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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

DEVELOPMENT OF THEORETICAL AND PRACTICAL BASES FOR WAVE-DEFORMATION RELATIONS DURING HYDROCARBON FILTRATION IN POROUS MEDIA

Speciality: 2003.01 – Mechanics of liquids, gases and plasma

Field of science: Mechanics

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GENERAL OVERVIEW OF THE WORK

Relevance of the topic and level of development.

Minerals are very important for the development and maintenance of the benefits of society. For at least a century, one of the most important raw material bases is oil, consisting mostly of various types of hydrocarbons, gas, water and other possible impurities.

Due to the occurrence of hydrocarbon reserves at significant depths underground (from 1 to 5, ultra-deep up to 10 kilometers), in caverns, porous soils, etc. generally called reservoirs, there is a need for exploration to analyze the properties of these reservoirs to estimate reserve volumes, preferred extraction methods, etc. One of these main methods is seismic exploration. On the other hand, the landscape was constantly changing, earthquakes occurred, and the reservoirs experienced significant deformations and pressures from the overlying rocks. Also, during the process of fluid production, the ratio of rock and reservoir pressure in reservoirs changes, which is accompanied by significant deformations.

The interaction of the parameters of wave velocities, pressures and deformations, and densities almost directly affects the nature of the filtration flow of fluids, since these parameters mainly affect the porosity of the medium. The opposite is also true: the nature of the flow of the filtered fluid, and its physical and chemical properties, change the elastic deformation properties of the soil through which the pressure wave passes. Nevertheless, it is possible and important to grasp some patterns of these processes.

The study of the processes of propagation of pressure waves, the formation of structures in rocks as a result of long-term changes in stress, their influence on the filtration of fluids and, conversely, the influence of fluid filtration on the structures of rocks contribute to a better understanding and study of mechanical processes in the environment. The solution to these issues is relevant, since the assessment of liquid filtration processes in a porous medium due to reservoir deformations plays a very important role in assessing the practical basis of the relationships that arise here.

There are a large number of studies that take into account deformation processes in problems of fluid filtration in reservoirs. However, a number of issues of the mutual influence of waves and deformations in reservoirs have been insufficiently studied. Therefore, we can say that the chosen direction is relevant.

Object and subject of research.

The object of study of the dissertation work is waves and the filtration process in porous fluid-saturated media. The subject of the research is the propagation of various waves in soils, their mathematical models, the development of experimental and theoretical foundations of filtration processes and the analysis of the obtained numerical results.

Goal of the work. The purpose of the dissertation work is to develop the theoretical and practical foundations of wave-deformation relationships during the filtration of hydrocarbons in porous media to create more advanced models that take into account the influence of deformations on the propagation of waves in deep-lying soil environments, as well as to determine the influence of rock consolidation on the processes of fractality and filtration in these collectors.

Basic provisions submitted for defense.

1. A connection has been established between the parameters of deformation and the elastic properties of the medium.

2. The regularities of the stress-strain state of the soil during the passage of cyclic shock waves through the rock material are studied.

3. A connection has been established between changes in stresses and displacements behind the shock wave front in rocks with different characteristics.

4 Possible influences of fluid flow under the fractal law of their filtration on the consolidation of a rock layer are considered.

5. The influence of consolidation pressure on the fractality of the filtration law in fluid-saturated porous media is considered.

6. An assessment was made of the nature of the propagation of pressure waves in a porous medium in the case of the action of a nonlinear law of fluid filtration.

7. The possibility of controlling hydrodynamic instability in order to increase the fluid saturation of the produced pore fluid by the method of increasing cyclic action in the porous material of the rock formation has been established.

Scientific novelty of the research.

1. The nature of the mutual influence of waves-strains in the formation is determined.

2. The regularities of the stress-strain state of the soil during the passage of shock waves through it were studied.

3. An assessment was made of the influence of pressure waves on the deformation and filtration properties of fluid-saturated soils.

4. The mutual influence of the consolidation of the soil layer under the fractal law of fluid filtration and consolidating rock pressure on the filtration of fluids in the formation, taking into account fractality, was studied.

5. The possibility of controlling hydrodynamic instability in the process of fluid displacement under increasing cyclic action in a porous medium has been studied.

Research methods.

To solve the problems, numerical methods for solving equations of mathematical physics, equations of elastic dynamics and a non-classically linearized approach, equations of underground hydrodynamics, equations of the theory of consolidation and fractality were used.

Reliability of results. The reliability of the theoretical and experimental results obtained in the work follows from the fact that they are based on the general laws and equations of continuum mechanics and are ensured by strict mathematical conclusions, estimates, and the choice of numerical methods, the high efficiency of which in constructing a numerical model is due to the use of a priori information about the properties of the solution, comparison of solutions to problems obtained by various methods, qualitative and quantitative agreement of theoretical results with experimental data and with the results of the work of other authors.

Practical value. The results obtained in the dissertation work expand and deepen the theoretical knowledge about the movement of soils during the propagation of shock waves, and also create the basis for solving problems associated with regulating the processes of filtration and displacement of fluids in heterogeneous and fractured-porous oil formations.

Approval and presentation.

The main scientific results and the progress of the dissertation work were presented and discussed at scientific seminars and meetings at the Department of Theoretical Mechanics and Continuum Mechanics of the Mechanics and Mathematics Faculty at Baku State University, the Department of Fluid and Gas Mechanics at the Institute of Mathematics and Mechanics. Additionally, they were presented at national and international conferences, including the Republican Scientific Conference "Mathematics, Mechanics, and Their Applications" dedicated to the 98th anniversary of National Leader Heydar Aliyev (Baku, 2021); the International Scientific Conference "Modern Problems of Mechanics and Mathematics" dedicated to the 110th anniversary of Academician Ibrahim Ibrahimov (Baku, 2022); the International Scientific Conference "Mathematical Analysis and Its Applications in Modern Mathematical Physics" (Uzbekistan, Samarkand, 2022); the International Scientific Conference "2nd International Conference on Engineering and Applied Natural Sciences" (Turkey, Konya, 2022); the Republican Scientific Conference "Theory of Functions, Functional Analysis, and Their Applications" dedicated to the 110th anniversary of Academician Ibrahim Ibrahimov (Baku, 2022); and the International Scientific Conference "Modern Problems of Mathematics and Mechanics" dedicated to the 100th anniversary of National Leader Heydar Aliyev (Baku, 2023).

Author's personal contribution.

The main results of the dissertation work were independently obtained by the author with the assistance of the academic advisor.

The development of the main scientific results was carried out with the direct participation of the author, either as the primary researcher or as the responsible executor.

Publications.

Based on the materials of the dissertation work, six articles were published in republic and international scientific journals, one of which is indexed in "Scopus," and another journal is indexed in both "Scopus" and "Web of Science" and one in “zbMath”. Additionally, six abstracts related to this work were published, including two abstracts in international foreign conferences.

Accuracy of obtained results.

The accuracy of the results obtained is ensured by the correctness of setting and solving problems based on the laws of linear and nonlinear elastodynamics using non-classical linearization, known filtration laws and soil consolidation laws. The accuracy during the numerical solution for the corresponding speed parameters and pressure parameters is determined by the selected scale of steps for changing the deformation parameter and the time parameter.

Name of the institution where the dissertation work was conducted.

The work was performed at the Department of Theoretical Mechanics and Continuum Mechanics of the Faculty of Mechanics and Mathematics at Baku State University.

Structure and volume of the dissertation (in characters, specifying the volume of each structural unit separately).

This dissertation consists of an introduction, three chapters, a conclusion, and a bibliography.

The total volume of the dissertation work is 233,854 characters (title page – 461 characters, table of contents – 1,393 characters, introduction – 29,568 characters, first chapter – 56,713 characters, second chapter – 42,035 characters, third chapter – 104,140 characters).

This work is presented in 162 pages and includes 52 figures, including graphical representations; 11 tables, and a bibliography containing 126 scientific publications.

DISSERTATION CONTENTS

In the first chapter of the dissertation, the features of wave propagation in soils are studied. Here we briefly talk about the types of waves and their speeds when propagating through porous - granular soil.

In the field of wave propagation in porous media and soils, much attention was paid to Ya.B.Zeldovich, A.H.Mirzajanzade, Yu.A.Amenzade, A.N.Guz, G.G.Guliev, S.D.Akbarov, R. M.Sattarov, O.L.Kuznetsov, J.K.Agalarov, R.Yu.Amenzade, M.A.Bayot, B.G.Kholodar, A.A.Ilyushin, A.G.Averbukh, R.Dobriy, G.Kolski , A.V. Ilyushonok, V.I. Tereshenkov, R.K. Khismatulin, and others.

A wave refers to the vibration of soil particles. In the general case, particles in such an oscillation describe an elliptical trajectory, with energy transferred from particle to particle in a plane parallel to the focal length. In a particular case, particles can move along a round trajectory or along a small straight line. Thus, in total, the movement of such particles leads to the fact that the soil is deformed in the longitudinal and/or transverse direction when a wave passes through it. Ground motions can be quite complex due to the interaction of waves with soft surface sediments and the generation of surface waves. The characteristics of wave propagation depend on the individual properties and parameters of the materials that make up these media: the rigid skeleton and the liquid. Axial and transverse deformations can occur when waves propagate parallel to the horizontal or vertical axis of the reservoir. Shear deformations often occur if the cross-section of the reservoir is less than the shear wavelength. Moreover, in fluid-saturated porous soils, transverse waves can either die out completely or significantly lose their intensity, since, obviously, the shear modulus in liquids and gases is zero. And all the work on the propagation of transverse waves is assigned to the solid skeletal frame of such soil and depends on its properties.

Next, the mutual influence of the parameters of stress waves and elastic parameters of the medium is considered. Since the material of the deep-lying soil of the earth actually does not have an initial state, because the initial state was formed millions of years ago, then by the initial state of the soil we mean its current state. Therefore, here it is necessary to apply the method of a non-classically linearized approach. The application of this approach to the range of problems under consideration has a certain methodological advantage compared to the direct use of general nonlinear theory. First of all, the nonlinear problem can be reduced to linear (non-classical) and well-studied mathematical problems. In the case of homogeneous deformed states, simple analytical formulas are obtained, in the structure of which the contributions of linear (it is these components that should be called elastic properties of the medium) and nonlinear influences on the quantitative indicators of the parameters of elastic properties are explicitly identified. Elastic potentials are used to connect deformation parameters with elastic properties. These potentials, roughly speaking, imply the distribution law of the internal elastic energy of a material with certain given elastic parameters under a given deformation. To obtain analytical formulas expressing the change in the ratio of the elastic wave propagation parameter to the density or deformation parameter, these elastic potentials should be used. In other words, the elastic potential can be defined as a value identical to the free energy of the body. One of the simple and relatively convenient elastic potentials is the harmonic type potential. It has the following form (A.N. Guz, 2004[1]; G.G. Guliev, 2018[2]):

$$\Phi^S = \frac{1}{2}\lambda S_1^2 + \mu S_2 \qquad (1)$$

[1] Guz, A.N. Elastic waves in bodies with initial (residual) stress. // A.N.Guz. - Kiyev: ASK, -2004. -300p.

[2] Guliev, G.G. On the elastic parameters of deformed media. // Structural Engineering & Mechanics, -2018. 67(1), – p-p. 53-67.

where λ, μ – are Lame parameters, S_1, S_2 – invariant system that depend on deformation parameter.

Furthermore, from here, the main formulas relating the parameters of longitudinal and share wave velocities to the deformation parameter of the form were derived:

$$C_{l_{x_1}} = C_{l_0}\sqrt{\frac{\rho_0}{\rho}(1+2\varepsilon_0)^{\frac{5}{2}}}$$

$$\frac{C_{S_{x_2}} = C_{S_{x_3}} = C_{S_{x0}}\sqrt{\frac{\rho_0}{\rho}(1+2\varepsilon_0)^{\frac{3}{2}}}\cdot}{\cdot\left[(1+2\varepsilon_0)\left(\frac{2-\nu}{1-2\nu}\right)-\sqrt{1+2\varepsilon_0}\left(\frac{1+\nu}{1-2\nu}\right)\right]} \quad (2)$$

$$\frac{P_0}{\mu} = \frac{2+2\nu}{1-2\nu}\left(\sqrt{1+2\varepsilon_0}-1\right)$$

The $\frac{P_0}{\mu}$ – parameter describes pressure.

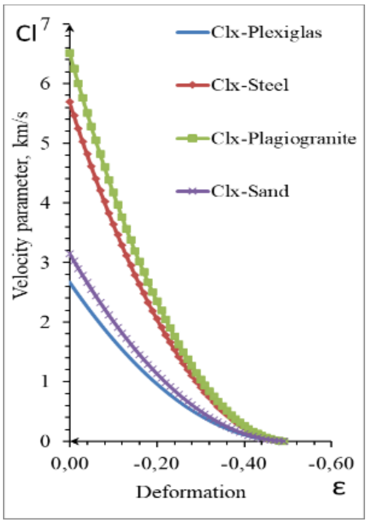


Figure 1.
Variation of the longitudinal wave propagation parameter Clx for different materials as a function of the deformation magnitude ε.

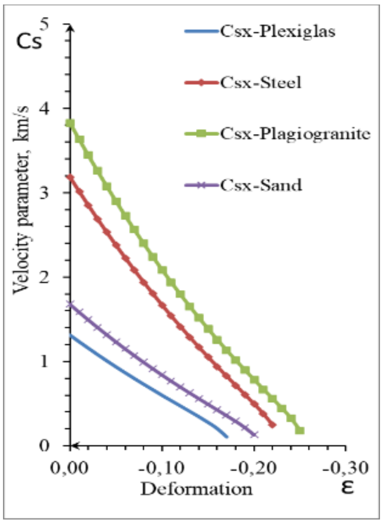


Figure 2.
Variation of the share wave propagation parameter Csx for different materials as a function of the deformation magnitude ε.

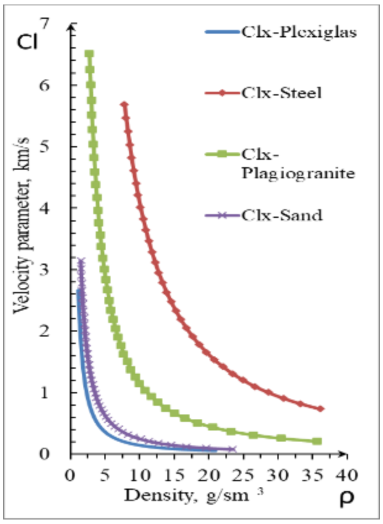


Figure 3.
Variation of the longitudinal wave propagation parameter Clx for different materials as a function of density ρ values.

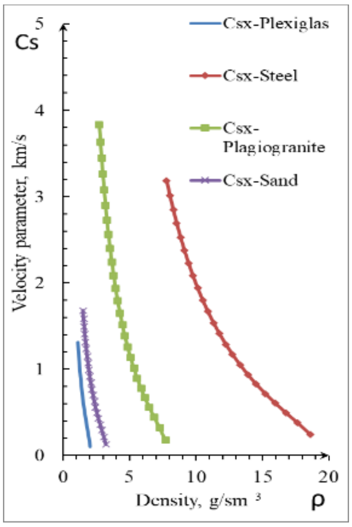


Figure 4.
Variation of the share wave propagation parameter Csx for different materials as a function of density ρ values.

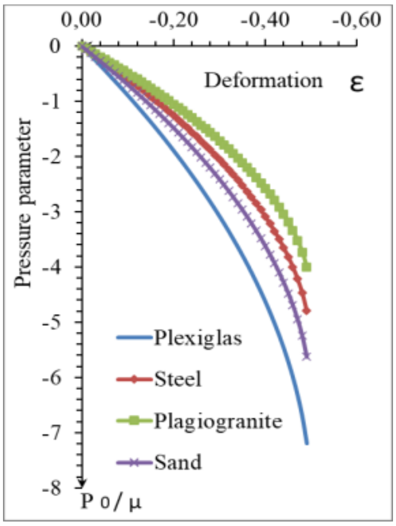


Figure 5.
Change in the pressure parameter P_0/μ for various materials as a function of the deformation magnitude ε.

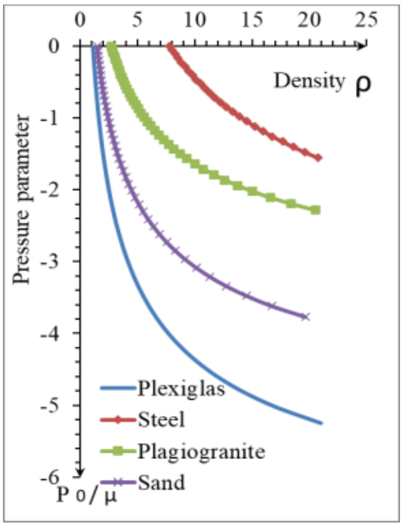


Figure 6.
Change in the pressure parameter P_0/μ for various materials as a function of the density values ρ.

These formulas, valid for the harmonic type of elastic potential, were compared with potentials of the quadratic type and the Murnaghan potential carried out by other researchers using constants for specific materials, namely: plagiogranite, steel and plexiglass. When using all of the above types of elastic potentials, there is a general decrease in the values of specific velocity parameters and an increase in pressure values with an increase in the deformation parameter. Specific critical speed values differ only quantitatively in the case of using the appropriate type of potential (Fig. 1-6). **In the second chapter,** the stress-strain state of the soil when shock waves pass through it is studied. Basic concepts are given and the features of the propagation of shock effects on the soil environment are considered.

In the field of soil behavior under dynamic loads, much attention was paid by M.B. Akhundov, N.B. Rasulova, E.A. Voznesensky, V.T. Trofimov, V.G. Kovalenko, O.V. Zerkal, V.A. Kalachev, A.M. Kapitonov, E.N. Samarin, A.D. Sashurin and others.

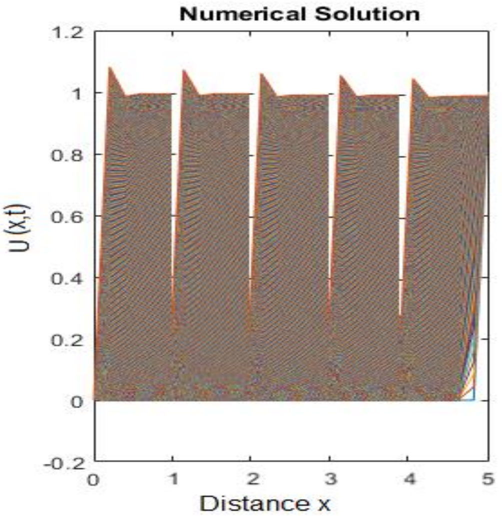
A shock wave, acting on soft soil, causes large stresses in it and movements of particles of the medium. After passing, the shock wave changes the structure and properties of the soil. Therefore, determining the stresses and displacements behind the shock wave front can suggest new parameter values for deformed soil. A shock wave is characterized by a sharp compressive effect on the medium and a longer-lasting, relative to its compressive effect, discharge zone. Those. A feature of the propagation of a shock wave in the soil is the dynamic effect that changes relatively quickly over time. Dynamic impact is understood as a load of any type created by a force of variable magnitude and direction. The forms of soil reaction to dynamic impact can be reduced to several options: 1. fatigue failure, 2. dynamic compaction, 3. reduction in strength, which is expressed in the form of partial softening or complete loss of stability. The mechanisms of dynamic instability of soils can be divided into the following types: fatigue (accumulation of damage under the influence of alternating cyclic stresses), thixotropy (decrease in viscosity and/or strength characteristics (liquefaction of soils) under alternating cyclic action), quasi-thixotropy (the same as thixotropy, but with lower frequency impacts and longer duration of time, slow-flowing thixotropy), dilatation and dilatant thixotropic effects (increase in rock volume (porosity) under saturation or other impact, such as wave), dynamic dilatancy (increase in porosity under shear impacts). Thus, the study of the propagation of shock waves in media remains an important aspect of modern science and technology. When a shock wave propagates in solid media (such as soils or rocks) under the influence of high pressure, the rock layers are compressed (i.e. change their structure), compressed and, turning into a plastic state, move in the radial direction from the center of shock wave propagation . As a result, depending on the magnitude of the pressure, the porosity of the soil, the following may occur: a displacement zone, a zone of compacted rock, and at relatively low pressures or at a distance from the center of the shock wave, where compressive stresses become less than the tensile strength of the material, but are still high displacement of particles of the medium, a zone of rupture of the medium appears. When shock disturbances propagate in a soil mass, many characteristic areas arise that differ from each other in the deformation mechanisms implemented in this process. Thus, an important task arises of determining the nature of the influence of pressure on the movement of particles of the medium, because, obviously, the porosity of a substance directly depends on the location of solid impermeable particles of the soil skeleton. And, since the pressure-induced movement of particles of the medium leads to a restructuring of the internal structure of the soil and the emergence of a stressed state of the medium (both the rock skeleton and the fluid contained in it), the nature of the movements and stresses in the medium should be clarified. Next, we determine the distributions of displacements and normal stresses behind the front of the shock wave; for this we assume, again, that the pressure on the ground is due to the propagation of the shock wave in it.

In some sufficiently small region of the medium, one-dimensional shock waves can be considered propagating along the x axis as a plane front. The boundary is affected by pressure P(t). When a certain pressure value P*, critical for a given soil, is reached, the ratio of pressure P(t) and deformation ε can be expressed in linear form (P(t)=kε, k-coefficient, tangent of the angle of inclination between the values of parameters P and ε). According to the simplified soil model proposed by A.Yu. Illinsky[3], A.P. Sinitsin[4] obtained an equation for determining the law of propagation of the front of a shock wave x=S(t), propagating in undisturbed soil without taking into account elastic disturbances. Considering the case when P>P*, and the soil with the initial density ρ0 and the speed of propagation of elastic waves α0 is pressed to a new state, a characteristic equation is obtained showing the relationship between the pressure parameter and deformation. Based on this, the law of

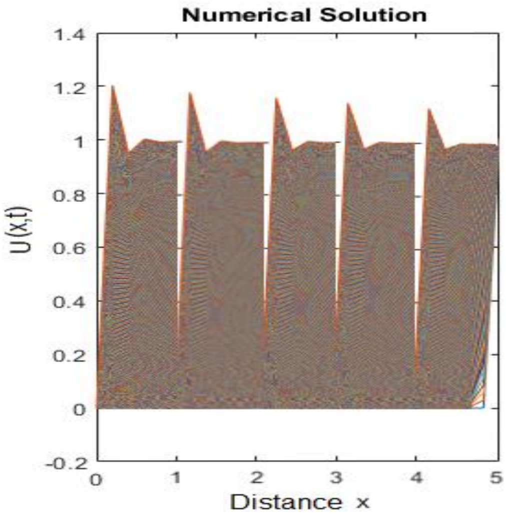
[3] Illinsky, A.Yu. On the plane movement of sand // Ukrainian mathematical journal, -1954. -№4, Т.У.П. -p-p.430-441.

[4] Sinitsyn, A.P. Propagation of waves in a hardened elastic-plastic layer// Proceedings of the All-Union Symposium on the Propagation of Elastic-Plastic Waves in Continuum Media. -Baku: AN Azb.SSR, -1966, -6p.

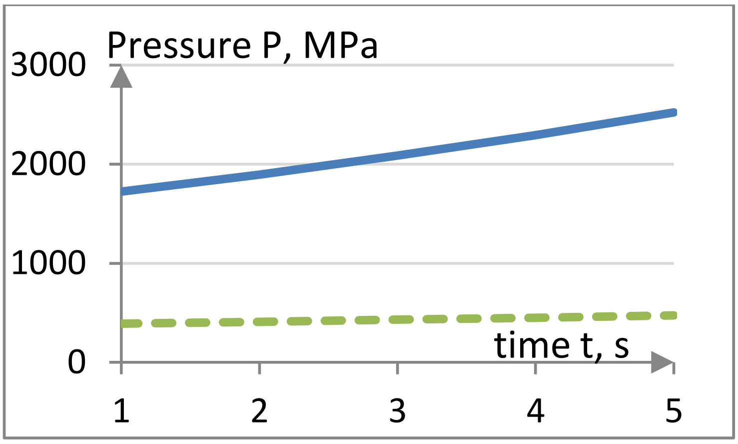
wave front propagation of the form is determined (U.S. Sarimsakov, 1988[5]):



1. Displacement distribution for dolomite



2.Displacement Distribution for sandstone



3. Time-dependent Peak Pressure Distribution for Dolomite (Solid Line) and Sandstone (Dotted Line).

Figure 7. *Distribution of Displacements and Maximum Pressures During the Passage of Cyclic Shock Waves.*

$P(t)=\rho_0 \varepsilon_* \frac{d(S\dot{S})}{dt} + f(S,\dot{S},\ddot{S},\dddot{S},\alpha,\rho,\varepsilon_*,\theta_*)$, Knowing this equation, one can find the distribution of displacements and stresses behind the shock wave front. The function f represents the nature of the pressure-deformation relationship. Continuing the analogy, the

[5] Sarimsakov, U.S., Babichev, A.I., Elmuratov, I.T. Determination of displacements and stresses behind the front of an intense shock wave in soft soil //- Samarkand: Mathematical modeling of problems of applied mathematics. Ministry of Higher and Secondary Special Education of the Uz.SSR., - 1988. –p-p.113-116.

obtained equation is generalized for subsequent n waves passing through the same medium, taking into account the natural influence of the previous wave on the same medium.

$$[P(t)]_{n+1} = \rho_n(\theta_*)_n \frac{d((S)_n(\dot{S})_n)}{dt} + \rho_n(\theta_*)_n \frac{1-(\varepsilon_*)_n}{{\alpha_n}^2} \cdot$$
$$\cdot [(1-(\varepsilon_*)_n)\left((\ddot{S}^2)_n(S^2)_n + (\dddot{S})_n(\dot{S})_n{(S)_n}^2\right) +$$
$$+3(\ddot{S})_n{(\dot{S})_n}^2(S)_n + {(\dot{S})_n}^4] \qquad (3)$$

Thus, based on this generalized equation, numerical assessments of displacement distribution and stress changes were carried out using specific numerical parameters for materials most commonly encountered in fluid-saturated reservoirs – dolomite and sandstone. The results show the existence of a range within which density, pressure, and soil displacement change.

The third chapter studies the influence of pressure waves on the deformation and filtration properties of fluid-saturated soils. An assessment was made of the nature of the consolidation of the soil layer under the fractal law of fluid filtration in view of the fact that the structure of the soil massif can be represented as fractal. This representation quite accurately reflects and replaces the complex real structure of the soil with an arbitrary location and size of pores, multi-layered and multi-phase, etc.

A.Kh.Mirzajanzadeh, R.N.Bakhtizin, A.Kh.Shakhverdiev, M.M.Gasanov, B.Kh.Khuzhayorov, G.M.Panakhov, O.V. were engaged in the field of fractal properties and soil porosity research. Iselidze, V. Bokhayenko, A. Gladky, A. V. Mashchenko, A. B. Ponomarev, V. Bulavatsky, J. K. Tsugawa, Y. A. Pachepsky, J. Zhang, Zh. Hui, D. Yong- Fen and others.

Consolidation of water-saturated soils implies the process of bringing soil particles closer together and reducing the volume of pores, accompanied by the displacement of the fluid contained in these pores. A two-dimensional equation for the consolidation of two-phase, completely water-saturated soil is obtained when the fractal nature of the geometry of the pore space is taken into account only in Darcy's filtration law (G.M. Panakhov, 2023[6]):

$$\frac{\partial H}{\partial t} = C_v \left(\frac{\partial^2}{\partial x^2} \left(D^{\alpha}_{a_+} H \right)(x) + \frac{\partial^2}{\partial z^2} \left(D^{\beta}_{0_+} H \right)(z) \right) \tag{4}$$

Where $C_v = \frac{(1+e)(1+\xi)k}{2\gamma a}$ is called a coefficient of consolidation.

An assessment of the influence of the fractality of the filtration law on the consolidation process was conducted for the following initial parameter values: $H_0 = 1, \beta = 0.5$ (fractality of filtration law), $\beta = 0$ (classic filtration law), $\Delta t = 0.01$, $\Delta z = 0.1$, $k_* = 20$, $n = 10$. The obtained results are reflected in the graphs (fig.8):

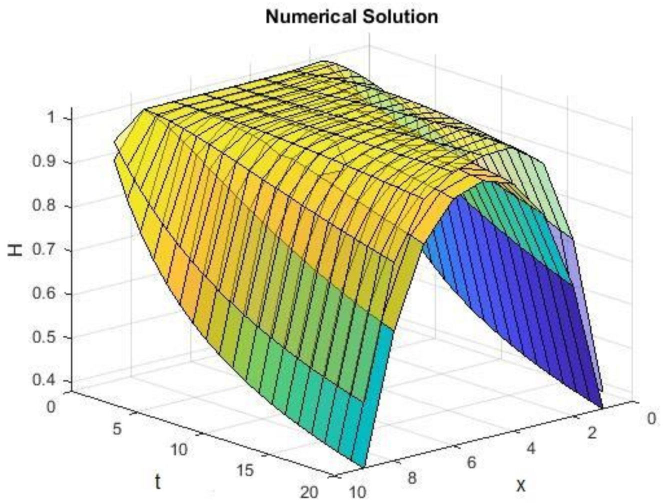


Figure 8. *The graphs illustrating the influence of the fractality of the filtration law on the consolidation of the soil layer. The top graph is constructed taking into account the fractality of the filtration law.*

The obtained results indicate that accounting for the fractality of the filtration law leads to an overestimation of the calculated values of the pressure function, and this overestimation becomes more significant over time. The greatest difference reached approximately 22% for corresponding points on the graph..

Next, it is necessary to assess the influence of soil consolidation, for example, due to the prolonged influence of

[6] Panahov, G.M., Museibli, P.T., Sultanov, B.N. Effect of soil consolidation on the fractality of the filtration law. // -Poland: International Journal of Applied Mechanics and Engineering. The Journal of University of Zielona Góra. -2023. v.28, №1, -p.84-94.

overlying rock pressure on the studied soil, causing changes in the structure of the skeleton and, consequently, the pore space. Rock pressure transmitted to fluid-saturated soil is divided between the fluid in its pores and the solid soil structure, consisting of mineral particles surrounded by a fluid film and connected to each other. The stress in the soil structure causes significant compression of this structure and affects its shear resistance. The size of fractal particles under fractal filtration law also plays a significant role in determining the stress-strain state of the soil. An increase in the maximum particle size leads to a more pronounced influence on the shear stress. Over time, during the operation of hydrocarbon reservoirs, the effective pressure increases, leading to a reduction in pore space and a significant decrease in the porosity coefficient. Changes in porosity and the volume of pores filled with fluid directly affect the rate of pressure reduction.

As a result, fractal structures of fluid filtration in soils with creep properties are obtained. Assessing the influence of consolidation on the fractality of filtration structure, during the processes of pore branching in the soil, is associated with solving the corresponding tasks. Establishing regularities in the influence of soil mass consolidation on the fractality of the structure requires obtaining a generalized and solvable system of equations. However, it is natural that the obtained generalized system of equations will be mathematically complex both in terms of its solution and analysis. Therefore, it is necessary to resort to simplifying transformations and apply relatively simple numerical methods. If we assume that the fluid is practically incompressible and express the pressure function in a linear form, we can once again describe the consolidation equation taking into account the fractality. Making the corresponding simplifications and arriving at the inverse problem to find the consolidation coefficient - C_v with known initial and final data of consolidation functions, we will need to solve an auxiliary problem. Thus, using this coefficient, we obtain the equation of the influence of consolidation on the fractality of the fluid filtration law in the soil.

$$H_{i,k+1} = H_{i,k} + \frac{(H_{i,m}-H_{i,m-1})(f_{i+1,k}-3f_{i,k}+3f_{i-1,k}-f_{i-2,k})}{(f_{i+1,m-1}-3f_{i,m-1}+3f_{i-1,m-1}-f_{i-2,m-1})} \quad (5)$$

Numerical assessments have been conducted for the parameters $k = 0$ and $0 \leq i \leq n$; $H_{i,k} = H_0 = 1$, $k \neq 0$ and $i = 0$ or $i = n$; $H_{i,k} = 0, \beta = 0.5$ (fractality of filtration law), $\Gamma(1-\beta) \approx 1{,}77245, \Delta t = 0.01$, $\Delta x = 0.1$, $k_{max} = m = 20$, $i_{max} = n = 10$. As a result, characteristic graphs were obtained, from which it follows that under the given initial and boundary conditions, as well as the consolidation law, the change in fractality has an almost increasing oscillatory nature with increasing depth. There is also a sharp, exponential increase in the absolute value of the fractal pressure function.

The following graphs (fig.9) of changes in fractality were obtained for this soil consolidation law.

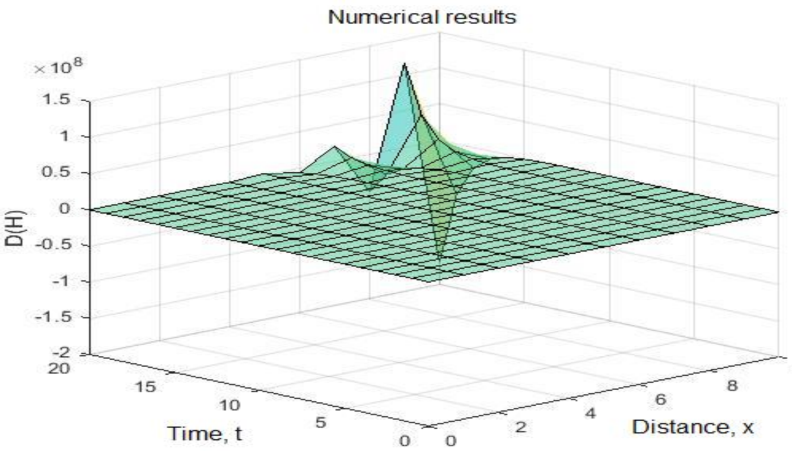


Figure 9. *Graphs depicting changes in fractality over space and time under this consolidation law.*

The consolidation coefficient decreases from approximately 0.670677598 at the initial point to approximately 0.425436241 at the final point. This decrease in this case can be roughly approximated as a continuously decreasing nearly straight line (approximation formulas are provided in Figure 10).

In general, it can be said that homogeneous soil under these initial parameters and constant pressure from overlying rock will expand its fractal structure. Although the graphs show unlimited growth, in real soils, the process quickly diminishes due to energy dissipation, non-uniform layered structure, foreign inclusions in the soil, elasticity, and compressibility of both the skeleton and the fluid, as well as a whole range of other factors.

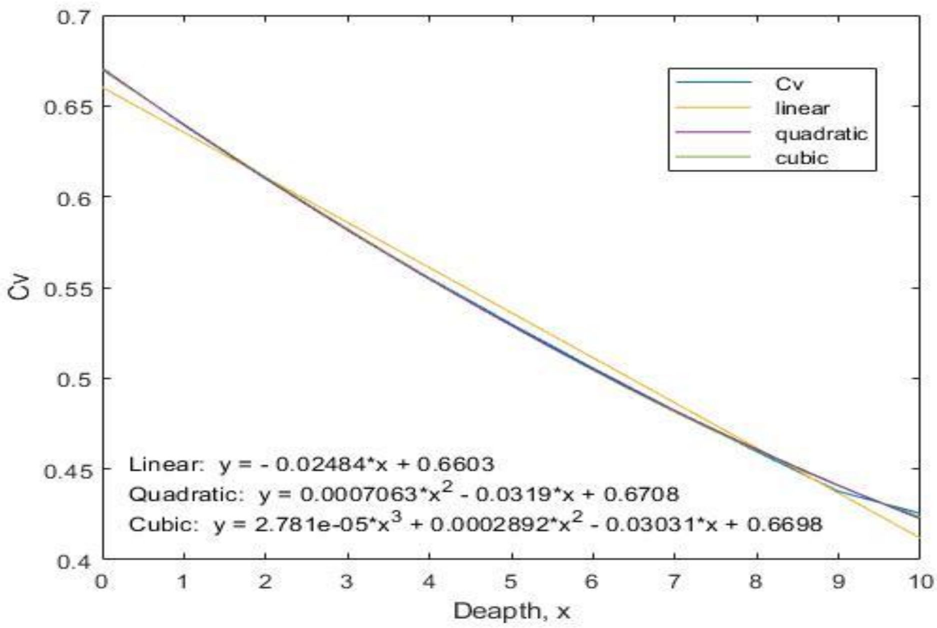


Figure 10. *Change in the consolidation coefficient over distance x.*

Attempting to account for all factors exponentially complicates the formulation and solution of the problem. It seems much more expedient to use empirical data with simplified models of soil behavior. Furthermore, it is interesting to study the propagation of a pressure wave in a porous medium in the case of a nonlinear filtration law. As is known, drop liquids such as oil and water, which are most commonly encountered in industrial and domestic practice, are incompressible substances. This means that their relative volume change is only a fraction of a percent. However, when gas bubbles dissolved in the fluid are released, which can change their volume by several times, acting like small springs, as well as the elastic properties of the rock itself – all of the above together can significantly affect the fluid filtration processes. Let's assume that there is a sudden increase in pressure at a certain point in the reservoir with a certain value . Due to the elastic properties of the reservoir and inertia; the influence of one volume of pressurized fluid on another volume of undisturbed fluid takes some time. Thus, there is a gradual increase in pressure, layer by layer, which can be interpreted as an advancing pressure wave. Let's try to consider the distribution of pressures in the reservoir during the propagation of this advancing pressure wave. By conducting this research, we hope to obtain characteristic graphs. We will consider two cases: 1 - In the first case, let's assume that the fluid filtration process follows a linear law – Darcy's law(G.B.Pykhachev,1973[7]).

$$\frac{\partial(mp)}{\partial t}+div(\rho \upsilon)=0; \upsilon=\frac{K_f}{\gamma}\frac{\Delta P}{\Delta L}=-\frac{k}{\mu}grad(p) \quad (6)$$

This holds true for low fluid flow velocities, and naturally, there is no mention of turbulence or any cavitation disruptions in the continuous fluid. After a series of transformations, we arrive at the differential equation.

$$\frac{\partial p}{\partial t}=\chi\frac{\partial^2 p}{\partial x^2}; p(x,0)=0; p(0,t)=\Delta p; \lim_{x\to\infty} p(x,t)=0 \quad (7)$$

According to Leontiev[8], this problem (7) reduces to an ordinary differential equation, the solution of which has the form:

$$p(x,t)=\Delta p\left[1-erf\left(\frac{x}{2\sqrt{t\chi}}\right)\right] \quad (8)$$

By applying specific numerical data for the coefficients of dolomite and moderately viscous oil, we obtain pressure distribution graphs where we observe a gradual increase in pressure with time for each successive spatial point (figure 11).

The second case - Now let's consider a nonlinear filtration law in the form of the Forchheimer-Dupuit nonlinear filtration law.

$$\frac{\Delta P}{\Delta L}=\frac{\mu}{k}\upsilon+\frac{\rho}{k_\rho}\upsilon^2 \quad (9)$$

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[7] Pykhachev, G.B. Underground Hydraulics/ G.B. Pykhachev, R.G.Isayev. - Moscow: Nedra, -1973. – 354p.

[8] Leontyev, N. E. Fundamentals of filtration theory/ N.E. Leontyev, -Moscow: Maks Press, - 2017. - 87p.

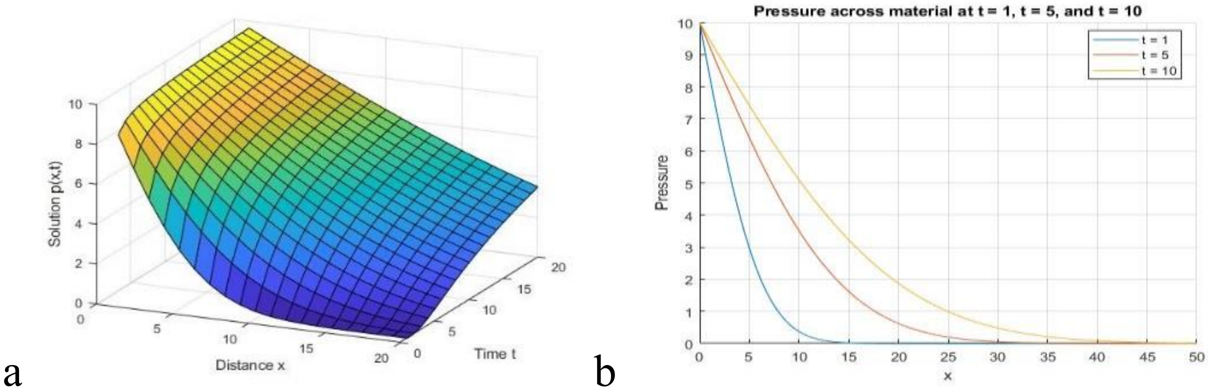


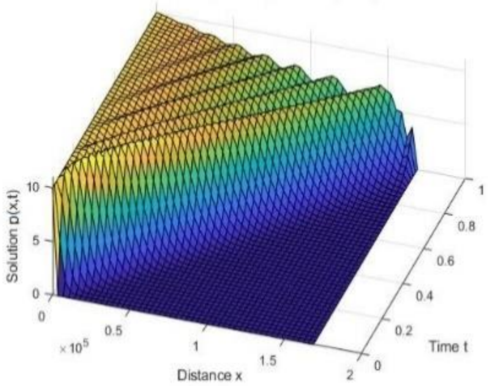
Figure 11. *Pressure distribution by coordinates and time with Darcy's linear filtration law. a-three-dimensional view, b-two-dimensional view of the nature of the pressure change.*

Also, after transformations, for the same numerical coefficients of the materials, we obtain the expression:

$$\frac{\partial(m(p)\rho(p))}{\partial t}=\frac{\partial}{\partial x}\left(\frac{\rho(p)\left(-\frac{\mu(p)}{k(p)}\pm\sqrt{\frac{\mu^2(p)}{k^2(p)}+4\frac{\rho(p)}{k_\rho(p)}grad(p)}\right)}{2\frac{\rho(p)}{k_\rho(p)}}\right) \qquad (10)$$

This expression describes several distinct graphs with a clear wave-like structure caused by nonlinearity (figure 12).

In this case, the pressure oscillates with damping around the Δp value , gradually approaching this value. Moreover, the increase in pressure at the initial and nearby points occurs with a slight delay. At the same time, it turns out to be slightly higher than the true Δp. value . In the first scenario, with distance, there is a sharp increase in pressure to Δp, followed by a gradual decrease to zero. In the second scenario, the pressure gradually increases and, with oscillations, decreases for the given moment in time t . The further the coordinate x, the smaller the value of the pressure oscillations becomes at this moment in time until it reaches the constant value Δp Next, the instability of displacement under hydrodynamic action in a porous medium was studied.



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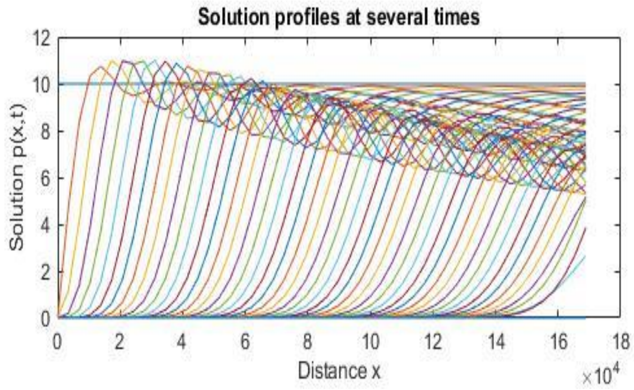

b

Figure 12. *Distribution of pressure in terms of coordinates and time when applying the non-linear Forchheimer-Darcy filtration law. a-three-dimensional view, b-two-dimensional view of the nature of the pressure change.*

The nonequilibrium state during displacement is more pronounced when oil breaks through with water injected into a porous medium. For example, the displacement of oil from a homogeneous reservoir using a polymer solution whose viscosity is almost the same as the viscosity of the oil being displaced, usually assumed to be piston displacement, may encounter instability at a certain stage, taking into account the adsorption of the polymer on the walls of the reservoir and a number of other physico-chemical processes. The manifestation of non-stationary effects is also possible in relatively macro-homogeneous oil-saturated formations. Due to high values of viscous instability (the ratio of oil and water viscosity), local breakthroughs of injected water occur, which is reflected in the indicators of anhydrous and current oil recovery of the reservoir. In a reservoir at high values of residual oil saturation, significant interfaces are formed between the oil and water phases. This hydrodynamic situation can be used to implement non-stationary methods of influencing the formation during its flooding. One of the effective methods for producing heavy, highly viscous oils is to stimulate the formation using hydrodynamic methods. The problem arises of controlling the hydrodynamic situation at the displacement front by creating conditions to enhance the effect of involving oil trapped in the pores into the general flow, both by reducing surface tension and by overcoming capillary pressure at a distance from the pressure source. According to the works of other authors, on the one hand, with a decrease in filtration rate, the size of pore channels with immobile oil increases. At the same time, breakthrough tongues appear on the repression front. Structures of increasing complexity are formed, as happens with fractals. On the other hand, with an increase in the applied pressure and filtration rate, an increasing number of pore channels cease to participate in displacement. To overcome capillary pressure at a certain distance from the well, it is necessary to generate additional “local” pressure. In heterogeneous formations, the efficiency of cyclic waterflooding is higher than that of conventional waterflooding. This is due to the fact that under flooding conditions of a heterogeneous formation, the residual oil saturation of the formation sections with deteriorated reservoir properties is significantly higher than the oil saturation of the main flooded part of the formation. With increasing pressure, the elastic forces of the formation and liquid contribute to the penetration of water into areas of the formation with worse reservoir properties, while capillary forces retain the water that has penetrated into the formation with a subsequent decrease in formation pressure. Under the influence of pressure changes, liquids are redistributed in horizontal layers and the displacing liquid is introduced into the pores by overcoming capillary forces. When solving the problem of increasing the coverage of zones with deteriorated reservoir properties, but saturated with hydrocarbons, you can rely on the method of cyclic stimulation of the reservoir. Thus, a technological solution is proposed to increase the efficiency of the process of covering the displacement of oil with water by creating a cyclically increasing hydrodynamic pressure, which makes it possible to overcome the resistance of capillary forces along the strike of the injection zone. Figure 13 shows the results of experimental studies in displacement modes at constant injection pressure with cyclically increasing injection pressure. Figure 14, in turn, shows the change in the degree of oil saturation of the porous medium during the displacement process when applying the proposed solution to the reservoir model.

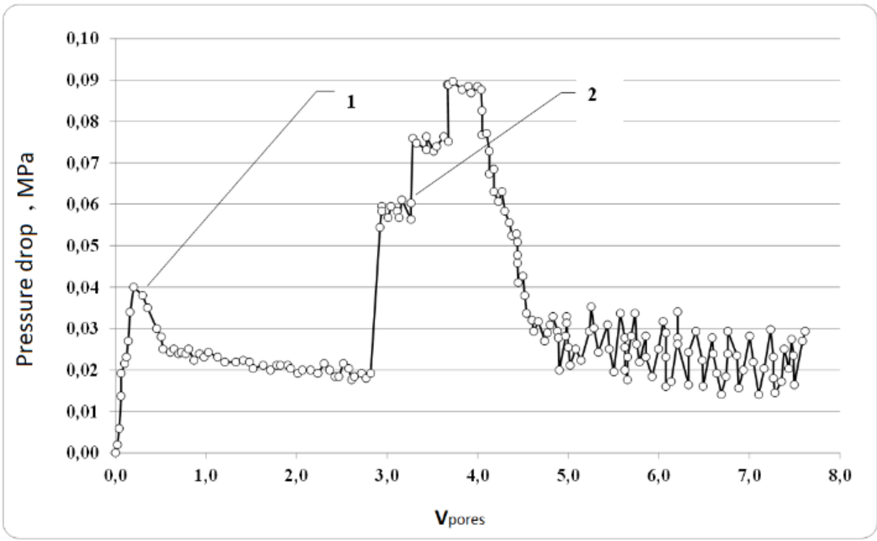

Figure 13. *The dynamics of displacing oil with an aqueous solution of soda ash under different hydrodynamic pressure regimes were investigated: 1 - displacement under constant injection pressure; 2 - displacement under increasing injection pressure. In all cases, displacement was carried out by continuous injection of solutions in an amount equal to three times the pore volume..*

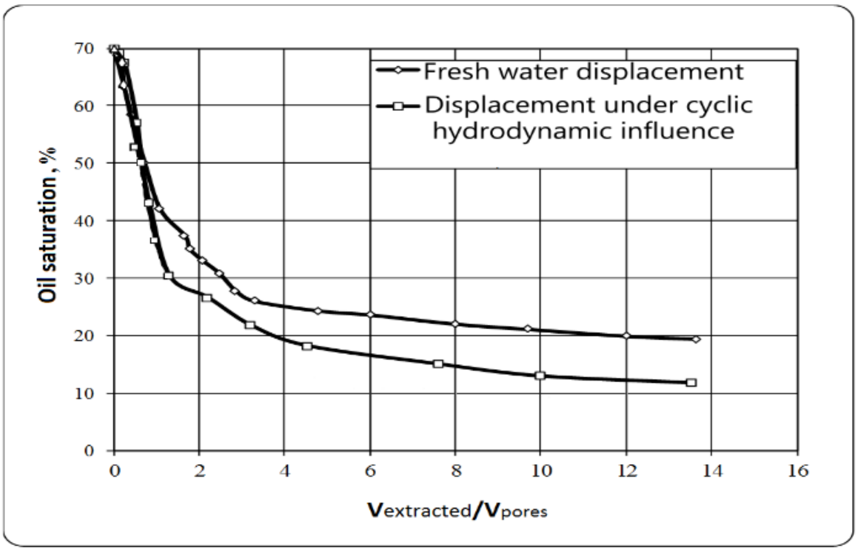

Figure 14. *The change in oil saturation of the porous medium during displacement.*

To assess the effect of accelerating water infiltration into heterogeneities based on an approximating linear equation with constant coefficients, obtained from experiments on displacement with water and aqueous solutions, and with subsequent transformations, we arrive at the general equation relating the filtration rate and the step function of the pressure parameter, which increases abruptly by an amount *h* at time *t*. The redistribution of pressure after injection at a constant rate until the next cycle takes the form of radial filtration. Obtaining the pressure value at any point in the reservoir at an arbitrary moment in time can be achieved by integrating the equation that describes pressure, which takes the form of a partial differential parabolic equation. Solving such an equation directly is quite challenging. Therefore, you can resort to the method of successive change of stationary states. As a result, discrete pressure values are obtained for a given coordinate with injection rates of 4.6×10^{-7} m/s and 0.12×10^{-5} m/s. Thus, the calculated example makes it possible to estimate the pressure distribution along the strike of the reservoir in discrete sections of the reservoir as the displacement front advances and, thereby, the time frame for changes in the hydrodynamic pressure drop.

This makes it possible to determine the duration and stages of injection pressure regulation in order to achieve the expected hydrodynamic effect and, as a result, increase the oil flow to the gallery of production wells. Field experimental studies were carried out. As observations of the main operational indicators of the surrounding (reacting) wells at the technology implementation site have shown, in general, for all wells there is a positive reaction to the hydrodynamic impact - for most production facilities there is an increase or stabilization of the average daily oil flow rate.

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Personal contribution of the author.

The main results of the dissertation work were obtained by the author independently with the help of a supervisor. The main scientific developments were carried out with the direct participation of the author or with his participation as a responsible executor.

The main conclusions of the work

1. A connection has been established between the parameters of deformation and the elastic properties of the deformable medium from various types of elastic potentials.

2. For the first time, the stress-strain state of the soil when cyclic shock waves pass through it is considered.

3. The dynamics of changes in stresses and displacements behind the front of cyclic shock waves propagating in soil media was assessed.

4. Displacements during the passage of pressure waves for soils with different properties were numerically estimated.

5. Solutions to problems of soil consolidation under the fractal law of filtration and problems of fractal filtration arising due to the consolidating effect of rock pressure are given.

6. The problem of propagation of pressure waves in a porous medium in the case of a nonlinear filtration law is considered.

7. The influence of pressure waves on the deformation and filtration properties of fluid-saturated rocks was assessed.

8. The possibilities of regulating displacement instability under increasing cyclic hydrodynamic influence in a porous medium are shown.

Main results of the work were published in following papers:

1. Sultanov, B.N. Elastic parameters of materials at high and ultrahigh deformations in case of their description by harmonic potential.// -Baku: Transactions of ANAS, issue mechanics, ser. of phys.-tech. & math. science. -2019. v.39, issue 7, -p.52-62.
2. Sultanov, B.N. The compare of elastic parameters of materials and their wave propagation velocities at extremely high deformations in the case of harmonic potential in initially strained

media.// -Baku: Transactions of ANAS, issue mechanics, ser. of phys.-tech. & math. science. -2020. v.40, issue 8, -p. 44-50.
3. Султанов, Б.Н. Сравнение упругих параметров материалов при экстремально высоких деформациях. // Республиканская виртуальная научная конференция «Математика, механика и их приложения», посвященная 98-летию Общенационального Лидера Гейдара Алиева, -26-27 мая -2021, -Баку: -с.185-186.
4. Sultanov, B.N. Pressure wave propagation in a porous medium in the case of a nonlinear filtration law. // -Bakı: "Genc Tetqiqadci" Elmi-praktiki jurnal. Fizika-riyaziyyat və texnika elmləri, -2022. v.VIII, №1, -s.6 – 12.
5. Panahov, G.M., Sultanov, B.N.Cyclic shock wave in soil and determination of changes in stresses and displacements. // -Baku: Transactions of ANAS, issue mechanics, ser. of phys.-tech. & math. science. -2022. v.42, issue 7, -p.44-52.
6. Sultanov, B.N. Propagation of cyclic shock waves in a soil and determination of changes in displacements. / "Funksiyalar nəzəriyyəsi, funksional analiz və onların tətbiqləri", akademik İbrahim İbiş oğlu İbrahimovun anadan olmasının 110-illiyinə həsr olunmuş Respublika elmi konfransı, -28-29 noyabr -2022, -s.463-466.
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9. Panahov, G.M., Sultanov, B.N. Fractal modeling of consolidation under filtration in water saturated soils. / "Modern Problems of Mathematics and Mechanics" Proceedings of the İnternational scientific conference devoted to the 110-th anniv. of

academician İbrahim İbrahimov, -Bakı: -29 June - 1 July, -2022, -p.171-172.
10. Panahov, G.M., Museibli, P.T., Sultanov, B.N. Effect of soil consolidation on the fractality of the filtration law. // -Poland: International Journal of Applied Mechanics and Engineering. The Journal of University of Zielona Góra. -2023. v.28, №1, -p.84-94.
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12. Panahov, G.M., Sultanov, B.N. Influence of cyclic shock waves on reservoir structures and formations. / “Modern problems of Mathematics and Mechanics” International conference dedicated to the 100-th anniv. of National Leader Heydar Aliyev, -Baku: - 26-28 Aprel, -2023, -p.322-324.

Personal work of the plaintiff in works performed in collaboration:

[1], [2], [3], [4], [6]- done independently.
[9], [12]- formulation of the problem, conducting research and interpreting the results.
[5], [7], [8], [10], [11]- the contribution of the co-authors is equal.

Applicant's signature:

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