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INVESTIGATION OF THE EFFECTS ON THE BOUNDARY LAYERS IN HETEROGENEOUS FLUID HYDRODYNAMICS

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Applicant: **Parviz Tofig oglu Museibli**

ABSTRACT

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The work was performed at the department of "Fluid and gas mechanics" of the Institute of Mathematics and Mechanics of the Azerbaijan National Academy of Sciences.

Scientific supervisor:

corr.-member of NASA, dr. of tech. sci.
Geilani Minhaj oglu Panahov

Official opponents: corr.-member of NASA, dr.of tech. sci.,prof
Garib Isaq oglu Jalalov

dr. of ph. and math. sci. prof.
Asaf Dagbeyi oglu Zamanov

cand. of ph. and math. sci. assoc. prof.
Musraddin Musa oglu Tagiyev

One time dissertation council BFD 3.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics of Azerbaijan National Academy of Sciences.

Chairman of the Dissertation council:

corr.-member of NASA, dr.of ph. and math. sci., prof.
Misir Jumail oglu Mardanov

Scientific secretary of the Dissertation council:

cand. of ph. and math. sci. assoc. prof.
Mahsati Akif kizi Rustamova

Chairman of the scientific seminar: dr. of tech. sci, prof.

Khanlar Mehvali oglu Hamzayev

GENERAL CHARACTERISTICS OF THE WORK

Rationale of the theme and development degree. At a time when the specific weight of oil and gas in the Azerbaijani oil sector is growing, ensuring their efficient operation and reliable transportation can be considered one of the important issues.

On the other hand, since all the hydrocarbons recovered in the operation of the oilfields are characterized by heterogeneity, it is necessary to clarify and regulate the complex technological problems that arise here in order to maintain efficiency at the project level.

The complexity of the manifestations arising in the joint flows of fluid mixtures differs significantly in comparison with single-phase systems. The question at issue is that they are not studied as much as necessary. To ensure the normal operation of industries, all related problems must be solved in a complex way.

Without knowing the relevant laws, it is impossible to fundamentally project the technological processes going on in transport systems and flows.

It is of particular importance to study the laws of changing the basic hydrodynamic parameters characterizing the flows of fluid-gas mixtures through pipes.

Recent studies in gas-liquid flow and filtration have shown that there is a significant difference between a two-phase flow and a single-phase flow, even at low flow rate. In the joint flows of these, systematic studies on the boundary layers were not carried out.

Ensuring effectiveness in the flows of mixtures is one of the main factors in hydrodynamic indicators. The adjustment and control of these and other parameters ensures the resolution of issues related to the state of flow modes.

Gas-liquid mixtures used in industrial areas are often in the form of dispersed systems. In such systems, depending on the time in different environments, changes occur in the boundary layers at phase boundaries, which, in turn, by creating interactions, results in the emergence of separate phases, velocity gradients in flows, pressure and temperature changes, heat flow exchanges in gas-liquid systems and various distributions in the environment.

On the other hand, in many cases, streams with gas mixtures are characterized by liquid layers. The gas flows in such layers are able to mix with different liquid layers. At a sufficiently high flow velocity, waves form on the surface of the layer.

One of the main features of gas-liquid mixtures in the form of disperse systems is the formation of a phase layer in the medium that changes over time. All these formations can affect the processes of heat and mass transfer in gas-liquid systems and regulate their distribution, intensification and cessation in the medium.

In this regard, many researchers, including the works of A.KH. Mirzajanzade, R.I. Niqmatulin, RN Bakhtizin, V.Ye. Gubin, İ.M. Ametov, R.M Sattarov, AG Gumerov, G.M. Panahov, G.I. Jalalov, A.Kh.Shahverdiev, M.M.Hasanov and others can be mentioned.

In the bubble flow mode, the dimensions of the gas phase are smaller than the characteristic dimensions of the system and are distributed individually in the liquid. This mode occurs when there are a small number of gas mixtures in the flow. These, in turn, combine in the medium and accumulate in the form of gas balls, which are characterized by periodic oscillations along the axis of the pipe.

In other cases, large gas bubbles separated from the liquid are located at a certain distance from each other and consist of mixtures filled with small bubbles in the form of dispersions. In this case, a change in the velocity of the gas phase and, accordingly, chaotic movement leads to their collision. In the conditions of the combination of bubbles, there is an increase in their size, which can be compared with the radius of the cross-section in the tubes.

To these directions, A.KH. Mirzajanzade, R.I. Nigmatulin, R.Y. Amanzade, F.B. Nagiyev, I.O. Protodyakonov, Van Wijk, S.I. Stralan and others researchers have given more space to their work.

Cases of formation and growth of gas bubbles occur under different conditions with some regularities. The study of the law reflected in the dissertation work, some features of gas separation are modeled and the physical dimensions are explained.

These or other issues solved in the dissertation work are actualized due to the investigation of the difficulties that may arise in the flow and filtration and the search for ways to regulate them, and the further investigation of the solutions and making the right decisions.

Object and subject of research. The study of the effects on the boundary layers in heterogeneous fluid hydrodynamics and their effects on hydrodynamic parameters.

Goal and tasks of the research. The aim of the work is to study the role of internal influences of boundary layers on the hydrodynamic parameters in heterogeneous fluid flows.

Investigation methods. The solution of the set problems was carried out theoretically and experimentally by building models, processing of actual operational indicators on flow and filtration, using mathematical statistics and software tools.

The basic aspects to be defended. Study of the role of electrokinetic processes in boundary layers in heterogeneous fluid flows in the hydrodynamics of these systems.

The study of effect of electrokinetic processes occurring in boundary layers on the gas formation dynamics, on the propagation of hydroshock, nonlinear waves and mixtures flows .

Nonequilibrium conditions in filtration and flows, methods of their regulation, pulsating flows in mixtures and study of electrokinetic problems.

Scientific novelty of the research. In the dissertation work the following main results were obtained:

1. The degree of dependence of density amplitude, wave propagation velocity and mixture velocity on volume concentration of bubbles in pulsating flows of two-phase viscous bubbling liquids with elastic semi-infinite tubes was evaluated.

2. For the first time it was shown that the influence of electrokinetic parameters on gas formation dynamics in the flows of gas-liquid mixtures, there was an inverse correlation between the

increase in the electrical conductivity of the system and the expansion of the bubble radius.

3. The influence of electrokinetic process on nonlinear wave propagation in gaseous liquids was theoretically justified, wave amplitude reduction was found by increasing the difference in potentials on the basis of established simulation program.

4. Theoretical evaluation of the propagation of hydroshock waves in the dynamics of formation of gas bubbles in liquids, the importance of regular expansion of bubbles in the medium to reduce wave amplitude and the impact of shock in flow has been taken.

5. For the first time, the change of the radius of the bubble in the flows was studied jointly with depending on the difference of pressures and the difference of potentials, the differences of pressures could be replaced with the difference of potentials and the evaluations were made.

6. The mathematical model of the effect of the electric field generated in the flow on hydraulic characteristics of the two-phase mixtures were given, taking into account the electric conductivity of the liquid, the slipping speed of the mixture on the pipe wall was greater than other parameters.

7. Electrokinetic effects on fluid flow in porous medium were studied, electro viscosity and slipping effects were detected and this process was modeled.

Theoretical and practical value of the study. Since the considered issues, for proper control of flow and filtration, are optimizing the main technological parameters of non-stationary processes occurring on the boundary layers, gas generation dynamics, hydrocarbon and non-linear wave propagation , such cases increase the possibility of regulation in hydrodynamics of systems in a timely manner.

With such proposals and methods, work is practical importance, as extensive research is being conducted to look for important electrokinetic effects that may arise in flows and have not yet been considered, and that these have lead to the right solutions in oil and gas production and transportation processes.

Approbation and application. The main scientific results of the dissertation were regularly discussed and approved at the scientific seminar of the department "Fluid and gas mechanics" of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan. The work was performed at the conference dedicated to the 88th anniversary of academician A.KH.Mirzajanzade (Ufa, 2016), at the international conference “Theoretical and applied problems of mathematics (Sumgait, 2017), at the International Conference "Modern Problems of Mathematics and Mechanics" dedicated to the 80th anniversary of academician Akif Hajiyev (Baku 2018), at the IX International Conference dedicated to the 100th anniversary of Tbilisi State University, Georgian Society of Mathematicians (Georgia, 2018) at the International Scientific Conference dedicated to the 90th anniversary of academician A.KH.Mirzajanzade (Baku, 2018), at the International Conference "Modern Problems of Mathematics and Mechanics" dedicated to the 60th anniversary of the Institute of Mathematics and Mechanics (Baku, 2019).

Personal contribution of the author The author of the dissertation is responsible, except setting some issues in the work, for the expression of solutions in the main solutions as a whole, participation in experiments, program design and calculations.

Publications of the author. 7 articles and 5 theses were published in the publications recommended by HAC under President of the Republic of Azerbaijan

Institution where the dissertation work was executed. The work was performed at the department of "Fluid and gas mechanics" of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan.

Structure and volume of the dissertation (in signs, indicating the volume of each structural subsection separately) General volume of work is – 213947 signs (title page – 386 signs, table of contents – 2391 signs, introduction – 15070 signs, chapter I – 64000 signs, chapter II – 50000 signs, chapter III– 80000 signs, cconclusions – 2100 signs). The dissertation work consists of list of references 172 names and 30 figures.

THE MAIN CONTENT OF THE DISSERTATION

The dissertation consists of introduction, three chapters, list of references.

In the introductory part of the dissertation, information on the rationale of the theme, the goal and tasks of the research, scientific novelty, theoretical and practical value of the study was given and substantiated.

Studies have confirmed that the formation of a separate phase, such as gas bubbles that affect the hydraulic parameters, is not only due to external influences and their nature, but also as a result of electrical potential caused by internal structural changes and friction in the flow process.

The issues set forth here have broadened theoretical and practical aspects in the mentioned direction and reflected mathematical modeling in the dynamics of gas formations.

The process is written in the form of a system of equations that combines all the parameters of influence in itself, taking into account the difference of potentials caused by friction in the liquid mixture. The solution of the problem is done by numerical method and supplied with the program.

Regulation parameters were found by solving the problems of formation of important boundary layer effects and wave propagation in the pulsating flows of bubbly fluid flowing in elastic semi-cylindrical pipe.

The change of the radius of the bubble depending on the difference in pressure and potentials was studied together and it was found possible to replace one another and the evaluations were made, mathematical expression was obtained based on the presence of the sliding effect formed at the boundary of gasless viscous liquid and gasless viscous fluid mixture.

As a result of the studies, it was also found that the difference of potentials in capillaries was altered in the velocity profile, visual drawings were obtained with the simulation program.

The conducted research will find its place in the study of the regularities of hydrodynamics and wave dynamics and in the solution of problems arising in a number of application areas, including hemodynamics, chemical engineering, oil and gas industry and aviation.

Chapter 1 is devoted to the study of non–stationary States in heterogeneous fluid flows.

In 1.1, hydrodynamic instabilities in fluid systems were investigated, appropriate non – stationary processes in flow and filtration and solutions of their regulation problems were given.

Numerous studies conducted in recent years have shown that the flows of multicomponent systems are accompanied by a number of instabilities.

In the traditional approach to such systems, all effects, as a rule, in the calculation of hydrodynamic properties, are not taken into account. As is known, ignoring only one factor leads to significant discrepancies between theoretical and experimental results. Therefore, as the interest in studying the diversity of multicomponent systems has increased, many researchers have begun to focus their work on these areas.

It should be noted that during the study of the effects of additional multiple factors, it is not possible to accurately determine the characteristics of the flow processes from the initial data confusion and the lack of measuring equipment. Therefore, this leads to difficulties in conducting research on models and an increase in errors. In this regard, in addition to deterministic methods, the importance of empirical study of multicomponent system models has been reflected.

The justified choice of model parameters requires the study of the main regularities of the difficulties arising from the action of this environment and their effect on the hydrodynamic properties of various component systems. Since the study of such problems is important and actual, a wide space has been allocated to this direction in the work of dissertation.

The construction of flow models of multicomponent systems is primarily related to the determination of the internal structure of the object under study and the evaluation of the selected model parameters. Non-stationary situations arising in the joint flow of gas and liquids and the resulting transformations actualize the search for ways of impact of external areas.

In this direction, many researchers, including the works of A.KH. Mirzajanzade, R.I. Niqmatulin, RN Bakhtizin, I.O. Protodyakonov, I.E. Lyublinskaya, İ.M. Ametov, R.M Sattarov, R. Clift, J.R. Grace, M.E. Weber, G.M. Panahov, M.M Hasanov, and others can be mentioned.

In this study, both theoretical and experimental studies of the splitting and coalescence of gas particles during movement in different liquids were given, and the roles of physical fields in the influence to the processes were shown to be fundamental.

The study of the effect of alternating electric current and magnetic fields on mass exchange in liquid and gas bubbles can help to solve some problems in the oil, transport and chemical industries, which these systems always meet. Factors affecting surface stress changes in gas-liquid systems may include surface tension in interphase, temperature difference and electric field, in which an additional flow may occur along the boundary in the phases during mass exchange process (Marangoni effect)[1]. The movement of the substance along the interphase surface varies depending on the nature of their flow relative to each other. In the latter case, the coefficient of surface tension forms small pulses.

Gas bubbles formed around saturation pressure in isothermic conditions lead to a decrease in hydraulics resistance in flow and filtration, while separations of more "large" sizes or in the form of free bubbles lead to an increase in a certain amount of resistance.

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[1] И.О. Протодьяконов, И.Е. Люблинская // Гидродинамика и массообмен в системах газ-жидкость, Л: Наука, 1990, 349 с.

This process manifests itself more often in pipes of variable diameter and in non-isothermal conditions. The regulation of such cases is very expedient both technologically and economically.

Since it is important to examine these and find the necessary aspects, the study carried out a survey of appropriate non-stationary processes in this direction **in 1.2**.

As mentioned, in the flows of multicomponent systems are waveforms in the separation layer between the components, mass exchange processes, pulsations in the flow pressure, etc. they are observed with a number of such formations.

A number of unsolved problems in oil mining mechanics are related to the flow of heterogeneous fluids, and the changes in the composition are distinguished by the fact that such changes can be important factors for the study of the composition itself and in contact with artificial additives, regulation of technological processes.

An exceptional role in the technological process is played by the influence of complex dispersive liquids on the state of structural instability. In gaseous liquids, cement solutions, blood and etc. such cases also occur.

The model put in this direction has shown that in the initial pressure gradient $\Delta\theta > \Delta P$, is not flows in the pipe $v = 0$, that is, the equilibrium point is created. In the case of $\Delta\theta < \Delta P$, the stable is broken $v \neq 0$ and a new point of equilibrium $v = v_*$ is formed.

On the other hand, as is clear, in heterogeneous gaseous liquids, peculiar irregularities arise from the influence of the incoming gas.

In this direction, conducted with the introduction of gas into the dispersive systems in laboratory experiments, it was noted that the pressure recovery curves were observed with dynamic oscillation in case of sudden retention of the steady flow. Here, first the hydraulics shocks are formed, and then the pressure wave is restored in the opposite direction of the flow, and in the next stage, vice versa.[2] [3]

The main cause of such pressure changes was assessed as the result of the effects on the contact of dispersion systems with gas in the composition of the liquid.

Therefore, even in the gas-free solution, this is not the case. These experiments were conducted and substantiated by the authors, creating several steady rate. It seems that gas-mixed fluid systems are characterized by non-stationary processes.

The change of laminar regime, manifested mainly in heterogeneous systems, occurs from various distributions as a result of a violation of mass and boundary forces in the operating zone. In this case, a complex mixture-convex flow arises.

In such flows and filtration (originating from thermobaric conditions), migration occurs, which not only plays a role as a source for non-stationary processes, but also ensures their regulation.

Many processes occurring here, including diffusion and adsorption of fluid components, are the result of a violation of additivity in the porous media system.

It is characterized by interdependence of processes, phase permeability, interaction in sorption and surface diffusion, which can lead to changes in kinetic and boundary layers in adsorption in the porous medium.

The process of sorption is considered as the migration of components from the amorphous phase to the medium.

When adsorption occurs in the filtration and flow of heterogeneous systems, temperature and pressure changes always have a direct effect on this process . Therefore, it is important to look at them in a mutual way.

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[2] Мирзаджанзаде А. Х. Этюды о моделировании сложных систем нефтедобычи: Нелинейность, неравномерность, неоднородность / А. Х. Мирзаджанзаде, М. М.Хасанов, Р. Н. Бахтизин - Уфа: Гилем, 1999.

[3] Шахвердиев А.Х., Панахов Г.М., Аббасов Э.М. Регулирование нестационарных процессов в сложных дисперсных системах // Нефтяное Хозяйство, 2004, №11, с. 59 - 61

The process of sorption, at different values of *T* and *P*, can be either good or bad, depending on the proportions of enthalpy and volume adsorption. However , the separation coefficient $\alpha_{i,j}^{S}$ in *T* and *P* changes, is more stable than the equilibrium stability of sorption.

Such flows of mixtures lead to the formation of velocity gradients, pressure and temperature changes, various distributions in the medium in gas - liquid systems. In many cases, in flows with gas mixtures, liquid layers are formed, and gas mixes in these layers, creating waves.

In 1.3, the issues of pulsating flows of two-phase viscous bubbly fluids with elastic semicircular tubes in this direction were considered.

Here, theoretical and experimental studies have been carried out and the issues of flow of two-phase flows have been resolved, taking the two-phase medium consisting of a mixture of gaseous liquids as an important example of the free media.

It is necessary to keep in mind that such environments differ from other two-phase environments by the fact that the heat capacity of the carrier phase is much greater than the heat capacity of the dispersed phase due to predominant mass content of carrying phase in unit volume. In this regard, the liquid can be considered as a thermostat having a constant temperature . Following, we put the following assumptions, which are the basis of the theory used here to describe the flow of bubble mixtures by the methods of continuum mechanics, which simplify the formulation and solution of the problem without distorting the essence of the phenomenon:

-in each elementary macrovolume of bubbles are present in the form of spherical inclusions of the same radius r_0, and the volume concentration of bubbles is low (the mixture is monodispersed), and the value of r_0 is much smaller than the characteristic dimensions of the problem;

-direct interactions and collisions of bubbles with each other can be neglected;

-the merge processes (coagulation), crushing and formation of new vesicles are absent;

-speed of the bubbles and the carrier phase are the same;

-bubbles have neutral buoyancy, i.e. do not settle and do not float;

-the viscosity of the carrier phase is much greater than the viscosity of gas bubbles (for example, the viscosity of water is 10 times greater than the viscosity of air) and therefore the viscosity of the mixture practically does not depend on volume content of bubbles.

As part of the assumptions we write the momentum equation:

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \qquad (1)$$

and the rheological equation of state of the mixture:

$$p = c^2 \rho + \frac{\xi}{\rho_0} \frac{\partial \rho}{\partial t} \qquad (2)$$

For a one-dimensional approximation the continuity equation for a tube with variable cross section can be obtained based on the following physical considerations. Select in the space filled with the mixture elementary volume *S(x)dx*, where $S(x) = \pi R^2(x) -$ cross-sectional area of the tube. Calculate the difference between the rate of the liquid flowing through a time dt through the opposite plane at a distance dx:

$$\left\{ \left[Su + \frac{\partial}{\partial x}(Su)dx \right] - Su \right\} dt = \frac{\partial}{\partial x}(Su)dxdt$$

On the other hand, the extra flow rate is due to deformation of the pipe walls and shown as:

$$L\frac{\partial w}{\partial t}dxdt$$

where $L(x) = 2\pi R(x) -$ the length of its circumference. For a compressible medium it is necessary to take into account the charge associated with the reduction of its density:

$$-S\frac{1}{\rho_0}\frac{\partial \rho}{\partial t}dxdt$$

Thus, the continuity equation is finally written in the form:

$$S\frac{1}{\rho_0}\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(Su) + L\frac{\partial w}{\partial t} = 0 \qquad (3)$$

In equations (1) - (3) $u(x,t) -$ the flow rate of the mixture, $p(x,t) -$ hydrodynamic pressure, $\rho(x,t) -$ density of mixture; $w(x,t) -$ the radial throw of the wall, $c^2 -$ is the square of the equilibrium sound velocity.

$$c^2 = \frac{1}{\alpha_{20}(1-\alpha_{20})}\left(\frac{\rho_{10}}{\rho_{10} - \rho_{10}}\right)\frac{\rho_0}{\rho_{10}} \qquad (4)$$

$$\rho_0 = \alpha_{10}\rho_{10} + \alpha_{20}\rho_{20}, \ (\alpha_{10} + \alpha_{20} = 1) \qquad (5)$$

$$\xi = \frac{4}{3}\frac{\mu(1-\alpha_{20})}{\alpha_{20}} \qquad (6)$$

$\xi -$ is the bulk viscosity, where $\mu -$ dynamic viscosity of the carrier phase. Here $\alpha_{20} -$ volume fraction of bubbles, $\rho_{10}, \rho_{20} -$ the densities of the carrier and the dispersed phase, $p_0 -$ configurable static pressure. An *0* index means the value in the equilibrium state. It should be noted that in the linear setting the equilibrium α_{20} are using instead of the current bulk concentration of α_2, and this approach assumes a priori of the presence of bubbles . If the volume fraction of bubbles is sufficiently small, the medium can be thought as homogeneous. The peculiarity of such liquid with $\rho_{20} << \rho_{10}$ is that

$$\rho_0 = \alpha_{10}\rho_{10} + \alpha_{20}\rho_{20} \approx \alpha_{10}\rho_{10} \approx \rho_{10} \qquad (7)$$

This allows with a sufficient degree of accuracy to rewrite formulas (4) and (6) as follows:

$$c^2 = \frac{\rho_0}{\alpha_{20}\rho_{10}}, \quad \xi = \frac{4}{3}\frac{\mu}{\alpha_{20}} \qquad (8)$$

In this case, as it follows from the first formula (8), compression of the mixture occurs due to the gas component.

Now, to closure equations (1) - (3) we write the equation of motion of the tube, believing it is linear elastic, that the ratio of wall thickness to the radius and that the tube is rigidly attached to the environment, causing it cannot move along its axis. Under these conditions, it is sufficient to use the following equation:

$$p = \frac{hE}{\left(1-\nu^2\right)R^2(x)}w + \rho_* h \frac{\partial^2 w}{\partial t^2} \qquad (9)$$

where ρ_* – the density of wall, E – Young's modulus, ν – Poisson's ratio.

The second term in (9) expresses the inertia of the tube wall. This effect is usually considered negligible in the present case it can be ignored. So we write down

$$w = \frac{\left(1-\nu^2\right)R^2(x)}{hE}p \qquad (10)$$

So, equations (1) – (3) and (10) represent a closed system of hydroelasticity, which can be used to describe the evolution of small perturbations in a tube of variable cross section containing a gas-liquid medium.

The results shows that the dependence of the density amplitude $|\rho|/\rho_0$, the velocity of wave propagation c and the velocity of the mixture $||u|/h\omega$ by volume content of bubbles [1]. The corresponding value for the hydrodynamic pressure and the displacement is not specified, since they do not depend on the size α_{20} .

In Chapter 2, the effects of electrokinetic formations on non – stationary processes in flows have been investigated . Here, the internal electrokinetic processes occurring in flows and filtrations, the effects of electrokinetic parameters on gas formation dynamics and nonlinear wave propagation in the flows of gas – liquid mixtures, as well as the role of gas bubbles formation dynamics in the propagation of hydroshock waves in liquids have been resolved.

In 2.1, experimental studies have been conducted in this direction since assessing the electrical properties of mixtures and adjusting hydrodynamic parameters it is important to use these properties.

As is known, there are processes in streams that cannot be explained by the accepted theory of underground hydraulics and hydrodynamics. These include discrepancies in the Darcy law, anomalies in technological processes in various pipelines and oil fields.

In the research carried out by academician A.Kh.Mirzajanzade and other scientists, it was important to explain these phenomena in fluid flow and filtration by extra – electrophysical processes not related to hydraulics and gravitational forces. Since the effects of electrophysical phenomena on flows are multifaceted, it is appropriate to study some aspects of them. Here, first of all, the electrical properties of the solutions with the addition of electrolytes in static cases were studied and evaluated by means of a conductometer. It was determined that the electrical conductivity of various amount NaCl and isopropyl alcohol solutions added to the water, which are known properties of the composition, is based on the nature and concentration of the electrolyte.

In the second half of the second chapter, the effects of electrokinetic parameters on gas generation dynamics in the flows of gas – liquid mixtures were investigated.

In contrast to the other studies, it was investigated the influence of potential difference on the formation of gas bubbles and fluid dynamics in these studies. Theoretically, this study had been investigated in the following sequence. The electric field is determined from the following expression:

$$\vec{E} = -\vec{\nabla}\varphi$$

$$\int_l E dl = \varphi(l_1) - \varphi(l_2) \tag{11}$$

E – electric field;

φ – difference of potential;

Taking into account the motion equation and the equilibrium condition, the expression for the evolution of the bubble of gas becomes:

$$R\frac{d^2R}{\partial t^2} + \frac{3}{2}\left(\frac{dR}{\partial t}\right)^2 + \frac{\sigma E^2}{\rho} R \frac{dR}{dt} + 2\frac{\Sigma}{R} = \frac{P_2(t) - P_\infty}{\rho} \tag{12}$$

$$R_{t=0} = 1, \frac{dR}{\partial t} = 0$$

Σ – coefficient of surface tension, P_∞ – pressure of fluid in infinity, $R(t)$)– bubbles radius. $R(t)$ are unknown coefficients.

Solving the obtained equations system by the numerical method of Runge- Kutta, we estimated the change in the radius of gas bubbles [5].

The results of changing the radius depending on the value of potential in the liquid with gas mixture are shown in Figure 1:

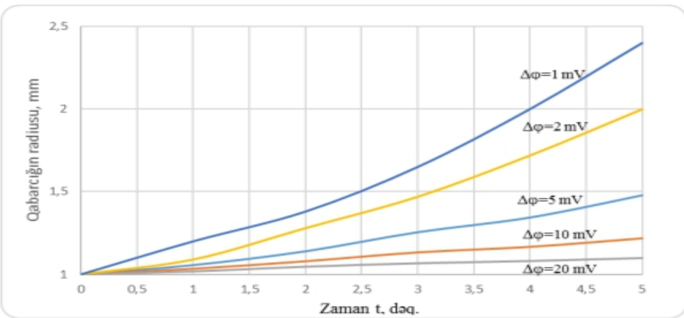

Figure1. Gas bubble size vs. time depending on the differences of potential

It was found that the value of potentials generated by fluid friction is increased when the radius of existing gas bubbles decreases. In this turn, the regulation of fluid rate and maintaining gas bubbles' initial form is allowed at the fixed $\Delta\varphi -$ value [5].

In 2.3., the effects of electrokinetic processes on the propagation of nonlinear waves in gaseous liquids were investigated. Here, the propagation of longitudinal nonlinear waves is studied taking into account the electrokinetic process that occurs during the flow in the liquid with gas bubbles.

The problem of transport of two-phase flows is traditionally solved, bringing to system of differential equations connecting both liquid and gas phases. However, assuming that in this case the gas content is insignificant, a single-speed model can be used [4].

Suppose that the characteristic linear dimension of the perturbation is much greater than the distance between the bubbles and, in addition, the distance between the bubbles is different from the radius of the bubble. In this case, the motion of such a system can be considered in a homogeneous approach: the entire mixture is a continuous medium which density is approximately equal to the density of the liquid, and the compressibility is determined only by the compressibility of the gas.[5]

We introduce the notation:

- ρ_1^0, ρ_2^0 - respectively, the true density of the liquid and gas,
- $\alpha_1, \alpha_2 -$ respectively, volume concentrations of liquid and gas,
- $P_1, P_2 -$ respectively, pressure in the liquid and inside the bubbles,
- $n -$ is the number of bubbles per unit volume of the mixture.

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[4]Нигматуллин Р. И. Основы механики гетерогенных сред. / Р. И. Нигматуллин - М.: Наука, 1978. - 336с.

[5] Накоряков В.Е., Соболев В.В., Шрейбер И.Р. Длинноволновые возмущения в газожидкостной смеси // Изв.АН СССР, МЖГ, 1972, № 5, с. 71-76.

Equation of continuity and momentum of a gas - liquid medium in the homogeneous approximation

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho U)}{\partial x}=0 \qquad (13)$$

$$\rho\frac{\partial U}{\partial t}+\rho U\frac{\partial U}{\partial x}+\frac{\partial P}{\partial x}=0 \qquad (14)$$

The *Rayleigh-Lamb* equation for the radial motion of a gas bubble in an infinite incompressible fluid excluding phase transitions and surface tension, taking into account the electric field

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2=\frac{1}{\rho_1^0}\left(P_2-P_1-\frac{4\mu}{R}\frac{dR}{dt}-\frac{2}{3}\sigma ER\frac{dR}{dt}\right) \qquad (15)$$

After the simplifications, we obtain an equation for velocity disturbations

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial z}-\eta\frac{\partial^2 U}{\partial z^2}+\beta\frac{\partial^3 U}{\partial z^3}=0 \qquad (16)$$

Where

$$\eta=\frac{\left(\frac{4\mu}{R_0}+\frac{2}{3}\sigma ER_0\right)R_0}{6\alpha_1\alpha_2\rho_1^0};\ \beta=\frac{R_0^2c}{6\alpha_1\alpha_2}$$

Equation (16) is the classical *Burgers-Korteweg-de-Vries* equation.
We have derived the nonlinear differential equation for the long weakly nonlinear waves in the bubbly liquid (Eq. (16)). This nonlinear differential equation is classical *KdV–Burgers* equation with respect to the potentials difference parameter.
We have investigated numerically the nonlinear wave process described by Eq. (16) as well.
It was defined that the more potential difference increase, the less radius of bubbles decrease [5]. Accordingly this result, we demonstrate that when potential difference increases, wave attenuates gradually.

Plots of this solution at the radius R_0=0.0001 m are presented in Fig.2

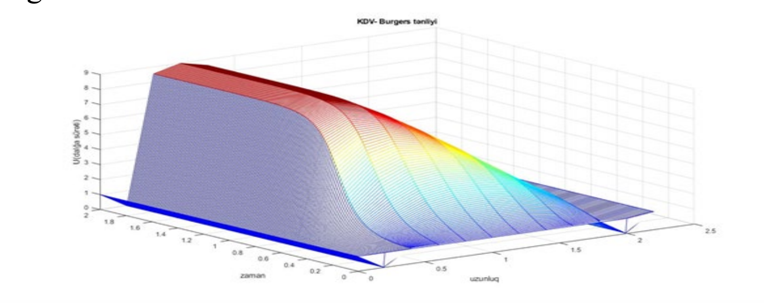

Fig. 2. The description of propagation of nonlinear waves at the R_0=0.001m , $\Delta\varphi$ =0.002V

At the next stage, using values of σ, $\Delta\varphi$ and we solve obtained equation at R_0=0.00008m in the same state that are presented in Fig. 3.

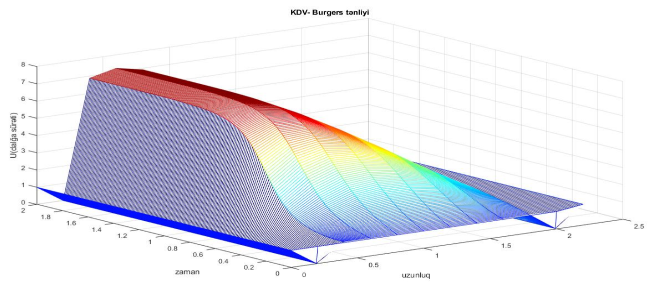

Fig.3. The description of propagation of nonlinear waves at the R_0=0.00008m, $\Delta\varphi$ =0.003V

From Fig. 3 it can be seen, that the amplitude of waves slowly decreases when the radius of bubbles while the potential differences increase.

In 2.4, the effects of dynamics of gas bubbles formation in liquids on the propagation of hydroshock waves were investigated.

The formation and expansion of gas bubbles in the system consisting of liquid-gas mixtures directly affects the propagation area (amplitude) of waves formed in different flow modes of gas separations. Given the importance of these issues, in order to study them deeper, the effect of gas bubbles expansion on the amplitude of waves has been theoretically investigated.

First of all, the processes of propagation and formation of waves formed in the flow of gas-liquid mixtures with transported pipes were considered. As is known, the mathematical expressions of the equations of motion and discontinuity of the fluid is written as follows:

$$C^2 \frac{\partial M}{\partial x} + f \frac{\partial P}{\partial t} = 0 \tag{17}$$

$$f \frac{\partial P}{\partial x} + \frac{\partial M}{\partial t} = \frac{\rho \lambda}{2D} fw|w|. \tag{18}$$

Here: $c = \sqrt{K/\rho}, M = \rho f w;$ $w-$the average velocity of the liquid, $\rho-$the density of the liquid; $K-$ the module of elasticity; $D-$the diameter of the tube; $\lambda-$ the coefficient of resistance to hydraulics.

The amplitude of the waves caused by gas separations is expressed as follows in the works of the authors:[6] [7]

$$A = \sqrt{\frac{f(R_0)}{f(R_q)}} \frac{1}{1 + \int_{R_0}^{R_q} \xi(R_q) dR_q} \tag{19}$$

Here, $\xi(R_q) = \frac{\lambda \sqrt{\pi} w_0}{8\sqrt{f(R_q)c'}}$, $c'-$ was a wave speed in gas separations, it is determined as follows:

$$c' = \frac{cp}{\sqrt{\rho^2 c^2 RTM + p^2}} \tag{20}$$

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[6] Фокс Д.А. Гидравлический анализ неустановившегося течения в трубопроводах / Д.А.Фокс - М., 1981.- 400

[7] Г. Н. Ледовский, С. В. Самоленков, О. В. Кабанов, Эффективность систем защиты оборудования нефтеперекачивающих станций при повышенных волнах давления // Научный журнал «Записки Горного института» , 2013, Т-206, с. 99-102

If we consider all these expressions in the equation (19), then we can write the equation that characterizes the amplitude of the wave as follows:

$$A = \frac{f(R_0)}{f(R_q)} \frac{1}{1 + C\int_{R_0}^{R_q} \frac{1}{\sqrt{f(R_q)}} dR_q} \tag{21}$$

Here $-C = \frac{\lambda\sqrt{\pi}w_0}{8c}\sqrt{1+\frac{KM}{p}}$ –. By solving the equation (21) at the thermobaric conditions at different prices of the radius, the area of propagation of the wave is determined.

Change of wave amplitude depending on the radius of the bubble was mentioned in Figure 4.

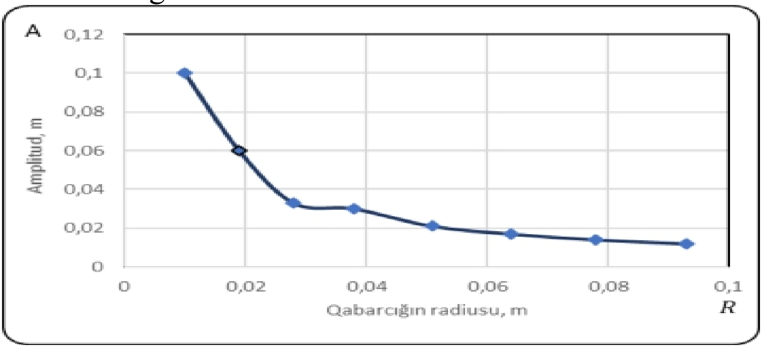

Fig. 4. Change of wave amplitude depending on the radius of the bubble

As can be seen from the figure, with an increase in the radius, the wave amplitude decreases, that is, a decrease in the “hardness” of the liquid leads to a decrease in the hydroshock.

It can be concluded from the research that the implementation of measures for the regular expansion of gas bubbles in the fluid to reduce wave amplitude and related shock effects on the pipes is one of the important conditions[6].

In chapter 3, the effects of boundary layer formation on hydrodynamic parameters in flow and filtration of liquid mixtures was investigated.

The effectiveness in the flows of mixtures is mainly due to the regulation of flow rate by difference of pressure and process of formation internal influence on the boundary layers.
To investigate such processes more deeply, the effects of boundary layers on hydrodynamic parameters in flows of non-Newtonian fluids were investigated **in 3.1**.
Problems subject to decision, is the determination of flow rate and effective viscosity $\eta(\tau)$ of fluid with the destructed structure.
The effect of perturbations intensity on the process of bonds destruction in a dynamic equilibrium occurs nonmonotonically. It is found, that, under the sticking effect at the flow of thixotropic oil in a circular pipe, there is arbitrarily low stresses at the shear flow, that lead to the identical destruction behavior. This anomaly of a structured system allows to associate the coefficients of rheological equations with the characteristics of the destruction process [2].
In 3.2, the effects of internal formations, which formed on the boundary layers, on the hydrodynamic parameters in the flows were investigated.
Taking into account the studies of the gas bubbles dynamics through various physical parameters, regulation of process of bubble formation through the potential difference parameter was considered in this section.
The *Rayleigh-Plesset* equation (12) is used to model effect of static electric field on the change in bubble radius R with time t in fixed thermobaric conditions.
Note that the static electric field formed by friction in the liquid is defined by equation (11).
Here gas pressure is expressed as follows.

$$\frac{dp_2}{dt} = -3\gamma_a \frac{p_2}{R} w_R$$

Here, p_2 – the pressure inside the bubble, γ_a - adiabat indicator, w_R - the velocity change of the bubble radius, R - the radius of the bubble.
First, the dynamics of formation of gas bubbles at different values of the ΔP pressure difference (at the smallest values of potential

difference) were investigated. The equation (1) is solved by the *Runge-Kutta* numerical method, Δφ= 0.5mV, Σ = 0.0002 N/m (Fig. 5).

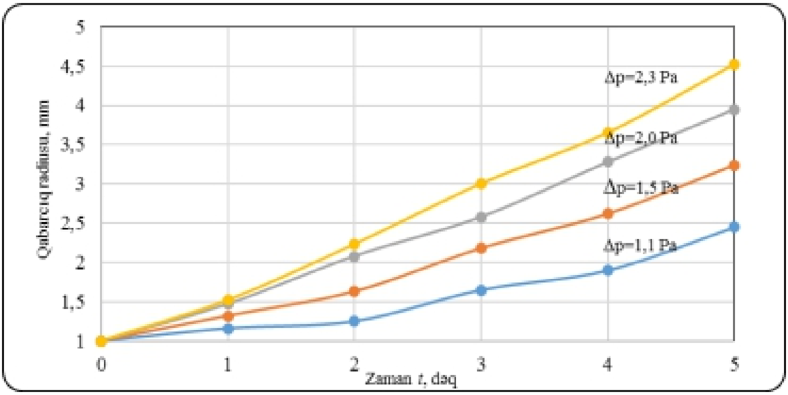

Fig. 5 Radius of bubble vs. time under different pressure drop

The expansion radius of bubbles increases as the pressure difference increases(Fig.5). Then, this equation was solved by changing the potential difference in the constant value of the pressure drop Δ*P*, and the following results were obtained.

On the contrary, we can see from this that as the value of potential difference increases, the radius of the bubble decreases gradually.

Given all this, it has been established that the regulation of the formation of gas bubbles at certain values of pressure difference can be regulated by the potential difference parameter.

In [5] and Chapter2, the electrical conductivity of the fluid under different concentrations of electrolytes is investigated. Depending on the concentration of electrolytes the value of potential difference arising in the fluid at different flow rates was determined.

That is, the value of the potential difference in aqueous solutions of 0,1% *NaCl* (in a constant value of pressure difference) has been determined.

In this study, the dynamics of bubble formation in the gas-liquid system was studied, taking into account the potential difference parameter. In experiments the aqueous solution of *NaCl* was used.

Depending on the electrolytic concentration, the electric conductivity of the liquid and, accordingly, the electrostatic field arising from friction in fluid is determined. The effect of the electrostatic field on the dynamics of the bubble formation showed that the radius of gas bubbles and its expansion dynamics formed by the pressure drop can be regulated by the potential difference parameter (Fig. 6) [10]. For this purpose, we must concentrate mainly on the nature of the liquid phase and concentration of the added electrolyte.

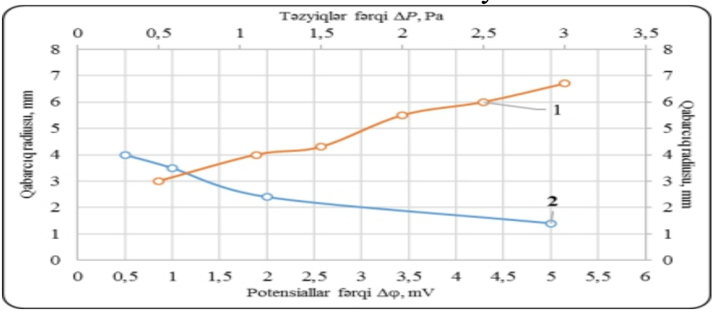

Fig. 6. Dependence of bubble radius vs. pressure drop and potential difference (1-pressure drop, 2-potential difference)

The problem of mathematical modeling of the effect of the electric field generated in the flow of two-phase mixtures on the hydraulic characteristics was considered in 3.3.

Let us investigate the motion of mixtures consisting of gaseous liquid and viscous liquid in a horizontal cylindrical tube. Here, we accept gas-liquid mixture as a leading phase which moving parallel to the walls of the pipe and gas separated (non-gas) viscous liquid as the second phase.

In the present study, the separation of gas bubbles from the liquid caused by effects of electrostatic field was investigated. It is assumed that the gas bubbling move towards the walls of the tube and form a certain gaseous fluid layer. The main purpose here is to examine the adhesive properties and slip velocity of the layer formed between the gaseous liquid and non-gas liquid.

The authors investigated the effects of electrostatic field arising from friction between the particles in the gas-liquid mixture and how it affects formation of gas bubbles [5]. Accordingly, the possibility of regulating the formation of gas bubbles through impact of the electrostatic field was determined. Based on these results, the role of the electrostatic field formed from the friction of particles between the two fluids is taken into consideration in this study.

Here, the effects of the electrostatic field arising from friction of particles in the flow on the properties of the layer between the gas fluid and the wall, the slip velocity between two fluids are investigated.

At the same time, the adhesion feature of the layer between the wall and the fluid dependence on the nature of the mixture and the character of the flow is determined. The study also provides a dependency between the slipping velocity and the distribution velocity of the career phase. Considering the sliding parameter, the distribution of the media velocity is noted.

Let us consider the problem of the steady-state motion of mixtures consisting of viscous media in a horizontal cylindrical tube. Suppose that each phase of the mixture is incompressible, the reduced densities are constant, and the flow of the mixture has axial symmetry. Then the equation of motion of a two-phase mixture with an averaged term has the form

$$f_1\sigma\left(\frac{d^2u_1}{dr^2}+\frac{1}{r}\frac{du_1}{dr}\right)+K_m\left(u_2-u_1\right)=\frac{2\tau_0 f_1}{R}+f_1\frac{\partial p}{\partial x}, \qquad (22)$$

$$f_2\mu\left(\frac{d^2u_2}{dr^2}+\frac{1}{r}\frac{du_2}{dr}\right)+K_m\left(u_1-u_2\right)=f_1\frac{\partial p}{\partial x}.$$

Here u_1 and $u_2 -$ the velocity of the phases, $\sigma,\mu -$ the parameter characterizing the electrical conductivity and dynamic viscosity of the liquid, $K_m -$ the coefficient of mutual resistance between the phases in the mixture, f_1 and $f_2 -$ the volume of the phases, $p -$ the pressure, $\tau_0 -$ sliping coefficient on the contact surface.

The boundary conditions for the carrier phase are the no slip conditions, and for the second phase we accept the conditions of constant flow,

$$r = R \quad u_1 = 0, \tag{23}$$

$$\int_0^R u_2 r dr = \frac{R^2 V_2}{2} = const. \tag{24}$$

To solve the system of equations (22) under condition (23), (24), and successively performing some calculations and transformations, we obtain the corresponding equations for u_1 and u_2 in the form

$$f_1\eta\left(\frac{d^2u_1}{dr^2} + \frac{1}{r}\frac{du_1}{dr}\right) - m^2u_1 = Ar^2 + B - Cu_2(R), \tag{25}$$

$$f_2\mu\left(\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr}\right) - m^2u_2 = Ar^2 + D - Cu_2(R).$$

For slip velocity we have

$$u_2 = -\frac{R^2}{2(f_1\sigma + f_2\mu)}\left(\frac{N}{4} + \frac{f_2\tau_0}{R}\right)$$

Here is a comparison of the change of the slip velocity in the wall of the pipe, depending on the electric conductivity and structural viscosity of the liquid. As a result of the study, it was found that, given the electrical conductivity of the liquid, the slip velocity of the mixture on the wall of the tube is greater than that of the other parameter [11].

In 3.4, the study of boundary layer electrokinetic processes on hydrodynamic parameters in liquid mixtures filtration was investigated.

The study of simplified the Navier-Stokes equation in a stationary state with a net charge density and without taking into account inertial conditions assuming that the fluid is incompressible.

The paper offers a numerical estimation of the velocity distribution profile in a circular capillary $(r = x = y)$ for different values of p_e

and $\nabla\varphi$ with and without the electrokinetic factor. The equation is solved numerically in MATLAB to obtain the velocity profile, the pressure gradient in this case is assumed to be constant $\nabla p = const.$ The graphs show that the velocity distribution plot near the capillary wall in the boundary layer has different values. At constant pressure and changing the gradient of the potential flow (charge density), the velocity on the capillary walls is not zero (Fig. 7 and 8) [12].

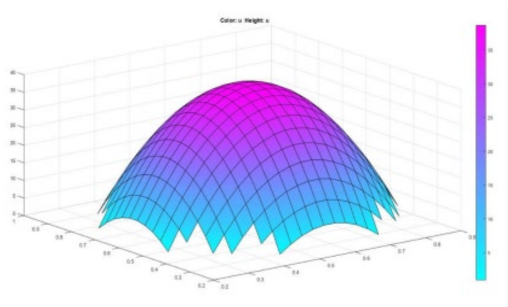

Fig.7. Velocity distribution on capillary walls($\nabla p = const.$)

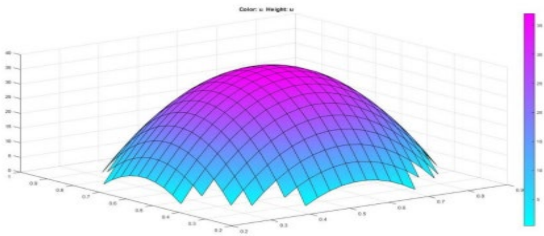

Fig. 8. Velocity distribution on capillary walls (depending on $\nabla\varphi$)

On the other hand, many other studies have shown a change in medium viscosity depending on the value of the electropotential. The formula obtained on the basis of the studies is written in the form of the dependence mentioned below.

$$\frac{\mu}{\mu_e} = \left(1 - \frac{\beta_* \varphi^2}{2\pi^2 r^2 m}\right)^{-1}$$

Based on this expression, estimates on the change in viscosity, depending on the radius changes of the capillary and potential, are shown in Figure 9 and 10. Here, a nonlinear variation of the $\frac{\mu}{\mu_e}$ – ratio depending on these parameters is observed.

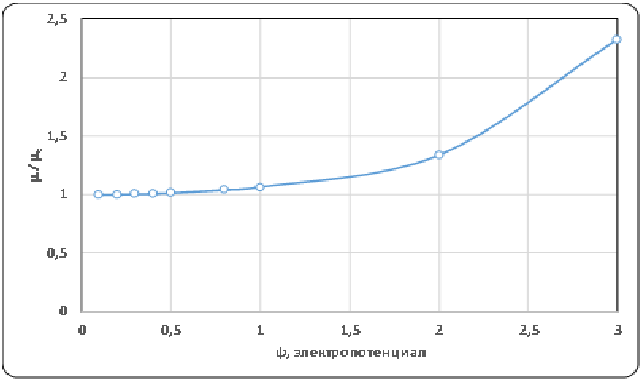

Fig. 9. Variation of the $\frac{\mu}{\mu_e}$ – ratio depending on electropotential

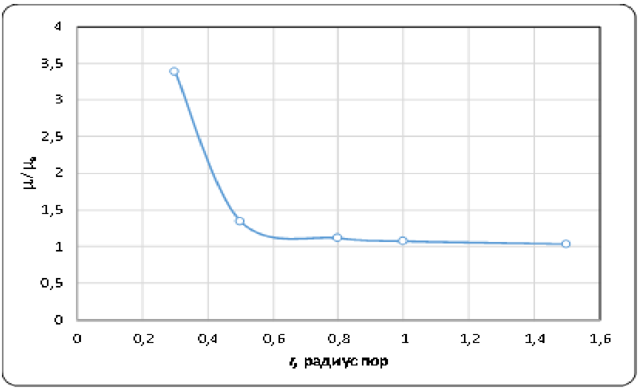

Fig. 10. Variation of the $\frac{\mu}{\mu_e}$ – ratio depending on porosity radius

Conducted studies may be important conditions for regulating the process of fluid filtration in the porous medium. To this end, electrokinetic effects on fluid flow in porous environments were investigated, electroviscosity and slipping effects were detected and process modeled.

The obtained effects are great importance and can be used in the development of oil fields.

In conclusion the author expresses his deep gratitude to his supervisor corr.-member of NASA, doc. of tech. sc. G. M. Panahov and employees of the department for the constant attention and useful advices.

Conclusions

The main results of the dissertation work are the followings:

1. Non-stationary conditions in hydrodynamic parameters in heterogeneous fluid flows have been investigated and it is possible to regulate mass exchanges, electrical conductivity, gas separations by external influences.
2. The degree of dependence of density amplitude, wave propagation velocity and mixture velocity on volume concentration of bubbles in pulsating flows of two-phase viscous bubbling liquids with elastic semi-infinite tubes was evaluated.
3. Electrokinetic properties in mixtures were evaluated, it was determined that their properties depend on concentration of electrolytes non-linear. It has been experimentally confirmed that the electric field in the flow of these systems at the difference of constant pressures is also changed non-linear.
4. For the first time it was shown that the influence of electrokinetic parameters on gas formation dynamics in the flows of gas – liquid mixtures, there was a inverse correlation between the increase in the electrical conductivity of the system and the expansion of the bubble radius.

5. The influence of electrokinetic process on nonlinear wave propagation in gaseous liquids was theoretically justified, wave amplitude reduction was found by increasing the difference in potentials on the basis of established simulation program.
6. Theoretical evaluation of the propagation of hydroshock waves in the dynamics of formation of gas bubbles in liquids, the importance of regular expansion of bubbles in the medium to reduce wave amplitude and the impact of shock in flow has been taken.
7. In the process of destruction of structural bonds of non-Newtonian thixotropic liquids (oil), the intensity of perturbation is non-monotonous.
8. For the first time, the change of the radius of the bubble in the flows was studied jointly with depending on the difference of pressures and the difference of potentials, the differences of pressures could be replaced with the difference of potentials and the evaluations were made.
9. The mathematical model of the effect of the electric field generated in the flow on hydraulic characteristics of the two-phase mixtures were given, taking into account the electric conductivity of the liquid, the slipping speed of the mixture on the pipe wall was greater than other parameters.
10. Electrokinetic effects on fluid flow in porous medium were studied, electro viscosity and slipping effects were detected and this process was modeled.

The main results of the dissertation work were published in the following works:

1. Amanzade, R.Y. Pulsating flow of two-phase viscous bubbly fluid in an elastic semi-infinite cylindrical tapering tube / R.Y.Amanzade, G.M.Panahov, E.M.Abbasov, P.T.Museibli// Transactions of NAS of Azerbaijan, Issue Mechanics, -2015, XXXV, №7, -p. 22 - 31

2. Panahov, G.M. Wall effects under non-Newtonian fluid flow in a circular pipe / G.M. Panahov, E.M. Abbasov, P.T. Museibli [et.al] // Transactions of NAS of Azerbaijan, Issue Mechanics, -2016, XXXVI, №7, - p. 68 - 73.
3. Panahov, G.M., Museibli, P.T. Influence of the electrostatic potential on the dynamic of gas evolution // "Riyaziyyatın nəzəri və tətbiqi problemləri" Beynəlxaq elmi konfransın materialları. -Sumqayıt: Sumqayıt Dövlət Universiteti, -2017, -s. 122.
4. Panahov, G.M., Museibli, P.T. The study of internal exposure on the fluid hydrodynamics // "Riyaziyyat və Mexanikanın müasir probemləri" Akif Hacıyevin 80 illiyinə həsr olunmuş beynəlxalq konfrans materiallari, -Bakı: -2017, -s.171.
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11. Museibli, P.T. Mathematical modelling of effect of electrostatic field formating on hydraulic characteristics of two-phase mixtures flow // Advances and applications in mathematical sciences, -2019, vol.19, №1, -pp. 21 – 32
12. Panahov G. M., Abbasov E. M. , Yuzbashiyeva A.O., Museibli P.T. Flow control of fluids through porous media based on electrokinetic effects // Transactions of NAS of Azerbaijan, Issue Mechanics, -2020, XL, №7, p. 28 – 36.

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Address: AZ 1141, Baku, B.Vahabzadeh, 9.

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