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ABSTRACT

of the dissertation for the degree of Doctor of Science

SOME TASKS OF STATICS AND DYNAMICS FOR DETERMINING THE LOADING CAPACITY OF STRUCTURE ELEMENTS AND SOLID BODIES TAKEN INTO ACCOUNT OF THE INFLUENCE OF THE EXTERNAL ENVIRONMENT

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INTRODUCTION

Relevance of the topic and degree of development

Since the middle of the last century, directions have emerged and are now widely developing, establishing a close connection between the mechanics of materials and structure with other areas of physics, thanks to which the physical questions of the deformability of continuous media and the strength of solids have received important development. The first works on the mechanical theory of creep and the problem of buckling and loss and stability of thin-walled structural elements belong to N.M. Belyaev (1943), N. Hoff (1951), Yu. N. Rabotnov (1957,1966), L. M. Kachanov (1960), N.N.Malinin (1959), S.A. Shesterikov(1957,1963), A.S. Volmir (1962), I.G. Teregulov (1962,1966), V.I. Rozemblyum (1954), A.M. Lokoshenko (2008), L.M. Kurshin (1961, 1963), G.V. Ivanov (1961,1963) and their followers. In our republic, since 1970, problems of creep of metals and hereditarily elastic media, as well as issues of buckling and determination of the load-bearing capacity of structural elements and solids bodies, devoted to the study of M.F. Mehdiev, R.Yu. Amenzadeh, S.D. Akbarov, M.Kh. Ilyasov, A.N. Alizade, L.H. Talybly, F.S. Latifov, A.D. Zamanov, G.G. Aliev, M.B. Akhundov, L.F. Fatullaeva and others.

Spatial dynamic problems of a solid deformable body, hydrodynamics, elastodynamics and hydroelasticity in general, this is one of the most complex classes of problems in continuum mechanics. However, the mathematical difficulties of solving boundary value problems in a three-dimensional formulation often led to the need to involve various hypotheses and simplifying assumptions. This significantly simplifies the formulation and solution of problems, but always imposes significant restrictions on the scope of applicability of the solutions obtained.

These remarkable achievements in the field of solving mathematical problems related to fundamental issues of continuum mechanics and engineering are still relevant today, and many of these results have been applied to the solution of seismic problems. Studies of natural vibrations of a multilayer hollow sphere and a multilayer hollow cylinder within the framework of the three-dimensional linearized theory of deformable bodies with initial inhomogeneous stresses and the theory of elastic waves were carried out by S.D. Akbarov and A.N. Guz.

For fundamental research, the relevance of solving spatial problems is determined by the fact that the stress-strain state is almost always three-dimensional in nature. The relevance of this kind of large-scale research for engineering applications is determined by the fact that in problems of the strength of materials and the load-bearing capacity of structural elements, data are mainly used on experimental values of the stress-strain state (SSS) in local areas (in zones of sharp changes in the geometric shape of structural elements, in places applications of uneven load, etc.). Obtaining this kind of reliable and complete information is associated with the use of methods and specific results of spatial tasks.

Object and subject of research.

Objects that are considered in this dissertation and for which the very formulation of the problem of determining the bearing capacity has practical meaning, there will be structural elements - rods, plates and shells. When presenting general issues of stability under creep with the construction of variational principles, as well as for studying problems of dynamics, the main objects are solid deformable elastic and inelastic bodies with finite and infinite geometric dimensions. Equations for more specific objects, as is known, can always be obtained from the general equations of the theory of elasticity, plasticity, and creep, introducing the corresponding kinematic and static hypotheses and applying variational principles. Deformable solids, in the form of hollow multilayer spheres and hollow multilayer cylinders and structural elements - multilayer rods and multilayer shells (ring) form a piecewise homogeneous elastic and inelastic medium, composed of a finite number of homogeneous parts different in shape and physical properties, connected into one solid body in one way or another. The connection of dissimilar parts can be either natural or artificial. The latter always serve the purpose of strengthening the load-bearing capacity of structures and are often used in engineering practice.

In the case of connecting a solid body from dissimilar parts with other elastic characteristics, such as composites and nanostructures, solving the problem becomes much more difficult.

In the first, second and third chapters of the dissertation, it is additionally assumed that the contacting surfaces of the bodies do not lag each other due to deformation, i.e., during deformation, the elements contact each other along their entire common surface.

The work examines some static and dynamic problems in the mechanics of structures and deformable solids, determines their stress-strain state and evaluates their load-bearing capacities.

The subject of the dissertation research is: 1. **Static problem** - constructing a variational principle for solid anisotropic bodies and structural elements during creep, considering the influence of external physio-chemical fields, mechanical influences, and damageability of the material. 2. **Dynamic problem** - determination of the main wave dynamic characteristics for hollow multilayer spherical and cylindrical bodies containing compressible liquids inside during free vibrations, considering various boundary and contact conditions.

Goal of the work

The purpose of the dissertation work is to create mathematical models of certain classes of static and dynamic problems in the mechanics of a deformable solid and structural elements, determine their load-bearing capacity considering the influence of surrounding external fields and obtain accurate and approximate analytical solutions in a three-dimensional formulation.

Research methods

Extended variational principles were implemented in the works of A.J.Wang, W.Prager [1][1], J.L. Sanders, G.D. Mac Comb, F.R. Shlechte [2][2], K.Washizu [3][3] and others, where variational principles were formed for the boundary value problem, where it is assumed that

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[1] Wang A.J. Termal and creep effects in work-hardening elastic-plastic solids / W. Prager // Journal of the Aeronautical Sciences, –1954. V.21. №5, –p.343-344.

[2] Sanders J. L. A variational theorem for Creep with applications to plates and columns / H. G. McComb, F.R. Schlechte // NASA Report, –1957. –p.134.

[3] Washizy K. Variational principles in continuum mechanics // University of Washington College of Engeneering.Department of Aeronauticai Engineering, –1962.Report 62-2

stresses and displacements are known now of time. Knowing the rate of change of surface forces, the rate of change of displacements on the surface, the rate of change of mass forces and the relationship between the rates of stress and strain, we find the rate of stress and displacement occurring in the body. In this case, the creep strain rate can be considered as the initial strain rate. As is known, in quasi-static processes, given mass forces, surface forces and displacements change so slowly over time that the inertial terms in the equations of motion are neglected. Thus, in a quasi-static problem, considering the distributions of stresses and displacements in the body at the initial moment of time as given, we will determine the time derivatives of stresses and displacements; here time will play the role of a parameter.

Within the framework of the three-dimensional linearized theory of deformable bodies with inhomogeneous initial stresses and the theory of elastic waves, S.D. Akbarov et al. developed a discrete-analytical method, which is used to study the natural vibrations of a solid hollow sphere and a cylinder.

Natural vibrations of a solid hollow sphere and a multilayer hollow sphere and a multilayer hollow cylinder containing compressible inviscid fluids, within the framework of three-dimensional linearized theory of deformable bodies with initial inhomogeneous stresses, hydrodynamics, as well as the theory of ideal elastic waves and within the framework of exact equations of the theory of elasticity with discontinuous kinematics contact conditions were studied by the Fourier method.

Main provisions submitted for defence:

- a constructed mixed variational principle for creep problems, taking into account the influence of external physic-chemical fields on the bearing capacities of solids and structural elements in a geometric linear and nonlinear formulation.

-modified mixed variational principle for composite materials, considering the damageability of materials.

- an analytical solution obtained by a discrete analytical method for problems of the dynamics of a sphere with non-uniform initial stresses in a three-dimensional formulation.

- analytical solution to the problem of the propagation of elastic waves in a non-uniformly prestressed hollow multilayer sphere and a hollow multilayer cylinder filled with a compressible fluid of free vibrations.

-solutions to the problem of establishing the influence of initial non-uniform stresses on the dynamic characteristics of a multilayer hollow sphere and a hollow cylinder filled with a compressible ideal fluid.

-solution for determining the dynamic characteristics of a layered hollow sphere in the presence of non-ideal contact conditions.

-solution of an axisymmetric problem of the propagation of elastic waves in a hydroelastic system - a cylindrical shell and a viscous compressible fluid with spherical gas bubbles.

Scientific novelty of the research

The scientific novelty of the work is as follows:

- a mixed variational principle was formulated and proven to determine the bearing capacity of elastoplastic deformable anisotropic solids and structural elements made of structural metals based on the kinetic theory of Yu.N. Rabotnov, who are under the influence of physical and chemical external environments.

For dynamic problems of natural vibrations of an isotropic elastic sphere with inhomogeneous initial stresses within the framework of a piecewise homogeneous body, using three-dimensional linearized equations and relations of elastodynamics, a discrete analytical method is proposed, and analytical solutions are obtained.

The solution to the corresponding equations of motion of a three-layer hollow sphere containing a stationary compressible inviscid fluid and a multilayer hollow cylinder, inside of which a compressible inviscid fluid flows at a constant longitudinal velocity, having initial non-uniform electrostatic stresses arising under the action of radial compression forces uniformly distributed on the outer and inner surfaces, are presented through Helmholtz potentials.

Analytical solutions to the problem of studying the influence of inhomogeneous initial stresses on dynamic stresses have been obtained.

reactions of a hollow three-layer sphere with non-ideal contacts between layers, using exact three-dimensional equations and relations of elastodynamics.

Theoretical and practical significance of the study

The theoretical value of the dissertation includes:

-variation principle for determining the load-bearing capacity of a three-dimensional body and structural element during creep, taking into account the influence of external physical and chemical fields and damage;

-analytical solutions obtained by the discrete analytical method for problems of the dynamics of a sphere with inhomogeneous initial stresses based on the linearization of the three-dimensional theory of stability of elastic bodies;

-analytical solutions to problems on the propagation of elastic waves in a multilayer hollow sphere and a hollow cylinder filled with liquid with non-uniform initial stresses, obtained in a three-dimensional formulation;

- determination of the dynamic characteristics of a layered hollow sphere in the presence of non-ideal contact conditions;

-study of the problem of the propagation of elastic waves in a hydroelastic system - a cylindrical shell and a viscous compressible fluid with spherical gas bubbles.

The results of the analysis of the causes of man-made disasters and accidents of critical structures and structures show that destruction and disasters could have been avoided if the necessary means of non-destructive testing and diagnostics of the condition of materials and the structure were available. In applied research and practical work, methods are needed for mathematical modelling of the processes of loss of bearing capacity of a material and establishing the expected residual life of structures, which is the practical significance of the results obtained in the work.

Reliability of research results

The reliability of the results obtained is ensured by the mathematical correctness of the problems posed, obtaining solutions to the problems using rigorous analytical methods, the results of numerical calculations, and a comparison of the final analytical and numerical results in particular cases with those known in the literature. In the calculations, MATHLAB is used and uses a modern licensed package for mathematical calculations and the corresponding graphs are built. The results obtained are confirmed cases with their coincidences with the known results of other authors.

Approbation of work

The main results of the dissertation work were reported and discussed at the following conferences:

- Республиканская конференция «Актуальные проблемы теоретической и прикладной математики» посвящённое 100 летию академика М. Расулова, Шеки, 2016;
- Международная научно-техническая конференция «Актуальные проблемы прикладной математики, информатики и механики», Воронеж, 2017;
- Республиканская конференция «Актуальные проблемы математики и механики посвящённое 100 летию чл. корр. НАНА, профессора Г. Т. Ахмедова, Баку, 2017;
- XVII International Conference «Dynamical System Modelling and Stability Investigation», Ukraine, Kiev, 2017;
- International Conference Geoinformatics, Ukraine, Kiev, 2018;
- International Conference «Modern problem mechanics and mathematics» dedicated to the 90th anniversary of academician A. Kh. Mirzajanzade, Baku, 2018;
- Республиканская научная конференция «Актуальные проблемы математики и механики» посвящённое 95 летному юбилею Общенационального Лидера Азербайджана Г. А. Алиева, 2018;

8th Internasional Conference on «Appleid Analysis and Mathematical Modeling», Istanbul Gelisim University, 2019;

- Республиканская научная конференция «Актуальные проблемы математики и механики» посвящённое 96 летному юбилею Общенационального Лидера Азербайджана Г. А. Алиева, 2019;
- 21 Ulusal Mekanik Kongresi TUMTMK, Türkiye, Nigde, 2019;
- 8th International Euroasian Conference on mathematical sciences and applications, Baku, 2019;
- Республиканская научная конференция «Актуальные проблемы математики и механики» посвящённое 97 летному

юбилею Общенационального Лидера Азербайджана Г. А. Алиева, 2020;
- 7th International Conference on Control and Optimization with industrial applications, Baku, 2020;
- Республиканская научная конференция «Актуальные проблемы математики и механики» посвящённое 98 летному юбилею Общенационального Лидера Азербайджана Г. А. Алиева, 2021;
- Республиканская научная конференция «Актуальные проблемы математики и механики» посвящённое 99 летному юбилею Общенационального Лидера Азербайджана Г. А. Алиева, 2022;
- International Conference «Modern Problem of Mathematics and Mechanics» devoled to the 110th anniversary of academician Ibrahim Ibrahimov, Baku, 2022;
- 8th International Conference on Control and Optimization with industrial applications, Baku, 2022;
- «Funksiyalar nəzəriyyəsi, funksional analiz və onların tətbiqləri» mövzusunda Respublika konfransı, Bakı, 2022;
- На научных семинарах Механико-математического факультета БГУ март 2021, ноябрь 2022, февраль 2023.
- На расширенных семинарах кафедры Теоретическая механика и механика сплошной среды 2020,2021,2022, 2023.

Organization of work execution

The dissertation work was completed at the Department of Theoretical Mechanics and Continuum Mechanics of Baku State University.

Scope and structure of the dissertation

Structure and volume of the dissertation (in characters, indicating the volume of each structural unit separately). The total volume of the dissertation work is 421,229 characters (title page - 758 characters, table of contents - 2758 characters, introduction - 52,735 characters, first chapter - 118,533 characters, second chapter - 39,775 characters, third chapter - 66,224 characters, fourth chapter - 140,446 characters). The list of used literature consists of 204 titles.

CONTENT OF THE DISSERTATION

The introduction substantiates the relevance of the problem considered in the work, formulates the purpose of the work and the objectives of the research, shows the scientific novelty, theoretical and practical significance of the results obtained.

The first chapter examines the influence of external fields and influences on the load-bearing capacities of structural elements and solids during creep. For mathematical modelling and solution, direct methods of mathematical analysis are used, a mixed-type variational method is developed for three-dimensional theory, and the Rayleigh-Ritz method is used to determine the stress-strain state of inhomogeneous anisotropic elastoplastic bodies under the action of a neutron flux at small and terminal deformations.

In paragraph 1.1, a variational method of a mixed type of plasticity theory is formed for inhomogeneous and composite bodies under irradiation, with independent variation of the fields of displacement speeds and stresses. It is assumed that the physical and mechanical characteristics of the medium depend on the radiation dose. A modification of this variational theorem is given for the case of a composite material, when in a heterogeneous medium different phases (inclusions) are clearly expressed.

Note that the change in volumetric deformation during neutron irradiation occurs quite slowly, because of which, when assessing the stress-strain state, dynamic effects can be neglected, and the duration of irradiation in time t can be considered as a parameter. Then, with a constant irradiation intensity, as a parameter characterizing the formation process, along with time, d the irradiation dose can be taken, determined by the formula $d = n\nu t$, where n ($1/см^3$) is the number of neutrons per unit volume of flow, and is the average flow velocity (d the unit of measurement is neutron($1/см^2$)) In this regard, the dot above the values will mean differentiation with respect to . If we now accept that $\dot{e}_{ij}^{(1)}$ the strain rate satisfies an elastic-plastic law such as flow theory, and also that θ both mechanical characteristics and volumetric expansion are functions of coordinates x^k and radiation dose d, then taking the rule of summation over repeating indices, we can write

$$\dot{e}_{ij}^{(1)} = \left\{C_{ijkm}\left(x^k, d\right)\sigma^{km}\right\}^{\bullet}\left(i, j, k, m = \overline{1,3}\right), \tag{1.1}$$

$$\dot{e}_{ij}^{(2)} = \dot{\theta}\left(x^k, d\right)\delta_{ij}, \tag{1.2}$$

Let a body of volume V be given in three-dimensional Euclidean space, bounded by a sufficiently smooth surface S. On some part of it, only $\overline{u}_k$ the components of the displacement vector are specified S_u, and on the remaining part S_σ - the loads $\overline{T}^k$, and $S = S_u \cup S_\sigma$
.
Let us introduce the functionality

$$J = \int_V \left\{\dot{\sigma}^{ij}\dot{e}_{ij} - \frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}\dot{\sigma}^{km} - \dot{C}_{ijkm}\sigma^{km}\dot{\sigma}^{ij} - \dot{\theta}\delta_{ij}\dot{\sigma}^{ij}\right\}dV -$$
$$- \int_{S_\sigma} \dot{\overline{T}}^i \dot{u}_i dS - \int_{S_u} \dot{T}^i\left(\dot{u}_i - \dot{\overline{u}}_i\right)dS. \tag{1.3}$$

When writing functional (1.8), the starting point, on the one hand, was the variational principle of Sanders, McComb and Schlechte in the theory of creep, and on the other hand, its modification for the case of elastoplasticity. Section 1.2 provides a variational method for solving the problem of the limit state of a multilayer rigidly reinforced nonlinear-elastic rod under creep with a modification of the mixed variational principle for the case of heterogeneous media.

Within the framework of geometrically nonlinear theory, for a nonlinear-elastic anisotropic material from physical laws

$$\varepsilon_{ij}^{v} = f_{ij}\left(\sigma^{sn}\right) \tag{1.4}$$

we get the final form of the functional

$$J = \sum_m \int_{V_m} \left\{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i\dot{u}^k\nabla_j\dot{u}_k - \frac{1}{2}\left(\dot{\varepsilon}_{ij}^{v} + 2\dot{p}_{ij}\right)\dot{\sigma}^{ij}\right\}dV -$$
$$- \int_{S_u} \dot{T}^i\left(\dot{u}_i - \dot{u}_i^0\right)dS - \int_{S_\sigma} \dot{T}^{i0}\dot{u}_i dS \tag{1.5}$$

It is assumed that the rod, rectangular in length l and thickness, is composed of alternating layers of different thicknesses $2h$, divided parallel to its side faces s. We denote the thickness of each layer by δ_k, then $\delta_1+\delta_2+...+\delta_s=2h$. Based on a physical hypothesis, we accept a theory in which the creep rate is described with the dependence:

$$\dot{p}=A_{k+1}\sigma^m,\ \ a_k\le z\le a_{k+1},\ \ [k=0,1,...(s-1)]\ . \tag{1.6}$$

Let us define the instantaneous deformation for the package as a whole in the form of a single equality

$$e^v=\frac{\sigma}{E_{k+1}}\left\{1+\left(\frac{\sigma}{\sigma_{k+1}^0}\right)^n\right\}\qquad a_k\le z\le a_{k+1} \tag{1.7}$$

Here is $n-$the nonlinearity index, which takes even values (2,4,6,...), E_{k+1} and σ_{k+1}^0 – the elastic modulus and proportionality limit of the $k-$ layer, respectively. In (1.11) and (1.12) the notation is introduced

$$a_k=-h+\sum_{j=0}^{k}\delta_j\quad(\delta_0=0) \tag{1.13}$$

Let us now consider the stability of the selected rod under creep, centrally compressed by force $T=const$. Assuming that one transverse dimension of the rod is equal to unity, considering the nonlinearity of only the deflection and relation (1.7), we write the functional used as

$$J=\int_0^l\int_{-h}^{h}\left(\dot{\sigma}\dot{\varepsilon}+\frac{1}{2}\sigma\dot{w}_{,x}^2\right)dxdz-\frac{1}{2}\int_0^l\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}\frac{\dot{\sigma}^2}{E_{k+1}}\left[1+(n+1)\left(\frac{\sigma}{\sigma_{k+1}^0}\right)^n\right]dxdz-$$
$$-\int_0^l\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}A_{k+1}\dot{\sigma}\sigma^m dxdz. \tag{1.9}$$

Assuming that the stress distribution along the thickness is linear and that the hypothesis of flat sections is fulfilled, we formulate boundary conditions corresponding to the case of rigid pinching of both ends.

For subsequent application of the Rayleigh-Ritz method when calculating the critical buckling time, it is necessary to specify the bending and moment shapes. As the first eigenfunctions satisfying the boundary conditions, we have

$$w(x,t) = a(t)\sin\frac{\pi x}{l}\sin\pi(1-\frac{x}{l}),$$
$$M(x,t) = b(T)\cos\frac{\pi x}{l}\cos\pi(1-\frac{x}{l}) \tag{1.10}$$

The stationarity of the functional (1.9) leads to a system of two ordinary differential equations, after which we obtain:

$$d\tau / d\eta = \left(-\frac{\pi^2\xi^2}{2} + \omega\phi_2 + \omega^{n+1}\sum_{i=0}^{n} K_n^i \phi_{i+2}^v \eta^i\right) / \left(\omega^{n+1}\sum_{j=0}^{m} K_m^j \phi_{j+1}^c \eta^j\right), \tag{1.11}$$

Where $\eta = a/h$ and $\tau = A_1 E_1^m t$ are the independent variables, and $K_n^i, \varphi_2, \varphi_{i+2}^v, \varphi_{j+1}^c, \xi, \omega$ dimensionless quantities. Equation (1.11) must be supplemented with the initial condition

$$\eta(0) = \eta^v, \tag{1.12}$$

in which η^v –the value of the deflection that occurs immediately after the application of a load ω.

Let , $s=3$, $E_1 = E_3$, $\delta_1 = \delta_3$, $A_1 = A_3$, $\sigma_1^0 = \sigma_3^0$, denoted by $\mu = A_2/A_1$, $a = E_2/E_1$, $\beta = \delta_2/\delta_1$, $\gamma = \sigma_1^0/\sigma_2^0$, $\lambda = E_1/\sigma_1^0$. and considering $\lambda = 3\cdot 10^2$, $\xi = 5\cdot 10^{-2}$, that Tables 1-3 give numerical values depending on $\alpha(\gamma = 0{,}25; \beta = 4), \gamma(\alpha = 0{,}25; \beta = 4)$, with $\eta_0 = 10^{-1}$ and $w = 3{,}11\cdot 10^{-5}$. This choice eliminates instantaneous buckling of the rod, because with the accepted system parameters it is equal to $7{,}3\cdot 10^{-5}$

Table 1.1. Dependence of deflection value $-\eta^v$ on parameter α

α	0,25	1,25	2,25	3,25
η^v	0,118	0,120	0,122	0,125

Table 1.2. Dependence of deflection value $-\eta^v$ on parameter γ

γ	0,25	1,25	2,25	3,25
η^v	0,118	0,128	0,139	0,148

Table 1.3. Dependence of deflection value $-\eta^{v}$ on parameter β

β	0,5	1,5	2,5	3,5
η^{v}	0,145	0,138	0,131	0,122

In Figure 1.1. The dependences of the critical time on the values of τ_{kp}, α, γ, β and μ are given. The case of a homogeneous rod corresponds to the values when $\alpha=\gamma=\beta=\mu=1$. Then $\eta^{v}=0{,}129$ and at $w=3{,}11\cdot10^{-5}$, $(w_{kp}=6{,}72\cdot10^{-5})$ $\tau_{kp}=4{,}16\cdot10^{10}$.

Section 1.3 presents a study of buckling of a multilayer thin-walled shell during creep under the influence of a distributed load. The task of determining the stress-strain state (SSS) in structural elements during creep, considering geometric nonlinearity, made of composites and various nonlinear elastic materials, and interconnected through full adhesion, is mathematically complex. The problem is doubly nonlinear, and it may be necessary to study solutions to nonlinear boundary value problems with discontinuous coefficients. To solve the problem using the variational method in combination with the approximate Ritz method, we define the approximating functions for the deflection and bending moment in the form

$$w=a_0(t)+a(t)\cos l\theta, \qquad M=b(t)\cos l\theta \tag{1.13}$$

Due to the law of plane sections, the total deformation will be:

$$\varepsilon=\frac{w}{R}+\frac{1}{R^2}\left(\frac{\partial w}{\partial\theta}\right)^2+\frac{1}{R^2}\frac{\partial^2 w}{\partial\theta^2}r \tag{1.14}$$

From the stationary condition of the functional, we obtain a differential equation relating the dimensionless time to the dimensionless deflection, the parameter of which is the dimensionless load ω.

$$\frac{d\tau}{d\eta}=\frac{-c_1l^2+\frac{9c_2}{4}\zeta^3\varphi_2\omega+\frac{9(n+1)}{2^{n+1}}\zeta^{n+3}\omega^{n+1}\sum_{i=0}^{n}3^iC_n^ic_{i+2}\eta^i\varphi_{i+1}}{\frac{3}{2^{m+1}}\zeta^{m+2}\omega^m\sum_{i=0}^{n}3^jC_m^jc_{j+1}\eta^j\varphi_{j+1}} \tag{1.15}$$

The differential equation with the initial condition for the deflection will constitute the Cauchy problem.

We assume that the acting compressive load q, uniformly distributed over the surface, is less than the critical Euler force q_{kr}, i.e., $q < q_{kr}$. In this case, the shell, because of instantaneous elastic deformation under load q, takes on a new position. This position must be stable, otherwise stability for this structural element is lost within the limits of elasticity, without the occurrence of creep deformation. Among the kinematic possible positions of the shell, the unstable state corresponds to the state within the limits of elasticity in which the functional

$$J = R\int_0^{2\pi}\int_{-h}^{h}\left\{\dot{\sigma}\dot{\varepsilon} + \frac{1}{2R^2}\sigma\left(\frac{\partial\dot{w}}{\partial\theta}\right)^2\right\}dr\,d\theta - \tag{1.16}$$

$$-\frac{R}{2}\int_0^{2\pi}\sum_{k=0}^{s-1}\frac{1}{E_{k+1}}\int_{a_k}^{a_{k+1}}\dot{\sigma}^2\left[1+(n+1)\left(\frac{\sigma}{\sigma_{k+1}^0}\right)^n\right]dr\,d\theta + R\int_0^{2\pi}\dot{w}\,d\theta$$

takes a stationary value. We will approximate the deflection and bending moment in the same mode as for creep (1.13), but with amplitudes depending on the load parameter.

$$w = a_0(q) + a(q)\cos\ell\theta,\;\; M = b(q)\cos\ell\theta \tag{1.17}$$

The phenomenon of loss of stability with buckling of the shell during elasticity occurs when the load reaches a critical value and remains unchanged. We differentiate stresses within the limits of elasticity in relation to the monotonically changing load q, so that from the condition of stationarity of the functional, the corresponding loss of elastic stability with buckling, the following differential equation is obtained in dimensionless quantities

$$\frac{d\omega}{d\eta} = \frac{-\pi l^2 + \frac{9\pi}{4}\zeta^3\phi_2\omega + \frac{9(n+1)}{2^{n+2}}\zeta^{n+3}\omega^{n+1}\sum_{i=0}^{n}3^i c_n^i c_{i+2}\eta^i\phi_{i+2}}{-\frac{9\pi}{4}\zeta^3\eta\phi_2 + \frac{3}{2^{n+2}}\zeta^{n+3}\omega^n\sum_{i=0}^{n}3^i c_n^i c_{i+1}\eta^i\phi_{i+1} + \frac{9(n+1)}{2^{n+1}}\zeta^{n+3}\omega^n\sum_{i=0}^{n}3^i c_n^i c_{i+2}\eta^i\phi_{i+2}} \tag{1.18}$$

Suppose that in a three-layer shell the facing layers are made of the same material, with the same thickness, then $E_1 / E_3, B_1 = B_3, \delta_1 = \delta_3, \sigma_1^0 = \sigma_3^0$. To numerically implement the solution to the problem with compiling a table and constructing characteristic graphs, we introduce the following dimensionless quantities $\xi = R/h, \alpha = E_1/E_2, \beta = \delta_2/\delta_1, \gamma = \sigma_1^0/\sigma_3^0, \mu = B_2/B_1$, $\lambda = E_1/\sigma_1^0$, with values $\xi = 20, \beta = 2, \gamma = \sigma_1^0/\sigma_3^0$, $\lambda = 3$. Equation (1.18) is solved numerically using the Runge-Kutta method, taking the value for the initial dimensionless deviation $\eta_0 = 0{,}1$. From the condition that the numerator equals zero, we obtain the value of the critical load. The solution to the elastic problem will correspond to the number

$$\eta(0) = 0{,}1273. \qquad (1.19)$$

The Cauchy problem for equation (1.15) - (1.19) is also solved by the Runge-Kutta method for various values of dimensionless parameters. For values of the dimensionless critical time, corresponding to the moment of loss of stability with buckling, we will have (at $m = 5$):

n	$\alpha = \gamma = 1.75, \mu = 2$	$\alpha = \gamma = \mu = 1$
2	$\tau_{cr} = 3.92 \times 10^4$	$\tau_{cr} = 3.42 \times 10^4$
4	$\tau_{cr} = 3.15 \times 10^3$	$\tau_{cr} = 2.6 \times 10^3$
6	$\tau_{cr} = 342$	$\tau_{cr} = 300$

In paragraph 1.4, the variational principle is proven for determining the bearing capacity of solids and structures during creep, considering the effects and influence of external physicochemical fields. It is assumed that instantaneous elastoplastic deformation $\varepsilon_{ij}^{(1)}$, creep deformation p_{ij} and deformation resulting from irradiation $\varepsilon_{ij}^{(2)}$ occur in the material, so that covariant the components of the total deformation will be

$$\varepsilon_{ij} = \varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)} + p_{ij} \qquad (1.20)$$

However, the phenomenon of creep is accompanied by the process of accumulation of damageability of the material and the generalized theory formed by Yu.N. Rabotnov[4] in the form of the concept of the

[4] Работнов Ю.Н. Ползучесть элементов конструкций / М. Наука, 1966. -752 с.

equation of mechanical state represents an opportunity for their joint description if damageability parameters are introduced as structural parameters in the kinetic relationships of creep.

To model the processes of non-stationary corrosion, long-term strength, and their relationships for a structurally stable material at a given time, we can write:

$$\dot{p}_{ij} = \dot{p}_{ij}(\sigma^{\alpha\beta}, T, q_1, ..., q_N) \qquad (1.21)$$

During deformation, the structural parameters change according to the following non-integrable equations:

$$dq_i = a^i_{rs} dp_{rs} + b^i_{rs} d\sigma_{rs} + \varphi^i dt + d^i dT, (i = 1,2,...,n) \quad (1.22)$$

Here $a^i_{rs}, b^i_{rs}, \varphi^i, d^i$ are functions of $p_{ij}, \sigma^{rs}, T, t$ and $q_1, q_2, ..., q_N$ also of parameters, i.e. $\varphi^i = \varphi^i(p_{ij}, \sigma^{\alpha\beta}, T, t, q_1, q_2, ..., q_N)$ etc.

Radiation training, according to experimental evidence, causes a significant decrease in the ductility of steels and nickel alloys, but at significant doses of radiation (up to 10^{22} neutrons/cm2) they retain the plasticity zone at normal and moderately elevated temperatures.

For the deformation rate we will have the utility

$$\dot{p}_{ij} = \dot{p}_{ij}(\varepsilon_{\alpha\beta}, \sigma^{\alpha\beta}, \omega, c) \qquad (1.23)$$

kinetic equation of diffusion considering corrosion of the environment

$$\dot{c} = div(D\nabla c) - kc \qquad (1.24)$$

kinetic equation of damage

$$\dot{\omega} = \varphi(\sigma^{\alpha\beta}, \omega, c) \qquad (1.25)$$

Here $D = D(\sigma^{\alpha\beta}, \omega, c)$, $k = const$ the characteristic rate of a chemical reaction, kc -is the rate of decomposition of chemical bonds under the influence of an aggressive chemical environment.

Among the many effects caused by neutron irradiation, issues related to volumetric expansion and changes in the physical and mechanical properties of the body h occupy a special place. If the flux intensity nv does not depend on time, then the total neutron flux $N = nvt \exp \mu(z - h/2)$ neutron/cm2 will pass through a unit area of a plate with thickness after time t. In the case when deformations and

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displacements are constrained for some reason, then internal forces and stresses arise in the deforming body, and the components of the deformation tensor change completely.

$$\dot{\varepsilon}_{ij}^{(1)} = \left\{C_{ijkm}(x^k, d)\sigma^{km}\right\}^{\bullet} \quad (i, j, k, m = \overline{1,3}); \quad \dot{\varepsilon}_{ij}^{(2)} = \dot{\theta}(x^k, d)\delta_{ij},$$

Finally, for the components of the total strain rate tensor we will have

$$\dot{\varepsilon}_{ij} = \left\{C_{ijkm}\sigma^{km}\right\}^{\bullet} + \dot{p}_{ij} + \dot{\theta}\delta_{ij}. \tag{1.26}$$

Note that when $d = 0$ we have $\theta = 0$, and the covariant components of the fourth-rank tensor C_{ijkm} for the anisotropic case are the physical and mechanical characteristics of the material of the unirradiated body. In metals and alloys, as well as in structures made from them, because of irradiation at high temperatures, the processes of creep and accumulation of damage are accelerated, depending on the type of stress state. Neglecting dynamic effects, let us consider the equilibrium of a deformable solid body V with a volume and limited by a sufficiently smooth closed surface S. On some part of it S_u, only the components of the displacement vector $\bar{u}_i$ are specified, and on the remaining part S_σ - the load $\bar{T}^j$. Since the variational theorem will be used to solve problems of buckling of thin-walled structural elements, finite strain relations and nonlinear equilibrium equations are used here[5]. Then the geometrically nonlinear theory of equilibrium of an elastoplastic body in a chemically active medium during creep under the influence of a neutron flux will be described by means of the following boundary value problem

$$\nabla_j\left\{\sigma^{ij}(\nabla_i u^k + \delta_i^k)\right\} = 0, \quad (k = \overline{1,3}), \quad \dot{\varepsilon}_{ij} = \left\{C_{ijkm}\sigma^{km}\right\}^{\bullet} + \dot{p}_{ij} + \dot{\theta}\delta_{ij},$$

$$2\varepsilon_{ij} = \nabla_i u_j + \nabla_j u_i + \nabla_i u^k \nabla_j u_k, \quad T^i = \bar{T}^i, \ x^k \in S_\sigma, \tag{1.27}$$

$$u_i = \bar{u}_i, \quad x^k \in S_u, \quad \dot{c} = div(D\nabla c) - kc, \quad \dot{\omega} = \varphi(\sigma^{\alpha\beta}, \omega, c).$$

Here $T^k = \sigma^{ij} n_j\left(\nabla_i u^k + \delta_i^k\right)$, $S = S_\sigma \cup S_u$.

[5] Амензаде Ю.А. Теория упругости / М. Высшая школа, 1976. – 272с. [228-231]

In some variational principles, based on varying the tensors of the rates of change of stress and strain, a more general assumption is made about the law of elastic-plasticity for instantaneous deformation.

$$\dot{\varepsilon}_{ij}^{(1)} = \left\{C_{ijkl}(x^k, d)\sigma^{kl}\right\}^{\bullet}, \quad x^k \in E_3$$

Deformation during creep of solids and structural elements leads to a change in shape, redistribution of stresses and loss of stability. Therefore, creep is one of the important factors in structural analysis at high temperatures.

In three-dimensional Euclidean space, we will consider the process of creep in an elastoplastic anisotropic medium, which is subject to irradiation by a neutron flux. In the case of a complex stress state, the dependences of the stress components on the strain components in all stages of deformation must be known. These dependencies are established in the theories of plasticity and creep.

In this work, the load-bearing abilities of elements of a thin-walled structure and solid bodies under creep under the influence of external physical fields and influences are investigated. A mixed-type variational principle for creep is formulated for geometrically nonlinear problems of plastic bodies, considering damage, the diffusion process, and irradiation with a neutron flux within the same functional.

$$J = \int_V \{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i\dot{u}^k\nabla_j\dot{u}_k - \frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}\dot{\sigma}^{km} - \dot{C}_{ijkm}\dot{\sigma}^{km}\dot{\sigma}^{ij} - (\dot{\varepsilon}_{ij}^{(2)} + 2\dot{p}_{ij})\dot{\sigma}^{ij} + \lambda_\omega(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\varphi) +$$
$$+ \lambda_c[\frac{1}{2}\dot{c}^2 - \dot{c}div(D\nabla c) - kc\dot{c}]\}dV - \int_{S_\sigma}\dot{\bar{T}}^i\dot{u}_i dS - \int_{S_u}\dot{T}^i(\dot{u}_i - \dot{\bar{u}}_i)dS \qquad (1.28)$$

In the functional, the independent variable quantities $\dot{\sigma}^{ij}, \dot{u}_i, \dot{\omega}$ are $\lambda_\omega = \lambda_\omega(\sigma^{\alpha\beta}, \varepsilon_{\alpha\beta})$ and $\lambda_c = \lambda_c(\sigma^{\alpha\beta}, \varepsilon_{\alpha\beta})$ weight functions, which are selected depending on the type of interpolation functions to refine the approximations. In order to overcome the difficulty of solving the variational problem using direct methods, refusing to satisfy the kinetic equations (1.24) and (1.25) exactly, we replace them with approximate integral relations

$$\int_V \lambda_\omega[\dot{\omega} - \varphi(\sigma^{\alpha\beta}, p_{ij}, \omega)]^2 dV \approx 0, \quad \int_c \lambda_c[\dot{c} - div(D\nabla c) + kc]^2 dV \approx 0,$$

where functions of damageability and levels of concentration of an aggressive environment are sought in the form

$$\omega = \sum_{k=1}^{p} a_k(t)\psi_k(x_j); \quad a_k(0) = 0; \; c = \sum_{k=1}^{m} c_k(t)\eta_k(x_j); \quad c_k(0) = 0;$$

$\eta_k(0) = 0, d\,\eta_k(x)/dx\big|_{x=0} = 0$, which is a shifted initial-boundary condition for the function (at turns into a second-order parabolic differential equation with constant coefficients).

Thus, this creep problem is reduced to a system of differential equations with a known system procedure, i.e. with an algorithm that allows you to approach the mathematical solution of these problems. Many different direct methods for constructing approximate solutions are possible, as well as direct methods for qualitative analysis of the existence and uniqueness of a solution or the derivation of a priori estimates for this problem.

The statement is proven that the stationary conditions of the functional (1.28) are equivalent to a system of equations and relations, which is a mathematical model of the problem of determining the true stress-strain state of an elastoplastic continuous medium irradiated by a neutron flux with damage in physicochemical fields in the process of creep at finite deformations.

Let us find the variation of the functional (1.28) in a curvilinear coordinate system. Considering that the variation operator δ acts on the speed of quantities, then we obtain:

$$\delta J = \int_V \{\dot{\varepsilon}_{ij}\delta\dot{\sigma}^{ij} + \dot{\sigma}^{ij}\delta\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i\dot{u}^k\delta(\nabla_j\dot{u}_k) + \frac{1}{2}\sigma^{ij}\nabla_j\dot{u}_k\delta(\nabla_i\dot{u}^k) -$$

$$-\frac{1}{2}C_{ijkm}\dot{\sigma}^{km}\delta\dot{\sigma}^{ij} - \frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}\delta\dot{\sigma}^{km} - \dot{C}_{ijkm}\sigma^{km}\delta\dot{\sigma}^{ij} - (\dot{\varepsilon}^{(2)}_{ij} + \dot{p}_{ij})\delta\dot{\sigma}^{ij} + \lambda_m(\dot{\omega} - \varphi)\delta\dot{\omega} +$$

$$+ \lambda_c[\dot{c} - div(D\nabla c) - kc]\delta\dot{c}\}dV - \int_{S_\sigma}\dot{\bar{T}}^i\delta\dot{u}_i dS - \int_{S_u}[(\dot{u}_i - \dot{\bar{u}}_i)\delta\dot{T}^i]dS. \quad (1.29)$$

This took into account the fact that, by definition, volumetric deformation is a function of coordinates and irradiation dose, and the rate of creep deformation in the general case depends on stress, temperature, time and structural parameters, therefore $\delta\dot{\theta} = 0$,

$\delta \ddot{p}_{ij} = 0,$, and to satisfy the boundary conditions $\delta \dot{T}^i = 0$, on S_σ the equalities $\delta \ddot{u}_i = 0$ on S_u are accepted. , respectively. Since the tensor C_{ijkl} does not depend on speed, the following relations hold true:

$$\delta C_{ijkm} = 0, \quad \delta \dot{C}_{ijkm} = 0, \quad \delta \dot{\varepsilon}_{ij}^{(1)} = C_{ijkl} \delta \dot{\sigma}^{kl},$$

and, equality $C_{ijkm} \dot{\sigma}^{km} \delta \dot{\sigma}^{ij} = C_{ijkm} \dot{\sigma}^{ij} \delta \dot{\sigma}^{km}$

From the symmetry of the stress tensor we obtain the equality of the third and fourth terms in (1.29). Using the Gauss-Ostrogradsky formula, taking into account the symmetries of the stress tensor and carrying out several mathematical projections, we obtain a variation of the functional, where the terms are collected at the same independent variations:

$$\delta J = \int_V \{[\varepsilon_{ij} - (\varepsilon_{ij}^{(1)} + p_{ij} + \theta \delta_{ij})]^{\bullet} \delta \dot{\sigma}^{ij} - \nabla_j \left[\sigma^{ij} \left(\delta_i^k + \nabla_i u^k \right) \right]^{\bullet} \delta \dot{u}_k +$$

$$+ \lambda_\omega (\dot{\omega} - \phi) \delta \dot{\omega} + \lambda_c [\dot{c} - div(D \nabla c) - kc] \delta \dot{c}\} dV +$$

$$+ \int_{S_\sigma} \left[\sigma^{ij} \left(\delta_i^k + \nabla_i u^k \right) n_j - \bar{T}^k \right]^{\bullet} \delta \dot{u}_k dS - \int_{S_u} \left(\dot{u}_k - \dot{\bar{u}}_k \right) \delta \dot{T}^k dS = 0$$

Taking into account the main lemma of the calculus of variations and formula (1.26), from the condition of vanishing, as the Euler equations in a curvilinear coordinate system, we obtain a system of complete relations for the boundary value problem (1.21).

Modified variational principle for a composite body.

It is known that the strength of real solids and structural elements is several orders of magnitude less than the theoretical strength of an ideal crystal lattice of structural metals, corresponding to the simultaneous rupture of all intermolecular bonds, which is usually associated with the existence of lattice defects. The strength of most polymers for structural purposes does not exceed 100–150 MPa, and Young's modulus of elasticity is 3–4 GPa. Further improvement of the mechanical properties of polymers through the design of the chemical structure of molecules is not promising, so we must look for other ways to improve the elastic-strength characteristics of structural

polymers, among which the transition from polymers to nanocomposites is currently considered the most promising.
Let us now move on to the description of the composite material. When formulating the contact boundary value problem, we assume that the body V_k consists of K elements. An element with a number occupies a volume with a surface S_k. We assume $S_k = S_k^{(1)} \cup S_k^{(2)}$, that where is $S_k^{(1)}$ the boundary of the volume that does not have common points with $V_k^{(2)}$, a is the boundary of the volume $S_{k\sigma}^{(2)}$ is part of the common boundary of the body $T^{\alpha(0)}$. Let forces be given on the surface, $S, S_k^{(1)}$ and $S_k^{(2)}$ displacements $u_\alpha^{(0)}$ on the remaining surface. Let us assume that the surfaces are sufficiently smooth.
The theory of composite media used is based on the following premises:
- during the deformation process, the elements contact each other along their entire common surface;
-deformations are finite (geometrically nonlinear);
- the conditions of complete adhesion are met on the contact surfaces.

Further, we will proceed from the fact that the materials of different elements are different and their physical and mechanical properties are described according to an elastoplastic law such as flow theory. Then the geometrically nonlinear equilibrium theory is described by the following boundary value problem:

$$\nabla_j \left\{ \sigma_{(k)}^{ij} \left(\delta_i^\alpha + \nabla_i u_{(k)}^\alpha \right) \right\} = 0, \quad \left(\alpha = \overline{1,3} \right) \tag{1.30}$$

$$\dot{\varepsilon}_{ij(k)} = \left\{ C_{ijmn(k)} \sigma_{(k)}^{mn} \right\}^{\bullet} + \dot{p}_{ij(k)} + \dot{\theta}_{(k)} \delta_{ij}, \tag{1.31}$$

$$\dot{c} = div(D\nabla c) - kc, \quad \dot{\omega} = \varphi(\sigma^{\alpha\beta}, \omega, c).$$

$$2\varepsilon_{ij(k)} = \nabla_i u_{j(k)} + \nabla_j u_{i(k)} + \nabla_i u_{\alpha(k)} \nabla_j u_{(k)}^\alpha, \tag{1.32}$$

$$u_{i(k)} = \bar{u}_{i(k)}, \quad S_{ku}, \tag{1.33}$$

$$T_{(k)}^\alpha = \bar{T}_{(k)}^\alpha, \quad S_{k\sigma}, \quad \text{где } T_{(k)}^\alpha = \sigma_{(k)}^{ij} n_j \left(\nabla_i u^\alpha + \delta_i^\alpha \right) \tag{1.34}$$

It is important to note here that in the most general case, according to the general formulation of the problem

$$\bar{u}_{i(k)} = \begin{cases} u_{i(k)}^{(0)} & \forall S \in S_{ku}^{(2)}, \\ u_{i(k)}^{(00)} & \forall S \in S_{ku}^{(1)}, \end{cases} \text{ and } \bar{T}_{(k)}^{\alpha} = \begin{cases} T_{(k)}^{\alpha(0)} & \forall S \in S_{k\sigma}^{(2)}, \\ T_{(k)}^{\alpha(00)} & \forall S \in S_{k\sigma}^{(1)}, \end{cases}$$

To the given equations (1.30) - (1.34), conjugation conditions on $S_k^{(1)}$. With complete adhesion between adjacent elements on the interface surface, we have continuity of displacements, where in expanded form the absence of a jump in the corresponding quantities has the form

$$u_{i(k)}^{+} = u_{i(k)}^{-}, \quad T_{(k)}^{i+} = T_{(k)}^{i-}. \tag{1.35}$$

Here, the "+" and "–" signs indicate the values of the functions at the junction points when approaching them to the right and left of the contact line.

As before, again select an arbitrary element with volume V_k . Following (1.28), we write the corresponding functional for this volume:

$$J_k = \int_{V_k} \{\dot{\sigma}_{(k)}^{ij} \dot{\varepsilon}_{ij(k)} + \frac{1}{2} \sigma_{(k)}^{ij} \nabla_i \dot{u}_{(k)}^{\alpha} \nabla_j \dot{u}_{(k)\alpha} - \frac{1}{2} C_{ijkm} \dot{\sigma}_{(k)}^{ij} \dot{\sigma}_{(k)}^{km} - \dot{C}_{ijkm} \dot{\sigma}_{(k)}^{km} \dot{\sigma}_{(k)}^{ij} -$$
$$- (\dot{\varepsilon}_{ij(k)}^{(2)} + \dot{p}_{ij(k)}) \dot{\sigma}_{(k)}^{ij} + \lambda_{\omega} (\frac{1}{2} \dot{\omega}^2 - \dot{\omega}\varphi) + \lambda_c [\frac{1}{2} \dot{c}^2 - \dot{c}\, div(D \nabla c) - kc\dot{c}]\} dV -$$
$$- \int_{S_\sigma} \dot{\bar{T}}_{(k)}^{i} \dot{u}_{i(k)} dS - \int_{S_u} \dot{T}_{(k)}^{i} (\dot{u}_{i(k)} - \dot{\bar{u}}_{i(k)}) dS \tag{1.36}$$

Here S_{ku} and $S_{k\sigma}$ - are the sections of the boundary where displacements $\bar{u}_{i(k)}$ and forces $\bar{T}_{(k)}^{i}$ are considered known or given by formulas (1.39) and (1.40). Now let's move on to generalizing the functional for the entire volume when the body is composed of K elements. It is easy to see that in this case the surface integrals in (1.36) cancel each other and the functional finally takes the form

$$J=\sum_{k=1}^{K}\int_{V_k}\left\{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij}+\frac{1}{2}\sigma^{ij}\nabla_i\dot{u}^m\nabla_j\dot{u}_m-\frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}\dot{\sigma}^{km}-\dot{C}_{ijkm}\dot{\sigma}^{km}\dot{\sigma}^{ij}-\right.$$

$$\left.-(\dot{p}_{ij}+\dot{\theta}\delta_{ij})\dot{\sigma}^{ij}+\lambda_{\omega}(\frac{1}{2}\dot{\omega}^2-\dot{\omega}\varphi)+\lambda_c[\frac{1}{2}\dot{c}^2-\dot{c}div(D\nabla c)-kc\dot{c}]\right\}dV-$$

$$-\int_{S_u}\dot{T}^i(\dot{u}_i-\dot{\bar{u}}_i)dS-\int_{S_\sigma}\dot{\bar{T}}^{\alpha}\dot{u}_{\alpha}dS.$$

The second chapter, using the discrete analytical method proposed by S.D. Akbarov[6] and co-authors, provides solutions to the problem of the dynamics of a hollow sphere with inhomogeneous initial stresses, and it consists of five paragraphs.

Paragraph 2.1 is devoted to the results of important studies over the past period and showing the state of the issue, highlighting the purpose of the study, with the justification of the influence of non-uniform initial stresses on the dynamic response of layered hollow and solid spheres. In Section 2.2, the problem of the dynamics of a hollow sphere with non-uniform initial stresses is formulated.

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[6] Akbarov S.D. Dynamics of pre-strained bi-material elastic systems: linearized three-dimensional approach / Springer-Heidelberg. New York, 2015. P-1004 [15-19]

A hollow sphere with an inner radius b and an outer radius a is considered under the assumption that this sphere is loaded with uniformly distributed normal forces with intensities on both the outer p and q inner surfaces, respectively.

Let us present a mathematical formulation of the problem of the dynamics of a hollow sphere with initial inhomogeneous stresses given within the framework of linearized field equations.

$$\sigma_{rr}^{(0)}=-p\frac{a^3(r^3-b^3)}{(a^3-b^3)r^3}-q\frac{b^3(a^3-r^3)}{(a^3-b^3)r^3},\ \sigma_{\theta\theta}^{(0)}=\sigma_{\phi\phi}^{(0)}=-p\frac{a^3(2r^3+b^3)}{2(a^3-b^3)r^3}+q\frac{b^3(2r^3+a^3)}{2(a^3-b^3)r^3},$$

$$\sigma_{r\theta}^{(0)}=\sigma_{r\phi}^{(0)}=\sigma_{\theta\phi}^{(0)}=0, \qquad (2.1)$$

The sphere with these initial stresses is assumed to receive a dynamic harmonic excitation over time, and it is required to determine how these initial stresses affect the dynamic behavior (e.g., natural frequencies) of the sphere.

According to the three-dimensional linearized theory of elastic waves in initially stressed bodies, these equations in spherical coordinates (r,θ,ϕ) can be represented as follows.
Equations of motion:

$$\frac{\partial t_{rr}}{\partial r}+\frac{1}{r}\frac{\partial t_{\phi r}}{\partial \phi}+\frac{1}{r\sin\phi}\frac{\partial t_{\theta r}}{\partial \theta}+\frac{1}{r}(2t_{rr}-t_{\phi\phi}-t_{\theta\theta}+t_{\phi r}ctg\phi)=\rho\frac{\partial^2 u_r}{\partial t^2},$$
$$\frac{\partial t_{r\phi}}{\partial r}+\frac{1}{r}\frac{\partial t_{\phi\phi}}{\partial \phi}+\frac{1}{r\sin\phi}\frac{\partial t_{\theta\phi}}{\partial \theta}+\frac{1}{r}(2t_{r\phi}+t_{\phi r}+(t_{\phi\phi}-t_{\theta\theta})ctg\phi)=\rho\frac{\partial^2 u_\phi}{\partial t^2}, \quad (2.2)$$
$$\frac{\partial t_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial t_{\phi\theta}}{\partial \phi}+\frac{1}{r\sin\phi}\frac{\partial t_{\theta\theta}}{\partial \theta}+\frac{1}{r}(2t_{r\theta}+t_{\theta r}+(t_{\phi\theta}+t_{\theta\phi})ctg\phi)=\rho\frac{\partial^2 u_\theta}{\partial t^2}.$$

Here

$$t_{rr}=\sigma_{rr}+\sigma_{rr}^{(0)}\frac{\partial u_r}{\partial r},\ t_{\varphi\varphi}=\sigma_{\varphi\varphi}+\sigma_{\varphi\varphi}^{(0)}\left(\frac{u_r}{r}+\frac{\partial u_\varphi}{r\partial\varphi}\right),\ t_{r\varphi}=\sigma_{r\varphi}+\sigma_{rr}^{(0)}\frac{\partial u_\varphi}{\partial r},$$
$$t_{\theta\theta}=\sigma_{\theta\theta}+\sigma_{\theta\theta}^{(0)}\left(\frac{u_r}{r}+ctg\varphi\frac{u_\varphi}{r}+\frac{1}{r\sin\varphi}\frac{\partial u_\theta}{\partial\theta}\right),\ t_{\varphi r}=\sigma_{\varphi r}+\sigma_{\varphi\varphi}^{(0)}\left(\frac{\partial u_r}{r\partial\varphi}-\frac{u_\varphi}{r}\right),$$
$$t_{r\theta}=\sigma_{r\theta}+\sigma_{rr}^{(0)}\frac{\partial u_\theta}{\partial r},\ t_{\theta r}=\sigma_{\theta r}+\sigma_{\theta\theta}^{(0)}\left(\frac{1}{r\sin\varphi}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r}\right),\ t_{\varphi\theta}=\sigma_{\varphi\theta}+\sigma_{\varphi\varphi}^{(0)}\frac{\partial u_\theta}{r\partial\varphi},$$
$$t_{\theta\varphi}=\sigma_{\theta\varphi}+\sigma_{\theta\theta}^{(0)}\left(\frac{1}{r\sin\varphi}\frac{\partial u_\varphi}{\partial\theta}-ctg\varphi\frac{u_\theta}{r}\right). \quad (2.3)$$

Elasticity ratios:

$$\sigma_{rr}=\lambda\left(\varepsilon_{rr}+\varepsilon_{\theta\theta}+\varepsilon_{\phi\phi}\right)+2\mu\varepsilon_{rr},\ \sigma_{\theta\theta}=\lambda\left(\varepsilon_{rr}+\varepsilon_{\theta\theta}+\varepsilon_{\phi\phi}\right)+2\mu\varepsilon_{\theta\theta},$$
$$\sigma_{\varphi\varphi}=\lambda\left(\varepsilon_{rr}+\varepsilon_{\theta\theta}+\varepsilon_{\phi\phi}\right)+2\mu\varepsilon_{\phi\phi},\ \sigma_{r\theta}=2\mu\varepsilon_{r\theta},\ \sigma_{\theta\phi}=2\mu\varepsilon_{\theta\phi}, \quad (2.4)$$
$$\sigma_{r\phi}=2\mu\varepsilon_{r\phi}.$$

Relationships between deformations and displacements:

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r},\ \varepsilon_{\theta\theta}=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{1}{r}u_r,\ \varepsilon_{\phi\phi}=\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial\phi}+\frac{1}{r}u_r+\frac{1}{r}u_\theta ctg\theta,$$
$$\varepsilon_{r\theta}=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r}\right);\ \varepsilon_{\theta\phi}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_\phi}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial\phi}-\frac{u_\phi}{r}ctg\theta\right); \quad (2.5)$$
$$\varepsilon_{r\phi}=\frac{1}{r}\left(\frac{\partial u_\phi}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial\theta}-\frac{u_\phi}{r}\right).$$

In (2.2) and (2.3) $t_{rr},...,t_{\theta\phi}$ are the components of the Kirchhoff stress tensor in a spherical coordinate system.
As boundary conditions that are satisfied on the inner and outer end surfaces of the hollow sphere, we take the following homogeneous conditions:

$$t_{rr}\big|_{r=a}=0,\ t_{r\theta}\big|_{r=a}=0,\ t_{rr}\big|_{r=b}=0,\ t_{r\theta}\big|_{r=b}=0,\ t_{r\phi}\big|_{r=b}=0. \qquad (2.6)$$

It is possible to formulate inhomogeneous boundary conditions instead of the conditions given in (2.6), and the corresponding initial conditions for non-stationary dynamic problems can also be formulated. Note that in the case when there are no initial stresses in the sphere, i.e. in the case where $p=q=0$, the above formulation coincides with the corresponding one in the framework of classical linear elastodynamics. Section 2.3 is devoted to the selection and development of a method for solving the problem. According to the expressions in (2.1), the system of equations in (2.2) - (2.5) is an equation with variable coefficients, the analytical solution of which, in the general case, is very difficult and in many cases impossible. As a rule, until recently, the system of equations (2.2) -(2.5) were solved numerically using various numerical methods, but in the case under consideration there is the following peculiarity in relation to the variable coefficients: these coefficients depend only on the coordinate r. This feature of the coefficients allows us to use the discrete analytical method, developed and used in the works of Akbarov et al., to solve these equations. Section 2.4 provides a discretization of the solution domain and the derivation of equations for the functions included in the classical Lamé decomposition. To use the discrete analytical method, the interval $r\in[a,b]$ is divided into a certain number of subintervals. To do this, we introduce notations and divide the interval , where the notations $R_1=b$ and $R_2=a$ are introduced, are divided into subintervals , where $U_{k=1}^{N}\left[R_{1k},R_{2k}\right]=\left[R_1,R_2\right], R_{1k}=R_1+(k-1)(R_2-R_1)/N$ and $R_{2k}=R_1+k(R_2-R_1)/N$. Within each subinterval $[R_{1k},R_{2k}]$, the functions $\sigma_{rr}^{(0)}(r),\sigma_{\theta\theta}^{(0)}(r)$ and $\sigma_{\phi\phi}^{(0)}(r)$ are taken as constants, which are equal to the values $r_k = R_{1k} + (R_{2k} - R_{1k})/2$, i.e. are written in the form $\sigma_{rr}^{(0)}(r_k), \sigma_{\theta\theta}^{(0)}(r_k)$ and $\sigma_{\phi\phi}^{(0)}(r_k)$ instead of the corresponding functions. After this discretization of the interval $[R_{1k}, R_{2k}]$, the solution to the full system of equations (2.2)-(2.5) is performed within each subinterval separately, in which the variable coefficients $\sigma_{rr}^{(0)}(r), \sigma_{\theta\theta}^{(0)}(r)$ and $\sigma_{\phi\phi}^{(0)}(r_k)$ in the system of equations (2.2) and (2.3) are replaced by constants $\sigma_{rr}^{(0)}(r_k), \sigma_{\theta\theta}^{(0)}(r_k)$ and $\sigma_{\phi\phi}^{(0)}(r_k)$, where $r_k = R_{1k} + (R_{2k} - R_{1k})/2$. On the mating surfaces between subintervals, the continuity conditions for the force and displacement vectors are satisfied. Denoting the thickness of each sublayer by $h_k = (R_1 - R_2)/N$ and introducing an additional superscript for quantities related to the (k)th sublayer, these continuity conditions can be written as follows

$$
\begin{gathered}
t_{rr}^{(1)}\Big|_{r=R_1} = 0,\ t_{r\theta}^{(1)}\Big|_{r=R_1} = 0,\ t_{r\varphi}^{(1)}\Big|_{r=R_1} = 0, \\
t_{rr}^{(1)}\Big|_{r=R_1+h} = t_{rr}^{(2)}\Big|_{r=R_1+h},\ t_{r\theta}^{(1)}\Big|_{r=R_1+h} = t_{r\theta}^{(2)}\Big|_{r=R_1+h},\ t_{r\varphi}^{(1)}\Big|_{r=R_1+h} = t_{r\varphi}^{(2)}\Big|_{r=R_1+h}, \\
u_{r}^{(1)}\Big|_{r=R_1+h} = u_{r}^{(2)}\Big|_{r=R_1+h},\ u_{\theta}^{(1)}\Big|_{r=R_1+h} = u_{\theta}^{(2)}\Big|_{r=R_1+h},\ u_{\varphi}^{(1)}\Big|_{r=R_1+h} = u_{\varphi}^{(2)}\Big|_{r=R_1+h}, \\
t_{rr}^{(N-1)}\Big|_{r=R_1+(N-1)h} = t_{rr}^{(N)}\Big|_{r=R_1+(N-1)h},\ t_{r\theta}^{(N-1)}\Big|_{r=R_1+h} = t_{r\theta}^{(N)}\Big|_{r=R_1+(N-1)h}, \\
t_{r\varphi}^{(N-1)}\Big|_{r=R_1+(N-1)h} = t_{r\varphi}^{(N)}\Big|_{r=R_1+(N-1)h},\ u_{r}^{(N-1)}\Big|_{r=R_1+(N-1)h} = u_{r}^{(N)}\Big|_{r=R_1+(N-1)h}, \\
u_{\theta}^{(N-1)}\Big|_{r=R_1+(N-1)h} = u_{\theta}^{(N)}\Big|_{r=R_1+(N-1)h},\ u_{\varphi}^{(N-1)}\Big|_{r=R_1+(N-1)h} = u_{\varphi}^{(N)}\Big|_{r=R_1+(N-1)h}, \\
t_{rr}^{(N)}\Big|_{r=R_2} = 0,\ t_{r\theta}^{(N)}\Big|_{r=R_2} = 0,\ t_{r\varphi}^{(N)}\Big|_{r=R_2} = 0.
\end{gathered}
\tag{2.7}
$$

Section 2.5 is devoted to solving the system of equations (2.1) - (2.5). To solve the system of equations (2.2) - (2.5) for the −th sublayer, we use the following classical Lamé expansion, according to the monograph by Eringen et al.

$$u_r^{(k)} = \frac{\partial \Phi^{(k)}}{\partial r} + \frac{\partial^2 (r\chi^{(k)})}{\partial r^2} - r\nabla^2 \chi^{(k)}, u_\theta^{(k)} = \frac{1}{r}\frac{\partial \Phi^{(k)}}{\partial \theta} + \frac{1}{\sin\theta}\frac{\partial \psi^{(k)}}{\partial \phi} + \frac{1}{r}\frac{\partial^2 (r\chi^{(k)})}{\partial \theta \partial r}$$

$$u_\phi^{(k)} = \frac{1}{r\sin\theta}\frac{\partial \Phi^{(k)}}{\partial \phi} - \frac{\partial \psi^{(k)}}{\partial \phi} + \frac{1}{r\sin\theta}\frac{\partial^2 (r\chi^{(k)})}{\partial \phi \partial r}, \qquad (2.8)$$

Substituting expressions from (2.8) into equations (2.2) - (2.5) for the k sublayer, and performing cumbersome mathematical transformations, we obtain the following equations for the functions $\Phi^{(k)}, \psi^{(k)}, \chi^{(k)}$.

$$\mu\nabla^2 \begin{Bmatrix} \psi^{(k)} \\ \chi^{(k)} \end{Bmatrix} + \sigma_{rr}^{(0)}(r_k)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\begin{Bmatrix} \psi^{(k)} \\ \chi^{(k)} \end{Bmatrix}\right)\right] + \sigma_{\theta\theta}^{(0)}(r_k)\left[\frac{ctg\theta}{r^2}\frac{\partial}{\partial \theta}\begin{Bmatrix} \psi^{(k)} \\ \chi^{(k)} \end{Bmatrix} + \right.$$

$$\left. + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\begin{Bmatrix} \psi^{(k)} \\ \chi^{(k)} \end{Bmatrix} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}\begin{Bmatrix} \psi^{(k)} \\ \chi^{(k)} \end{Bmatrix}\right] = \rho\frac{\partial^2}{\partial t^2}\begin{Bmatrix} \psi^{(k)} \\ \chi^{(k)} \end{Bmatrix}$$

$$(\lambda + 2\mu)\nabla^2 \Phi^{(k)} + \sigma_{rr}^{(0)}(r_k)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\Phi^{(k)}\right)\right] +$$

$$\sigma_{\theta\theta}^{(0)}(r_k)\left[\frac{ctg\theta}{r^2}\frac{\partial}{\partial \theta}\Phi^{(k)} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\Phi^{(k)} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}\Phi^{(k)}\right] = \rho\frac{\partial^2}{\partial t^2}\Phi^{(k)}, \qquad (2.9)$$

Here

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \Delta_{\theta,\phi}, \ \Delta_{\theta,\phi} = \frac{ctg\theta}{r^2}\frac{\partial}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}. \qquad (2.10)$$

In paragraph 2.6. Analytical solutions of equations for the functions included in the Lamé expansion for the problem of natural oscillations are obtained. In accordance with well-known physical and mechanical considerations, the functions $\psi^{(k)}, \chi^{(k)}$ and $\Phi^{(k)}$ are presented

$$\varphi^{(k)}(r,\theta,\phi,t) = F_\varphi^{(k)}(n,r)P_n^m(\cos\theta)\cos m\phi e^{i\omega t},$$

$$\psi^{(k)}(r,\theta,\phi,t) = F_\psi^{(k)}(n,r)P_n^m(\cos\theta)\cos m\phi e^{i\omega t}, \qquad (2.11)$$

$$\chi^{(k)}(r,\theta,\phi,t) = F_\chi^{(k)}(n,r)P_n^m(\cos\theta)\cos m\phi e^{i\omega t},$$

where $P_n^m(\cos\theta)$ in expression (2.11) denotes the associated Legendre functions with the m order and the n harmonic.

Thus, substituting expressions (2.11) into equations (2.9) and (2.10), we obtain the following equations for the functions $F_{\phi}^{(k)}(n,r)$, $F_{\psi}^{(k)}(n,r)$ and $F_{\chi}^{(k)}(n,r)$

$$\frac{d^2F_{\psi;\chi}^{(k)}(n,r)}{dr^2}+\frac{2}{r}\frac{dF_{\psi;\chi}^{(k)}(n,r)}{dr}+\left((\lambda^{(k)})^2-\frac{v_n^{(k)}(v_n^{(k)}+1)}{r^2}\right)F_{\psi;\chi}^{(k)}(n,r)=0,$$

$$\frac{d^2F_{\Phi}^{(k)}(n,r)}{dr^2}+\frac{2}{r}\frac{dF_{\Phi}^{(k)}(n,r)}{dr}+\left((\gamma^{(k)})^2-\frac{\eta_n^{(k)}(\eta_n^{(k)}+1)}{r^2}\right)F_{\Phi}^{(k)}(n,r)=0,$$

(2.12)

Here

$$(\lambda^{(k)})^2=\rho\omega^2/(\mu+\sigma_{rr}^{(0)}(r_k)),\ v_n^{(k)}=-\frac{1}{2}+\sqrt{\frac{1}{4}+\alpha^{(k)}n(n+1)},$$

$$(\gamma^{(k)})^2=\rho\omega^2/(\lambda+2\mu+\sigma_{rr}^{(0)}(r_k)),\ \eta_n^{(k)}=-\frac{1}{2}+\sqrt{\frac{1}{4}+\beta^{(k)}n(n+1)},$$

$$\alpha^{(k)}=\frac{(\mu+\sigma_{\theta\theta}^{(0)}(r_k))}{(\mu+\sigma_{rr}^{(0)}(r_k))}\ \ \beta^{(k)}=\frac{(\lambda+2\mu+\sigma_{\theta\theta}^{(0)}(r_k))}{(\lambda+2\mu+\sigma_{rr}^{(0)}(r_k))}.$$

(2.13)

Thus, solutions to equation (6.2) are represented through spherical Bessel functions as follows.

$$F_{\psi}^{(k)}(n,r)=C^{(k)}j_{v_n^{(k)}}(\lambda^{(k)}r)+D^{(k)}y_{v_n^{(k)}}(\lambda^{(k)}r),$$
$$F_{\chi}^{(k)}(n,r)=E^{(k)}j_{v_n^{(k)}}(\lambda^{(k)}r)+G^{(k)}y_{v_n^{(k)}}(\lambda^{(k)}r), \qquad (2.14)$$
$$F_{\Phi}^{(k)}(n,r)=A^{(k)}j_{\eta_n^{(k)}}(\gamma^{(k)}r)+B^{(k)}y_{\eta_n^{(k)}}(\gamma^{(k)}r),$$

Here $j_{\alpha}(cr)=\left(\frac{\pi}{2cr}\right)^{\frac{1}{2}}J_{\alpha+1/2}(cr),\ y_{\alpha}(cr)=\left(\frac{\pi}{2cr}\right)^{\frac{1}{2}}Y_{\alpha+1/2}(cr)$. (2.15)

In (2.15), the functions $J_{\alpha+1/2}(cr)$ and $Y_{\alpha+1/2}(cr)$ are Bessel functions of the first and second kind with non-integer order. In the case when $\sigma_{rr}^{(0)}(r_k)=\sigma_{\theta\theta}^{(0)}(r_k)=\sigma_{\phi\phi}^{(0)}(r_k)=0$, according to which $\alpha^{(k)}=\beta^{(k)}=1$, the expressions in (2.14) coincide with the corresponding ones obtained in the classical case. Using relations (2.14), (2.11). (2.8), (2.5) and (2.4), expressions are obtained for the displacements and for the components of the stress tensor. To simplify the writing of the resulting expressions, two sets of complete orthogonal functions in $[0,\pi]$ are introduced, defined as follows[7]:

$$X_{nm}(\theta)=P_n^m(\cos\theta),\ Y_{nm}(\theta)=n\cot\theta P_n^m(\cos\theta)-\frac{n+m}{\sin\theta}P_{n-1}^m(\cos\theta). \tag{2.16}$$

Using the notation in (2.16), we write the following expressions for the required functions:

$$\begin{aligned}
u_r^{(k)}&=\frac{1}{r}\left\{A^{(k)}u_{11}^{(k)}+B^{(k)}u_{12}^{(k)}+E^{(k)}u_{31}^{(k)}+G^{(k)}u_{32}^{(k)}\right\}X_{nm}(\theta)\cos m\phi e^{i\omega t},\\
u_\theta^{(k)}&=\frac{1}{r}\{\left[A^{(k)}v_{11}^{(k)}+B^{(k)}v_{12}^{(k)}+E^{(k)}v_{31}^{(k)}+G^{(k)}v_{32}^{(k)}\right]Y_{nm}(\theta)+\\
&+\left(C^{(k)}v_{21}^{(k)}+D^{(k)}v_{22}^{(k)}\right)\frac{m}{\sin\theta}X_{nm}(\theta)\}\cos m\phi e^{i\omega t},\\
u_\phi^{(k)}&=\frac{1}{r}\{\left[A^{(k)}v_{11}^{(k)}+B^{(k)}v_{12}^{(k)}+E^{(k)}v_{31}^{(k)}+G^{(k)}v_{32}^{(k)}\right]\frac{-m}{\sin\theta}X_{nm}(\theta)\\
&+\left(-C^{(k)}v_{21}^{(k)}-D^{(k)}v_{22}^{(k)}\right)Y_{nm}(\theta)\}+\sin m\phi e^{i\omega t},
\end{aligned} \tag{2.17}$$

[7] Guz A.N. Dynamics of an elastic isotropic sphere of an incompressible material subjected to initial uniform volumetric loading // IAM, vol.21, No8, 1985, pp.738-746

$$\sigma_{rr}^{(k)}=\frac{2\mu^{(k)}}{r^2}\left[A^{(k)}T_{111}^{(k)}+B^{(k)}T_{112}^{(k)}+E^{(k)}T_{131}^{(k)}+G^{(k)}T_{132}^{(k)}\right]X_{nm}(\theta)\cos m\phi e^{i\omega t},$$

$$\begin{aligned}
\sigma_{r\theta}^{(k)}&=\frac{2\mu^{(k)}}{r^2}\{\left[A^{(k)}T_{411}^{(k)}+B^{(k)}T_{412}^{(k)}+E^{(k)}T_{431}^{(k)}+G^{(k)}T_{432}^{(k)}\right]Y_{nm}(\theta)\\
&+\left(-C^{(k)}T_{421}^{(k)}-D^{(k)}T_{422}^{(k)}\right)\frac{m}{\sin\theta}X_{nm}(\theta)\}\cos m\phi e^{i\omega t},
\end{aligned}$$

Where $k=1,2,...,N.$ As noted above, here N-is the number of subintervals into which the solution region is divided relative to the radial coordinate r, and this number is determined in accordance with the convergence of the numerical results. Thus, substituting expressions (2.17) considering the function with coefficients expressed through the Bessel functions into the boundary and contact conditions (2.6) and (2.7), we obtain two unrelated systems of homogeneous algebraic equations. The first (second) system contains unknowns $A^{(k)}, B^{(k)}, E^{(k)}$ and $G^{(k)}$ ($C^{(k)}$ and $D^{(k)}$). Equating to zero the determinant of the matrix of coefficients of the first and second groups separately, the following equations are obtained for determining the frequency of natural vibration.

$\det(\alpha_{q_1q_2}) = 0,\ q_1; q_2 = 1,2,...,4N$ (for spheroidal vibration). (2.18)

$\det(\delta_{p_1p_2}) = 0,\ p_1; p_2 = 1,2,...,2N$ (for torsional vibrations). (2.19)

The expressions for the components of the matrices $(\alpha_{q_1q_2})$ and $(\delta_{q_1q_2})$ can be easily determined from the expressions given in (2.17). Section 2.7 contains the results of numerical examples, analysis of the influence of initial stresses on the natural frequencies of a hollow ball. Numerical results are obtained by solving equations (2.18) (for the spheroidal mode) and (2.19) (for the torsional mode), and this solution is obtained numerically using the algorithm we developed in MATLAB and the corresponding PC programs using the method of dividing a segment in half (bisection method). The results refer to dimensionless natural frequencies, denoted as $\Omega = \omega a / \sqrt{\mu / \rho}$, and were obtained for various values of the coefficients $b/a,\ p/\mu$ and q/μ. Here the last two relations characterize the initial stresses in a hollow sphere, and the numerical results differ in vibration harmonics and in the sequence of roots in each harmonic.

The third chapter of the dissertation, consisting of five paragraphs, reflects the results of a study of the natural vibrations of a multilayer hollow sphere in cases where there is a preliminary inhomogeneous stress state in the sphere filled with a compressible fluid. The first paragraph provides a brief overview of those works that contain research over the past thirty years. The second paragraph is devoted to the formulation of problems and the mathematical formulation of the problem. A multilayer hollow sphere is considered, the centre of which is connected to the spherical $Or\theta\phi$ and Cartesian $Ox_1x_2x_3$ coordinate systems. It is assumed that before the vibration of the hydroelastic system described above, normal compression forces are uniformly distributed on the inner and outer end surfaces of the sphere with intensity $\tilde{q}$ and $\tilde{p}$, respectively. As a result of the action of these external forces, non-uniform initial stresses arise in the sphere. The study is described by three-dimensional linearized equations of elastic wave theory and linearized hydrodynamic equations of a barotropic inviscid compressible fluid. Three-dimensional linearized equations of inviscid fluid flow and continuity equations:

$$\rho_0 \frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} p, \qquad \frac{\partial \rho}{\partial t} + \rho_0 \nabla_n v^n = 0. \tag{3.1}$$

Here $$\vec{\nabla} p = \frac{\partial p}{\partial r}\vec{e}_r + \frac{1}{r\sin\phi}\frac{\partial p}{\partial \phi}\vec{e}_\phi + \frac{1}{r}\frac{\partial p}{\partial_\theta}\vec{e}_\theta$$

$$\nabla_n v^n = \frac{1}{r^2}\frac{\partial\left(r^2 v_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} \tag{3.2}$$

Three-dimensional linearized equations and relationships for wave propagation in elastic bodies with inhomogeneous initial stresses are given in this dissertation work in the second chapter (2.2) - (2.5). For the problem under study, these equations are performed separately in each layer of the multilayer sphere. It is assumed that between adjacent layers (at the contact surface between a solid and a liquid) ideal contact conditions (corresponding to the conditions between a liquid and a sphere) are observed.

Conditions for compatibility between the liquid and the inner end layer of the sphere:

$$v_r\big|_{r=b}=\left.\frac{\partial u_r^{(1)}}{\partial t}\right|_{r=b},\ v_\theta\big|_{r=b}=\left.\frac{\partial u_\theta^{(1)}}{\partial t}\right|_{r=b},\ v_\phi\big|_{r=b}=\left.\frac{\partial u_\phi^{(1)}}{\partial t}\right|_{r=b},\ t_{rr}^{(1)}\Big|_{r=b}=-p\big|_{r=b},$$

$$t_{r\theta}^{(1)}\Big|_{r=b}=0,\ t_{r\phi}^{(1)}\Big|_{r=b}=0 \tag{3.3}$$

Boundary conditions on the outer end surface of the outer layer of the sphere:

$$t_{rr}^{(m)}\Big|_{r=a}=0,\ t_{r\theta}^{(m)}\Big|_{r=a}=0,\ t_{r\phi}^{(m)}\Big|_{r=a}=0\,. \tag{3.4}$$

Contact conditions between the layers of the sphere:

$$t_{rr}^{(1)}\Big|_{r=H^{(1)}}=t_{rr}^{(2)}\Big|_{r=H^{(1)}},\ t_{r\theta}^{(1)}\Big|_{r=H^{(1)}}=t_{r\theta}^{(2)}\Big|_{r=H^{(1)}},\ t_{r\phi}^{(1)}\Big|_{r=H^{(1)}}=t_{r\phi}^{(2)}\Big|_{r=H^{(1)}},$$

$$u_r^{(1)}\Big|_{r=H^{(1)}}=u_r^{(2)}\Big|_{r=H^{(1)}},\ u_\theta^{(1)}\Big|_{r=H^{(1)}}=u_\theta^{(2)}\Big|_{r=H^{(1)}},\ u_\phi^{(1)}\Big|_{r=H^{(1)}}=u_\phi^{(2)}\Big|_{r=H^{(1)}},$$

$$t_{rr}^{(2)}\Big|_{r=H^{(2)}}=t_{rr}^{(3)}\Big|_{r=H^{(2)}},\ t_{r\theta}^{(2)}\Big|_{r=H^{(2)}}=t_{r\theta}^{(3)}\Big|_{r=H^{(2)}},\ t_{r\phi}^{(2)}\Big|_{r=H^{(2)}}=t_{r\phi}^{(3)}\Big|_{r=H^{(2)}}\cdots$$

$$u_\theta^{(m-1)}\Big|_{r=H^{(m-1)}}=u_\theta^{(m)}\Big|_{r=H^{(m-1)}},\qquad u_\phi^{(m-1)}\Big|_{r=H^{(m-1)}}=u_\phi^{(m)}\Big|_{r=H^{(m-1)}},$$

$$t_{rr}^{(m)}\Big|_{r=a}=0,\ t_{r\theta}^{(m)}\Big|_{r=a}=0,\ t_{r\phi}^{(m)}\Big|_{r=a}=0, \tag{3.5}$$

where $H^{(1)}=b+h^{(1)},...,H^{(m)}=b+h^{(1)}+h^{(2)}+,...,+h^{(m)}=a$, …,
This concludes the formulation of the problem of natural vibration by considering the initial stresses $\sigma_{rr}^{(i)0}(r),\sigma_{\theta\theta}^{(i)0}(r)$ and $\sigma_{\phi\phi}^{(i)0}(r)$, which are determined from the solution of the corresponding static problem formulated within the framework of the linear theory of elasticity. This static problem relates to the determination of the stressed state in a multilayer hollow sphere in the case when uniformly distributed normal forces with intensity act on the internal q and p external

surfaces of this sphere. In accordance with the problem for a single-layer sphere, we use the following representation

$$u_r^{(i),0}(r)=A^{(i)}r+\frac{B^{(i)}}{r^2},\ u_\theta^{(i),0}=0,\ u_\phi^{(i),0}=0,\ \sigma_{rr}^{(i),0}(r)=A^{(i)}-2\frac{B^{(i)}}{r^3},$$

$$\sigma_{\theta\theta}^{(i),0}(r)=\sigma_{\phi\phi}^{(i),0}(r)=A^{(i)}+\frac{B^{(i)}}{r^3}. \qquad (3.6)$$

The unknown constants in (3.6) are determined from the following boundary $A^{(i)}$ and $B^{(i)}$ contact conditions

$$\sigma_{rr}^{(1),0}(r)\Big|_{r=b}=-\tilde{q},\ \sigma_{rr}^{(1),0}\Big|_{r=H^{(1)}}=\sigma_{rr}^{(2),0}\Big|_{r=H^{(1)}},\ u_r^{(1),0}\Big|_{r=H^{(1)}}=u_r^{(2),0}\Big|_{r=H^{(1)}},$$

$$\sigma_{rr}^{(2),0}\Big|_{r=H^{(2)}}=\sigma_{rr}^{(3),0}\Big|_{r=H^{(2)}},\ u_r^{(2),0}\Big|_{r=H^{(2)}}=u_r^{(3),0}\Big|_{r=H^{(2)}},$$

$$\sigma_{rr}^{(m-1),0}\Big|_{r=H^{(m-1)}}=\sigma_{rr}^{(m),0}\Big|_{r=H^{(m-1)}},$$

$$u_r^{(m-1),0}\Big|_{r=H^{(m-1)}}=u_r^{(m),0}\Big|_{r=H^{(m-1)}},\ \sigma_{rr}^{(m),0}(r)\Big|_{r=a}=-\tilde{p}. \qquad (3.7)$$

The solution of linear algebraic equations (3.7) is carried out on a PC using a well-known solution algorithm in MATLAB. In the third paragraph, the hydrodynamic equation is solved with the introduction of potential $\Phi^{(f)}$.

$$p(r,\theta,\phi,t)=-\frac{\rho_0}{a_0^2}\frac{\partial\Phi^{(f)}}{\partial t},\quad v_r(r,\theta,\phi,t)=\frac{\partial\Phi^{(f)}}{\partial r},\ldots. \qquad (3.8)$$

Where $\Phi^{(f)}$ the potential satisfies the following equation

$$\left(\Delta+\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right)\Phi^{(f)}=0,$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\mathrm{ct\,g}\theta}{r^2}\frac{\partial}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}, \qquad (3.9)$$

We represent $\Phi^{(f)}$ the potential in the form of the following series

$$\Phi^{(f)} = \sum_{n=0}^{\infty}\sum_{m=-n}^{+n} \varphi_{nm}^{(f)}(r,t)\,\chi_n^m(\theta,\phi), \quad \chi_n^m(\theta,\phi) = P_n^m(\cos\theta)\cos m\phi, \qquad (3.10)$$

where $P_n^m(\cos\theta)$ - is the associated Legendre function with order m and with harmonics n. Using the representation $\varphi_{nm}^{(f)}(r,t) = R_{nm}(r)\cos\omega t$, we obtain a differential equation solution that will look like:

$$R_n(r) = K\sqrt{\frac{\pi}{2\Omega_a r}}\,J_{n+\frac{1}{2}}\left(\Omega_a r\right), \quad \Omega_a = \frac{a\omega}{a_0} \qquad (3.11)$$

here $J_{n+\frac{1}{2}}$ -is the Bessel function of the first kind with order $(n+1/2)$ and K-is the unknown constant. Substituting solution (3.11) into equations (3.8) and (3.10), we obtain expressions for the fluid pressure and for the radial component of the fluid velocity vector.

$$p = -\omega\rho_0 K\sqrt{\frac{\pi}{2\Omega_a r}}\cos\omega t\sum_{n=0}^{\infty}\sum_{m=-n}^{+n} J_{n+\frac{1}{2}}\left(\Omega_a r\right)P_n^m(\cos\theta)\cos m\phi,$$

$$v_r = K\sqrt{\frac{\pi}{2\Omega_a r}}\sin\omega t\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}\left\{\frac{n}{r}J_{n+\frac{1}{2}}\left(\Omega_a r\right) - \Omega_a J_{n+\frac{3}{2}}\left(\Omega_a r\right)\right\}P_n^m(\cos\theta)\cos m\phi \qquad (3.12)$$

Now let's consider solving elastodynamics problems that correspond to the system of equations (2.2) - (2.5) and note that for this purpose we use the discrete analytical method given in the second chapter. According to this method, i-each layer of the sphere

$\left\{H^{(i)} \le r \le H^{(i+1)}\right\}$ is divided $n^{(i)}$ into numerical sublayers $\left\{H^{(i)} \le r \right.$ $\left. \le R_1^{(i)}\right\}, \{R_1^{(i)} \le r \le R_2^{(i)}\}, \ldots, \left\{R_{n^{(i)}-1}^{(i)} \le r \le R_{n^{(i)}}^{(i)}\right\}$ where $R_{n^{(i)}}^{(i)} = H^{(i+1)}$ and in each of them the initial stresses are assumed to be constant. Thus, the system of equations (2.2) and (2.3) with variable coefficients is reduced to the corresponding system of equations with constant coefficients, which are satisfied separately in each sublayer $\left\{H^{(i)} \le r \right.$ $\left. \le R_1^{(i)}\right\}$. To solve the latter, we use the Lamé expansion for (i_k) displacements. Substituting the expansion formulas in equations (2.2) and (2.3), rewritten for the sublayer, we obtain the following equations for the Helmholtz potentials $\Phi^{(i_k)}$, $\psi^{(i_k)}$ and $\chi^{(i_k)}$. To solve the above equations, we present the functions $\Phi^{(i_k)}$, $\psi^{(i_k)}$ and $\chi^{(i_k)}$ as follows:

$$\Phi^{(i_k)}(r,\theta,\varphi,t) = \cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n} F_{\phi}^{(i_k)}(n,r)P_n^m(\cos\theta)\cos m\varphi \ ,$$

$$\psi^{(i_k)}(r,\theta,\varphi,t) = \cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n} F_{\psi}^{(i_k)}(n,r)P_n^m(\cos\theta)\sin m\varphi \ , \quad (3.13)$$

$$\chi^{(i_k)}(r,\theta,\varphi,t) = \cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n} F_{\chi}^{(i_k)}(n,r)P_n^m(\cos\theta)\cos m\varphi \ .$$

After substituting expression (3.13) into the equations for the Helmholtz potentials, we obtain the following equations:

$$\frac{d^2F_{\Psi;\chi}^{(i_k)}(n,r)}{dr^2} + \frac{2}{r}\frac{dF_{\Psi;\chi}^{(i_k)}(n,r)}{dr} + \left((\delta^{(i_k)})^2 - \frac{\nu_n^{(i_k)}(\nu_n^{(i_k)}+1)}{r^2}\right)F_{\Psi;\chi}^{(i_k)}(n,r) = 0$$

$$\frac{d^2F_{\Phi}^{(i_k)}(n,r)}{dr^2}+\frac{2}{r}\frac{dF_{\Phi}^{(i_k)}(n,r)}{dr}+\left((\gamma^{(i_k)})^2-\frac{\eta_n^{(i_k)}(\eta_n^{(i_k)}+1)}{r^2}\right)F_{\Phi}^{(i_k)}(n,r)=0$$

(3.14)

The solution to equations (3.14) is determined similarly to the solution given in the second chapter, with the only difference being that instead of a layer k, a sublayer will appear i_k. And they will also contain unknown constants $C_n^{(i_k)}$, $D_n^{(i_k)}$, $E_n^{(i_k)}$, $G_n^{(i_k)}$, $A_n^{(i_k)}$ and $B_n^{(i_k)}$ to determine which we use not only the conditions in (3.3) – (3.5), but also additional conditions for the continuity of forces (i.e. stresses acting on the interface between adjacent sublayers) and displacement vectors between the above sublayers. The result is the following expressions for the mentioned displacements and stresses.

$$u_r^{(i_k)}(r)=\cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}u_{rn}^{(i_k)}(r)X_{nm}(\theta)\cos m\phi \ ,$$

$$u_\theta^{(i_k)}(r)=\cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}\left[u_{\theta 1n}^{(i_k)}(r)Y_{nm}(\theta)+u_{\theta 2n}^{(i_k)}(r)\frac{m}{\sin\theta}X_{nm}(\theta)\right]\cos m\phi$$

$$u_\phi^{(i_k)}(r)=\cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}\left[u_{\phi 1n}^{(i_k)}(r)\frac{-m}{\sin\theta}X_{nm}(\theta)+u_{\phi 2n}^{(i_k)}(r)Y_{nm}(\theta)\right]\sin m\phi$$

$$\sigma_{rr}^{(i_k)}(r)=\cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}\sigma_{rrn}^{(i_k)}(r)X_{nm}(\theta)\cos m\phi \ ,$$

$$\sigma_{r\theta}^{(i_k)}(r)=\cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}\left[\sigma_{r\theta 1n}^{(i_k)}(r)Y_{nm}(\theta)+\sigma_{r\theta 2n}^{(i_k)}(r)\frac{m}{\sin\theta}X_{nm}(\theta)\right]\cos m\phi$$

$$\sigma_{r\phi}^{(i_k)}(r)=\cos(\omega t)\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}\left[\sigma_{r\phi 1n}^{(i_k)}(r)\frac{m}{\sin\theta}X_{nm}(\theta)+\sigma_{r\phi 2n}^{(i_k)}(r)Y_{nm}(\theta)\right]\sin m\phi$$

(3.15)

Here $X_{nm}(\theta)=P_n^m(\cos\theta)$, $Y_{nm}(\theta)=n\cot\theta P_n^m(\cos\theta)-\frac{n+m}{\sin\theta}P_{n-1}^m(\cos\theta)$.

The result is a system of homogeneous linear algebraic equations for unknown constants. The characteristic equation related to the spherical vibration mode can be represented as follows

$$\det\left(\alpha_{q_1q_2}\right)=0 \tag{3.16}$$

$q_1;q_2=1,...,4k^{(1)};4k^{(1)}+1,4k^{(1)}+2,...,4k^{(1)}+4k^{(2)};...;4k^{(1)}+$

$+4k^{(2)}+...+4k^{(m-1)}+1,...,4k^{(1)}+4k^{(2)}+....+4k^{(m-1)}+4k^{(m)}$

However, this equation for the torsional vibration mode has the following form

$$\det\left(\delta_{p_1p_2}\right)=0, \tag{3.17}$$

$p_1;p_2=1,...,2k^{(1)};2k^{(1)}+1,2k^{(1)}+2,...,2k^{(1)}+2k^{(2)};...;2k^{(1)}+$

$+2k^{(2)}+...+2k^{(m-1)}+1,...,2k^{(1)}+2k^{(2)}+...+2k^{(m-1)}+2k^{(m)}$

In (3.16) and (3.17) $k^{(i)}$, indicates the number of sublayers in the i layer of the sphere.

The fourth paragraph of the third chapter is devoted to numerical results and discussion:

- the presence of liquid inside a multilayer hollow sphere leads to a decrease in the values of the natural frequency of oscillations of the hydroelastic system in relation to the natural frequency of oscillations of the same hollow sphere; - the magnitude of this influence depends not only on the mechanical and geometric parameters of the hydroelastic system under consideration, but also on the number of harmonics, and on the ordinal number of the roots; - the influence of non-uniform initial stresses on the fundamental frequency of natural oscillations (the first root of the frequency equation) of the hydroelastic system is insignificant, however, these initial stresses, in general, lead to a decrease in the indicated frequency; - a decrease in the thickness of the hollow sphere leads, in general, to an increase in the influence of initial stresses on the frequency of the fundamental mode.

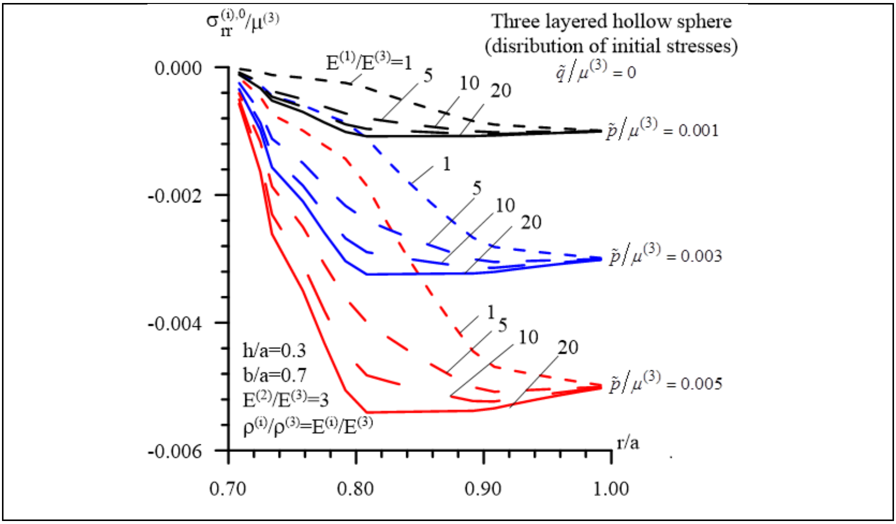

Fig. 3.2. Initial voltage distribution

In the fifth section, the influence of initial non-uniform stresses on the frequencies of natural vibrations of a hollow infinite cylinder filled with a compressible inviscid fluid moving along the cylinder axis at a constant speed filling the inside of the cylinder is investigated. Accurate three-dimensional linearized equations and relations of the theory of elastic waves and linearized equations of motion of barotropic inviscid fluids are used. The formulation of boundary conditions on the outer surface of the cylinder and compatibility conditions between the cylinder and the liquid on the inner surface of the cylinder is presented. Specific formulations are made for the axisymmetric case and general aspects of methods for solving the formulated problems are considered.

In the fourth chapter of the dissertation, within the framework of a model of a piecewise homogeneous body using exact three-dimensional equations and relations of elastodynamics, the influence of imperfect contact between the layers of a three-layer hollow sphere on the natural frequencies of this sphere is studied. The first paragraph of the fourth chapter substantiates the conditions for non-ideality or imperfection of contacts, which remained unexplored until recently. The second paragraph gives a mathematical formulation of the problem and selects a solution method. The study of natural vibrations of a three-layer sphere is presented using exact three-dimensional equations and relations of elastodynamics, including equations of motion, elasticity relations and Cauchy formulas.

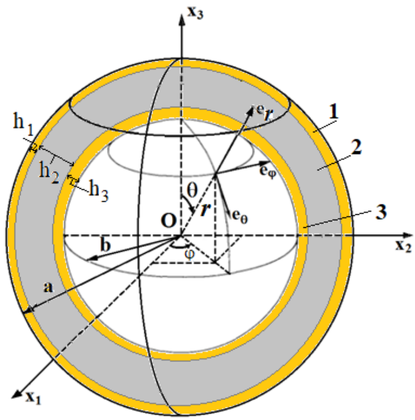

Fig. 4.1. Geometry of a three-layer sphere

The following boundary conditions are set on the outer and inner front surfaces of the sphere:

$$\sigma_{rr}^{(1)}\Big|_{r=a}=0,\ \sigma_{r\theta}^{(1)}\Big|_{r=a}=0,\ \sigma_{r\varphi}^{(1)}\Big|_{r=a}=0,\ \sigma_{rr}^{(3)}\Big|_{r=b}=0,$$
$$\sigma_{r\theta}^{(3)}\Big|_{r=b}=0,\sigma_{r\varphi}^{(3)}\Big|_{r=b}=0, \qquad (4.1)$$

We assume that the conditions of continuity of the force vector are satisfied at the interfaces between the layers, i.e. the following relations hold:

$$\sigma_{rr}^{(1)}\Big|_{r=a-h_1}=\sigma_{rr}^{(2)}\Big|_{r=a-h_1},\sigma_{r\theta}^{(1)}\Big|_{r=a-h_1}=\sigma_{r\theta}^{(2)}\Big|_{r=a-h_1},\ \sigma_{r\varphi}^{(1)}\Big|_{r=a-h_1}=\sigma_{r\varphi}^{(2)}\Big|_{r=a-h_1},$$

$$\sigma_{rr}^{(2)}\Big|_{r=a-h_1-h_2} = \sigma_{rr}^{(3)}\Big|_{r=a-h_1-h_2}, \quad \sigma_{r\theta}^{(2)}\Big|_{r=a-h_1-h_2} = \sigma_{r\theta}^{(3)}\Big|_{r=a-h_1-h_2},$$

$$\sigma_{r\varphi}^{(2)}\Big|_{r=a-h_1-h_2} = \sigma_{r\varphi}^{(3)}\Big|_{r=a-h_1-h_2}. \qquad (4.2)$$

The contact condition regarding movement on the surfaces between layers is non-ideal, and this non-ideality is mathematically modeled as the following six equalities:

$$u_r^{(1)}\Big|_{r=a-h_1} - u_r^{(2)}\Big|_{r=a-h_1} = \frac{F_1 h_1}{\mu_1}\sigma_{rr}^{(1)},$$

$$u_\theta^{(1)}\Big|_{r=a-h_1} - u_\theta^{(2)}\Big|_{r=a-h_1} = \frac{F_2 h_1}{\mu_1}\sigma_{r\theta}^{(1)},\ldots,$$

$$u_\varphi^{(2)}\Big|_{r=a-h_1-h_2} - u_\varphi^{(3)}\Big|_{r=a-h_1-h_2} = \frac{F_6 h_2}{\mu_2}\sigma_{r\varphi}^{(2)} \qquad (4.3)$$

Let us introduce the following notations: $h^{(12)}$ ($h^{(23)}$) - thickness of the adhesive transition layer between main layers 1 and 2 (between main layers 2 and 3), $K^{(12)}$ ($K^{(23)}$) and $\mu^{(12)}$ ($\mu^{(23)}$) moduli of volumetric and shear elasticity of the material of the transition layer between main layers 1 and 2 (between main layers 2 and 3). For real cases, we must proceed from the assumption of anisotropy of the rigidity properties of the transition layer material and, in accordance with this assumption, write:

$$F_1 = \frac{h^{(12)}}{h_1}\frac{\mu_1}{E_r^{(12)}}, \quad F_4 = \frac{h^{(23)}}{h_2}\frac{\mu_2}{E_r^{(23)}}, \quad F_2 = F_3 = \frac{h^{(12)}}{h_1}\frac{\mu_1}{\mu_{r\varphi}^{(12)}}$$

$$F_5 = F_6 = \frac{h^{(23)}}{h_2}\frac{\mu_2}{\mu_{r\varphi}^{(23)}} \qquad (4.4)$$

Formula (4.4) was introduced based on physical and mechanical considerations and generalizations of existing formulas. Moreover, in (4.4) through constants $E_r^{(12)}$ and $\mu_{r\varphi}^{(12)}$ ($E_r^{(23)}$ and $\mu_{r\varphi}^{(23)}$) the values of the rigidity of the material of the transition layer in the radial and azimuthal directions are indicated. Instead, formulas (4.4) can also be taken as a modified form of the Winkler and Pasternak model for beds with finite thickness. The solution to the elastodynamics equations, as in the two previous chapters of the dissertation, is constructed through the Helmholtz representation, in which the potentials $\phi^{(k)}(r,\varphi,\theta,t)$, $\chi^{(k)}(r,\varphi,\theta,t)$ and $\psi^{(k)}(r,\varphi,\theta,t)$ satisfy the wave equations, the solutions of which are selected in the following form

$$\phi^{(k)}(r,\theta,\varphi,t)=\left[A^{(k)}j_n(\alpha^{(k)}r)+B^{(k)}y_n(\alpha^{(k)}r)\right]P_n^m(\cos\theta)\cos m\varphi e^{i\omega t}$$

$$\psi^{(k)}(r,\theta,\varphi,t)=\left[C^{(k)}j_n(\beta^{(k)}r)+D^{(k)}y_n(\beta^{(k)}r)\right]P_n^m(\cos\theta)\sin m\varphi e^{i\omega t}$$

$$\chi^{(k)}(r,\theta,\varphi,t)=\left[E^{(k)}j_n(\beta^{(k)}r)+F^{(k)}y_n(\beta^{(k)}r)\right]P_n^m(\cos\theta)\cos m\varphi e^{i\omega t}$$

In the formulas $\alpha^{(k)}=\omega/c_1^{(k)}$, $\beta^{(k)}=\omega/c_2^{(k)}$, $A^{(k)},\dots,F^{(k)}$ – unknown constants, ω are the frequency of harmonic oscillations of the sphere, c_1^k and c_2^k are the velocities of longitudinal and transverse waves.

So, equating to zero the determinant of the matrix of coefficients of the system of equations related to the unknowns $A^{(k)}$, $B^{(k)}$, $E^{(k)}$ and $F^{(k)}$,as well as to the unknowns $C^{(k)}$ and $D^{(k)}$ separately, we obtain the frequency equation for the spheroidal and torsional modes of vibration, which can be formally represented in the following forms:

$$\det\left(\gamma_{q_1q_2}(F_1,F_2,\dots,F_6)\right)=0,\ q_1;q_2=1,2,\dots,12 \quad (4.5)$$

$$\det\left(\delta_{p_1p_2}(F_2,F_3,F_5,F_6)\right)=0,\ p_1;p_2=1,2,\dots,6 \quad (4.6)$$

Note that the explicit form of the expression γ_{q1q2} and δ_{p1p2} can be easily established from the formula for displacements and stresses, and functions included in the expressions, as well as from the boundary (4.1) and contact (4.2) and (4.3) conditions.

As a result of numerical analysis, it is established that the imperfection of contact relationships between the layers of the sphere leads to a significant decrease in the values of natural frequencies. To select the values of the ratios of elastic moduli and material densities of the layers of the sphere, it is based on the corresponding selections of the test problem, as well as on well-known mechanical engineering considerations. Considering the significance of the influence of non-ideal contact conditions on the value of natural frequencies of a three-layer sphere and the possible applications of the obtained results not only in the dynamics of layered materials, but also in geomechanics, there was a need to continue research for high harmonics of vibrations and for subsequent roots of frequency equations.

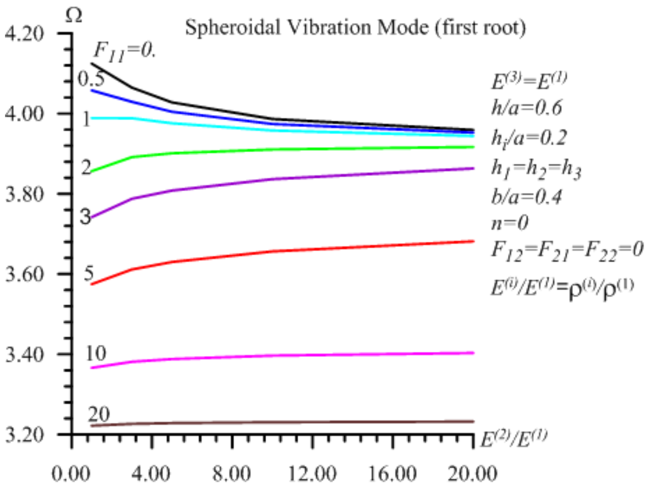

Fig. 4.2. Influence of non-ideal contact conditions between the upper and middle layers for $E^{(2)}/E^{(1)}$ different values of spheroidal frequencies of natural oscillations $F_{12} = F_{21} = F_{22} = 0$

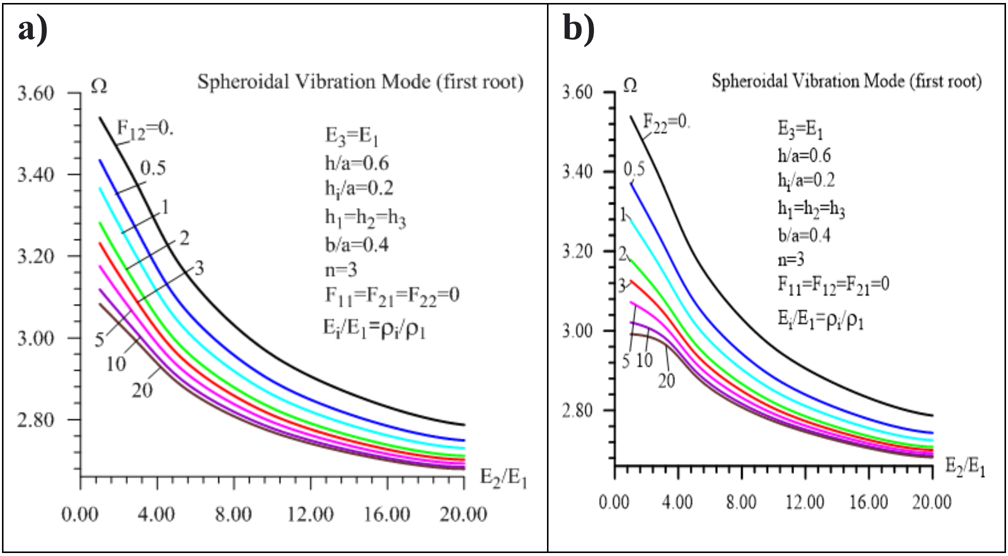

Fig.4.3. The influence of imperfect contact conditions of various options between layers on the frequencies Ω of oscillations of a spheroidal shape for harmonics $n=3$

In the fifth paragraph of the fourth chapter, a mathematical model of the wave motion of a hydroelastic system is constructed - a cylindrical shell and a compressible viscous liquid with spherical gas bubbles; wave processes in shells with liquid interacting with each other are studied. This interaction is often highly dependent on the deformation of the shell itself. Within the accepted assumption that small disturbances have formed in the two-phase liquid, we write the linearized Navier–Stokes equation, the continuity equation of the medium in Euler coordinates.

The linearized equation for pressure in a Newtonian fluid is obtained in the form:

$$\frac{1}{a_f^2}\frac{\partial^2 p}{\partial t^2}=\nabla^2\left(p+\frac{4\mu}{3\rho_0 a_f^2}\frac{\partial p}{\partial t}\right). \qquad (4.7)$$

For thin shells, the validity of the Kirchhoff–Love hypothesis is accepted. The axisymmetric case of shell motion in the Lagrangian coordinate system is considered and the known equations of shell motion are written. It should be noted that in a cylindrical shell filled with a two-phase single-velocity liquid-gas medium, which is initially at rest, according to the coordinate transformation formula, the Euler and Lagrange coordinates coincide. To obtain a closed mathematical system of hydroelasticity equations, boundary and contact conditions of the problem are drawn up, which are a mathematical model of a steady-state wave process. For unknown functions of the system of hydroelasticity equations p, u_r, u_x, w_r and w_x dynamic and kinematic contact conditions are compiled.

$$p_t = -\sigma_{rr}\big|_{r=R-h/2} = -\left(-p - \frac{2}{3}\mu div\vec{u} + 2\mu\frac{\partial u_r}{\partial r}\right),$$

$$q = -\sigma_{rx}\big|_{r=R-h/2} = -\mu\left(\frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial r}\right). \tag{4.8}$$

The conditions for kinematic contact on this surface will also be satisfied:

$$u_r\big|_{r=R-h/2} = \frac{\partial w_r}{\partial t}\bigg|_{r=R-h/2}, \qquad u_x\big|_{r=R-h/2} = \frac{\partial w_x}{\partial t}\bigg|_{r=R-h/2}. \tag{4.9}$$

We write the solution to the linearized wave equation (4.7) of a viscous liquid–gas medium as follows:

$$p = \mathrm{Re}\left\{p^*(r)e^{i(kx+\omega t)}\right\} \tag{4.10}$$

Having written the last dependence in equation (4.7) and performed simple mathematical transformations, the result is the Bessel equation, the solution of which, taking into account the limited pressure, is

$$p^* = p_0 J_0(\lambda r) \tag{4.11}$$

After setting the components of the velocity vector of a two-phase fluid u_r and u_x, taken in the same mode as for pressure, in the linearized equation of motion, the expressions are obtained:

$$u_r^* = AJ_1(\beta r) - \frac{p_0\lambda\left(1 + i\dfrac{\omega\mu}{3\rho_f a_f^2}\right)}{\mu\left(\beta^2 - \lambda^2\right)}J_1(\lambda r)$$

$$u_x^* = BJ_0(\beta r) - \frac{ip_0 k\left(1 + i\dfrac{\mu\omega}{3\rho_f a_f^2}\right)}{\mu(\beta^2 - \lambda^2)} J_0(\lambda r)\,.$$

Here $\beta^2 = -\left(k^2 + i\dfrac{\rho_f \omega}{\mu}\right)$ - complex number.

In the obtained solutions, the quantities, p_0, A and B are unknown numbers, which are determined from the contact conditions of the problem by solving the differential equations of shell motion. Additionally, we get more conditions

$$B = -\frac{i\beta}{k} A,$$

thus eliminating one of the four remaining unknowns. After the transition to dimensionless quantities, the resulting expressions for pressure and fluid velocity will be:

$$\bar{p}^* = \bar{p}_0 J_0(\bar{\lambda}\bar{r}),$$

$$\bar{u}_r^* = J_1(\bar{\beta}\bar{r})\bar{A} - i\frac{\bar{\lambda}\bar{\omega}J_1(\bar{\lambda}\bar{r})}{\bar{K} + \bar{\lambda}}\bar{p}_0,$$

$$\bar{u}_x^* = i\frac{\bar{\beta}}{K} J_0(\bar{\beta}\bar{r})\bar{A} - i\frac{\bar{K}\bar{\omega}J_0(\bar{\lambda}\bar{r})}{\bar{K} + \bar{\lambda}}\bar{p}_0,$$

$$\bar{\beta}^2 = -\left(K^2 + i\frac{\bar{\omega}}{\bar{\mu}}\right),$$

$$\bar{\lambda} = -K^2 + \frac{\bar{\omega}^2}{\left(1 + i\dfrac{4\bar{\mu}\bar{\omega}}{3}\right) - K^2}.$$

The shell equation in dimensionless form will look like this:

$$\begin{aligned} \frac{\partial^2 \bar{w}_r}{\partial \bar{t}^2} &= \bar{p}_t\left(1 - \frac{\bar{h}}{2}\right) + \frac{1}{\eta}\frac{\partial^2 \bar{w}_r}{\partial \bar{x}^2} - \bar{w}_r - \nu\frac{\partial \bar{w}_r}{\partial \bar{x}}, \\ \frac{\partial^2 \bar{w}_x}{\partial \bar{t}^2} &= \bar{q}\left(1 - \frac{\bar{h}}{2}\right) + \frac{\partial^2 \bar{w}_x}{\partial \bar{x}^2} + \nu\frac{\partial \bar{w}_r}{\partial \bar{x}}. \end{aligned} \tag{4.12}$$

We will look for a solution in the form:

$$\bar{w}_r = \bar{w}_r^* e^{i(K\bar{x} + \bar{\omega}_t \bar{t})} \qquad \bar{w}_x = \bar{w}_x^* e^{i(K\bar{x} + \bar{\omega}_t \bar{t})}$$

Contact conditions for tangential and normal pressures in dimensionless form:

$$\bar{p}_t = -\bar{\sigma}_{rr}\Big|_{\bar{r}=1-\bar{h}/2} = -\left[-\bar{p}^* - \frac{2\mu}{3}\left(\bar{u}_x^*(iK) + \frac{\bar{u}_r^*}{\bar{r}} \right) + \frac{4\bar{\mu}}{3}\frac{\partial \bar{u}_r^*}{\partial \bar{r}} \right] \Bigg| e^{i(K\bar{\chi}+\omega t)},$$

$$\bar{q} = -\bar{\sigma}_{rx}\Big|_{\bar{r}=1-\bar{h}/2} = -\bar{\mu}\left(\frac{\partial \bar{u}_x^*}{\partial \bar{r}} + (iK)\bar{u}_r^* \right) \Bigg| e^{i(K\bar{\chi}+\omega t)},$$

$$\bar{p}_t = \bar{p}_t^* e^{i(K\bar{\chi}+\omega t)},$$

Or $$\bar{q} = \bar{q}^* e^{i(K\bar{\chi}+\omega t)}$$

Substituting the formulas for displacement and pressure into equation (4.12) and solving for w_r^* and w_x^* we obtain

$$\bar{w}_r^* = \bar{p}_{t-}^* \frac{1-\bar{h}/2}{\Phi} + \bar{q}^* \frac{\theta}{\Phi},$$

$$\bar{w}_x^* = \bar{p}_t^* \frac{\left(-iK\left(1-\bar{h}/2\right)\nu\right)}{\Phi\left(-K^2+\bar{\omega}_t^2\right)} + \bar{q}^* \frac{\left(-i\nu\theta K + \Phi\left(1-\bar{h}/2\right)\right)}{\Phi\left(-K^2+\bar{\omega}_t^2\right)}, \quad (4.13)$$

The conditions of kinematic and dynamic contact (4.8) form a system consisting of two complex linear homogeneous algebraic equations.

$$c_{11}\bar{A} + c_{12}\bar{p}_0 = 0,$$
$$c_{21}\bar{A} + c_{22}\bar{p}_0 = 0. \quad (4.14)$$

The unknows $\bar{A}$ and $\bar{p}_0$ which are complex quantities, must be equal to a nontrivial solution to equation (4.14). It is known that the the determinant of the coefficient matrix is equal to zero, gives a nontrivial solution to the equations of the system/

The result is a dispersion relation. Solving the dispersion equation leads to the determination of a constant unknown complex wave number K. The coefficients of the equation K consist of a constant wave number and dimensional physical, mechanical, geometric, kinematic and dynamic parameters of the fluid and shell.

In the case of a process that is stationary in time t and decaying along the coordinate χ, the real frequency is known ω, and the required one is the complex wave number K. In contrast to natural oscillations, we will agree to call these oscillations steady-state oscillations.

In the cases of $K_i < 0$, $K_R > 0$ and $K_i > 0$, $K_R < 0$ the phases of oscillations and the excitation amplitude in the direction of propagation of the speed of phase change correspond to the mode of damped oscillations. $L \to 0$, $K_R \to \infty$ short waves and $L \to 0$, $K_R \to \infty$ - long waves characterize the limiting state of the process.
The solution to the dispersion equation, which is a complex algebraic equation, was obtained by a numerical method using a special MATLAB program on a personal computer (PC).

CONCLUSIONS

1.1. For the three-dimensional theory of deformable solids, a mixed variational type principle has been developed to determine the stress-strain state of inhomogeneous anisotropic elastoplastic bodies during creep under the action of neutron fluxes at finite deformations, taking into account damageability and diffusion. The work presents a modification of the established principle for the case of a composite material and for structures with nanotubes, when in a heterogeneous medium various phase inclusions are clearly expressed.
2.1-A discrete-analytical method is proposed for solving dynamic problems of a hollow sphere with inhomogeneous initial stresses, when the initial stresses are symmetrical relative to the center of the sphere and depend only on the spherical radial coordinate. The essence of the developed method is to divide the spherical layer into a certain number of corresponding spherical sublayers, in each of which the initial stresses are uniform, and try to find an analytical solution for the field equations inside each sublayer separately.
3.1- numerical results on the influence of the existence of fluid and the influence of inhomogeneous initial stresses on the natural vibrations of the hydroelastic system under consideration are presented and discussed.
4.1. Within the framework of the model of a piecewise homogeneous body using exact three-dimensional equations and relations of elastodynamics, the influence of imperfect contact between the layers of a three-layer hollow sphere on the natural frequencies of this sphere

was studied. The case is considered when non-ideal contact relationships relate only to displacements. Numerical results related to the spheroidal and torsional modes of vibration are separately considered, and it is established that the non-ideal contact relationships between the layers of the sphere lead to a significant decrease in the values of natural frequencies.
4.2. It has been shown that the dynamic properties of a hydroelastic system in technological and natural processes and living organisms depend on the interaction of solid deformable shells and liquid with bubbles. For shells containing liquid with spherical bubbles, the effects of viscosity on the dynamic characteristics of wave propagation are assessed. The shape and frequency of oscillations generated in a dynamic system, shell–liquid, are determined.
Fundamental iteration method was used to calculate eigenvalues and eigenfunctions to represent field quantities using MATLAB software.

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