and the nonlinear – crystal placed resonators as well as three and four frequency interactions of optical waves in metamaterials were studied respectively.

Aims and objectives of the research. The aim of the study is to theoretically investigate the transformations of laser radiation frequencies taking into account phase changes in metamaterials, nonlinear optical crystals, compare the obtained results with the results available for light transmitters, as well as solve various issues (parametric interaction of three and four waves, harmonics inside the resonator and cascade scheme generation with, non-stationary change of frequencies, influence of the inhomogeneity of the refractive index on the frequency conversion) analysis of the efficiency of frequency conversion with the aim of finding the optimal values of the optimal parameters - that is, the length of the nonlinear crystal, the optimal phase relationship that yields to the effective conversion of frequencies, and the intensities of pump and idler waves is to do.

To achieve this goal, the following tasks were performed:

- Analytical calculation of the conversion efficiency of the pump wave into a signal wave at low and high frequencies and the amplification factor of the signal wave in the CIA for the case when the energy flow vectors of the pump wave and idler wave are directed against the energy flow vector of the signal wave in a metamaterial with quadratic nonlinear susceptibility and the dependences on various parameters of the problem determination.

- Obtaining analytical expressions for the conversion efficiency into the signal wave frequency, amplification factor of signal wave, reflective index of metamaterial as well as threshold intensity of pump wave in the CIA when the phase and group velocities are opposite at signal frequencies in the metamaterial with cubic nonlinearity. Searching the possibility of compensation of signal wave losses by the losses of pump and idler waves, determining the dependence of the signal wave amplification factor on the idler wave intensity, finding optimum values of the wave number difference (WND) for the maxima of reflective index

- Analytical calculation the spectral density and energy of ultrashort pulses of second harmonic generation and sum frequency generation in the first and second approximations of dispersion theory for the case of non-stationary parametrical interaction of light pulses in metamaterial, determination the dependence of spectral density of a signal wave on the length of medium, intensity of pump, characteristic lengths and frequency modulation parameter of idler wave.

- Analytical calculation the third harmonic intensity and coherent length in ZnO:Er nanocomposite films by taking into account the phase changes of interacting waves, comparison of theoretical and experimental results, determination the dependences of third harmonic intensity on the concentration of ZnO:Er nanoparticles, thickness of films and the pump intensity.
- Calculation the intensities and durations of the signal pulse of second harmonic and sum frequency generation in optical fibers with a weak and strong inhomogeneities of refractive index when the idler pulse posses Gauss profile and comparison with the case of homogeneous medium.
- Analytical calculation the frequency conversion efficiencies for the second harmonic and cascade third harmonic generation in Fabry-Perrot internal cavity by taking into account the phase variations and finding optimum parameters of the problem.
- Comparison the second harmonic generation in nonlinear regime, effects of losses on the second and third harmonic efficiencies in various approximations. Theoretical search of the determining the dispersion of refractive index of plasma on the base of second harmonic generation with dispersion interferometer.

Research methods.

Constant field (CFA) and constant intensity approximations (CIA) are widely, used as research methods in nonlinear optics. İn contrast to the constant field approximation, in the constant intensity approximation only the real part of the complex amplitude of the pump wave remains constant, and no restriction is imposed on the imaginary part.

Basic theses for defense.

1. İn the metamaterial, an increase in the conversion efficiency into the signal wave in the CIA at low frequencies of the pump wave depending on the intensity of the idler wave, the change of the location of the minima with the change of the wave number difference (WND) expressing through the hyperbolic tangent at the optimal value of the WND, and receiving maximum value at the optimum power of the pump wave.

2. İn the constant intensity approximation, to have for amplification factor of signal wave higher values than unit for optimum values of difference in wave numbers at low frequencies of pump wave in case when there is no an idler wave at the input of metamaterial.

3. An existence the dependences on the intensities of both pump and idler waves as well as the total length of the metamaterial for the amplification factor and conversion efficiency of the signal wave at higher frequencies of pump wave, during the four frequency parametric interaction of waves. Under phase - matching conditions efficiency reaches infinite larger values at optimum length of metamaterial.

4.Having reach the signal amplification factor to the resonance values at definite medium length and pump intensity, a non-monotonically increase of signal conversion efficiency depending on coordinate, dependence of threshold value of pump intensity on the nonlinear coupling coefficients, loss parameters and idler wave intensity, decrease in reflective index with increase of idler wave intensity under phase-matching and compensation of signal wave losses by the losses of other waves in metamaterials during four frequency stationary parametrical interaction of waves.

5.For the signal pulse spectral density in metamaterial to be higher by a factor of one order compared to the ordinary nonlinear medium at central frequency of spectrum, narrowing of the spectrum of signal wave with increase in the ratio of nonlinear length to the quasi-static length, being the spectrum symmetric under more less values of nonlinear length compared to the dispersion spreading length, during the non-stationary interaction of long pump pulse with the short idler of Gauss profile with quadratic phase modulation.

6. Decrease in the signal pulse spectral density with increase of the phase modulation factor of idler pulse and expanding of central maximum, transfer of energy from the central maximum to the side maxima with increase in the idler pulse intensity, obtaining relatively higher values of energy not at the output of metamaterial, but at its input.

7. An increase in the third harmonic intensity with increase of both pump intensity and concentration of dopant, decrease of coherent length with increase in dopant concentration, decrease in third harmonic intensity with increase the thickness of films, increase in third

harmonic intensity with increase in pump intensity in ZnO: Er nanocomposite films.

8. Variation in the durations of second harmonic and sum frequency pulses as a result of a weak and strong inhomogeneity compared to homogeneous media in the optical fiber of second order nonlinearity.

9.The optimum phase condition and optimum length of nonlinear crystal being dependent on the pump wave intensity to reach maximum value of efficiency of conversion into second harmonic, a widening the synchronism curves plotted in CIA as compared to the curves in CFA, to be more effective the cascade third harmonic generation under equality of intensities of pump and second harmonic intensities in internal resonator.

10. Decrease in the efficiencies of conversion into second and third harmonics with increase in losses, the results of CIA to be closed to the exact solution compared to the CFA in the nonlinear regime of second harmonic generation.

The scientific novelty of the research. The frequency conversions by taking into account of phase changes of interacting waves in metamaterials, nonlinear crystals with second and third order nonlinearities and optical fibers are theoretically investigated and for the first time:

· The conversion efficiency to the frequency of signal wave and the amplification factor of the signal wave are analytically calculated in the CIA for the three frequency stationary parametric interaction at low and high frequencies of the pump wave in the metamaterial with nonlinear susceptibility, the optimal length of the metamaterial at high frequencies as a function of the intensities of the pump and idler waves, infinite increase in the amplification factor at the optimum value of the length are determined.

· It is shown that the threshold value of the pump wave intensity and optimum value of difference in wave numbers for maximum of reflective index in addition to other parameters also depend on the idler wave intensity during four frequency interaction in metamaterials with third order nonlinear susceptibility.

· It is obtained analytical expression for the compensation of signal wave losses by the losses of contrary propagating pump and idler waves during four frequency interaction in metamaterials with third order nonlinear susceptibility.

· It was determined that the spectral density of signal pulse in metamaterial is higher for one order as compared to the ordinary medium, the spectrum becomes narrowed with increase in the ratio of nonlinear length to the quasi-static length, the higher values of energy is obtained not at the output but at the input as a result of non-stationary interaction of ultrashort pulses in metamaterial.

· The intensity of third harmonic generation and coherent length in ZnO:Er nanocomposite films were analytically calculated in CIA and the dependences of intensity on the changes of concentration of nanoparticles, the pump intensity and thickness of nano-films were determined.

· The intensities and durations of the second harmonic and sum frequency pulses were calculated in the CIA for the optical fibers with a weak and strong inhomogeneity of refractive index.

· The efficiencies of conversion into second harmonic and cascade third harmonic were calculated by employment CIA the optimum values of nonlinear medium and phase condition are determined in Fabry - Perrot cavity.

· It was shown that results of CIA are more close to the results of accurate solution as compared to the results of constant field approximation for the tasks of second harmonic generation in nonlinear regime and effect of linear losses on the generation of higher harmonics.

Theoretical and practical significance of the research. The theoretical significance of the work is that the CIA, which is more accurate than the CFA, has been applied to solve many problems. The used method can be applied in radio - physics, nonlinear acoustics and plasma physics, in addition to nonlinear optics problems.

· The method of compensating losses in metamaterials with the "non-linear interaction mechanism of waves propagating in opposite directions" can be applied to amplify signal waves by compensating losses in parametric frequency converters.

The practical significance of the work is as follows:

• "Third harmonic generation method" was developed for the study of ZnO: Er nanocomposite films in the CIA.

• It is possible to use the results obtained in this approach both in the estimation of the parameters of laser systems and in making calculations for the preparation and improvement of their devices.

• The obtained results for pulse durations, taking into account inhomogeneity in optical fibers, can be applied in optical communication technologies.

• The results obtained for metamaterials can be useful in the development of frequency converters whose frequency can be adjusted based on these materials.

• The obtained results can also be useful for the determination of the optical properties of the substance, including the refractive index and high-order susceptibilities.

Approbation and implementation. The main results of the research, included in the dissertation work, were discussed in the scientific seminars of the Department of Solid State Physics of BSU, and were presented at the following foreign and domestic conferences: Республиканская межвузовская научная конференция по физике, (Baku, 1992); Republican scientific conference "Physics-93", Baku, (1993); Final scientific conference of Azgosmeduniversiteta for 1995 (Baku, 1996); II Republican Scientific Conference "Actual Problems of Physics" (Baku, 2001, October 30-31); A.I. III Republican Scientific Conference "Actual Problems of Physics" dedicated to Mukhtarov's 85th anniversary (Baku 2004); VI Republican Scientific Conference "Modern problems of physics" (Baku, 2012,); International Scientific and Practical Conference (Prague, 2016); International Conference "Modern Trends in Physics" (Baku, 2017); XIII International Scientific Conference European research (Penza, Russia); "Problems of Physics and Astronomy" international scientific conference of graduate students and young researchers (Baku, May 24-25, 2018); International conference "Modern Trends in Physics" (Baku, May 1-3, 2019); Proceedings of VIII International Scientific and Practical Conference "Science and Practice: Implementation to modern society" (December 26-28, 2020, in Manchester, Great Britain); XXI General-Republic conference of graduate students and young researchers (May 20-21, 2021 Baku); Second International Conference on light and light-based Technologies

ICLLT-2, Gazi University, Turkey (May 27, 2021); Collection of research papers of Scientific and Practical Conference "Current issues of biomedical sciences", Kharkov International Medical University, Kharkov (2021); 8th International Conference on Control and Optimization with Industrial Applications (COIA 2022), Baku August 24-26, 2022.

By changing the intensity and frequency of pump wave, it is possible to prepare frequency-tunable parametric frequency converters based on metamaterials with quadratic nonlinearity. During four frequency interaction in metamaterials, it is possible to control the value of the amplification factor according to the resonances obtained by changing the loss coefficients depending on the intensity of the pump wave. By adjusting the characteristics of the metamaterial and the idler pulse, it is possible to control the overlap process of the counter-propagating wave packets and, therefore, the frequency conversion efficiency over a wide frequency range. The process of compensating signal wave losses in metamaterials with the losses of pump and idler waves propagating against it, can be used in the development of adjustable frequency converters based on metamaterials.

Published scientific works. 36 scientific works on the materials of the dissertation (7 of them in journals with a high impact factor indexed in the "Web of Science" database included in Clarivate Analytics) were published in the materials of 9 International conferences and in the form of 8 abstracts in local and foreign journals. 11 articles (of which 6 are not co-authored) were published in journals recommended by the Supreme Attestation Commission under the President of the Republic of Azerbaijan

The name of the organization where the research work was performed. The work was performed at "Optics and molecular physics" and "Solid state physics" departments of Baku State University.

The total volume of the dissertation with a sign indicating the separate volume of the structural sections of the dissertation.

The dissertation work is placed on 315 pages. The volume of the work consists of 73 pictures, introduction, 6 chapters, conclusions, a list of abbreviations and conventional signs from the bibliography of 266 names used. The volume of the dissertation (excluding gaps in the text and pictures, graphs, appendices and the bibliography) is 400487 characters. Title page - 412 c.(1 p.), Table of contents - 7298 c. (6 p.), Introduction - 45001 c. (22p.), Chap. I - 109973 c.(68 p.), Chap. II - 40231 c. (32 p.), Chap. III - 64430 c. (49 p.), Chap. IV - 41680 c. (33 p.), Chap. V – 40754 c. (32 p.), Chap. VI - 42450 c. (33p.), conclusions - 7933 c. (5 p.), references - 41953 c. (25 p.), abbreviations - 724 c. (1p.)

BASİC CONTENT OF THE WORK

In the **Introduction** an overview of the work done on the research topic is given, the work done on this topic is examined. It is clear that 3-4 decades ago, research in this direction was not completely accurate. The main advantages of the presented dissertation over other works on this topic are justified. The relevance of the topic, the purpose of the research, and a justified statement of the issues raised are also given. The object and subject of the research, the used research methods were explained. Scientific innovations are substantiated, the main provisions defended are given. The practical and theoretical significance of the obtained scientific results was shown, and recommendations were given on the use and application of these results. Approbation and application areas are given, as well as a brief explanation of the content of individual chapters. Also, the name of the organization where the dissertation work was performed, the approval of the work, the publications where the main results obtained in the work were published, the structure and volume of the dissertation are indicated.

In the **first chapter** a general overview of local and foreign scientific literature dedicated to the modern issues faced by researchers in the field of studying the three- and four-frequency interactions of laser radiation in metamaterials and nonlinear optical crystals was given. The results of the research works of leading scientific centers working in this field were analyzed. The main advantages of the presented dissertation over other works on this topic are justified. This chapter also provides a standard method for deriving a reduced system of equations describing the interaction of electromagnetic waves in quadratic nonlinear media. The non-stationary reduced equations of the first and second approximations of the dispersion theory were obtained by applying the slowly varying amplitude method from Maxwell's equations and using the material equations of the medium. The truncated equations obtained for homogeneous media are generalized for the cases of interaction of waves in inhomogeneous media and in a nonlinear cavity.

In the Figure 1 the dependences of reduced amplitude of second harmonic on the WND.

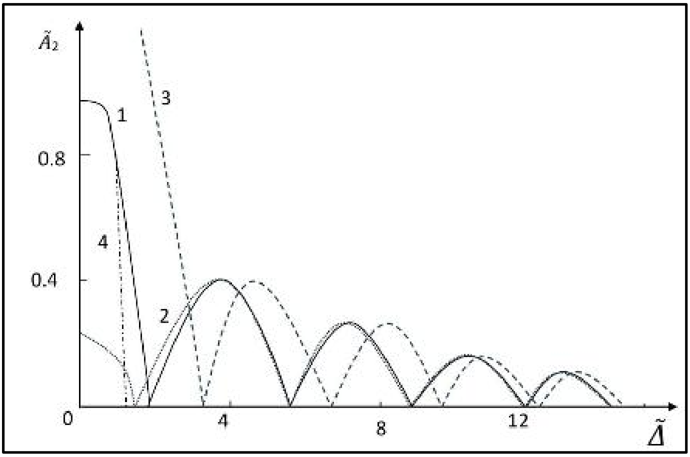

Figure 1. Dependence of dimensionless amplitude of second harmonic ($\tilde{A}_2 = A_2/A_{10}$) on the wave number difference $\tilde{\Delta} = \Delta z/2$ when $z/l_{n/l} = 2$: 1. Accurate solution[1]. 2. Constant intensity approximation[2] (CIA), 3. Constant field approximation[3], 4. Strong mutual interaction approximation[4].

As can be seen, when $\Delta > 2/l_{n/l.}$ calculations of accurate method and CIA give practically the same result, but they are significantly different from the result of CFA. Therefore, this approximation and the

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[1]Ахманов, С.А. Электромагнитные волны в нелинейных диспергируюших средах / Диссертация доктора физ.-мат. наук / С.А. Ахманов, – Москва: – 1967. – 439 с.

[2]Тагиев, З. А., Чиркин А. С. Приближение заданной интенсивности в теории нелинейных волн // Журнал экспериментальной и теоретической физики, − Москва: − 1977. 73, №4, − с. 1271-1282.

[3]Дмитриев, В. Г. Прикладная нелинейная оптика: Генераторы второй гармоники и параметрические генераторы света / В. Г. Дмитриев, Л. В. Тарасов. – Москва: Радио и связь, − 1982. − 352 с.

[4]Ибрагимов, Э. А. Приближение сильного взаимодействия в теории нелинейных волн / Э. А. Ибрагимов, Т. Усманов // ЖЭТФ, − Москва: − 1984. 86, №5, − с. 1618-1631.

exact solution are complementary. Therefore, it is appropriate to use CIA at large values of the difference of wave numbers.

The second chapter is devoted to the theoretical study of the generation of the second harmonic within the laser resonator and the third harmonic in the cascade scheme in the CIA.

A Fabry-Perrot resonator with a non-linear crystal inside increases the coherence length of the crystal by creating feedback through mirrors with a certain reflective coefficient.

In order to find the efficiency of the second harmonic frequency conversion inside the laser resonator the system of truncated equations

$$dA_1/dz + \delta_1 A_1 = -i\gamma_1 A_2 A_1^* e^{i\Delta z},$$
$$dA_2/dz + \delta_2 A_2 = -i\gamma_2 A_1^2 e^{-i\Delta z}, \qquad (1)$$

where $A_j (j = 1,2)$- are the complex amplitudes of interacting waves, δ_j- coefficients of absorption, $\Delta = k_2 - k_1$- difference in wave numbers and $\gamma_j = 8\pi c_{eff}^2 \omega_j^2 |\varepsilon_j| / k_j c^2$- are the nonlinear wave coupling coefficients.

To calculate the complex amplitude of second harmonic in internal cavity we apply following boundary conditions:

$$A_1(z = 0) = A_1(l) e^{i\varphi_1(2d) + i\varphi_{r,1}},$$
$$A_2(z = 0) = A_2(l) e^{i\varphi_2(2d) + i\varphi_{r,2}}, \qquad (2)$$

where $\varphi_1(2d)$, $\varphi_2(2d)$ - phase shifts of pump and second harmonic waves in between of nonlinear crystal and second mirror, $\varphi_{r,1}$, $\varphi_{r,2}$ - are the phase shifts for the waves reflected from the second mirror, $z = 0$ corresponds to the enterance of the nonlinear crystal.

Solving the set of equations (1) for the complex amplitude of the second harmonic wave yields:

$$A_{2,çıxış}(z) = A_2(l)\left(a + be^{i\Psi} + ice^{i\Psi}\right) \times$$
$$\times\, e^{-\frac{\delta_2 + 2\delta_1 - i\Delta}{2} l + i\varphi_2(2d) + i\varphi_{r,2}}. \qquad (3)$$

Where

$$A_2(l) = -i\gamma_2 A_{10}^2 sinc\lambda z \qquad \lambda = \left(\Delta^2 + 8 l_{q/x}^{-2}\right)^{1/2} / 2,$$

$$a = cos\lambda_2 z, \qquad b = (\lambda_1/\lambda_2)cot\lambda_1 l \cdot sin\lambda_2 l,$$

$$c = \frac{\Delta}{2\lambda_2} sin\lambda_2 l, \qquad \lambda_1^2 = 2G_1^2 - (\delta_2 - 2\delta_1 - i\Delta)^2/4,$$

$$G_2^2 = \gamma_1\gamma_2 I_1(l), \qquad \lambda_2^2 = 2G_2^2 - (\delta_2 - 2\delta_1 - i\Delta)^2/4,$$

$$I_1(l) = A_1(l)A_1^*(l),$$

$$\Psi = \Psi' + \Delta l = \Delta l + 2\varphi_1(2d) - \varphi_2(2d) + 2\varphi_{r,1} - \varphi_{r,2},$$

$$\Psi' = 2\varphi_1(2d) + 2\varphi_{r,1} - \varphi_2(2d) - \varphi_{r,2}.$$

From the expression (3) for the second harmonic intensity we get:

$$I_{2out.} = I_2(l)[a^2 + b^2 + c^2 + 2a(bcos\Psi - csin\Psi)] \times$$

$$\times \exp(-2\delta l). \qquad (4)$$

It is seen from (4) that intensity of second harmonic is periodically varying depending on phase at the output of nonlinear medium and for the effective conversion of frequencies following optimum phase condition should be satisfied:

$$\Psi + arctg\left(\frac{\Delta}{\lambda} tg\lambda l\right) = n\pi, \quad n = 1, 2. \qquad (5)$$

The parameter λ in the optimum condition (5) depends on the pump intensity. When the condition of $\Psi = 2n\pi$ is satisfied, for the conversion efficiency we obtain:

$$h = \gamma_2(\gamma_1\lambda_1^2)^{-1} G_1^2 sin^2\lambda_1 l[(a + b)^2 + c^2] \times e^{-4\delta l}. \qquad (6)$$

It can be seen from this equation, that when taking into account reverse reaction of the second harmonic wave on the phase of the pump wave in the resonator, a qualitatively new picture is observed as compared to the CFA. In the dependence of the frequency conversion efficiency on the wave numbers difference, the coordinates of the minima of the harmonic wave intensity depend on the intensity of the pump wave; with an increase in the intensity of the pump wave, the minima of the frequency conversion efficiency shift to smaller values of the wave number difference; the efficiency of the second harmonic conversion depends on the intensity of the fundamental wave as well as length of the nonlinear medium and has a maximum. At small values of the frequency conversion efficiency, the optimal length ($\delta_j = 0$) corresponding to the maximum of the intensity of the crystal is calculated as follows[5].

$$l^{opt.} = \left(2G_1^2 + \frac{\Delta^2}{4}\right)^{-1/2} \cdot arcsin\left(\frac{8\lambda^2}{16\lambda^2 - \Delta^2}\right)^{1/2}. \quad (7)$$

In the CIA-the optimum length depends on the pump intensity. Figure 2 shows the dependence of the efficiency of the conversion into the second harmonic in the resonator on the intensity of the pump wave. Such a non-monotonic character of the dependence of $\text{h}(I_{10})$ in the CIA differs qualitatively from the result of CFA, as the efficiency of the conversion in the latter is monotonic with the increase of the intensity of the fundamental wave increases.

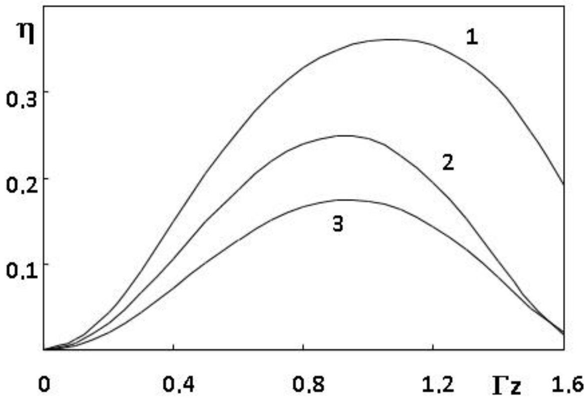

Figure 2. Dependence of frequency conversion efficiency into second harmonic ($\text{h}=I_{2,out.}/I_{10}$) on the dimensionless intensity of pump wave in a laser cavity ($\tilde{\Gamma} = \Gamma z$). $\delta z = 0.1$ (1, 3), 0 (2) və $\Delta z/2$=1.5 (1), 2 (2, 3).

When the length of the non-linear crystal is greater than the coherent length, at which the frequency conversion efficiency reaches its maximum, the energy of the newly generated wave is converted back

[5]Тагиев, З.А., Касумова, Р.Дж., Амиров, Ш.Ш. Теория внутрирезонаторной генерации второй гармоники в приближении заданной интенсивности // – Санкт-Петербург: Оптика и спектроскопия, – 1993. 75, №4, – с. 908-913.

into the energy of the main wave, and thus the frequency conversion efficiency decreases. The coherent length depends on the difference in wave numbers of the interacting waves.

Therefore, it is reasonable to use an arrangement consisting of several crystals placed in series to compensate the phase shifts that reduce the frequency conversion efficiency.

In order to generate the third harmonic with the cascade scheme, first the conversion to the second harmonic occurs in the second non-linear crystal, and then the third harmonic is generated in the form of a sum-frequency wave in the first crystal[6]. The unconverted part of the fundamental wave frequency in the first crystal and the second harmonic frequency generated in the second crystal are added: $\omega + 2\omega = 3\omega$. Since the energy of one quantum of the harmonic wave is twice as large as the energy of one quantum of the charge wave, in this process the energy of the second harmonic must be spent twice as much as the energy of the main harmonic. Hence it follows that the ratio of energies at the input of the second crystal should be 1:2 for high efficiency concentration of energy.

The following expression for the intensity of the third harmonic in the sum-frequency waveform is obtained from the solution of the system of reduced equations ($\delta_3 = \delta_1 + \delta_2$):

$$I_2 = \frac{\gamma_2'}{\gamma_1'} G^2 I_{10} \varkappa_1{}^{-2} sin^2 \varkappa_1 l_2 (a^2 + b^2 + 2abcos\Psi) \times$$

$$\times \exp(-2\delta_1 l_1), \qquad (8)$$

where

$$\varkappa_2^2 = 2\rho G^2, \qquad a = cos\varkappa_2 l_2, \qquad b = \rho^{-1/2} ctg\varkappa_1 l_2 sin\varkappa_2 l_2;$$

$$\Psi = 2\varphi_1(2d_2) - \varphi_2(2d_2) + 2\varphi_{r_1} - \varphi_{r_2},$$

$$\rho = cos\varkappa_1 l_2, \qquad G^2 = \gamma_1' \gamma_2' e^{-2\delta_1 l_1}.$$

When the optimum phase condition satisfies

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[6]Тагиев З. А. Каскадная генерация третьей гармоники в лазерном резонаторе / З. А. Тагиев, Р. Дж. Касумова, Ш. Ш. Амиров [и др.] // Квантовая электроника, – Москва: - 1994. 24, №10, – с. 968-970.

$$Y = 2\varphi_1(2d_2) - \varphi_2(2d_2) + 2\varphi_{r_1} - \varphi_{r_2} = 2\pi n, n = 0, 1, 2, \ldots$$

the frequency conversion efficiency into third harmonic becomes maximum and is given by

$$\eta_3 = I_3/I_{10} = \gamma_3^2 I_{10}^{-1} I_1 I_2 l_1^2 sinc^2 Г_1 l_1 \exp(-2\delta_3 l_1), \qquad (9)$$

where $Г_1^2 = G_1^2 + G_2^2 + \frac{\Delta^2}{4}$; $G_1^2 = \gamma_2\gamma_3 I_1$, $G_2^2 = \gamma_1\gamma_3 I_2$

The sum frequency conversion efficiency depends on the intensities of both the pump wave and the second harmonic wave. The presence of a maximum in the dependence of efficiency on the intensity testifies that the pump wave has an optimal intensity. At the same values of the absorption coefficients, the optimum length, corresponding to the maximum of conversion efficiency of the crystal decreases with the increase of the wave number difference.

In a dispersive medium since the pulse components of different frequencies propagate with different phase velocities $(c/n(\omega))$ when travel through the crystal or metamaterial it spreads out or undergoes deformation. The spatial analogue of this effect is the diffraction of a beam with a finite aperture. Calculations showed, that the duration of dispersion propagation pulses is 10^{-13}s. and has a significant role when it is smaller than that. When the linear losses in a nonlinear medium are small the three wave interaction in the second approximation of the dispersion theory is described by the following system of truncated equations[7].

$$\left(\frac{\partial}{\partial z} + \frac{1}{u_1}\frac{\partial}{\partial t} - i\frac{g_1}{2}\frac{\partial^2}{\partial t^2}\right)A_1(z,t) = i\gamma_1 A_3 A_2^* e^{i\Delta z},$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{u_2}\frac{\partial}{\partial t} - i\frac{g_2}{2}\frac{\partial^2}{\partial t^2}\right)A_2(z,t) = -i\gamma_2 A_3 A_1^* e^{i\Delta z}, \qquad (10)$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{u_3}\frac{\partial}{\partial t} - i\frac{g_3}{2}\frac{\partial^2}{\partial t^2}\right)A_3(z,t) = -i\gamma_3 A_1 A_2 e^{-i\Delta z},$$

where A_j (j=1–3) – are the complex amplitudes of the signal, pump

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[7]Ахманов, С. А. и Хохлов, Р. В. Проблемы нелинейной оптики. М.: ВИНИТИ, – Москва: 1964. – 295 с.

and idler waves respectively, u_j - are the group velocities of interacting waves, $\Delta = k_1 - k_2 - k_3$ difference in wave numbers of interacting waves, $g_j = \partial^2 k_j / \partial\omega_j^2$ – are the dispersions of group velocities, and $\gamma_1, \gamma_2, \gamma_3$ - are he nonlinear coupling coefficients, given in the set of equations (1).

If assume the amplitude of a pump to be constant ($A_2 = A_{20} = const.$) the boundary conditions for the complex amplitudes of interacting waves are as follows:

$$A_3^-(l) = R_3 A_3^+(l) e^{-i2k_3 l}; \qquad A_3^+(0) = R_{30} A_3^-(0);$$

$$A_1^+(0) = A_{10}(t); \qquad A_2^+(l) = A_{20}. \qquad (11)$$

By applying the Fourie transformations $A_{1,3}(\omega, t) = \int_{-\infty}^{+\infty} A_{1,3}(z, t) \times$ $\times\ e^{i\omega t} dt$ from the set of equations (10) for the spectral density of sum frequency pulse at the output of cavity we get:

$$S_3(\omega, z) = \frac{(1 - r_3^2) \cdot \gamma_3^2 S_{10}(\omega) I_{20} z^2 sinc^2 \mu z}{1 - 2 r_{30} r_3 c cos Y + r_3^2 c^2}, \qquad (12)$$

where

$$\mu^2 = \Gamma^2 + \frac{\beta^2}{4}, \quad c^2 = cos^2 \mu z + \frac{\beta^2}{4\mu^2} sin^2 \mu z,$$

$$\beta = \frac{1}{2}\omega^2 g - \omega n - \Delta, \ Y = \varphi + arctg\left(\frac{\beta}{2\mu} tg \mu z\right),$$

$$\varphi = \varphi_r + 2k_3 l + \frac{\Delta l}{2} + \frac{\omega^2}{4}(g_1 + g_3) - \frac{\omega n}{2} l, \ \varphi_r = \varphi_{10} + \varphi_1.$$

In the third chapter, the three - wave parametric interactions at the low and higher frequencies of pumping wave in quadratic nonlinear metamaterials, and the four-wave interaction in cubic nonlinear metamaterials are extensively investigated in the constant intensity approximation. In the three and four wave interactions that we have considered, the frequencies of the pump and idler waves fall into the region of positive values of the refractive index, and the frequencies of the signal wave fall into the region of negative values of the refractive index. For the solution of the truncated set of equations[8] when the dielectric and magnetic permeability in a medium have negative values at the frequency ω_1 of the signal wave, and positive values for idler and pump waves with frequencies ω_2 and ω_3 we use the following boundary conditions:

$$A_1(z = l) = A_{1l}\exp(i\varphi_{1l}),\ A_{2,3}(z = 0) = A_{20,30}\exp(i\varphi_{20,30}), \quad (13)$$

here $z = 0$ corresponds to the left input of metamaterial $A_{20,30}$, $\varphi_{20,30}$ - are the amplitudes and phases of both idler and pump waves in the left input of metamaterial respectively, A_{1l}, φ_{1l} - refer to the amplitude and phase of the signal wave at the input from right hand side ($z = l$) of the metamaterial respectively. At the boundary conditions above the wave vectors k_j of interacting waves are in the sense of positive direction of the z - axis.

Thus the five vectors in question (three wave vectors $\vec{k}_{1,2,3}$ and two Poyinting vectors $\vec{S}_{2,3}$ are opposite to the energy density vector $\vec{S}_1$ of signal wave propagating in opposite direction relatively other two waves. Applying the boundary conditions above we obtain following expression for the complex amplitude of signal wave:

$$A_1(z) = e^{-\frac{i\Delta z}{2}} \times$$

$$\times \left\{K \cdot cos/\,z + \left[\frac{i\gamma_1 A_{20}A_{30}e^{i(\varphi_{30}+\varphi_{20})}}{/} - iK\frac{\Delta}{2/}\right]cos/\,z\right\}, \quad (14)$$

$$A_1(z) = e^{-\frac{i\Delta z}{2}}\left\{K \cdot cos/\,z + \left[H - iK\frac{\Delta}{2/}\right]cos/\,z\right\}. \quad (14')$$

In the expression (14) there are following substitutions

$$K = \frac{A_{1l}e^{i\varphi_{1l}+\frac{i\Delta l}{2}} - \frac{i\gamma_1}{/}A_{20}A_{30}e^{i(\varphi_{30}+\varphi_{20})}sin/\,l}{cos/\,l + (i\Delta/2/)sin/\,l},$$

[8]Чиркин, А. С., Шутов, И. В. Параметрическое усиление световых волн при низкочастотной накачке в апериодических нелинейных фотонных кристаллах // Журнал экспериментальной и теоретической физики, − Москва: − 2009. 136, №4, − с. 639-649.

$$H = \frac{i\gamma_1 A_{20} A_{30} e^{i(\varphi_{30}+\varphi_{20})}}{l}, \qquad l = \sqrt{\Delta^2/4 - \Gamma_3^2 - \Gamma_2^2}$$

$$\Gamma_2^2 = \gamma_1\gamma_3 I_{20}, \qquad \Gamma_3^2 = \gamma_1\gamma_2 I_{30}, \qquad I_j = A_j \cdot A_j^*.$$

As can be seen from expression (14), since $\Gamma_3 > \Gamma_2$, this is a low-frequency case of the pump wave. In the CIA we get $l^{CIA}_{min.lower.freq.} = (\Delta^2/4 - \Gamma_3^2 - \Gamma_2^2)^{1/2}$. It can be seen, that the phase delay between the interacting waves has an acceptable minimum value. When the expression under the square root is positive, the parameter $l^{CIA}_{min.lower.freq.}$ in CIA becomes real, and we get the following relationship for the phase delay. $\Delta^{CIA} \geq 2(\Gamma_3^2 + \Gamma_2^2)^{1/2}$, at small values ($\Delta^{CIA} < 2(\Gamma_3^2 + \Gamma_2^2)^{1/2}$of the difference in wave numbers, the parameter $l^{CIA}_{min.lower.freq.}$ becomes a complex quantity, and a complex amplitude of signal wave is expressed through hyperbolic sine and hyperbolic cosine functions.

$$A_1(z) = e^{-\frac{i\Delta z}{2}} \times$$

$$\times \left\{ L \cdot cosh l\, z + \left[\frac{i\gamma_1 A_{20} A_{30} e^{i(\varphi_{30}+\varphi_{20})}}{l} - iL\frac{\Delta}{2l} \right] sinh l\, z \right\}, \qquad (15)$$

where

$$L = \frac{A_{1l} e^{i\varphi_{1l} + \frac{i\Delta l}{2}} - \frac{i\gamma_1}{l} A_{20} A_{30} e^{i(\varphi_{30}+\varphi_{20})} sinh l\, l}{cosh l\, l + (i\Delta/2l) sinh l\, l}. \qquad (15)$$

According to the equation (15) obtained for the complex amplitude, in the case of a low-frequency pumping wave, for the expression under the square root to be positive, it is required that the operating regime is far from the condition of phase synchronism. It should be noted that as the deviation from the condition of phase matching increases, more amplification of the wave at the given frequency is required. The given minimum possible value of deviation from phase synchronism depends on the intensities of both the charging wave (I_{30}) and the idle wave at the entrance of the metamaterial (I_{20}) through the parameters $\Gamma_{2,3}$. It is also possible to adjust the frequency by changing the intensity of the pumping wave.

For the idler and pump waves if $I_{20}/I_{30} = 0.1$ and $\Gamma_3 = 1\ cm^{-1}$ for the minimum deflection from phase matching $\Delta^{CIA} = 2\Gamma_3\sqrt{1 + I_{20}/I_{30}}$ =2.097768, when ratio equals $I_{20}/I_{30} = 0.5$ and $\Gamma_3 = 1\ cm^{-1}$ we get $\Delta^{CIA} = 2\Gamma_3\sqrt{1 + I_{20}/I_{30}}$ =2.44949.

The increase in the minimum deviation from the phase-matching condition is explained by the fact that the non-linear coupling coefficient, which is included in the expression under the square root through the parameter G_2 and takes into account the opposite effect of the signal wave on the phase of the filling wave, is different from zero. In CFA $\gamma_3 = 0$, the minimum phase deviation remains constant and is equal to $\Delta^{CFA} = 2.0$ since it does not depend on the intensity of the idler wave at the input.

At low frequencies of the pumping wave, the dynamics of the signal wave amplification process, as well as the energy conversion efficiency, are determined by two parameters; conversion efficiency of the signal wave frequency in the medium with length z - $h_1 = I_1(z)/I_{20}$ and signal wave amplification factor $A_{20} = 0$ $h_{amp.} = I_1^{out.}(z = 0)/I_1^{inp.}(z = l) = I_1(z = 0)/I_{1l}$. Consider at the frequency conversion for the case when the high frequency signal wave does not enter from the right input of the metamaterial ($A_{1l} = 0$). Then the parameter L in the formula (15) of the complex amplitude of the signal wave is as follows:

$$K^* = \frac{\frac{i\gamma_1}{l} A_{20}A_{30}e^{i(\varphi_{30}+\varphi_{20})} \sin l\, l}{\cos l\, l + i\frac{\Delta}{2l}\sin l\, l}.$$

In case of consideration the conversion efficiency into signal wave $h_1 = I_1(z = 0)/I_{20}$ given by:

$$h_1(z = 0) = \left[-i\gamma_1 A_{30}e^{i(\varphi_{30}+\varphi_{20})} \cdot \frac{\sin l\, L}{l\,(\cos l\, L + i(\Delta/2l)\sin l\, L)}\right]^2 =$$

$$= \frac{(\Gamma_3)^2 \sin^2 l\, L}{l^2\cos^2 l\, L + (\Delta^2/4)\sin^2 l\, L}. \qquad (16)$$

The value of the l parameter obtained in CIA sharply differentiates the dynamics of the processes in the cases of low-frequency

($I^{C\dot{I}A}_{low\ freq\ pump.}$) and high-frequency $I^{CIA}_{high\ freq\ pump.}$ charging wave. In the case of a high frequency pump wave, this process is analogous to the generation of a difference-frequency reverse wave, while in the case of a low frequency charge wave, it corresponds to the generation of a sum frequency reverse wave. It can be seen from the last statement that there are optimal values of both the filling wave intensity and the wave number difference, at which values the frequency conversion efficiency at the output of the metamaterial is maximum.

According to expression (16) above, when I_{20}/I_{30}=0.1 and Δ=2.097768 cm^{-1}, $I^{CIA}_{low\ freq.pump.} = \sqrt{\Delta^2/4 - \Gamma_3^2 - \Gamma_2^2} = 0$ the dependence $h_1(l)$ becomes horizontal (solid curve 3). The similar dependence obtained in CIA is agree with the result of accurate method given by Yacobi elliptical function.

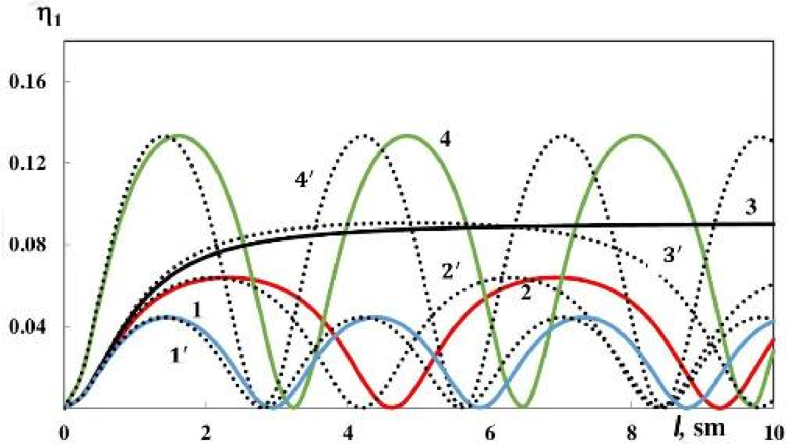

Figure 3. Dependence of signal wave conversion efficiency $h_1 = I_1(z)/I_{20}$ on the total length (l) of metamaterial. $I_{1\ell}$=0, Γ_3 =1 cm^{-1} at Δ =3 cm^{-1} (solid curves 1 and 4), 2.5 cm^{-1} (solid curve 2) and 2.097688 cm^{-1} (solid curve 3), I_{10}/I_{30} = 0.1 (solid curves 1-3) and 0.3 (solid curve 4). Results of CFA are the dotted curves.

With the increase in the intensity of the idler wave at the entrance of the metamaterial, the increase in the efficiency of frequency conversion (comparison of curves 1 and 4) and the displacement of the maxima and minima of the oscillations for the whole curves obtained in CIA (when $\Gamma_2 \neq 0$) are observed. The distance between two neighboring minima, or the period of oscillations, is easily determined. Such a shift is not observed in SAY, and the maxima and minima of the dotted curves (1 and 4) coincide when ($\Gamma_2 = 0$). When the intensity of the idler wave at the entrance of the metamaterial increases by 3 times, the frequency conversion efficiency increases approximately 3 times, i.e. from 4.4% to 13% increases.

From the expression (16) above, the efficiency $h_1(z)$ of the conversion at two different values of the intensity of the signal wave at the input. The calculated results $I_{20}/I_{30} = 0.1$ və $I_{20}/I_{30} = 0.2$ are given (curves 1-4) in Figure 4.

From the dependences, it can be seen that when the power of the charging wave is 2.85 W, the intensity of the signal wave at the right input of the metamaterial is doubled, and the amplification factor at the left output $I_1(z = 0)$ of the metamaterial doubles (compare curves 2 and 4). From here, by choosing a high intensity of the reflection (signal) wave at the entrance of the metamaterial, a signal wave with a greater intensity at the exit of the metamaterial can be obtained.

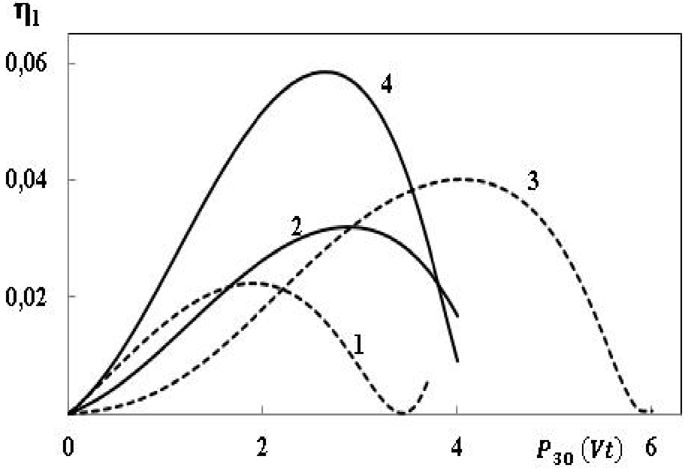

Figure 4. Dependence of signal wave conversion efficiency $h_1 = I_1(z)/I_{20}$ on the power of pump wave: $I_{1\ell}$ =0: (1-3 curves) and 0.2 (4th curve) və Δ =5 cm^{-1} (3rd curve), 5.5 cm^{-1} (2nd and 4th curves), 6 cm^{-1} (3rd curve).

At low frequencies of the filling wave, if the empty wave does not enter the left input of the medium ($A_{20} = 0$), we get the amplification factor of the signal wave at the output of the metamaterial from the formula (15):

$$h_{1,güc.} = \left|\frac{A_1(z = 0)}{A_1(z = l)}\right|^2 = \frac{1}{cos^2/l + (\Delta/2/)^2 sin^2/l}. \qquad (17)$$

By solving the number for the given intensities of the charge and idle waves, from the last expression we can find the equation that allows to find the optimal value of the difference of wave numbers

$$tg/l = /l[1 - (\Delta/2/)^2]. \qquad (18)$$

We consider the general case, i.e. the case where there are waves on both sides of the metamaterial. Then from (16) when $(\varphi_{1l,20,30} = 0)$ for amplification factor $h'_{amp.} = I_1^{out.}(z = 0)/I_1^{inp.}(z = l)$ we get[9]:

$$h'_{güc.}(z) = \frac{\left(cos\frac{\Delta l}{2} \cdot cos/z - \frac{\Delta}{2/} sin\frac{\Delta l}{2} sin/z\right)^2}{cos^2/l + (\Delta/2/)^2\, sin^2/l} +$$

$$+ \frac{\left[sin\frac{\Delta l}{2} \cdot cos/z + \frac{\Delta}{2/} cos\frac{\Delta l}{2} sin/z - w \cdot sin/(l - z)\right]^2}{cos^2/l + (\Delta/2/)^2\, sin^2/l}, \qquad (19)$$

where $w = (\gamma_1//)(A_{20}\, A_{30})/A_{1l}$.

The efficiency of the signal wave amplification process increases when the ratio of the intensities of the idler and signal waves at the entrance of the metamaterial is large.

A 5-fold increase in the intensity of the idler wave at the entrance of the metamaterial compared to the intensity of the signal wave leads to an increase in the amplification of the signal wave by about 20 times.

The dependence of the amplification factor of the signal wave on the length of the metamaterial is given in Figure 5 at four different values of the wave numbers difference.

Two values of difference in wave numbers are the roots of equation (18): $\Delta_{opt.1}$= 5.9484 cm^{-1} (2)), $\Delta_{opt.2}$=12.42 cm^{-1} (4), where Γ_3=1 cm^{-1} və ℓ =1 cm. It is seen from comparison of graphs that optimum

[9]Kasumova, R.J. Parametric interaction of optical waves in Metamaterials under low- frequency pumping / R.J. Kasumova, Sh.Sh. Amirov, Sh. A. Shamilova // Quantum Electronics, – 2017. 47, № 7, – p. 655-660.

values DWN for the signal wave amplification the condition ($h_{amp.} > 1$) is observed.

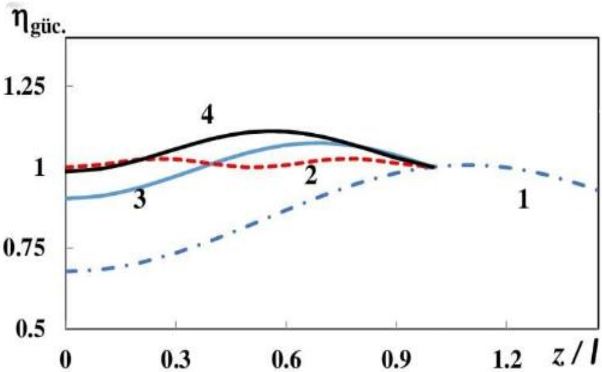

Figure 5. Dependence of amplificaton factor ($h_{amp.} = I_1^{output.}(z=0)/I_1^{input.}(z=l)$) on the dimensionless length of metamaterial: I_{20}= 0, l =1 cm, Γ_3 = 1 cm^{-1} and Δ =3.5 cm^{-1} (1st curve), 5 cm^{-1} (3rd curve), $\Delta_{opt.1}$= 5.9484 cm^{-1} (2nd curve) and $\Delta_{opt.2}$=12.42 cm^{-1} (4th curve). I_{1l}=0, Γ_3=1 cm^{-1}

In Figure 6 the dependence of the signal wave amplification on the total length of metamaterial for the absence of input idler wave is shown.

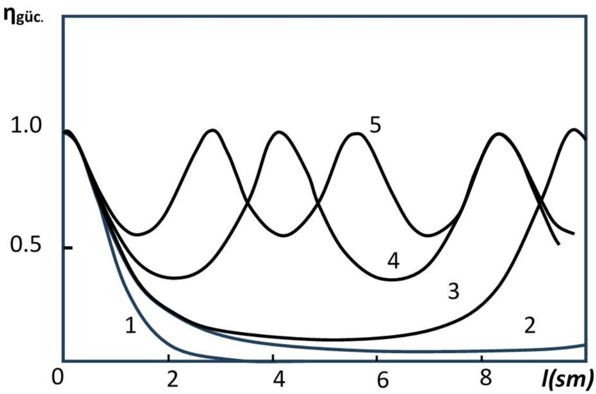

Figure 6. Dependence of amplification factor $h_{amp.} = I_1(z)/I_{1l}$ on the total length of metamaterial (l): I_{20} = 0 and Γ_3=1 cm^{-1}; Δ=0 (1st curve), 2.05 cm^{-1} (2nd curve), 2.1 cm^{-1} (3rd curve), 2.5 cm^{-1} (4th curve), 3 cm^{-1} (5th curve).

Oscillations are observed in dependences (curves 3-5) when the wave number difference (WND) of interacting waves is greater than its minimum possible value ($\Delta^{CIA} \geq 2\sqrt{\Gamma_3^2 + \Gamma_2^2}$)). However, when it is smaller than the minimum value, the dependences are expressed by hyperbolic sine and cosine functions, and oscillations are not observed (curves 1 and 2). As the wave number difference increases, the frequency of amplification coefficient oscillations increases and the depth of modulation decreases.

Dependences of amplification factor $h_{güc.} = I_1(z)/I_{1l}$ on the metamaterial length are shown in Figure 7. From the comparison of the dependences, it can be seen that the signal wave can be sufficiently amplified by changing the input intensities of both the charge wave and the idle wave.

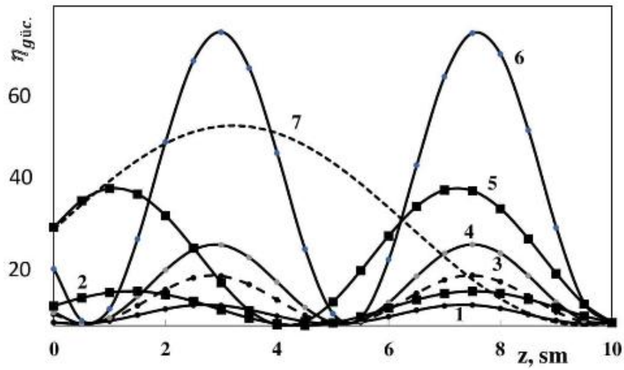

Figure 7. Dependence of amplificaton factor $h_{amp.} = I_1(z)/I_{1l}$ on the total length of metamaterial (Γ_3=1 cm^{-1}, l=10 cm, Δ=2.5 cm^{-1}): I_{20}/I_{30} = 0.1 (1, 3, 4 and 6th curves), 0.3 (2nd and 5th curves) and 0.5 (7th curve), I_{20}/I_{1l} = 1 (1st curve), 5 (3rd curve), 10 (2nd and 4th), 50 (5, 6 and 7th curves)

Increasing the idler input intensity by a factor of 5 relative to the signal intensity results in an approximately 20-fold increase in the signal gain (curves 4 and 6). By increasing the input intensity of the idle wave by 5 times compared to the intensity of the main wave, a slight increase in the amplification factor, i.e. approximately 1.5 times, is achieved (6-th and 7-th curves). It can be concluded that the increase of the amplification factor of the signal wave is mainly determined by the ratio of the intensities of the idle and signal waves at the entrance of the metamaterial. This is explained by the fact that the energies of the charging wave and the strong idler wave are more effectively transferred to the energy of the signal wave. In addition, from the comparison of curves 1, 3, 4, 6 and 2, 5, it can be seen that the maximum and minimum shifts of spatial beats are observed in the analysis of the CIA. At high frequencies of pump wave the l parameter in the formula (14) is expressed in CIA as $l^{CIA}_{high\ freq.pump.} = \sqrt{\Delta^2/4 + \Gamma_3^2 - \Gamma_2^2}$. As a result, in the case of a high-frequency charging wave, the expression under the square root sign remains positive at all values of the phase delay parameter up to zero. From the solution of the shortened system of equations corresponding to the high frequencies of the filling wave, we get for the complex amplitude[10] of the signal wave $A_1(z)$ when ($\delta_i = 0$):

$$A_1(z) = C\left[\frac{a\cos l\,l + A_{1l}e^{-(i\,\Delta L/2)+i\varphi_{1l}}}{\sin l\,l + ib\cos l\,l}(\sin l\,z + ib\,\cos l\,z) - a\cos l\,z\right], \quad (20)$$

where

$$C = e^{i\Delta z/2}, \qquad \Gamma_3^2 = \gamma_1\gamma_2 I_{30}, \qquad a = (2\gamma_1/\Delta)A_{30}A_{20}^{*}e^{i(\varphi_{10}-\varphi_{20})}$$

$$\Gamma_2^2 = \gamma_1\gamma_3 I_{20}, \qquad b = 2l/\Delta, \qquad l = \sqrt{\Gamma_3^2 - \Gamma_2^2 + \Delta^2/4}.$$

At high frequencies of the pump wave, the frequency conversion efficiency of the signal wave in the medium with length z is given by the formula $h_1 = I_1(z)/I_{20}$. If there is no signal wave at the output of the metamaterial $(z = 0)(A_{1l} = 0)$, the frequency conversion efficiency should be as follows:

$$h_1 = \frac{\Gamma_3^2\sin^2 l\,l}{(\Delta/2)^2\sin^2 l\,l + l^2\cos^2 l\,l}. \quad (21)$$

Then the phase-matching condition is met ($\Delta = 0$) for conversion efficiency from the formula (21) we get:

$$h_1 = \frac{\Gamma_3^2}{\Gamma_3^2 - \Gamma_2^2}\tan^2\sqrt{\Gamma_3^2 - \Gamma_2^2}\cdot l. \quad (22)$$

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[10]Kasumova, R.J. Phase effects of the parametric interaction in metamaterials / R.J. Kasumova, Z.H. Tagiyev, Sh.Sh. Amirov [et al.] // Journal of Russian Laser research, – Moscow: – 2017. 38, № 4, – p. 211-218.

If $\Gamma_3 \gg \Gamma_2$ then $h_1 = tan^2\Gamma_3 l$ Hence it is seen, that when the condition $\Gamma_3 l = \pi/2 + \pi m\ m = 0,1,2,$ is met the frequency conversion efficiency approaches infinity.

When there is not the idler wave $(A_{20} = 0)$ at the input from the left of nonlinear medium for the amplification factor of signal wave we get:

$$h_{güc.} = \left|\frac{sin/\,z + ibcos/\,z}{sin/\,l + ibcos/\,l}\right|^2. \qquad (23)$$

Figure 8 shows the dependence of the conversion efficiency on the non-unit length of the metamaterial at different values of the $\Gamma_3 l$ parameter for the condition of synchronism ($(\Delta = 0$). In a medium with ordinary quadratic nonlinearity, this dependence has a maximum at the coherence length of the medium. But in the "left" environment, this dependence has a monotonic character. As can be seen from Figure 8, the metamaterial acts as a mirror and reflects the signal wave from its surface at the left entrance to the interior of the material. For this reason, the efficiency of the frequency conversion is maximized at the input of the metamaterial, not at the output. In this way, for comparison, we also show the result of CFA at the value of $\Gamma_3 l$=1.82 (3rd curve) and it coincides with the result given for h_1.

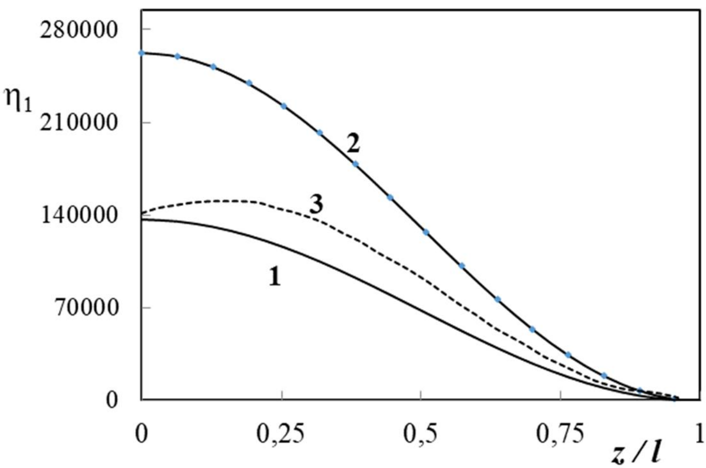

Figure 8. Dependence of signal wave frequency conversion efficiency $h_1 = I_1(z)/I_{20}$ on the reduced length of metamaterial (z/l ($\Delta = 0$, $I_{11}/I_{20} = 0.1$) and $\Gamma_3 l$: 1 – 1.57; 2 – 1.57009; 3 – 1.82.

When we compare the dependences, it is observed that enhancement in conversion efficiency (h_1) occurs when quantity $\Gamma_3 l$ approaches its resonance values (Fig. 9).

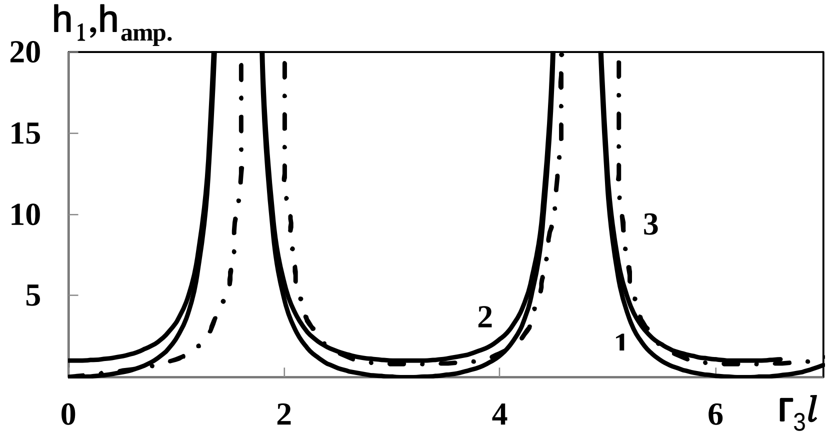

Figure 9. Dependence of signal wave frequency conversion efficiency $h_1 = I_1(z)/I_{20}$ (curve 1) and amplification factor $h_{amp.} = I_1(z)/I_{11}$ (2 and 3 curves) on the reduced length of metamaterial: ($\Delta = 0, I_{11}/I_{20} = 0.1$) Here 1st and 2nd graphs in CIA, 3-rd curve in CFA were plotted.

We see from comparison the curves 1 and 2 in Figure 10 that when $\Gamma_3 l_{opt.}$ approaches its optimum value the optimum value of length $l_{opt.}$ is given by the formula:

$$l_{opt.} = \frac{\pi/2}{\sqrt{\Gamma_3^2 - \Gamma_2^2 + \Delta^2/4}}. \qquad (24)$$

At optimum value of parameter $\Gamma_3 l_{opt.}$ the conversion efficiency to the signal wave increases by a factor of two. Under deflection from the phase-matching condition the conversion efficiency h_1 sharply decreases and reaches its theshold value (Fig. 10). Analysis has showed that the optimum value of wave number difference (WND) depends on the intensity of pump wave. When the value of the parameterΔl, which represents the WND, increases from π to 5, the frequency conversion efficiency decreases more than 2.5 times. In addition, when the value of $\Gamma_3 l_{opt.}$ – quantity decreases from 4.5 to 4.1, a shift of the efficiency maximum is observed.

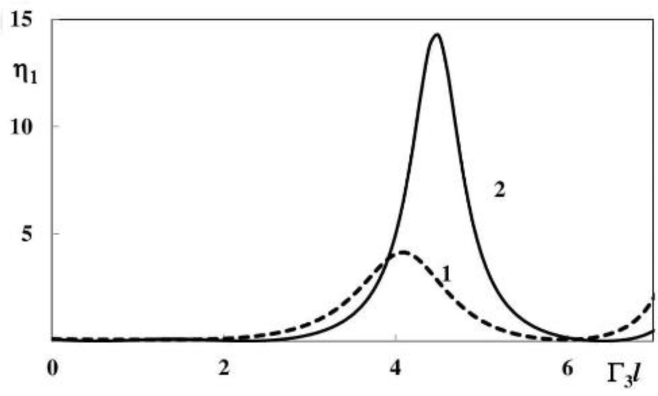

Figure 10. Signal wave frequency conversion efficiency $h_1 = I_1(z)/I_{20}$ as a function of dimensionless length $\Gamma_3 l$ of metamaterial. ($I_{1l}/I_{20} = 0.1$) və $\Delta l = 5$ (1st curve) 1-5; $\Delta l = \pi$ (2nd curve)

From a practical point of view, the creation of adjustable parametric frequency converters is of great interest. In the presence of a high-frequency strong wave, the creation of frequency converters based on metamaterial is carried out by gradually changing the frequency of the charging wave. Because the frequency of the signal wave can be adjusted only in a small area where the refractive index takes a negative value. The presence of such a frequency band is explained by the existing technology used in the preparation of metamaterials.

Since there are no experimental results for the nonlinear parametric interaction of waves in metamaterials, we perform a numerical evaluation for the frequency conversion efficiency in dielectric waveguides with quadratic nonlinearity. We are looking at a medium irradiated by a laser with a power of 1 W and a length of $l = 2$ cm If we take $\gamma_{1,2} = 1\ cm^{-1}Vt^{-1}$ for nonlinear connection coefficients, we get $\Delta l/2 = 2.6$ for the parameter of deviation from synchronism. The dependence of the conversion factor $h_1 = I_1(z)/I_{20}$ on the intensity of the strong wave for three different values of the input signal intensity I_{1l}/I_{20} =0.1, 0.2 and 0.5 is given in Figure 11 (curves 1 and 3).

It can be seen that when the intensity at the right input of the metamaterial increases by 5 times, the amplification factor at the left output increases by approximately 2 times (here, the optimal value of the

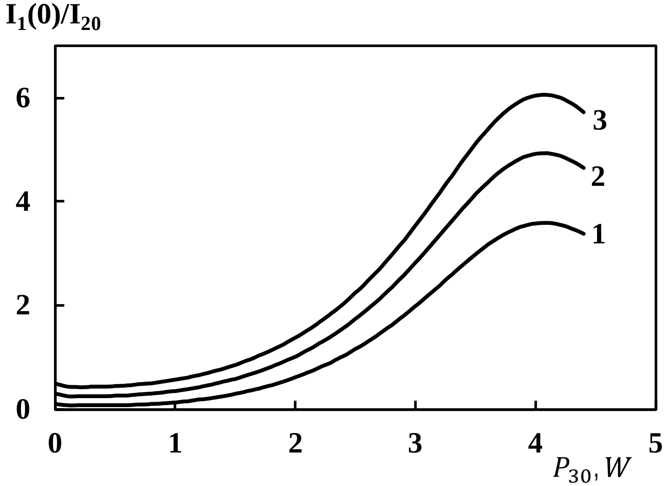

Figure 11. Signal wave frequency conversion efficiency $h_1 = I_1(z)/I_{20}$ as a function of pump wave power ($l = 2cm$): $\Delta l/2 = 2.6$ and $I_{1l}/I_{20} = 0.1$ (1), 0.3 (2) and 0.5(3).

power at the input is assumed to be 4 W). Thus, by choosing a large value of the signal wave at the right input of the metamaterial, it is possible to receive a reflection wave with a greater intensity at the left output.

Four-wave interaction in a medium with a negative refractive index was experimentally realized in a metal-dielectric-metal layered nanostructure. In the four-frequency interaction that we are looking at, the "left" environment is only for the signal wave of frequency ω_1; Dielectric permittivity ε_j and magnetic permeability $\mu_j$of the metamaterial at frequency ω_1 are negative ($\varepsilon_1 < 0$ and $\mu_1 < 0$) and at other frequencies $\omega_{2,3,4}$ are positive ($\varepsilon_{2,3,4} > 0$ and $\mu_{2,3,4} > 0$) The Poynting vectors of two strong waves and a weak wave of frequency ω_2 fall normal to the left surface of the metamaterial of length l and propagate in the positive direction of the z −axis. The energy flow of the signal wave is directed in the opposite direction in the metamaterial. Then the wave vectors $\vec{k}_{1,2,3,4}$ of the four waves coincide with the directions of the three energy flow vectors ($\vec{S}_{2,3,4}$) in the positive direction of the z −axis. Thus, during the four-wave interaction, seven vectors ($\vec{k}_{1,2,3,4}$ and $\vec{S}_{2,3,4}$) are opposite to the Poynting vector $\vec{S}_1$ of the opposite signal wave.

The four equations describing the four-frequency interaction are given as follows[11]:

$$\begin{aligned} dA_1/dz - \delta_1 A_1 &= -i\gamma_1 A_3 A_4 A_2^* e^{i\Delta z}, \\ dA_2/dz + \delta_2 A_2 &= i\gamma_2 A_3 A_4 A_1^* e^{i\Delta z}, \\ dA_3/dz + \delta_3 A_3 &= i\gamma_3 A_1 A_2 A_4^* e^{-i\Delta z}, \\ dA_4/dz + \delta_4 A_4 &= i\gamma_4 A_1 A_2 A_3^* e^{-i\Delta z}. \end{aligned} \tag{25}$$

Solving this set of equation by the boundary conditions in CIA

$$A_{2,3,4}(z = 0) = A_{20,30,40}, \qquad A_1(z = l) = A_{1l}. \tag{26}$$

Gives simplified formula[12] for the complex amplitude of signal wave:

$$A_1(z) = e^{-\frac{m}{2}z} \times$$

$$\times \left[\frac{A_{1l} e^{\frac{al}{2}} - \left(\frac{\delta_1}{Ι_1} A_{1l} - i\frac{b}{Ι} \right) sinΙ\, l}{cosΙ\, l + (m/2Ι) sinΙ\, l} \left(cosΙ\, z + \frac{m}{2Ι} sinΙ\, z \right) + D \right], \tag{27}$$

where

$$D = \frac{\delta_1 A_{1l} - ib}{Ι} sinΙ\, z, \qquad m = \delta_2 + \delta_3 + \delta_4 - \delta_1 - i\Delta,$$

$$Ι = \left[\gamma_1\gamma_2 I_{30} I_{40} - \gamma_1\gamma_3 I_{20} I_{40} - \gamma_1\gamma_4 I_{20} I_{30} - \left(\sum_{j=1}^{4} \delta_j - i\Delta \right)^2 \right]^{1/2},$$

$$b = \gamma_1 A_2^* A_{30} A_{40}.$$

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[11]Popov, A. K. Four - wave mixing, quantum control and compensating losses in doped negative-index photonic metamaterials / A. K. Popov, S. A. Myslivets, T. F. George, V. M. Shalaev // Optics Letters. - 2007. 32, №20, - p. 3044 - 3046.
[12]Kasumova, R.J. Four-wave mixing in metamaterials / R.J. Kasumova, G. A., Safarova, Sh.Sh. Amirov, A. R. Akhmadova // Russian Physics Journal, − Москва: − 2018. 61, № 9, - p. 1559-1567.

From the above boundary conditions (25) $A_{2,3,4}(z=0)=A_{20,30,40}$, $A_1(z=l)=A_{1l}$ for the value of signal wave at the left exit of metamaterial $A_1(z=0)$ we get

$$A_1(z=0) \geq A_1(z=l) = A_{1l}. \tag{28}$$

When the phase matching is met, according to (27) by accepting simplification $I_{30}=I_{40}=I_{Dol.}$ we obtain following conditions[13] for the losses and parameter λ:

$$\delta_2+\delta_3+\delta_4=\delta_1, \tag{29}$$

$$/\,l = 0, 2\pi, \ldots. \tag{30}$$

Last condition yields following for the threshold value of pump wave intensity

$$I_{Dol.}^{Astana} = \frac{\gamma_3+\gamma_4}{2\gamma_2} I_{20} + \sqrt{\left(\frac{\gamma_3+\gamma_4}{2\gamma_2} I_{20}\right)^2 + \frac{\delta_1^2}{\gamma_1\gamma_2}}. \tag{31}$$

As a result of the nonlinear parametric interaction of the waves, the amplitude of the signal wave can reach its constant value after passing through the medium once at certain threshold values of the parameters of the problem. The above condition (29) means that, as a result of the nonlinear interaction of waves propagating in forward and opposite directions, signal wave losses (δ_1) are compensated by losses of waves propagating in straight directions ($\delta_2, \delta_3, \delta_4$) resulting in a parametric amplification of the threshold value of the pump wave intensity. Threshold value of pump wave intensity is determined from the condition (30) above. The pump wave intensity threshold is a function of nonlinear coupling coefficients, dissipative losses, and idle wave intensity (I_{20}) He showed that the threshold value of the main wave intensity increases with the increase of dissipative losses and the intensity of the weak wave (I_{20}) The expression of the threshold value of the intensity is also influenced by the nonlinear coupling coefficients ($\gamma_{3,4}$)), which take into account the opposite effect of the newly formed wave on the phase of the filling wave. With increase of the nonlinear coupling coefficients, both

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[13]Kasumova, R. J., Amirov, Sh.Sh. On the theory of four-wave interaction in dissipative metamaterials // AJP "Fizika",– Baku: - 2022. 28, № 4, – p. 50-56.

the opposite effect and the threshold value of the intensity of the pump wave increase. The same result is obtained for the case where there is no idler wave at the entrance of the metamaterial.

When the condition of phase synchronism $(\Delta = 0)$ is satisfied and there is no idle wave at the input of the metamaterial ($A_{20} = 0$) we get the following expression for the amplification factor of the signal wave:

$$h_{1,güc.} = e^{-m_1 z} \times$$

$$\times \left\{ \frac{\left[e^{\frac{m_1}{2}} \left(cos I_1 z + \frac{m_1}{2 I_1} sin I_1 z \right) - \frac{\delta_1}{I_1} sin I_1 (l - z) \right]^2}{\left(cos I_1 l + \frac{m_1}{2 I_1} sin I_1 l \right)^2} \right\}, \quad (32)$$

where $I_1 = \sqrt{\gamma_1 \gamma_2 I_{30} I_{40} - \left(\sum_{j=1}^{4} \delta_j \right)^2 / 4}$.

If the losses of the environment are not taken into account, the expression of the amplification factor is simplified $h_{amp.1} = 1/cos^2 I'_1 l$, ($I'_1 = (\gamma_1 \gamma_2 I_{30} I_{40})^{1/2}$). It can be seen from this formula that the maxima of the amplification factor in the metamaterial are subject to periodic resonances satisfying the condition $I'_1 l = \pi/2 + \pi k$, $k = 0, 1, 2, \dots$

Figure 12 shows the dependence of the amplification factor $h_{amp.}$ on the total length (l) of the metamaterial when the synchronism condition is met ($\Delta = 0$) and $I_{20} = 0$.

From the comparison of the $h_{amp.}(l)$ curves for different values of the loss coefficient, it can be seen that as the losses in the metamaterial decrease, the radius of curvature of the resonance curves increases. From the comparison of the resonance curves for the values of the full length l of the metamaterial in the range of 40-65 nm, it is obtained that the resonance width increases with the increase of losses, that is, the quality of the circuit decreases, as in the case of the electric oscillation contour. Similar resonances are observed at certain values of the intensity of the controlling coherent pump wave for the amplification factor.

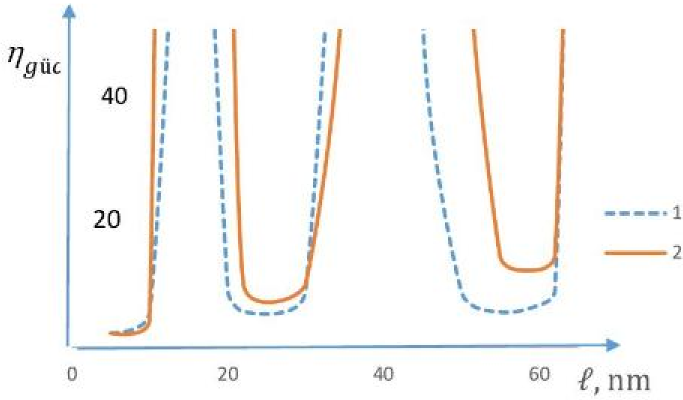

Figure 12. Dependence of signal wave amplification factor ($h_{amp.} = I_1(z=0)/I_{1l}$) on the total length ($l$) of metamaterial when $\Delta = 0$ and $I_{20} = 0$ $\delta_j = 2\cdot 10^4$ cm^{-1} (solid curve 1) and 10^4 cm^{-1} (dashed curve 2)

This shows that for the selected parameters of the problem, it is possible to calculate the value of the intensity of the driving field corresponding to the resonance increase of the amplification factor. From the dependences considered, it can be concluded that the observed resonances of the amplification factor can be adjusted by choosing the parameters of the problem, in particular, the loss and the intensity of the coherent field.

Figure 13 shows the dynamics of the amplification process $h_{amp.}(z)$ when waves propagate in a non-linear medium. Moving away from phase synchronism, the amplitude of the oscillations of the dependence of $h_{amp.}(z)$ decreases (curves 2, 3 and 5). Here also the dependence corresponding to the CFA is given (curve 4).

The difference in the dependences (curves 2 and 4) in both approximations is due to the fact that the coefficients $\gamma_3$and γ_4, which take into account the opposite effect of the pump wave on the signal wave, are different from zero in the constant intensity approximation. The analysis shows that by changing the values of the input intensities of the waves propagating in the forward direction, sufficient amplification of the signal wave propagating in the opposite direction can be achieved. In the CIA, when $\gamma_{3,4} \neq 0$ with the increase of the input intensity of the idle wave with frequency ω_2, the increase of the conversion

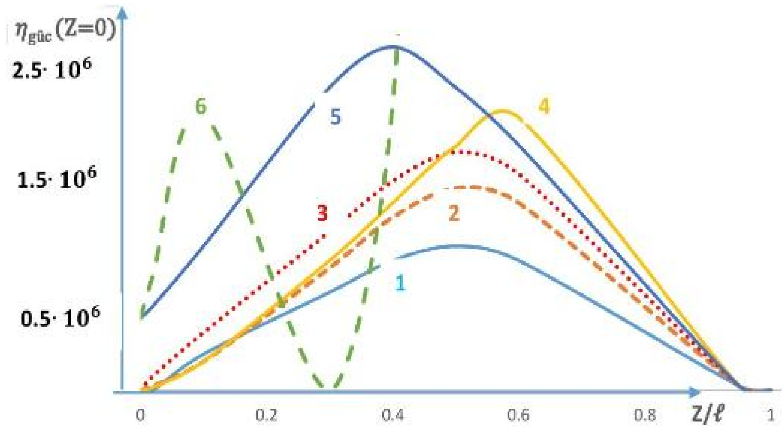

Figure 13. Dependence of amplification factor $h_{amp.}(z) = I_1(z)/I_{11}$ on the dimensionless length of metamaterial $I_{11} = 10^{-7} I_{30}$, total thickness $l = 20$ nm (1-5 curves), $l = 60$ nm (curve 6), $\Delta l/2$: 2.5 (1 and 5 curves), 2.8 (3rd curve), 3 (2 and 4 curves) and 8.4 (curve 6) $I_{20} = 0{,}3 \cdot I_{30}$ (1st curve) $I_{20} = 0{,}5 \cdot I_{30}$ (2-6 curves). Here solid 1-3 and 5-6 curves were plotted in CIA, dotted 4th curve is calculated in CFA.

efficiency (compare curves 1 and 2) and the displacement of the maxima and minima of oscillations are observed. The distance between two adjacent minima, i.e. the period of oscillations, can be easily determined. Such a shift is not observed in the CFA ($\gamma_{3,4} = 0$) that is, the maxima and minima of the corresponding curves coincide. The maximum conversion efficiency increases by 1.43 times at the values of the input intensity of the idler wave (frequency ω_2) in the interval $I_{20} = 0{,}3 \cdot I_{30} - 0{,}5 \cdot I_{30}$.

At the left exit ($A_{1l} = 0$) of metamaterial the reflective coefficient ($R = I_1(z = 0)/ I_{20}$) is determined as follows ($\delta_j = 0$):

$$R = \left(\frac{\gamma_1}{I_2} A_{30} A_{40} sin I_2 l\right)^2 / [cos^2 I_2 l + (\Delta/2 I_2)^2 sin^2 I_2 l]. \quad (33)$$

To obtain the optimum value of WND by the differentiation the above equation with respect to Δ parameter we get $sin I_2 l = cos I_2 l$. This equation we solve graphically and for the parameter $I_2 l$ obtain values $I_2 l$ =0, $3\pi/2, 5\pi/2$. From the expression of the I_2 the optimum value of WND corresponding to the central maximum of reflective coefficient we get

$$\Delta_{opt.,1} = 2\sqrt{\gamma_1\gamma_3 I_{20} I_{40} + \gamma_1\gamma_4 I_{20} I_{30} - \gamma_1\gamma_2 I_{30} I_{40}}.$$

Similar expression corresponding to the side maxima of reflective index is obtained as:

$$\Delta_{opt.,2} = 2\sqrt{4.7 + \gamma_1\gamma_3 I_{20} I_{40} + \gamma_1\gamma_4 I_{20} I_{30} - \gamma_1\gamma_2 I_{30} I_{40}}.$$

It can be seen from the obtained expressions that the quantities $\Delta_{opt.} l$ are equal to $/_2 l$ with the accuracy of expressions under the square root sign. Expressions under the square root, in turn, depend on the intensities of the pump wave and the wave at the frequency ω_2 should be noted that side maxima are observed at large values of the main wave intensities and in the absence of saturation in the wave field. With the increase of losses, the intensity of the signal wave at the left output of the metamaterial decreases, which leads to a decrease in the return coefficient at $z = 0$. It should be noted that side maxima are observed at large values of the main wave intensities and in the absence of saturation in the wave field. With the increase of losses, the intensity of the signal wave at the left output of the metamaterial decreases, which leads to a decrease in the return coefficient at $z = 0$. It should be taken into account that the observed dependence of R on losses is non-linear, so at the same time as exponential dependence, losses are also included in the sub-root expression through the $/_1$ parameter. In Figure 14, the maxima of the coefficient of reflection at the output of the metamaterial are observed at the values of the difference in wave numbers, which are the roots of the equation we obtained above. The maxima of the return coefficient are observed at the theoretically calculated values of $\Delta_{opt.}$. The main maximum is obtained at a certain distance around $\Delta = 0$, distance of which is determined by limits depending on the intensities of the pump wave and the wave of frequency ω_2 (idler wave) in the root expression.

Thus, we come to the conclusion that the increase in the amplification factor of the signal wave depends primarily on the ratio of the intensity of the wave of frequency ω_2 at the entrance of the metamaterial to the intensities of both pump waves$I_{20} / I_{30,40}$.

The **fourth chapter** is devoted to the study of the frequency conversion of ultrashort laser pulses in metamaterials in the first and second approximations of the dispersion theory.

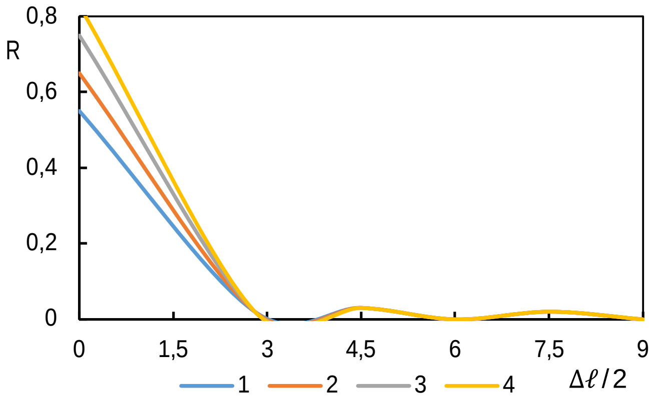

Figure 14. Dependence of reflection coefficient ($R = I_{1l}/I_{20}$) of metamaterial on the dimensionless wave number difference $\Delta l/2$ when $\delta_j = 0$: $l = 20\,nm$, I_{20}:$0.5 \cdot I_{30}$ (curve 1), $0.2 \cdot I_{30}$ (curve 3), $0.3 \cdot I_{30}$ (curve 4).

In the first approximation, we use the truncated system of equations (10). If we take into account that, in the input of the metamaterial there is a Gaussian profile idler pulse with quadratic phase modulation, then

$$A_{30}(t) = A_0 e^{-\frac{t^2}{2\tau^2} - i\gamma\frac{t^2}{2}}. \qquad (34)$$

In the first approximation of the dispersion theory, taking into account only the difference in group velocities, we get for the spectral density[14] of the signal frequency pulse at very small values of losses in the metamaterial ($\delta_i \approx 0$):

$$S_1(\omega, z) = (\gamma_1\gamma_3)^2 I_{20} \times$$

$$\times \frac{A_{20}^2\tau^2}{2\pi}\frac{1}{\sqrt{1+p}}\frac{(sina_1z - cosza_1z \cdot tga_1l)^2 e^{-\frac{m^2}{1+p}}}{a_1^2 + \left(\frac{\Delta - \nu\omega}{2}\right)^2 tg^2a_1l}, \qquad (35)$$

where $a_1 = \left(-G_2^2 - (\Delta - \nu\omega)^2/4\right)^{1/2}$, $\nu = 1/u_2 + 1/|u_1|$, $G_2^2 =$

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[14] Amirov, Sh.Sh. Spectrum of laser pulses in the first order dispersion theory // - Baku: AJP "Fizika", - 2021. 27, №2, - p. 3-7.

$\gamma_1\gamma_3 I_{20}$, $m = \omega\tau$ and $p = \gamma^2\tau^4$ – is the phase modulation factor.

Note that in the first approximation, the second derivatives of the complex amplitudes in the system of equations (10) are not taken into account..

The energy[15] of signal pulse is determined by the integral of expression (35) obtained for the spectral density

$$E_1 = K\int_{-\infty}^{+\infty} \frac{1}{\sqrt{1+p}} \frac{(sina_1 z - cosza_1 z \cdot tga_1 l)^2 e^{-\frac{m^2}{1+p}}}{a_1^2 + ((\Delta - \nu\omega)/2)^2\, tg^2 a_1 l} d\mu. \quad (36)$$

Here $K = cn\gamma_1^2 I_{30}\, I_{20}\tau^2 z^2/16\pi$

Since the dispersion length of a pulse is smaller than the length of the metamaterial, the necessity of applying the second approximation of the dispersion theory becomes important.

Generation of second harmonic, generation of sum frequency, etc. the non-stationarity of the processes is accompanied by the effects of propagation of radiation pulses due to group delay and dispersion.

These effects are observed for pulses whose duration is smaller than picoseconds. If the duration of the pumping pulse ($t_{duration.}$)) is smaller than the time of creation of a new pulse ($t_{creation}$), then the parametric generation of light cannot occur. When $t_{duration.} > t_{creation}$, parametric oscillations occur and parametric generation occurs in the quasi-stationary regime.

In the second approximation of the dispersion theory, the three-wave interaction is given by the system of equations (10). Applying Fourier transforms, we get the following expression for the spectral density of the signal wave[16]

$$S_1(\omega, z) = D\frac{(sin\lambda' z - tg\lambda' l \cdot cos\lambda' z)^2}{\lambda'^2 + (k^2/4)\, tan^2\lambda' l} \cdot \exp\left(-\frac{\omega^2\tau^2}{1 + C^2\tau^4}\right). \quad (37)$$

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[15]Amirov, Sh.Sh., Kasumova, R. J., Tagiev, Z. H. Energy of ultrashort pulses in metamaterials // Proceedings of International conference “Modern Trends in Physics“, - Baku: - may 01-03, - 2019, - p. 205-210.

[16]Kasumova, R. J., Amirov, Sh.Sh. Frequency transformation of ultrafast laser pulses in metamaterials / Rena J. Kasumova, Sh.Sh. Amirov // Superlattices and microstructures, – 2019. №126, – p. 49-56.

where $D = cn_1\gamma_1^2 I_{30} I_{20}\tau^2/16\pi^2$, və $k = \omega^2 g/2 - \omega\nu - \Delta$.

To analyze the shape of the signal waveform spectrum it is reasonable to work not with the dispersion propagation length $l_{d.}$ and quazi-static length l_ν, but with their ratios $l_{n/l}/l_{d.}$ və $l_{n/l}/l_\nu$ (here $l_{n/l} = \Gamma_2^{-1}$ – is the nonlinear length). Then expressions for λ' and k parameters in the last equation are given by:

$$\lambda' = \frac{1}{l_{q/x}}\left[\frac{1}{4}\left(\frac{1}{2}(\alpha - 1)\frac{l_{q/x}}{l_{d.}}\omega^2\tau^2 + \frac{l_{q/x}}{l_\nu}\omega\nu - \frac{\Delta}{G_3}\right)^2 - 1\right]^{1/2}$$

$$k = \frac{1}{l_{q/x}}\left[i\left(\frac{1}{4}(\alpha - 1)\frac{l_{q/x}}{l_{d.}}\omega^2\tau^2 + \frac{l_{q/x}}{l_\nu}\omega\nu - \frac{\Delta}{G_3}\right)\right], \alpha = \frac{g_2}{g_1}.$$

Curves of signal pulse spectral density $S_1'(\omega, z) = S_1(\omega, z)/D$ for various values of coordinate are given in Figure 15.

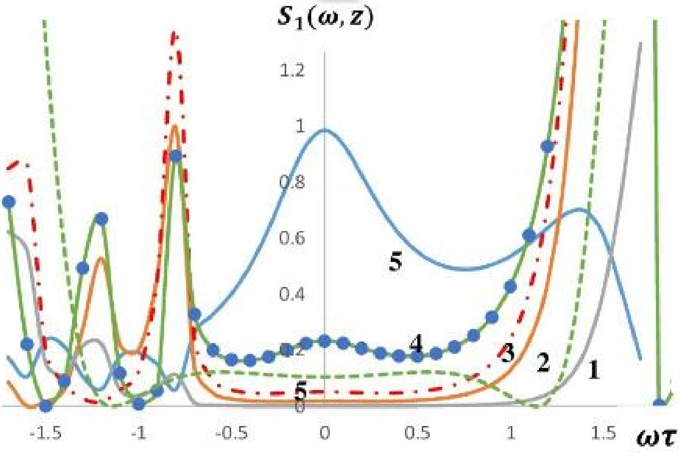

Figure 15. Dependence of relative spectral density $S_1'(\omega, z) = S_1(\omega, z)/D$ on the parameter $\omega\tau$ for total length $l = 2\,cm$ at different values of $z-$ coordinate ($p = C^2\tau^4 = 0, \Delta l_{nl} = 0, g_3 = 2g_1 = 0, l_{n/l}/l_{d.} = l_{n/l}/l_n = 3, I_{20} = 2W$): 1.8 cm (1st curve), 1.3 cm (2nd curve), 1 cm (3rd curve), 0.5 cm (4th curve), 0 cm (dashed line and solid 5th curves). Here 5-th solid green curve is calculated for ordinary nonlinear medium ($\nu = 0, u_3 = u_1$) other graphs are plotted for metamaterial.

When the excited wave propagates from the right input $(z = l)$ to the output $(z = 0)$ in the "left" environment, first the transverse maxima (second, third, etc.) increase, then the energies of the side components are transferred to the central maximum (comparison of curves 1-5). In the middle of the metamaterial$(l = z = 1cm)$, the conversion efficiency of the wave packets reaches a maximum, the first transverse maximum for the signal wave (3^{rd} curve) is observed, and then the central maximum is generated at the exit of the metamaterial $(l = z = 0)$ (5^{th} curve) curve. The conversion efficiency at the center frequency of the spectrum in the metamaterial is an order of magnitude higher than that of the conventional medium. When waves propagate in mutually opposite directions, the wave pulse propagated in one direction passes through the wave pulses propagated in the other direction. The interaction between the counter-propagating wave-fronts takes place over a long time interval.

Spectral density curves at different values of pump intensity are demonstrated in Figure 16. A non-symmetrical spectrum has side maxima. In the curves, it is observed that the strong wave intensity has an optimal value. In the given parameters of the case, the optimal intensity of the strong wave is close to 5W at the value $\omega\tau \approx -0.7$ (2^{nd}, 3^{rd} and 5^{th} curves). As the filling wave intensity increases, the energy of the central maximum is transferred to the side maxima (peaks) (2^{nd}, 3^{rd}, etc.). The dependences of the relative spectral density of the signal wave $S_1'(\omega, z)$on the parameter ωτ at different values of the phase and group velocity dispersion of waves propagating in straight and opposite directions are shown in Figure 17.

As can be seen, when $l_{n/l} \ll l_{d.}$) the ratio $l_{n/l}/l_{\nu}$ increases, the width of the central maximum decreases, that is, the spectrum of the signal pulse narrows (comparison of curves 1, 2 and 5). Such a decrease in the width of the spectrum also occurs with an increase in the spread due to dispersion (comparison of curves 3 and 4). At small values of the characteristic lengths $l_{d.}$and l_{ν} compared to the non-linear length of the medium $l_{n/l}$) or at large values of the ratios $l_{n/l} \ll l_{d.}$and $l_{n/l}/l_{\nu}$ the form of dependence becomes complicated (curve 4). In all dependencies, a symmetric picture is observed when there is no spread due to dispersion, and an asymmetric picture is observed when there is a spread due to dispersion. The analysis also showed that the ratio, $g_3/g_{1.}$ affects the speed of transformation of the initial pulse into a signal pulse. Note that when the coefficients g_1 and g_3 are equal, the interaction of waves occurs without propagation due to dispersion.

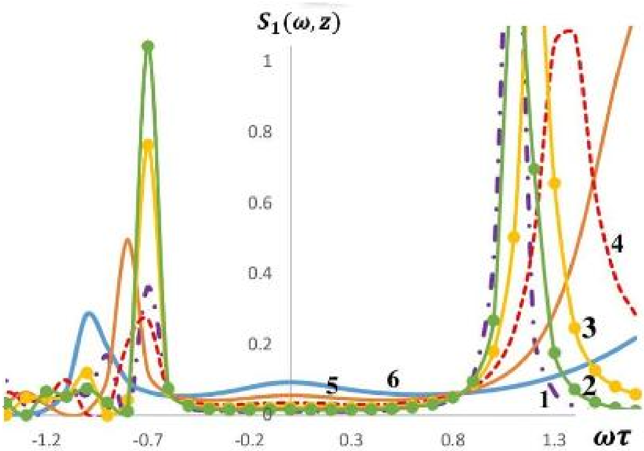

Figure 16. Dependence of relative spectral density $S_1'(\omega, z) = S_1(\omega, z)/D$ on the parameter $\omega\tau$ for total length $l = 2\ cm$ at different values of pump intensity (I_{20}) $(p = C^2\tau^4 = 2, \Delta l_{n/l} = 0, g_3 = 2g_1 = 0, l_{n/l}/l_{d.} = l_{n/l}/l_n = 3, z = 0.8\ cm)$: 6 W (1), 5 W (2), 4 W (3), 3 W (4), 2 W (5), 1 W(6).

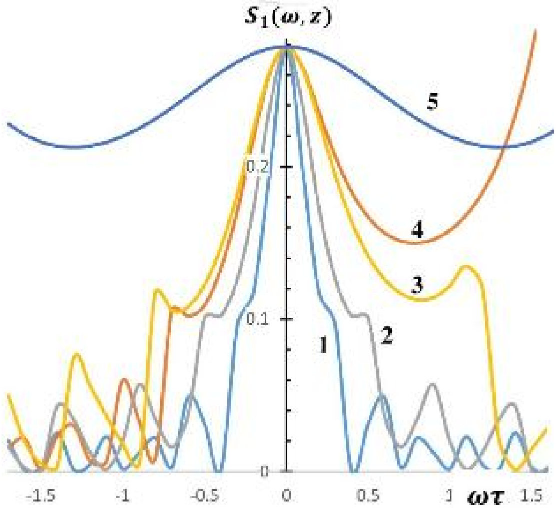

Figure 17. Dependence of relative spectral density $S_1'(\omega, z) = S_1(\omega, z)/D$ on the parameter $\omega\tau$ for total length $l = 2\ cm$ at different values of ratios of characteristic lengths. $(p = C^2\tau^4 = 3, \Delta l_{q/x} = 0, I_{20} = 2W, g_3 = 2g_1, l_{n/l}/l_d = l_{n/l}/l_\nu = 3, z = 0.2\ cm)$: $l_{n/l}/l_d = 0$, $l_{n/l}/l_\nu = 8$ (1st curve), $l_{n/l}/l_d = 0$, $l_{n/l}/l_\nu = 5$ (2nd curve).

The effect of the modulation coefficient ($p = C^2\tau^4$) of the idle wave with quadratic phase modulation on the relative spectral density of the signal wave can be seen in Figure 18.

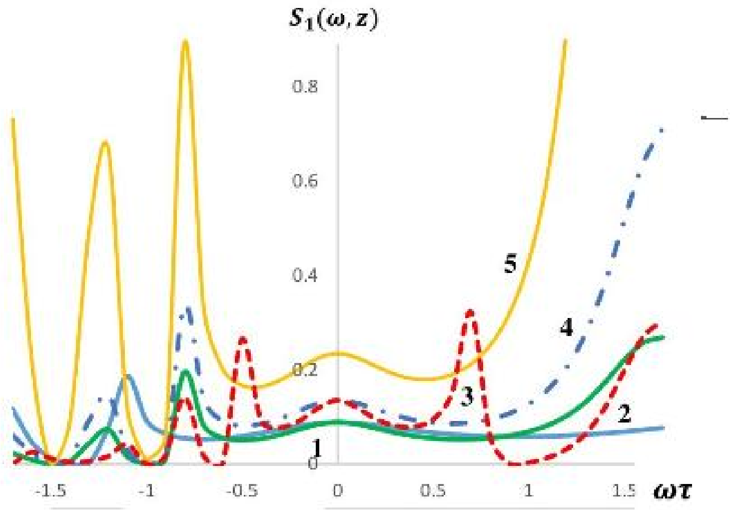

Figure 18. Dependence of relative spectral density $S_1'(\omega, z) = S_1(\omega, z)/D$ on the parameter $\omega\tau$ for total length $l = 2\ cm$ at different values of frequency modulation factor $p = C^2\tau^4$ for $\Delta l_{n/l} = 0, g_3 = 2g_1 = 0, l_{n/l}/l_d = l_{n/l}/l_\nu = 3, z = 0.8\ cm, I_{20} = 2W$): 6 (1st and 2nd curves), 2 (3rd and 4th curves) and 0 (5th curve)

The energy of the spectrum is transferred from the central maximum to the side maxima (comparison of curves 1, 2 and 5). At large values of the frequency modulation coefficient $p = C^2\tau^4$, an increase in the number of side peaks, in other words, the splitting of the signal pulse is observed (3 and 4, as well as (comparison of curves 1 and 2). It was determined that the frequency conversion efficiency decreases and the width of the central maximum of the spectrum increases with the increase of phase modulation of the idler wave at the entrance of the metamaterial. The frequency conversion efficiency in the metamaterial is an order of magnitude higher than that of the conventional medium. In the second approximation of the dispersion theory, the energy of the pulse with the signal frequency is obtained from the expression (37) obtained for the spectral density of the pulse:

$$E_1 = cnd/8\pi \times$$

$$\times \int_{-\infty}^{+\infty} D \frac{(sin\lambda' z - tg\lambda' l \cdot cos\lambda' z)^2}{\lambda'^2 + (k^2/4)tan^2\lambda' l} \cdot \exp\left(-\frac{\omega^2\tau^2}{1 + C^2\tau^4}\right) d\omega, \quad (38)$$

where $D = cn_1\gamma_1^2 I_{30} I_{20}\tau^2/16\pi^2$ and

$$\lambda' = \frac{1}{l_{q/x}}\left[\frac{1}{4}\left(\frac{1}{2}(\alpha - 1)\frac{l_{q/x}}{l_{d.}}\omega^2\tau^2 + \frac{l_{q/x}}{l_\nu}\omega\nu - \frac{\Delta}{G_3}\right)^2 - 1\right]^{1/2},$$

$$k = \frac{1}{l_{q/x}}\left[i\left(\frac{1}{4}(\alpha - 1)\frac{l_{q/x}}{l_{d.}}\omega^2\tau^2 + \frac{l_{q/x}}{l_\nu}\omega\nu - \frac{\Delta}{G_3}\right)\right], \alpha = \frac{g_2}{g_1}.$$

Analysis has shown that, large values of energy are obtained in phase synchronism of velocities and at certain values of characteristic lengths $l_{n/l}/l_d$ and not at the zero value of dispersion of syrup velocities. This shows that both the group velocity difference and the group velocity dispersion have a beneficial role from the point of view of energy, and a similar result is obtained for conventional nonlinear media.

Figure 19 shows the dependence of the signal wave energy on the introduced length $z/l_{n/l}$ of the metamaterial at different values of $l_{n/l}/l_\nu$, $l_{n/l}/l_d$ and $p = \gamma^2\tau^4$ quantities. In the metamaterial, since the direction of the signal wave stream is opposite to the propagation directions of the filling wave and the idle wave, as expected, large values of energy are observed at the entrance of the metamaterial and not at the exit.

The fifth chapter is devoted to the study of the influence of the non-stationary conversion of frequencies in non-homogeneous optical fibers on the intensity and duration of second harmonics and sum-frequency pulses in the constant intensity approximation.

For this case, in the second order dispersion theory, the truncated system of equations describing the generation of the second harmonic, taking into account the losses in the medium, is given as follows[17]:

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[17]Tagiev, Z. A. Theoretical studies on frequency doubling in glass optical fibers in constant intensity approximation / Z. A. Tagiev, R.J. Kasumova // Optics and Communications, – 2006. 261, – p. 258 - 265.

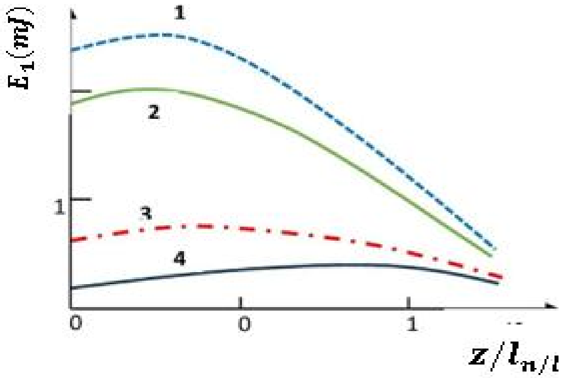

Figure 19. Dependence of energy density on the relative length $z/l_{n/l}$ at diffferent values of charackteristik lengths $l_{n/l}/l_v$ and $l_{n/l}/l_d$ for $(\Delta l_{n./l} = 3)$ and $p = \gamma^2\tau^4$ (1st, 3rd and 4th curves) and $p = 0$ (2nd curve): 1 – $l_{n/l}/l_n = 6$, $l_{n/l}/l_d = 0$; 2 – $l_{n/l}/l_n = 6$, $l_{n/l}/l_d = 0$; 3 – $l_{n/l}/l_n = l_{n/l}/l_d = 3$; 4 – $l_{n/l}/l_n = 0$, $l_{n/l}/l_d = 6$.

$$\frac{\partial A_1}{\partial z} + \frac{1}{u_1}\frac{\partial A_1}{\partial t} + \delta_1 A_1 =$$

$$= -i\gamma(|A_1|^2 + 2|A_2|^2)A_1 - i\beta_1 A_1^* A_2 exp[i\Delta_0 z + iY(z)], \quad (39)$$

$$\frac{\partial A_2}{\partial z} + \frac{1}{u_2}\frac{\partial A_2}{\partial t} + \delta_2 A_2 =$$

$$-i\gamma(|A_2|^2 + 2|A_1|^2)A_2 - -i\beta_2 A_1^2 exp[-i\Delta_0 z - iY(z)],$$

where A_1 - the complex amplitude of the pulse of frequency ω_1, A_2 - the complex amplitude of the pulse at frequency $\omega_2 = 2\omega_1$, $u_{1,2}$ - the group velocities at respective frequencies, $\delta_{1,2}$ - coefficients of absorption of interacting pulses, γ- mean value of nonlinear coupling coefficients due to self and cross modulation, $\beta_1 = \gamma_{SH}^*/2$, $\beta_2 = \gamma_{SH}$, $\gamma_{SH} = 3\omega_1\varepsilon_0^2\alpha_{\text{i}H} f_{112} c^{(3)^2}|E_{pump.}|^2|E_{SH}|$, ε_0 - dielectric permittivity of free space, $\alpha_{\text{i}H}$- constant, depending on microscopic processes, f_{112}- integral determined by the optical field mode's distribution, $E_{pump.}$- field of pulse with frequency ω_1-, $|E_{SH}|$-a weak field of second harmonic pulse with frequency $2\omega_1$; $\Delta_0 = k_2 - 2k_1 - \Delta(z)$, where Δ_0 and $\Delta(z)$ are the constant and variable parts of wave number difference and $Y(z) = \int_0^z \Delta(z')dz'$. For simplification the integral above we take $f_{ijk} \cong f_{112} \cong / A_{eff.}$, $(i, j = 1, 2)$ where $A_{eff.}$- is equal to the effective cross - sectional area of optical fiber.

In the quasi-static regime we solve set of equations (39) by the following boundary conditions:

$$A_1(z = 0) = A_{10}(t), \qquad A_2(z = 0) = 0, \tag{40}$$

Taking into account that, the medium process linear imhomogeneity $\Delta(z) = \alpha z$ and using the Weber's equation for the relative intensity[18] of second harmonic we obtain:

$$I_2(z)/I_{10} = (|\beta_2|z)^2 I_{10}(h) \cdot \exp(-4\delta_1 z)\left\{\left[1 - \frac{1}{3}|\alpha|z^2 + \right.\right.$$

$$\left.\left. + \frac{1}{30}(2|\alpha|z^4 - \Gamma^4 z^4)\right]^2 + \left[\frac{1}{3} - \frac{1}{10}|\alpha|z^2\right]^2 \Gamma^4 z^4\right\}. \tag{41}$$

When the pump pulse is of Gauss profile $(A_{10}(\eta) = A_{10}\, exp(-\eta^2/2\tau_1^2)$ expression for Γz in CIA is given by

$$\Gamma z = \sqrt{|\beta_1||\beta_2|I_{10}(h)}\, z = \sqrt{|\beta_1||\beta_2|I_{10}} z \cdot exp(-h^2/2\tau_1^2) = \Gamma(0)z$$

where $\Gamma(0) = \sqrt{|\beta_1||\beta_2|I_{10}}$, $I_{10} = A_{10} \cdot A_{10}^*$.

In case, when optical fiber possess a weak inhomogeneity ($|\alpha|z^2 < 1$) and interaction distance is small ($\Gamma z < 1$) from (39) we get

$$I_2(z) \approx [|\beta_2|I_{10}(h)z]^2\left(1 - \frac{2}{45}\Gamma^4 z^4\right) \cdot exp(-2\delta_2 z). \tag{42}$$

According to this equation duration of second harmonic pulse is given by the following equation

$$\tau_{2,inhom.} = \tau_1 \Big/ \sqrt{1 - \frac{4}{45}\Gamma^4(0)z^4}. \tag{43}$$

In the CFA (since $\Gamma = 0$) the last relationship is rewritten as

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[18]Kasumova, R.J. Laser pulse manipulation in optical fiber / R.J. Kasumova, Z.H. Tagiyev, Sh.Sh. Amirov // News of Baku State University, – Baku: – 2021. №1, – p. 72-82.

$\tau_{2,inhom.} = \tau_1$.

In the fiber with a strong inhomogeneity ($|\alpha|z^2 > 1$) and short interaction distance ($\Gamma z < 1$) from the equation (40) is obtained:

$$I_2(z) \approx [|\beta_2|I_{10}(h)z]^2 \cdot \Big(1 - \frac{2}{3}|\alpha|z^2 + \frac{11}{45}|\alpha|^2 z^4 - $$
$$- \frac{2}{45}|\alpha|^3 z^6 - \frac{1}{45}|\alpha|z^2\Gamma^4 z^4\Big) \cdot \exp(-2\delta_2 z). \quad (44)$$

To analyze the duration of second harmonic pulse from the formula (44) we get:

$$\frac{I_2(z)}{I_{10}} \approx \cdot \exp\left[\left(-\frac{h^2}{\tau_1^2}\right) \cdot \left(1 - \frac{2}{15}\Gamma^4(0)z^4\right)\right]. \quad (45)$$

In the fiber with a strong inhomogeneity duration of second harmonic pulse is given by

$$\tau_{2,inhom.} = \frac{\tau_1}{\sqrt{1 - \frac{2}{15}|\alpha|\Gamma^4(0)z^4}}$$

but in homogeneous medium $\tau_{2,inhom.} = \tau_1$. This fact is not observed in CFA (since $\Gamma = 0$). We can evaluate the pulse durations in the media with different inhomogeneites. Assume, $\Gamma(0) = 0.9$. Then we obtain in case of weak inhomogeneity $|\alpha|z^2 = 0.3$, and $\tau_{2,inhom.} = 1.0305\tau_1$; for the medium with strong inhomogeneity $|\alpha|z^2 = 3$ and $\tau_{2,inhom.} = 1.1644\tau_1$.

In this chapter, the influence of the inhomogeneity of the refractive index on the duration of the pulse of the sum frequency also was analyzed. Similarly to the case of second harmonic the following expression was obtained for the relative intensity of the sum frequency pulse when the idler wave is Gaussian[19,20]:

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[19]Kasumova, R. J., Amirov, Sh.Sh. Nonstationary sum frequency generation in inhomogeneous optical fiber // AJP "Fizika", − Baku: - 2022. 27, № 2, – p. 24-30.
[20]Kasumova, R. J., Amirov, Sh.Sh., G. A. Safarova, A. R., Ahmadova, N. V. Kerimli. Sum frequency generation in optical fiber in the constant intensity approximation // − Baku, The 8-th International Conference on control and optimization with industrial application, − 4-26 august, − 2022. − p. 273-275.

$$\frac{I_3(z)}{I_{10}} \approx I_2 exp\left\{-\frac{h^2}{\tau_1^2}\left[1-\frac{1}{30}\left(\Gamma_1^2(0)+\Gamma_2^2(0)\right)\Gamma_1^2(0)z^4\right]\right\}. \qquad (46)$$

As can be seen an increase of the duration of output sum frequency pulse is expressed by:

$$\tau_{3,inhom.} = \frac{\tau_1}{\sqrt{1-\frac{1}{30}\left(\Gamma_1^2(0)+\Gamma_2^2(0)\right)\Gamma_1^2(0)z^4}}. \qquad (47)$$

The rate of increase of the pulse duration has a weak dependence in the CFA.

$$\tau_{3,inhom.} = \frac{\tau_1}{\sqrt{1-\frac{1}{30}\Gamma_1^4(0)z^4}}. \qquad (48)$$

In the fiber with a strong inhomogeneity ($|\alpha|z^2 > 1$) and short interaction distance ($\Gamma z < 1$) the relative intensity is given by

$$I_3(z) \approx |\beta_3|^2 z^2 I_{10}(h)\cdot exp[-2(\delta_1+\delta_2)]\left[1-\frac{2}{3}|\alpha|z^2+\right.$$

$$\left.+\frac{11}{45}|\alpha|^2 z^4-\frac{2}{45}|\alpha|^3 z^6-\frac{1}{45\cdot 4}|\alpha|z^2(\Gamma_1^2+\Gamma_2^2)z^4\right]. \qquad (49)$$

This expression has the view in CFA

$$\frac{I_3(z)}{I_{10}} \approx |\beta_3|^2 z^2 I_{20}(h)\left[1-\frac{2}{3}|\alpha|z^2+\right.$$

$$\left.+\frac{11}{45}|\alpha|^2 z^4-\frac{2}{45}|\alpha|^3 z^6-\frac{1}{45\cdot 4}|\alpha|z^2\Gamma_1^4 z^4\right]. \qquad (50)$$

When refractive index of optical fiber is homogeneous, the intensity of sum frequency pulse is expressed by

$$\frac{I_3(z)}{I_{10}} \approx |\beta_3|^2 z^2 I_{20}(h) exp[-2(\delta_1+\delta_2)z]. \qquad (51)$$

When inhomogeneity satisfies condition $|\alpha|z^2 < 1.5$ from (49) for the relative intensity we get:

$$\frac{I_3(z)}{I_{10}} \approx I_{20}\ exp\left\{-\frac{h^2}{2\tau_1^2}\left[1-\frac{1}{45}|\alpha|z^2\left(\Gamma_1^2(0)+\Gamma_2^2(0)\right)\Gamma_1^2(0)z^4\right]\right\}.$$

Hence the duration of the sum frequency (ω_3) pulse is determined as follows:

$$\tau_{3,inhom.} = \frac{\tau_1}{\sqrt{1 - \frac{1}{45}|\alpha|z^2\left(\Gamma_1^2(0) + \Gamma_2^2(0)\right)\Gamma_1^2(0)z^4}}, \qquad (52)$$

However, for homogeneous fiber we get $\tau_{3,inhom.} = \tau_1$.

This dependence is weak in the CFA

$$\tau_{3,inhom.} = \frac{\tau_1}{\sqrt{1 - \frac{1}{45}|\alpha|z^2\Gamma_1^4(0)z^4}} \qquad (53)$$

In case of weak inhomogeneity when ($|\alpha|z^2 = 0.3$) we get,$\tau_{3,inhom.} = 1.0041\tau_1$, and for the strong inhomogeneity when ($|\alpha|z^2 = 1.5$) we get $\tau_{3,inhom.} = 1.01378\tau_1$ alırıq.

We can conclude, that with the increase in the degree of inhomogeneity of the optical system, the quality factor of the system, that is, the Q factor, decreases analogously to the Q factor of the optical resonator, which leads to the expansion of both second harmonic and sum frequency pulses generated in the system

The **sixth chapter** is devoted to the study of some applications of the constant intensity approximation. The generation of the second optical harmonic in the nonlinear mode, the effect on the efficiency of the conversion of the linear losses of the medium into higher harmonics, the conversion into the third harmonic in the erbium-doped zinc-oxide layers, and the dispersion interferometer used for determining the electron density of the plasma were analyzed theoretically**.**

We get the result of the constant amplitude approximation when $\gamma_1 = 0$ or ($G= 0$)) in the formula (3) obtained in the constant intensity approximation for the complex amplitude. It can be seen from that solution that, unlike the constant amplitude approximation, in the constant intensity approximation, the amplitude of the second harmonic wave takes its maximum value at a certain optimum ($I_{opt.}$) value of the wave intensity by means of the Γ-quantity included in$I$. This optimal value of the intensity, in turn, depends on other parameters, for example, the difference in wave numbers, the length of the nonlinear medium, etc. is a function.

In contrast to the constant amplitude approximation, the argument of the trigonometric function in the constant intensity approximation depends on the intensity of the filling wave through the formula $G^2 = \gamma_1\gamma_2 I_{10}$. When the condition $\delta_2 = 2\delta_1$ is satisfied for linear losses in the medium, the argument of the trigonometric function included in the expression of the complex amplitude $A_2(l)$in formula (3) is simplified:

$$\lambda z = \sqrt{2G^2 z^2 + \Delta^2 z^2/4}. \tag{54}$$

Unlike the apparently constant field approximation ($\gamma_1 = 0$) the period of the spatial beats of the second harmonic depends on the intensity of the filling wave. As the intensity of the filling wave increases, the period of the spatial beats decreases and therefore the width of the central maximum decreases.

In the equation above the $\gamma_1 = 0$ value corresponds to CFA. Hence, G→ 0 and $\lambda_m z = \Delta z/2$ is obtained. If denote the value of $\Delta -$ by Δ_m, corresponding to the maximum of second harmonic amplitude then $\Delta_m l/2 = \pi/2$ or $\Delta_m = \pi/l$. This equality is equivalent to the expression $\pi/2 = \sqrt{2G^2 z^2 + \Delta_m^2 z^2/4}$ in SIA. We denote $G = l_{n/l}^{-1}$ in the squared root and obtain equation for Δ_m at which amplitude of second harmonic becomes maximum[21].

$$\frac{\Delta_m l}{2} = \left[\frac{\pi^2}{4} - 2\left(\frac{l}{l_{n/l}}\right)^2\right]^{1/2}. \tag{55}$$

As can be seen the parameter $\Delta_m l/2$ characterizing wave number difference is a function of pump intensity in CIA, but not in CFA ($G^2 = \gamma_1\gamma_2 I_{10}, l_{n/l} = G^{-1}$). In Figure 20, the dependence of the maximum value of the difference in wave numbers (WND) on the etched length of the medium is given.

The analysis showed that the result of the CİA is the same as the

[21]Tagiev, Z.A. Generation of second optical harmonic in the nonlinear regime / Z. A. Tagiev, Sh.Sh. Amirov, N.V. Kerimli // Proceedings of International Conference "Modern Trends in Physics", – Baku: – 20-22 April, – 2017, - p. 395-397

result of the exact solution up to the value of $l/l_{n/l} \leq 0{,}7$. The result of the CFA approximation is close to the result of the exact solution only around $l/l_{n/l} = 0$. Also, when the length of the medium is equal to the nonlinear length ($l/l_{n/l} = 1$), the results of CIA and CFA differ from the result of the exact solution by 36.7% and 46%, respectively.

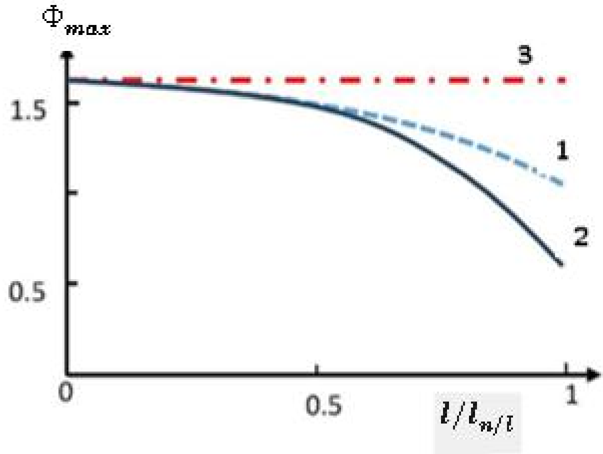

Figure 20. Dependence of maximum value of WND on the dimensionless length for the second harmonic generation 1. Accurate solution. 2. CIA. 3. CFA.

In this chapter, generation of the third harmonic in erbium-doped ZnO:Er thin films was theoretically studied and the obtained results were compared with the experimental results[22,23].

Concentration of dopant were introduced to the calculations of thirs harmonic intensity by use the experimental formula[24.]

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[22]Lamrani, M. A. Influence of roughness surfaces on third-order nonlinear- optical properties of erbium doped zinc oxide thin films / M. A. Lamrani, M. El Jouad, M. Addou [et al.] // Spectrroscopy Letters, − 2008. 41, − p. 292-298.

[23]Lamrani, M. A. Cathodoluminescent and nonlinear optical properties of undoped and erbium doped nanostructured ZnO films deposited by spray pyrolisis / M. A. Lamrani, M. Addou, Z. Sofiani [et al.] // Optics Communications, −2007. 277, − p. 196-201

[24]Kulyk, B Linear and nonlinear optical properties of ZnO//PMMA nanocomposite films / B. Kulyk, B. Sahraoui, O. Krupka [et al.] // Journal of Applied Physics, − 2009. №106, − p. 093-102.

Analytical analysis conducted for different thicknesses of ZnO:Er (5%) films (130 nm, 190 nm, 220 nm and 240 nm) showed that when the thickness of the film decreases from 240 nm to 130 nm, the intensity of the third harmonic is approximately 2.3 times increases (Fig. 21) Analytical expressions for the intensity of the third harmonic wave and the coherence length of the nanocomposite medium have been obtained. Expression obtained for the optimal length of the films in CIA corresponding to the maximum of the third harmonic conversion efficiency is given by ($\delta_3 = 3\delta_1$):

$$l_{coh.} = /_1^{-1} \operatorname{arctg}\left(\frac{/_1}{\delta_3}\right), \tag{56}$$

where $/_1^2 = 3\Gamma_3^2 + \Delta^2$.

This result, obtained in the CIA is differ from the result obtained in the CFA.

It was calculated that as a result of increasing the concentration of erbium dopant by 5%, the intensity of the third harmonic increases approximately 161.3 times, and the coherent length[25] calculated by formula (56) decreases from 650 nm to 609.97 nm. When the intensity of the pump changes in the range of 2GW/cm^2 - 2.4GW/cm^2, the intensity of the third harmonic increases by 1.69 times in ZnO layers without additives, and by 1.73 times in ZnO:Er(5%) fims. Compared to different thicknesses of the samples, the largest value of the third harmonic intensity was calculated for the layer with a thickness of 60 nm.

CONCLUSION

1. At low frequencies of the pump wave in the metamaterial, when the ratio of the intensities of the idler and pump waves at the entrance of the metamaterial increases by 5 times, the amplification factor of the signal wave increases by 20 times, and when the ratio of

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[25]Kasumova, R.J. On influence of temperature and doped concentrations on the frequency conversion efficiency in erbium doped zinc oxide films / R.J. Kasumova, S.R. Figarova, Sh.Sh. Amirov [et al.] // American Journal of Optics and Photonics, – USA: – 2016. 4, № 6, – p. 57-63.

the intensities of the idler and signal waves increases by 5 times, the amplification factor increases by 1.5 times, if the ratio of the idler wave intensity to the pump wave intensity is 3 times, the signal wave frequency conversion efficiency increases from 4.4% to 13%. At optimum power (2.85 W) of pump, with a 2-fold increase in the intensity of the signal wave at the input of the metamaterial, the amplification factor at the output also increases by a factor of 2.

2. At the optimum values of wave number difference (WND) $\Delta_{opt.,1}$=5.9484 cm^{-1} and $\Delta_{opt.,2}$=12.42 cm^{-1} at low frequencies of the pump wave, the amplification factor of the signal wave is greater than one ($h_{amp.} > 1$). At the value $\Delta = 2.097688\ cm^{-1}$ the dependence of the frequency conversion efficiency on the length of the nonlinear medium is expressed by a hyperbolic tangent. When the WND is greater than the minimum possible value ($\Delta^{CIA} > 2(\Gamma_3^2 + \Gamma_2^2)^{1/2}$) the dependences are expressed by oscillations of different frequencies, and when it is smaller, by hyperbolic sine and cosine functions.

3. At higher frequencies of the pump wave conversion efficiency increases by a factor of 2 at optimum length calculated by the formula $l_{opt.} = \pi/2/\sqrt{\Gamma_3^2 - \Gamma_2^2 + \Delta^2/4}$ obtained in CIA. İn contrast to ordinary nonlinear media (in such media, the conversion efficiency has a maximum), the conversion efficiency in the metamaterial increases monotonically, and the maximum efficiency is obtained at the input, not at the output of the metamaterial. When the conditions of phase matching ($\Delta = 0$) and $\Gamma_3 l = \pi/2 + \pi m$, $m = 0{,}1{,}2$ .... are satisfied the signal wave frequency conversion efficiency approaches infinity, and at the value $\Gamma_3 l_{opt.}$ the amplification factor becomes infinitely larger, exceeding the losses, and even the signal wave becomes to be generated. When the WND increases from π to 5, the efficiency of signal wave frequency conversion decreases by 2.5 times. At the optimum value of the pump wave power (≈ 4W), when the intensity of the signal wave at the input of the metamaterial (I_{1l}) increases by 5 times, the conversion efficiency at the output increases by almost 2 times. This shows that by increasing the intensity of the signal wave at the input, a higher level signal can be obtained at the output.

4. The amplification factor of the signal wave has periodic resonances, depending on the coordinate, with maxima satisfying the condition $I'_1 l = \pi/2 + \pi k,\ k = 0, 1, 2, ...$ when the condition of phase-matching during four-wave interaction is satisfied ($\Delta = 0$) and there is no idler wave at the entrance of the metamaterial ($I_{20} = 0$). In samples with a thickness of 20-80 nm, the width of the resonance curves increases with the increase of losses in the range of 40-65 nm. Resonances are also observed in the dependence of the amplification factor on the intensity of the pump. This allows to determine the intensities of the filler in the given parameters.

5. The threshold of pump intensity, obtained from the phase matching ($\Delta = 0$) and condition $A_1(z = 0) \geq A_1(z = l) = A_{1l}$, increases with increase in idler wave intensity, nonlinear coefficients of coupling and loss parameter during four frequency interaction in metamaterial under equal intensities of pump waves. The condition $\delta_2 + \delta_3 + \delta_4 = \delta_1$ obtained for the loss coefficients shows, that the loss of signal wave (δ_1) is compansated by the losses of two pump waves and the idler wave propagating in mutually opposite direction.

6. In CIA for the four-frequency interaction, it was obtained that in the experimental metamaterial films with a thickness of 20 nm, when the ratio of the intensity of the idler wave with frequency $\omega_2$to the intensity of the pump wave changes in the range $I_{20} = 0.3I_{30}$ and $I_{20} = 0.5I_{30}$, the amplification factor of the signal wave increases by a factor of 1.43 and in contrast to the CFA, the minima of the oscillations shift. In this case, it is possible to determine the distance between two adjacent minima or the period of oscillations. It was in CIA obtained that the optimum values of WND for the central maximum $\Delta_{opt.,1} = 2\sqrt{\gamma_1\gamma_3 I_{20} I_{40} + a}$ and for the side maxima $\Delta_{opt.,2} = 2\sqrt{4.7 + \gamma_1\gamma_3 I_{20} I_{40} + a}$ (here $\alpha = \gamma_1\gamma_4 I_{20} I_{30} - \gamma_1\gamma_2 I_{30} I_{40}$) of reflective index $R = I_1(z = 0)/I_{20}$ in addition to other parameters (found in CFA) depend also on the intensity (I_{20}) of the idler wave. Under phase matching conditions, an increase in reflective index takes place when the ratio (I_{20}/I_{30}) of intensities of idler and pump waves decreases.

7. The frequency conversion efficiency into signal wave was calculated in CIA as $\mathrm{h} \sim 10^{-7} - 10^{-8}$, and $\mathrm{h} \sim 10^{-6} - 10^{-7}$ for the values of WND $\Delta l/2 = 3.5$ and $\Delta l/2 = 1.68$ respectively in case of four frequency interaction in metamaterial when pump intensity equals $I_{30} = 5 \cdot W/cm^2$ and these results are agree with experimaental data.

8. In the second order dispersion theory, the spectral density of the signal wave is an order of magnitude greater than in the usual nonlinear medium at the central frequency and, depending on the coordinate, takes its largest value in the middle of the sample in the metamaterial. The spectral density of the signal pulse is maximum at the value of $\omega\tau \approx -0.7$ at the optimal power of the pump of 5 W, and with the increase of the intensity of the pump, the energy is transferred from the central maximum to the side maxima. As the ratio of the nonlinear length to the quasi-static length increases, the width of the spectrum decreases, with the increase of the ratio to the dispersion length, the spectrum broadens, and when the ratio to the dispersion length is equal to zero, the spectrum becomes symmetric. As the frequency modulation parameter increases, the spectral density decreases and the central maximum expands, energy is transferred from the central maximum to the side maxima. Large values of the delivered energy of the signal wave are obtained not at the output of the metamaterial, but at its input.

9. The intensity of the third harmonic and the coherence length of the samples in the erbium-doped ZnO fims were analytically calculated. It has been shown, that the number of additional polarization centers and therefore the nonlinear susceptibilities of the nanocomposite films also increase with the increase in the concentration of the erbium impurity. In the ZnO:Er(5%) nanocomposite, when the experimental intensity of the pump is equal 2 GW/cm^2, the relative intensity of the third harmonic wave has the largest value at the temperature of 450°C compared to other temperatures. When the concentration of erbium increases from 0%- to 5%- the intensity of the third harmonic wave increases by a factor of 161.83. At the optimal intensity value of 2 GW/cm^2 and temperature of 450°C, the coherent length for ZnO without dopant was 650 nm, and when 5%-Er impurity was added, it was decreased to 609.67 nm.

10. Analysis has showed that when intensity of pump wave increases in the range 2 GW/cm^2 – 2.4 GW/cm^2 – increase in intensity of third harmonic in pure ZnO samples was found by 1.69 times, but this factor in the ZnO:Er(5%) was found to be 1.73 times. The greatest value of intensity was calculated for the films of 60 nm thickness. For this case a transfer of energy from central maximum to the side maxima is observed. The intensity of the third harmonic is approximately increases by a factor of 2.3 in ZnO:Er(5%) films, when their thickness decreases from 240 nm to 130 nm.

11. In CIA the durations of the second harmonic and sum frequency pulses increase relative to the duration of the pump pulse with increase of the inhomogeneity of the refractive index, the results of the duration of the second harmonic pulse are $\tau_2 = 1.0305\tau_1$ in weak inhomogeneity, and $\tau_2 = 1.1644\tau_1$ in strong inhomogeneity. The duration of sum frequency pulse was calculated as $\tau_{3.} = 1.0041\tau_1$ and $\tau_{3.} = 1.01378\tau_1$ in fibers with weak and strong inhomogeneity respectively. In a homogeneous medium, the durations of the second harmonic and sum frequency pulses are the same as the duration of the pump pulse. These facts are not observed in CFA.

12. It has shown, that the optimal length of the nonlinear crystal corresponding to the maximum efficiency of conversion into the second harmonic intra-cavity laser depends on the intensity of the pump wave, the synchronism curves are wider than the curves obtained in the external resonator, the efficiency of cascade conversion into the third optical harmonic is maximum when the optimal phase condition is satisfied. The value of the WND corresponding to the maximum of the second harmonic conversion efficiency in the nonlinear regime depends on the intensity of pump wave. The values of the maximum of the WND in CIA and CFA differ from the result of the exact solution by 36.7% and 46%, respectively at equal lengths of the medium and the nonlinear length.

The list of published scientific works on the topic of the dissertation:

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