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ABSTRAKT

of the dissertation for the degree of Doctor of Science

DYNAMICS OF MOVING AND OSCILLATING-MOVING LOAD ACTING ON THE INNER SURFACE OF A HOLLOW CYLINDER SURROUNDED BY ELASTIC MEDIUM

Speciality: 2002.01– Mechanics of deformeble solid body

Field of science: Mechanics

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GENERAL CHARACTERISTICS OF THE WORK

Actuality of the work and degree of development. The construction of underground overpasses for high-speed transport communications, such as metro, tunnel, etc., as well as the creation of long layered gun barrels for firing high-speed bombs and missiles require studying the corresponding problems of elastodynamics. These studies are also required to ensure the safety of construction structures from the effects of vibration that occurs under trains moving through the subway and tunnels. This is a far incomplete list of problems that are related to the study of the dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium with finite and infinite thickness in the radial direction. This dissertation is devoted to these studies carried out in the framework of a piecewise homogeneous body model with the use of exact three-dimensional equations and relations of linear and linearized elastodynamics and this determines its relevance. The theoretical basis of conducted researches is the current level of elastodynamics advancement, which has been contributed by many scientists, including the research of J.G. Agalarov, S.D. Akbarov, Yu.A. Amenzadeh, A.N. Guz, A.A. Ilyushin, M.H. Ilyasov, V.D. Kubenko, M.F. Mekhtiyev, N.B. Rasulova, V.P. Tamuzh, L.X.Talybly, N.A. Shulga, H.A. Rakhmatulin, J.D. Achenbach, A.C. Eringen, E.S. Suhubi, C. Truestell, R.W. Ogden and a number of other researchers. It may be noted that among the dynamic problems of the elasticity theory, the problems related to the dynamics of moving loads acting on elastic structures take a special place and, as a result, we will consider the level of development in the relevant investigation area. It should be noted that the history of these studies begins with the study of the collapse causes of Chester bridge (England) in 1847, which caused for grave concern among civil engineers. Since then, relevant research in this direction has been continuing and developing. Classification of problems related to these studies can be carried out with respect to the geometric shape of elastic structures: (I) rods related problems; (II) plates related problems; (III) plane-layered systems related problems; and (IV) problems related to cylindrical layered systems.

Note that a review of research related to tasks I) and II) are mentioned in the paper (Quyang 2011, Moving load dynamic problems: A tutorial (with a brief overview). Mech. Syst. Signal Pr., 25. 2039-2060). A review of works related to problems III) was made in the monograph (Akbarov S. D. 2015) Dynamics of Pre-Strained Bi-Material Elastic Systems: Linearized Three-Dimensional Approach. Springer, New-York[1]. Based on this position, the thesis provides a detailed review of research related to type IV) problems and part of the problems related to type III) tasks, which is directly related to the topic of this dissertation.

Novelty and actuality of this research work, which is dedicated to the development of the dynamics theory of moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium, the development of analytical and numerical methods for solving relevant boundary value problems based on the piecewise-homogeneous bodies model with attraction of exact three-dimensional equations of elastodynamics and linearized elastodynamics, follow from these reviews.

The object and subject of the study. The object of the study is:

a) a hollow circular cylinder surrounded by an infinite elastic medium;

b) double-layer hollow circular cylinder.

The subject of the study is:

a) axisymmetric problems on the dynamics of a moving and oscillating-moving annular load acting inside a hollow cylinder surrounded by an infinite medium;

b) non-axisymmetric three-dimensional problems on the

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[1] Akbarov, S.D. Dynamics of Pre-Strained Bi-Material Elastic Systems: Linearized Three-Dimensional Approach / S.D. Akbarov. Heidelberg, New-York: Springer, - 2015. -1004p

dynamics of a moving and oscillating-moving locally distributed load acting inside a hollow cylinder surrounded by an infinite medium;

c) axisymmetric and non-axisymmetric three-dimensional problems on the dynamics of a moving load acting on the inner surface of a two-layer hollow cylinder;

(d) axisymmetric and non-axisymmetric problems of forced vibration of the "hollow cylinder + surrounding infinite medium" system and a two-layer hollow cylinder under the action of forces acting inside the cylinder that vary harmonically in time.

The purpose and objectives of the study. The aim of the work is to create a theory of the dynamics of a moving, vibrating and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an infinite and finite elastic medium based on a model of a piecewise homogeneous body with the use of exact three-dimensional equations of elastodynamics and a three-dimensional linearized theory of elastic waves in bodies with initial stresses (TLTEWIS).

The objectives of the study are as follows:

a) axisymmetric and non-axisymmetric three-dimensional problems on the forced oscillation of a system consisting of a hollow cylinder and an infinite elastic medium surrounding it, as well as on the forced oscillation of a two-layer hollow cylinder;

(b) axisymmetric and non-axisymmetric problems on the dynamics of a moving and oscillating-moving load acting inside a hollow cylinder surrounded by an infinite elastic medium, as well as corresponding problems related to a two-layer hollow cylinder.

Research methods. In the dissertation work, the following methods of mathematical physics are applied and developed:

a) transition to a time-invariant moving coordinate system;

b) Fourier transforms with respect to the axial coordinate;

c) Fourier series for representing the Fourier transforms of the desired quantities;

d) separation of variables of the desired quantities;

e) Sommerfeld contour for the development of a numerical algorithm for implementing the definition of the inverse Fourier transform;

f) creation of software module in the MATLAB for obtaining numerical results.

The main provisions submitted for defense.

In the dissertation, the theory of the dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium is developed on the basis of a model of a piecewise homogeneous body with the use of exact three-dimensional equations of elastodynamics and TLTEWIS, including:

a) the problems formulation and the development of analytical and numerical methods for solving the relating problems of the dynamic stress-strain state;

b) study of various classes of two-dimensional and three-dimensional problems on determining the influence of the velocity and vibration of a moving load on the distribution of stresses acting on the interface of the cylinder materials and the environment;

c) determination of the critical velocities of the moving and vibro-moving load and the effect of the problem parameters on the value of these critical velocities;

d) identification of «gyroscopic effects» (or «Coriolis acceleration effect») on the value of critical velocities and on the distribution of interface stresses;

e) study of forced vibration of the «hollow cylinder+ surrounded elastic medium» system;

f) study of the effect of homogeneous initial stresses on the value of critical velocities and interface stresses;

g) determination of the effect of non-axisymmetricity of moving and oscillating-moving loads on the value of critical velocities.

Scientific novelty. The scientific novelty of the results of the work is are:

in the statement of the research problem of the dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium;

in the development of methods for solving the related boundary value problems involving the Fourier transform in spatial coordinates and with the Fourier series expansion of these transformations;

in the development of algorithms and software to obtain specific numerical results on the critical speed and on the distribution of interface stresses;

in establishing a number of effects related to the vibration of the moving load and the non-axisymmetricity of the problems under consideration;

in determining the influence of the mechanical and geometrical properties of the hollow cylinder and of the surrounded medium on the values of the critical velocity and on the distribution of interface stresses.

Theoretical and practical significance of the researches. The theoretical significance of the work is that all the considered problems on the dynamics of a moving and vibro-moving load acting on the inner surface of a hollow cylinder surrounded by an infinite and finite elastic medium in the radial direction are solved for the first time in the framework of a piecewise homogeneous body model with the use of exact three-dimensional equations of elastodynamics and TLTEWIS.

The theoretical value of the obtained results and conclusions is confirmed by:

1) using exact three-dimensional equations and relations of the three-dimensional theory of elastodynamics and TLTEWIS in the framework of the piecewise homogeneous body model;

2) the correctness of the problem statements, the accuracy of the analytical solutions of the corresponding wave equations;

3) the consistency of the results obtained among themselves, physical and mechanical considerations, and known results by other authors in particular cases.

The practical significaunce of the results of this work is to create a theoretical basis for predicting and preventing catastrophic phenomena caused by moving and oscillating-moving objects in underground transport structures and in multi-layer gun barrels.

Approbation the work. Dissertation results were presented at the following conferences:

- The World Congress on Advances in Structural Engineering and Mechanics (ASEM17), 28 August - 1 September, 2017, Ilsan (Seoul), South Korea.
- VII International Joint Conference of Georgian Mathematical Union & Georgian Mechanical Union. Continuum Mechanics and Related Problems of Analysis dedicated to 125-th birthday anniversary of academician N. Muskhelishvili. Batumi (Georgia), Sept. 5-9, 2016.
- 1-st International Conference on Innovations in Natural Science and Engineering (ICINSE 2018), 3-6 January 2018, Turkish Republic of Northern Cyprus.
- The 6th International Conference on Control and Optimization with Industrial Applications Engineering (COIA-2018), 11-13 July 2018, Baku, Azerbaijan.
- 21 Ulusal Mekanik Kongresi, 02-06 Eylül 2019, Niğde Ömer Halisdemir Üniversitrsi (Turkey).

The obtained results were discussed in the General institute seminars of the Institute of Mathematics and Mechanics and also in the seminars of the departments of «Mechanics of deformeble solid body», «Wave Dynamics» and «Applied Mathematics».

The author's personal contribution consists in the formulation of specific tasks and the choice of the research direction. In addition, the development of methods for solving them, the numerical results obtained, as well as the formulated conclusions and engineering recommendations belong to the author personally.

Publications of the author. Publications in editions recommended by the Higher Attestation Commission under the President of the Republic of Azerbaijan-12 (7 of them – Web of Science), conference materials-1, abstracts – 5.

The name of the institution where the dissertation work was performed. The work was carried out in the department «Theory of Elasticity and Plasticity» of the Institute of Mathematics and Mechanics.

Structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion, a list of references and an appendix.

The total volume of the dissertation work is 435740 signs (title page-2000 signs, table of contents-2700 signs, introduction-30400 signs, first chapter-154700 signs, second chapter-163800 signs, third chapter-60000 signs, conclusion-22140 signs). The list of used literature consists of 114 titles. The dissertation contains 234 figures and 19 tables.

THE MAIN CONTENT OF THE DISSERTATION

Introduction provides an overview of research related to the problem under consideration. Here the topic and purpose of the dissertation are formulated, its relevance, novelty and reliability of the results obtained, and practical value are justified. The main content on chapters is summarized in the Introduction.

The first Chapter of the dissertation deals with the study of axisymmetric problems of dynamics of a moving and oscillating-moving ring load acting on the inner surface of a hollow cylinder surrounded by an elastic medium. The influence of homogeneous initial stresses in the cylinder and in the surrounded medium (caused by axial stretching or compression at infinity of the system under consideration) on the critical speed of the ring load is also considered[2]. The influence of non-ideal contact conditions between the cylinder and the environment on the critical velocity value and on interface stresses is also studied[3]. In addition, the problem of forced harmonic oscillation of the system under consideration is also investigated. An algorithm for numerical determination of the inverse Fourier transform, which is used throughout the dissertation

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[2] Babich, S.Y., Glukhov, Y.P., Guz, A.N. Dynamics of a layered compressible pre-stressed halfspace under the influence of moving load// - New York: International Applied Mechanics, - 1986, , 22, № 6. - p. 808-815

[3] Abdulkadirov, S.A. Low-frequency resonance waves in a cylindrical layer surrounded by an elastic medium/ Novosibirsk: Journal of Mining Science, - 1981, 80, - p. 229-234

work, is presented. Numerical results related to the critical velocity and the distribution of interface stresses are presented and analyzed.

Let's look at some fragments of the research made in Chapter 1 and for this purpose, we will highlight the problem of the dynamics of a oscillating-moving ring load acting on the inner surface of a hollow cylinder surrounded by an infinite elastic medium.

So, let's consider the system schematically shown in Fig. 1, according to which on the inner surface of a hollow cylinder of thickness h, surrounded by an infinite elastic medium, acts a oscillating-moving ring load moving along the axis of the cylinder with a constant velocity V. The cylindrical $Or\theta z$ and Cartesian $Ox_1x_2x_3$ coordinate systems are associated with this cylinder axis.

In all the statements in the dissertation and here, the upper index (2) indicates the values related to the hollow cylinder, and the upper index (1) - to the surrounded medium.

The study is carried out within the framework of a piecewise homogeneous body model with the use of exact equations of elastodynamics in the axisymmetric case, for which the following equations of motion, elasticity relations, and geometric relations take place:

$$\frac{\partial \sigma_{rr}^{(k)}}{\partial r} + \frac{\partial \sigma_{rz}^{(k)}}{\partial z} + \frac{1}{r}(\sigma_{rr}^{(k)} - \sigma_{\theta\theta}^{(k)}) = \rho^{(k)} \frac{\partial^2 u_r^{(k)}}{\partial t^2},$$
$$\frac{\partial \sigma_{rz}^{(k)}}{\partial r} + \frac{\partial \sigma_{zz}^{(k)}}{\partial z} + \frac{1}{r}\sigma_{rz}^{(k)} = \rho^{(k)} \frac{\partial^2 u_z^{(k)}}{\partial t^2}. \quad k = 1, 2. \qquad (1)$$

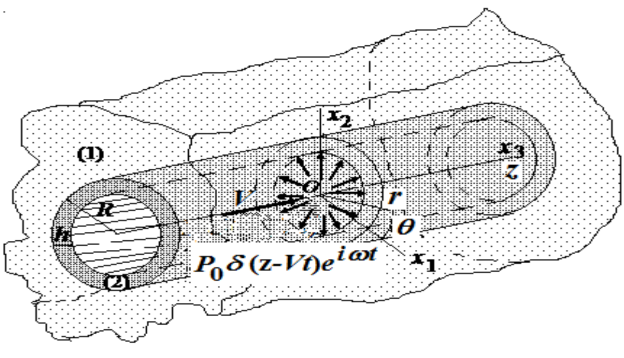


Fig. 1

$$\sigma_{rr}^{(k)} = \lambda^{(k)}\,\varepsilon^{(k)} + 2\mu^{(k)}\varepsilon_{rr}^{(k)},$$
$$\sigma_{\theta\theta}^{(k)} = \lambda^{(k)}\,\varepsilon^{(k)} + 2\mu^{(k)}\varepsilon_{\theta\theta}^{(k)},$$
$$\sigma_{zz}^{(k)} = \lambda^{(k)}\,\varepsilon^{(k)} + 2\mu^{(k)}\varepsilon_{zz}^{(k)},\quad \sigma_{rz}^{(k)} = 2\mu^{(k)}\varepsilon_{rz}^{(k)}. \qquad (2)$$
$$\varepsilon_{rr}^{(k)} = \frac{\partial u_r^{(k)}}{\partial r},\ \varepsilon_{\theta\theta}^{(k)} = \frac{u_r^{(k)}}{r},\ \varepsilon_{zz}^{(k)} = \frac{\partial u_z^{(k)}}{\partial z},$$
$$\varepsilon^{(k)} = \varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)},\quad \varepsilon_{rz}^{(k)} = \frac{1}{2}\left(\frac{\partial u_z^{(k)}}{\partial r} + \frac{\partial u_r^{(k)}}{\partial z}\right) \qquad (3)$$

According to Fig. 1, we write the following boundary conditions on the inner surface of the hollow cylinder:

$$\sigma_{rr}^{(2)}\Big|_{r=R-h} = -P_0\delta(z-Vt)e^{i\omega t},\quad \sigma_{rz}^{(2)}\Big|_{r=R-h} = 0. \qquad (4)$$

here $\delta(x)$ is the Dirac delta function, V is the speed of the ring load, and ω is the vibration frequency of this load. We assume that the condition of complete adhesion is satisfied on the interface of media:

$$\sigma_{rr}^{(1)}\Big|_{r=R} = \sigma_{rr}^{(2)}\Big|_{r=R},\ \sigma_{rz}^{(1)}\Big|_{r=R} = \sigma_{rz}^{(2)}\Big|_{r=R},$$
$$u_r^{(1)}\Big|_{r=R} = u_r^{(2)}\Big|_{r=R}.\ u_z^{(1)}\Big|_{r=R} = u_z^{(2)}\Big|_{r=R} \qquad (5)$$

We also assume that the following conditions are fulfilled:

$$\left|\sigma_{rr}^{(k)}\right|;\left|\sigma_{\theta\theta}^{(k)}\right|;\ \ldots\ ;\left|u_r^{(k)}\right|;\left|u_z^{(k)}\right| < M = const;$$
$$\text{as}\ \sqrt{r^2+z^2}\to\infty. \qquad (6)$$

Problem (1) - (6) is solved using the *Lame* representation for general solutions of the equations of elastodynamics, which for axisymmetric problems can be written as follows:

$$u_r^{(k)} = \frac{\partial\Phi^{(k)}}{\partial r} + \frac{\partial^2\Psi^{(k)}}{\partial r\partial z},\quad u_z^{(k)} = \frac{\partial\Phi^{(k)}}{\partial z} + \frac{\partial^2\Psi^{(k)}}{\partial r^2} + \frac{1}{r}\frac{\partial\Psi^{(k)}}{\partial r}, \qquad (7)$$

where the functions $\Phi^{(k)}$ and $\Psi^{(k)}$ satisfy the following equation:

$$\nabla^2\Phi^{(k)} = \frac{1}{\left(c_1^{(k)}\right)^2}\frac{\partial^2\Phi^{(k)}}{\partial t^2}0,\quad \nabla^2\Psi^{(k)} = \frac{1}{\left(c_2^{(k)}\right)^2}\frac{\partial^2\Psi^{(k)}}{\partial t^2},$$
$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2},$$
$$c_1^{(k)} = \sqrt{(\lambda^{(k)}+2\mu^{(k)})/\rho^{(k)}},\quad c_2^{(k)} = \sqrt{\mu^{(k)}/\rho^{(k)}}. \qquad (8)$$

We introduce a moving coordinate system associated with the moving load and defined by the following formulas[4]:

$$r' = r,\ z' = z - Vt, \tag{9}$$

Based on (9), all the required values are represented as $g(r', z', t) = \breve{g}(r', z')e^{i\omega t}$ further we omit (the strokes over r', z' and also dash over $\breve{g}$), whereby we get the following equation for $\Phi^{(k)}$ and $\Psi^{(k)}$:

$$\nabla^2\Phi^{(k)} - \frac{1}{\left(c_1^{(k)}\right)^2}\left(V^2\frac{\partial^2\Phi^{(k)}}{\partial z^2} - 2i\omega V\frac{\partial\Phi^{(k)}}{\partial z} - \omega^2\Phi^{(k)}\right) = 0,$$

$$\nabla^2\Psi^{(k)} - \frac{1}{\left(c_2^{(k)}\right)^2}\left(V^2\frac{\partial^2\Psi^{(k)}}{\partial z^2} - 2i\omega V\frac{\partial\Psi^{(k)}}{\partial z} - \omega^2\Psi^{(k)}\right) = 0. \tag{10}$$

In addition, in a moving coordinate system, the first boundary condition in (4) turns into the following condition:

$$\sigma_{rr}^{(2)}\Big|_{r=R-h} = -P_0\delta(z). \tag{11}$$

The rest of the conditions in (4) - (6) remain valid for the amplitudes of the desired values, as well.

To solve equation (10), an exponential *Fourier* transform is applied over the *z* coordinates, according to which, all the required values are represented as:

$$\left\{\Phi^{(k)}; \Psi^{(k)}; u_z^{(k)}; \dots; \sigma_{rz}^{(k)}\right\} =$$
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\Phi_F^{(k)}; \Psi_F^{(k)}; u_{zF}^{(k)}; \dots; \sigma_{rzF}^{(k)}\right\}e^{+isz}ds. \tag{12}$$

Using (12), we obtain the following equations for the *Fourier* transform $\Phi_F^{(k)}$ and $\Psi_F^{(k)}$:

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(s^2 - \frac{\mathrm{W}^2\left(c_2^{(2)}\right)^2}{\left(c_1^{(k)}\right)^2}\right)\right]\Phi_F^{(k)} = 0,$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(s^2 - \frac{\mathrm{W}^2\left(c_2^{(2)}\right)^2}{\left(c_2^{(k)}\right)^2}\right)\right]\Psi_F^{(k)} = 0, \tag{13}$$

[4] Achenbach, J.D., Keshava, S.P., Hermann, G. Moving load on a plate resting on an elastic half-space// USA: Trans ASME. Series of Engineering Journal of Applied Mechanics, - 1967, 34, № 4, - p. 183-189

Where

$$W = \Omega - \mathrm{sc}, \quad \Omega = \omega h / c_2^{(2)}, \quad \mathrm{c} = V / c_2^{(2)}. \qquad (14)$$

The solution of equations (13) is found in the following form:

$$\Phi_F^{(2)} = A_1 H_0^{(1)}(r_1) + A_2 H_0^{(2)}(r_1),$$
$$\Psi_F^{(2)} = B_1 H_0^{(1)}(r_2) + +B_2 H_0^{(2)}(r_2),$$
$$\Phi_F^{(1)} = C_2 H_0^{(2)}(r_{11}),$$
$$\Psi_F^{(1)} = D_2 H_0^{(2)}(r_{21}), \qquad (15)$$

where $H_0^{(1)}(x)$ and $H_0^{(2)}(x)$ are *Hankel* functions of the first and second kind with zero order and

$$r_1 = r\sqrt{W^2\delta_1^2 - s^2},\ \delta_1 = c_2^{(2)} / c_1^{(2)},\ r_2 = r\sqrt{W^2 - s^2},$$
$$r_{11} = r\sqrt{W^2\delta_2^2 - s^2},\ \delta_2 = c_2^{(2)} / c_1^{(1)},$$
$$r_{21} = r\sqrt{W_1^2 - s^2},\ \ W_1 = W\, c_2^{(2)} / c_2^{(1)}. \qquad (16)$$

Thus, substituting the solution (15) in the *Fourier* transform expression (7), and then substituting the *Fourier* transforms for displacements in the *Fourier* transforms of the relation (3) and (2), we get the expression for the *Fourier* transformations of stresses. Finally, substituting these expressions in the *Fourier* transform of the boundary conditions (4) and contact conditions (5), we obtain an equation for determining the unknown constants A_1, A_2, B_1, B_2, C_2 and D_2 included in the *Fourier* transform in the desired quantities. After finding these unknown constants, we completely determine the Fourier transforms of the desired quantities from the specified equations. To determine the originals of these transformations, the thesis we developed an algorithm based on the use of the *Sommerfeld* contour method for calculating integrals (12).

Based on the above, in particular cases, i.e. in cases where I. $\omega = 0,\ V > 0$. II. V=0, $\omega > 0$ and III. $V \cdot \omega > 0$ we can obtain results related to a moving ring load (Case I), to the forced oscillation of the system caused by a harmonic ring load (Case II) and to a oscillating-moving ring load (Case III). Note that in Case I, the critical velocities are determined from the corresponding dispersion curves, and the effect of non-ideality of contact conditions on these values is also considered. Moreover, the non-ideality of the contact conditions is expressed by replacing the last condition in (5) with the condition

$$\left(u_z^{(1)} - u_z^{(2)}\right)\Big|_{r=R} = \frac{FR}{\mu^{(1)}}\sigma_{rz}^{(1)}\Big|_{r=R}. \tag{17}$$

The specified type of non-ideality of the contact condition is called “shear-spring” type of non-ideality and F $(0 \le |F| \le \infty)$ is a parameter that characterizes the degree of deviation of the condition from the corresponding ideal conditions. In this case, the case F=0 corresponds to an ideal contact, and the case $|F| = \infty$ corresponds to a complete slip on the interface of media.

In case III, the so-called «gyroscopic effect» (or «Coriolis acceleration effect») appears. This fact appears due to the term $2\omega V$ in equation (10). Precisely, this effect breaks the symmetry and asymmetry of the distribution of stresses and displacements along the Oz axis relative to the point $z/h = 0$. In addition, in Case III, the critical velocities cannot be determined from the dispersion curves of the corresponding waves, and therefore, these velocities are determined through the relationship between the interface normal stress and velocity of the oscillating-moving load. In this case, the velocities for which the absolute values of the normal stress become infinite are taken as critical velocity. Let's consider some fragments of numerical results obtained in Chapter 1. Tables 1 and 2 show the values of the dimensionless critical velocity $V_{\text{кр}}/c_2^{(2)}$, obtained in Case I for the problem parameters shown in the titles of these tables. In addition, in Fig. 2 and 3 are graphs of the dependencies between the interface normal stress $V_{\text{кр}}/c_2^{(2)}$, and c($= V/c_2^{(2)}$). In Case I for the parameters of the problem shown in the same figures. Note that the conclusions drawn from these and other similar results, which are not given here, will be set out below.

Table 1.

The value of the dimensionless critical velocity $V_{кр}/c_2^{(2)}$ obtained for various values h/R for $\rho^{(1)}/\rho^{(2)} = 0.1; \nu^{(1)} = \nu^{(2)} = 0.25$ and $E^{(1)}/E^{(2)} = 0.35$ in cases when $F = 0$ (upper value) and $F = \infty$ (lover value)							
h/R							
0.5	0.2	0.1	0.05	0.04	0.0333	0.0125	0.01
$\frac{0.9355}{0.8809}$	$\frac{0.8642}{0.7642}$	$\frac{0.8437}{0.7311}$	$\frac{0.8360}{0.7186}$	$\frac{0.8347}{0.7166}$	$\frac{0.8339}{0.7154}$	$\frac{0.8317}{0.7120}$	$\frac{0.8315}{0.7116}$

Table 2.

The value of the dimensionless critical velocity $V_{кр}/c_2^{(2)}$ obtained for various values h/R for $\rho^{(1)}/\rho^{(2)} = 0.1; \nu^{(1)} = \nu^{(2)} = 0.25$ and $E^{(1)}/E^{(2)} = 0.05$ in cases when $F = 0$ (upper value) and $F = \infty$ (lover value)							
h/R							
0.5	0.2	0.1	0.05	0.04	0.0333	0.0125	0.01
$\frac{0.8261}{0.8101}$	$\frac{0.6176}{0.5876}$	$\frac{0.5291}{0.4900}$	$\frac{0.4885}{0.4437}$	$\frac{0.4821}{0.4360}$	$\frac{0.4781}{0.4314}$	$\frac{0.4690}{0.4205}$	$\frac{0.4683}{0.4196}$

We also present numerical results showing the frequency characteristics of the interface normal voltage $\sigma_{rr}h/P_0$ obtained for the relationship between $\sigma_{rr}h/P_0$ and Ω in the case when $\nu^{(1)} = \nu^{(2)} = 0.3$ and, $\rho^{(1)}\mu^{(2)}/\rho^{(2)}\mu^{(1)} = 1$ for different values of $E^{(1)}/E^{(2)}$ (Fig. 4 for $R/h = 10$ and for different values of R/h (Fig. 5 for $E^{(1)}/E^{(2)} = 0.5$ and 1.2). According to the above, these results relate to Case II, and in addition to this, we present examples from numerical results showing the influence of the «gyroscopic effect» on the above-mentioned frequency response obtained in Case III. These results are shown in Fig. 6, which were obtained for $E^{(1)}/E^{(2)} = 0.5$, $\rho^{(1)}/\rho^{(2)} = 0.5$, $\nu^{(1)} = \nu^{(2)} = 0.3$ for different values of $c(=V/c_2^{(2)})$ when $h/R = 0.05$.

The dissertation also presents a lot of other numerical results on the effect of the problem parameters on the dynamics of the system under consideration when it is axisymmetric under the action of a moving and oscillating-moving ring load. In Chapter 1, numerical results on the effect of initial homogeneous stresses on the values of critical velocities and on the distribution of interface stresses are also considered and obtained.

Chapter 2 of the dissertation is devoted to the study of three-dimensional non-axisymmetric problems on the dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an infinite elastic medium. These studies were carried out within the framework of a piecewise homogeneous body model, using exact three-dimensional equations and relations of elastodynamics. It was assumed that the loads acting on the inner surface of the cylinder are non-axisymmetrically distributed along the inner circumference of the cylinder cross-section and are point-centered relative to the cylinder axis. The following three cases are considered:

a) when these forces move along the axis of the cylinder with a constant velocity;

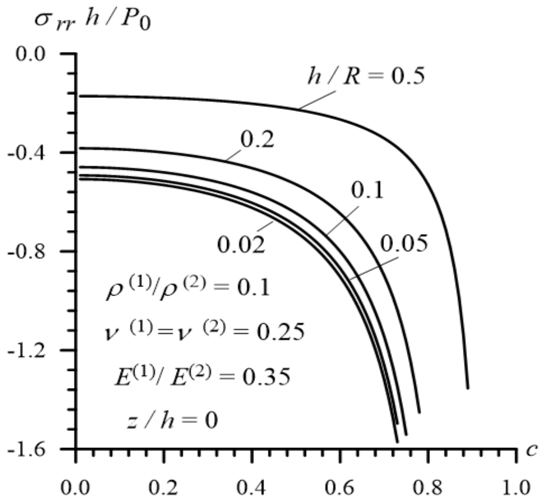


Fig. 2

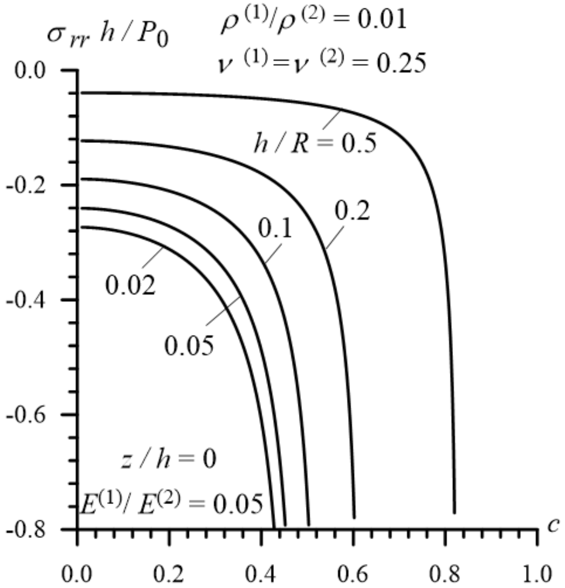

σ_rr h / P_0
ρ^(1)/ρ^(2) = 0.01
ν^(1)=ν^(2) = 0.25
h / R = 0.5
0.2
0.1
0.02
0.05
z / h = 0
E^(1)/ E^(2) = 0.05
c
0.0
-0.2
-0.4
-0.6
-0.8
0.0
0.2
0.4
0.6
0.8
1.0

Fig. 3

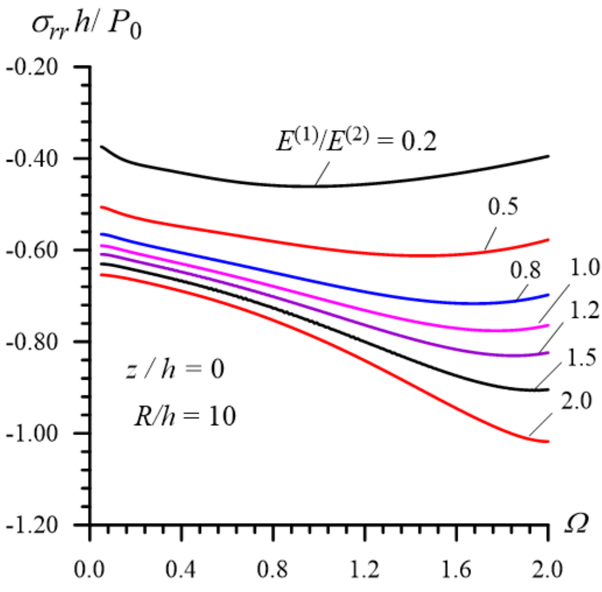

σ_rr h/ P_0
E^(1)/E^(2) = 0.2
0.5
0.8
1.0
1.2
1.5
2.0
z / h = 0
R/h = 10
Ω
-0.20
-0.40
-0.60
-0.80
-1.00
-1.20
0.0
0.4
0.8
1.2
1.6
2.0

Fig. 4

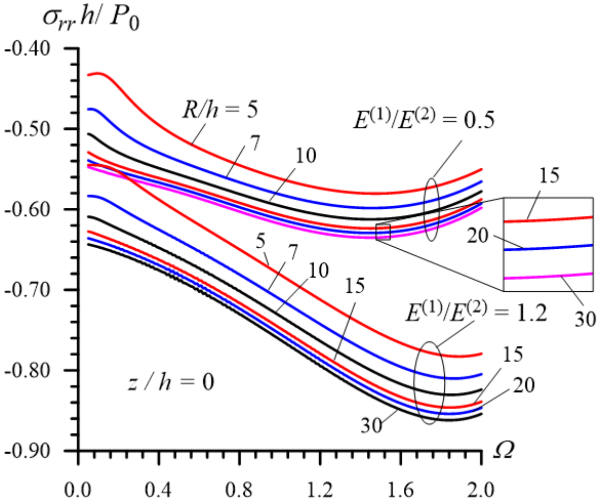


Fig. 5

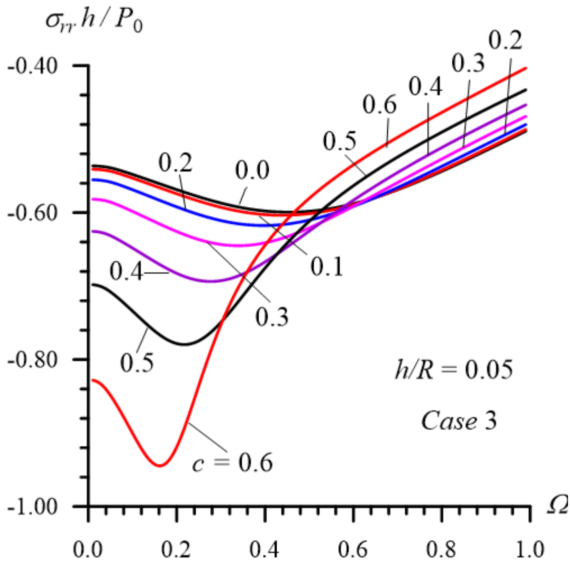


Fig. 6

b) when these forces change harmonically in time;

c) when these forces change harmonically in time and simultaneously move with a constant velocity along the axis of the cylinder.

The problems considered and investigated in Chapter 2 can also be considered as a generalization of the problems studied in Chapter 1 to the three-dimensional non-axisymmetric case. At the same time, methods for solving corresponding three-dimensional problems and an algorithm for obtaining specific numerical results are developed.

We present separate fragments that illustrate the formulation of three-dimensional problems and methods for solving them. For this purpose, consider a hollow cylinder of thickness h and denote by R the outer radius of the cross-section of this cylinder. Let us assume that this hollow cylinder is surrounded by an infinite medium and that forces distributed non-axisymmetrically around the circumference of the inner cross-section of the cylinder, vibrating harmonically in time, simultaneously move at a constant speed V, act on the inner surface of the cylinder. It is necessary to determine the dynamic stress-strain state in the system under consideration, caused by the above forces. Sketches of the system under study and the effective forces are shown in Fig. 7, according to which, the cylindrical $Or\theta z$ and Cartesian $Ox_1x_2x_3$ coordinate systems are connected to the central axis of the cylinder. In addition, according to the figure, it is assumed that the forces are distributed on an arc that corresponds to the central angle α, and the distribution of these forces along the Oz axis is a point-located and this point moves at a speed of V along this axis of the cylinder.

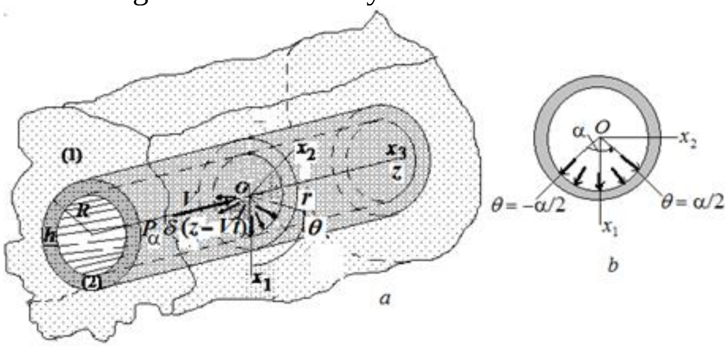


Fig. 7

As noted above, the study is carried out using three-dimensional equations and relations of elastodynamics written in a cylindrical coordinate system, which are given below:

Equation of motion:

$$\frac{\partial\sigma_{rr}^{(m)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}^{(m)}}{\partial\theta}+\frac{\partial\sigma_{rz}^{(m)}}{\partial z}+\frac{1}{r}(\sigma_{rr}^{(m)}-\sigma_{\theta\theta}^{(m)})=\rho^{(m)}\frac{\partial^2u_r^{(m)}}{\partial t^2},$$

$$\frac{\partial\sigma_{r\theta}^{(m)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(m)}}{\partial\theta}+\frac{\partial\sigma_{z\theta}^{(m)}}{\partial z}+\frac{2}{r}\sigma_{r\theta}^{(m)}=\rho^{(m)}\frac{\partial^2u_\theta^{(m)}}{\partial t^2},$$

$$\frac{\partial\sigma_{rz}^{(m)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{z\theta}^{(m)}}{\partial\theta}+\frac{\partial\sigma_{zz}^{(m)}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(m)}=\rho^{(m)}\frac{\partial^2u_z^{(m)}}{\partial t^2}. \qquad (18)$$

Elasticity relations:

$$\sigma_{rr}^{(m)}=\left(\lambda^{(m)}+2\mu^{(m)}\right)\frac{\partial u_r^{(m)}}{\partial r}+\lambda^{(m)}\left(\frac{\partial u_\theta^{(m)}}{r\partial\theta}+\frac{u_r^{(m)}}{r}\right)+\lambda^{(m)}\frac{\partial u_z^{(m)}}{\partial z},$$

$$\sigma_{\theta\theta}^{(m)}=\lambda^{(m)}\frac{\partial u_r^{(m)}}{\partial r}+\left(\lambda^{(m)}+2\mu^{(m)}\right)\left(\frac{\partial u_\theta^{(m)}}{\partial r}+\frac{u_r^{(m)}}{r}\right)+\lambda^{(m)}\frac{\partial u_z^{(m)}}{\partial z},$$

$$\sigma_{zz}^{(m)}=\lambda^{(m)}\frac{\partial u_r^{(m)}}{\partial r}+\lambda^{(m)}\left(\frac{\partial u_\theta^{(m)}}{\partial r}+\frac{u_r^{(m)}}{r}\right)+\left(\lambda^{(m)}+2\mu^{(m)}\right)\frac{\partial u_z^{(m)}}{\partial z},$$

$$\sigma_{r\theta}^{(m)}=\mu^{(m)}\frac{\partial u_\theta^{(m)}}{\partial r}+\mu^{(m)}\left(\frac{\partial u_r^{(m)}}{r\partial\theta}-\frac{1}{r}u_\theta^{(m)}\right),$$

$$\sigma_{z\theta}^{(m)}=\mu^{(m)}\frac{\partial u_\theta^{(m)}}{\partial z}+\mu^{(m)}\frac{\partial u_z^{(m)}}{r\partial\theta},$$

$$\sigma_{zr}^{(m)}=\mu^{(m)}\frac{\partial u_r^{(m)}}{\partial z}+\mu^{(m)}\frac{\partial u_z^{(m)}}{\partial r}. \qquad (19)$$

Note that in (18), (19) and further, $m=1$ means that the values relate to the surrounded medium, $m=2$ - to the hollow cylinder. According to Fig. 7, we write the following boundary conditions on the inner surface of the hollow cylinder:

$$\sigma_{rr}^{(2)}\Big|_{r=R-h}=\begin{cases} -P_\alpha\delta(z-Vt) \ \text{при} \ -\dfrac{\alpha}{2}\le\theta\le\dfrac{\alpha}{2} \ , \\ 0 \quad \text{при} \quad \theta\in\left([-\pi,\pi]-\left(-\dfrac{\alpha}{2},\dfrac{\alpha}{2}\right)\right), \end{cases}$$

$$\sigma_{r\theta}^{(2)}\Big|_{r=R-h}=0, \qquad \sigma_{rz}^{(2)}\Big|_{r=R-h}=0, \tag{20}$$

where P_α in (20) is determined from the following relation:

$$\int_{-\alpha/2}^{+\alpha/2} P_\alpha\,(R-h)\cos\theta d\theta=(R-h)P_0=const \ \Rightarrow$$

$$P_\alpha=P_0/2\sin(\alpha/2), \tag{21}$$

where the central α angle is shown in Fig. 7.

It is assumed that the condition of complete coupling is satisfied at the interface between the cylinder materials and the environment, which can be written as follows:

$$\sigma_{rr}^{(2)}\Big|_{r=R}=\sigma_{rr}^{(1)}\Big|_{r=R}, \quad \sigma_{r\theta}^{(2)}\Big|_{r=R}=\sigma_{r\theta}^{(1)}\Big|_{r=R},$$
$$\sigma_{rz}^{(2)}\Big|_{r=R}=\sigma_{rz}^{(1)}\Big|_{r=R},$$
$$u_r^{(2)}\Big|_{r=R}=u_r^{(1)}\Big|_{r=R}, \quad u_\theta^{(2)}\Big|_{r=R}=u_\theta^{(1)}\Big|_{r=R},$$
$$u_z^{(2)}\Big|_{r=R}=u_z^{(1)}\Big|_{r=R}. \tag{22}$$

The subsonic mode is considered, i.e. it is assumed that

$$V<min\left\{c_2^{(1)};\ c_2^{(2)}\right\}, \quad c_2^{(m)}=\sqrt{\mu^{(m)}/\rho^{(m)}}, \quad m=1,2. \tag{23}$$

The following attenuation conditions are also assumed to be satisfied:

$$\left\{\left|\sigma_{rr}^{(m)}\right|;\dots;\left|\sigma_{r\theta}^{(m)}\right|;\left|u_r^{(m)}\right|;\dots;\left|u_z^{(m)}\right|\right\}<M=const,$$
$$\text{as } \sqrt{r^2+(z-Vt)^2}\to+\infty. \tag{24}$$

With the above, we complete the mathematical formulation of the problem. Note that in the foregoing formulation the case of $\omega=0$ (where ω is the frequency of vibration of external forces) corresponds the problems of moving load, and in the case of $V=0$

corresponds the non-axisymmetric forced oscillation of the system under consideration.

To solve this problem, we use the A.N.Guz representations for general solutions of three-dimensional exact equations of elastodynamics. These representations have the following form:

$$u_r^{(m)} = \frac{1}{r}\frac{\partial}{\partial\theta}\Psi^{(m)} - \frac{\partial^2}{\partial r\partial z}\chi^{(m)},$$

$$u_\theta^{(m)} = -\frac{\partial}{\partial r}\Psi^{(m)} - \frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\chi^{(m)},$$

$$u_z^{(m)} = \left(\lambda^{(m)} + \mu^{(m)}\right)^{-1}\left(\left(\lambda^{(m)} + 2\mu^{(m)}\right)\Delta_1 + \right.$$

$$\left. + \mu^{(m)}\frac{\partial^2}{\partial z^2} - \rho^{(m)}\frac{\partial^2}{\partial t^2}\right)\chi^{(m)},$$

$$\Delta_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}, \quad m = 1,2. \qquad (25)$$

where the functions $\Psi^{(m)}$ and $\chi^{(m)}$ are the solutions of the following equations:

$$\left(\Delta_1 + \frac{\partial^2}{\partial z^2} - \frac{\rho^{(m)}}{\mu^{(m)}}\frac{\partial^2}{\partial t^2}\right)\Psi^{(m)} = 0,$$

$$\left[\left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right)\left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right) - \rho^{(m)}\frac{\lambda^{(m)} + 3\mu^{(m)}}{\mu^{(m)}\left(\lambda^{(m)} + 2\mu^{(m)}\right)} \times \right.$$

$$\left. \times \left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right)\frac{\partial^2}{\partial t^2} + \frac{\left(\rho^{(m)}\right)^2}{\mu^{(m)}\left(\lambda^{(m)} + 2\mu^{(m)}\right)}\frac{\partial^4}{\partial t^4}\right]\chi^{(m)} = 0. \qquad (26)$$

To solve equations (26), we proceed to the moving coordinate system defined by the following relations:

$$r' = r, \quad \theta = \theta', \quad z' = z - Vt \qquad (27)$$

When transition to a moving coordinate system $O'r'\theta'z'$, the operators $\partial/\partial r$, $\partial/\partial\theta$ and $\partial/\partial z$ in the above equations are replaced by the operators $\partial/\partial r'$, $\partial/\partial\theta'$ and $\partial/\partial z'$, respectively. Using the representations $g(r',\theta',z',t) = \overline{g}(r',\theta',z')e^{i\omega t}$ it turns out that in the moving coordinate system $O'r'\theta'z'$ the operators $\partial/\partial t$ are replaced by the operator $(V\,\partial/\partial z' - i\omega)$, hence the operators $\partial^2/\partial t^2$ and $\partial^4/\partial t^4$ - operators $(V\,\partial/\partial z' - i\omega)^2$ and $(V\,\partial/\partial z' - i\omega)^4$. Further, the equations and relations written in the moving coordinate system $O'r'\theta'z'$ are applied to the exponential Fourier transform in the z coordinates, i.e. we represent the amplitudes of the desired quantities in the form:

$$g(r,\theta,z,)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}g_F(r,\theta,s)\,e^{isz}dz, \qquad (28)$$

where, in (28) and further, the stroke above the coordinates and the dashes above the amplitude symbols are omitted. Moreover, the lower index *F* in (28) and further denotes the Fourier transforms of the amplitudes of the corresponding quantities. As a result of these procedures, the following equations are obtained for determining the transformations $\Psi_F^{(m)}$and $\chi_F^{(m)}$:

$$\left({\Delta_1}^2-s^2-\frac{\rho^{(m)}}{\mu^{(m)}}(h\omega-sV)^2\right)\Psi_F^{(m)}(r,\theta,s)=0,$$

$$[(\Delta_1-s^2)(\Delta_1-s^2)+$$

$$\rho^{(m)}\frac{\lambda^{(m)}+3\mu^{(m)}}{\mu^{(m)}(\lambda^{(m)}+2\mu^{(m)})}(\Delta_1-s^2)+(h\omega-sV)^2+$$

$$\left.+\frac{\left(\rho^{(m)}\right)^2}{\mu^{(m)}\left(\lambda^{(m)}+2\mu^{(m)}\right)}(h\omega-sV)^4\right]\chi_F^{(m)}(r,\theta,s)=0. \qquad (29)$$

Given the periodicity of solutions with respect to the angular coordinate θ, which follows from the geometry of the system under consideration, the functions $\Psi_F^{(m)}$ and $\Psi_F^{(m)}$ are represented as a Fourier series by θ coordinates in the following form:

$$\Psi_F^{(m)}(r,s,\theta)=\sum_{n=1}^{\infty}\Psi_{Fn}^{(m)}(r,s)\sin n\theta,$$

$$X_F^{(m)}(r,s,\theta)=\frac{1}{2}X_{F0}^{(m)}(r,s)+\sum_{n=1}^{\infty}X_{Fn}^{(m)}(r,s)\cos n\theta,$$

(30)

Substituting expression (30) in equation (29), we obtain the following equations for the coefficients of the Fourier series of the functions $\Psi_{Fn}^{(m)}(r,s)$, $X_{Fn}^{(m)}(r,s)$ $(n\geq 1)$ and $X_{F0}^{(m)}(r,s)$:

$$\left(\Delta_{1n}-(\zeta_1^{(m)})^2\right)\psi_{Fn}^{(m)}=0,$$

$$\left(\Delta_{1n}-(\zeta_2^{(m)})^2\right)\left(\Delta_{1n}-(\zeta_3^{(m)})^2\right)X_{Fn}^{(m)}=0,$$

$$\Delta_{1n}=\frac{d^2}{dr^2}+\frac{d}{rdr}-\frac{n^2}{r^2}, \qquad (31)$$

where

$$\left(\xi_1^{(m)}\right)^2 = s^2 - \frac{\rho^{(m)}}{\mu^{(m)}}(h\omega - sV)^2, \tag{32}$$

$\left(\xi_2^{(m)}\right)^2$ *and* $\left(\xi_3^{(m)}\right)^2$ are defined as the solution of the following biquadrate equation:

$$\left(\xi^{(m)}\right)^4 - \left(\xi^{(m)}\right)^2\Big[-\rho^{(m)}(h\omega - sV)^2 - s^2\left(\lambda^{(m)} + 2\mu^{(m)}\right) +$$

$$+\frac{\mu^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}}\left(-\rho^{(m)}(h\omega - sV)^2 - s^2\mu^{(m)}\right) +$$

$$s^2\frac{\left(\lambda^{(m)} + \mu^{(m)}\right)^2}{\lambda^{(m)} + 2\mu^{(m)}}\Big] +$$

$$+\left(\frac{-\rho^{(m)}(h\omega - sV)^2}{\lambda^{(m)} + 2\mu^{(m)}} - s^2\right)\left(-\rho^{(m)}(h\omega - sV)^2 - s^2\mu^{(m)}\right) = 0. \tag{33}$$

Thus, the solution of equation (33) is found in the following form.

For a hollow cylinder:

$$\psi_{Fn}^{(2)} = A_{1n}^{(2)} I_n(\zeta_1^{(2)} r) + B_{1n}^{(2)} K_n(\zeta_1^{(2)} r),$$

$$\chi_{Fn}^{(2)} = \Big[A_{2n}^{(2)} I_n(\zeta_2^{(2)} r) + A_{3n}^{(2)} I_n(\zeta_3^{(2)} r) +$$

$$+B_{2n}^{(2)} K_n(\zeta_2^{(2)} r) + B_{3n}^{(2)} K_n(\zeta_3^{(2)} r)\Big] i. \tag{34}$$

For the environment:

$$\psi_{Fn}^{(1)} = B_{1n}^{(1)} K_n(\zeta_1^{(1)} r),$$

$$\chi_{Fn}^{(1)} = \left[B_{2n}^{(1)} K_n(\zeta_2^{(1)} r) + B_{3n}^{(1)} K_n(\zeta_3^{(1)} r)\right] i. \tag{35}$$

In (34) and (35), the functions $I_n(x)$ and $K_n(x)$ are modified *n*-order Bessel functions with a purely imaginary argument of the first and second kind, respectively, and $i = \sqrt{-1}$. Writing an expression χ_F using "i" is associated with facilitating further mathematical procedures.

So, substituting the solution (34) and (35) in the representation (25), we get an expression for displacement transformations, and then substituting the expression of *Fourier* transformations of displacements in the *Fourier* transformations of elastic relations (19), we get an expression for the *Fourier* transformations of stresses. Finally, substituting the obtained expression into the *Fourier* transforms of the boundary forces (20) and the contact condition (22), we obtain the corresponding algebraic equation for determining the unknown constants $A_{20}^{(2)}, A_{30}^{(2)}, B_{20}^{(2)}, B_{30}^{(2)}, A_{1n}^{(2)}, B_{1n}^{(2)}, A_{2n}^{(2)}, B_{2n}^{(2)}, A_{3n}^{(2)}, B_{3n}^{(2)}, B_{1n}^{(1)}, B_{2n}^{(1)}$ and $B_{3n}^{(1)}$. Finding these unknowns, we completely determine all the coefficients in the series (30) from these equations.

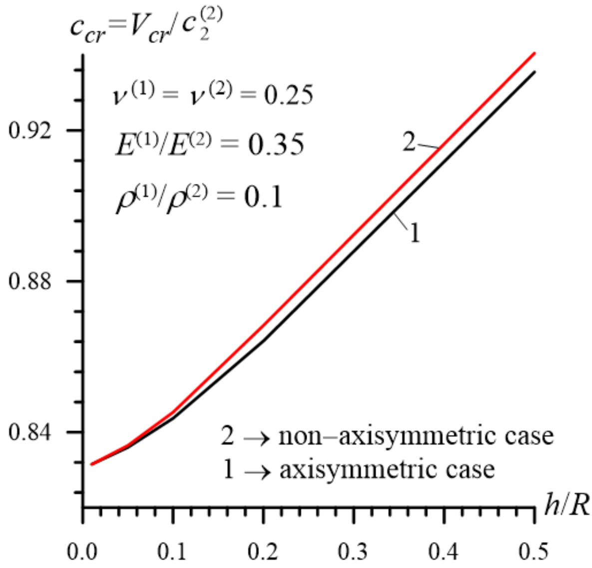
Applying the algorithm developed in the previous Chapter, we obtain numerical results related to the critical velocity and to the distribution of interface stresses.

Let's consider some fragments of the obtained numerical results.

Fig. 8 - Fig. 12 show graphs of the dependences of the dimensionless velocity of the $c_{кр}$ $(= V/c_2^{(2)})$ and the ratio h/R obtained for various pairs of materials of the hollow cylinder and the environment in the case when $\omega = 0$. These graphs show the effect of non-axisymmetry of moving loads on the critical velocity.

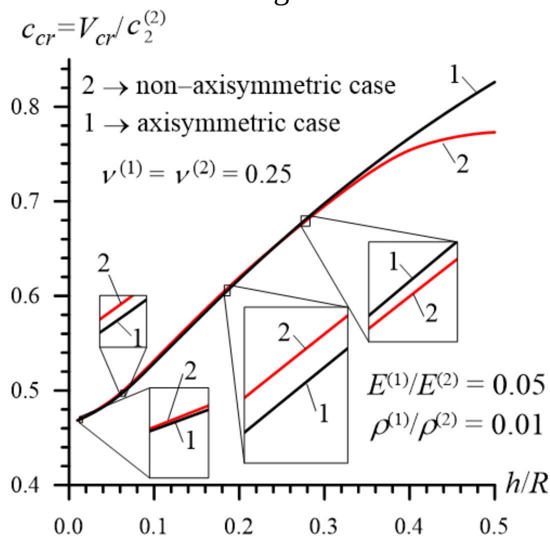
Fig. 13 and 14 show graphs of the relationships between the interface normal voltage $\sigma_{rr}h/P_0$ and the dimensionless speed of the $c_{кр}$ $(= V/c_2^{(2)})$, constructed for $h/R = 0.05$ and 0.1, respectively, obtained in the case when $\omega = 0$. When plotting these graphs, the voltage value σ_{rr} is calculated for $z/h = 0$, $\theta = 0$.

The thesis also provides many other numerical results not only for cases where $> 0, \omega = 0$, but also for the cases where $V = 0, \omega > 0$ and $\omega \cdot V > 0$. When all these results are obtained, the first 20 members are selected in the Fourier series. The conclusions drawn from these results will be presented below.



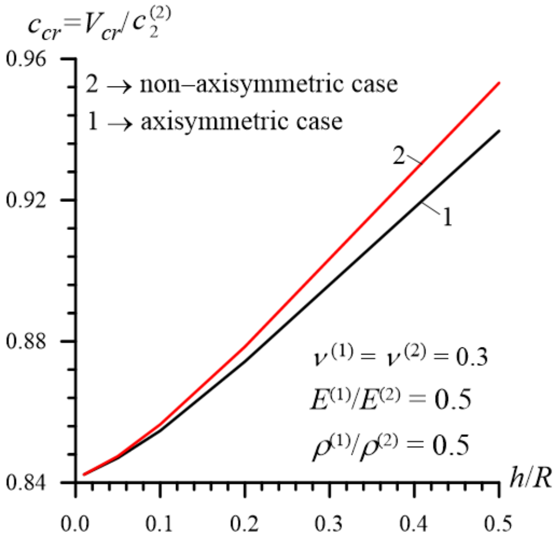
$c_{cr}=V_{cr}/c_2^{(2)}$
$\nu^{(1)} = \nu^{(2)} = 0.25$
$E^{(1)}/E^{(2)} = 0.35$
$\rho^{(1)}/\rho^{(2)} = 0.1$
0.92
0.88
0.84
2
1
2 → non–axisymmetric case
1 → axisymmetric case
h/R
0.0
0.1
0.2
0.3
0.4
0.5

Fig. 8



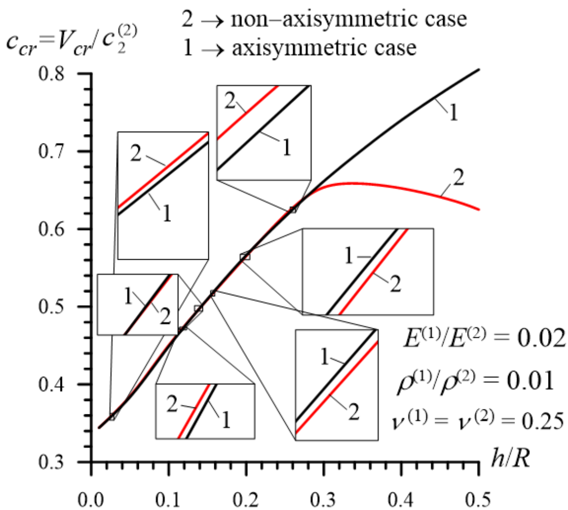
$c_{cr}=V_{cr}/c_2^{(2)}$
2 → non–axisymmetric case
1 → axisymmetric case
$\nu^{(1)} = \nu^{(2)} = 0.25$
$E^{(1)}/E^{(2)} = 0.05$
$\rho^{(1)}/\rho^{(2)} = 0.01$
0.8
0.7
0.6
0.5
0.4
h/R
0.0
0.1
0.2
0.3
0.4
0.5

Fig. 9



$c_{cr}=V_{cr}/c_2^{(2)}$
0.96
0.92
0.88
0.84
2 → non–axisymmetric case
1 → axisymmetric case
2
1
$\nu^{(1)} = \nu^{(2)} = 0.3$
$E^{(1)}/E^{(2)} = 0.5$
$\rho^{(1)}/\rho^{(2)} = 0.5$
h/R
0.0
0.1
0.2
0.3
0.4
0.5

Fig. 10



2 → non–axisymmetric case
1 → axisymmetric case
$c_{cr}=V_{cr}/c_2^{(2)}$
0.8
0.7
0.6
0.5
0.4
0.3
$E^{(1)}/E^{(2)} = 0.02$
$\rho^{(1)}/\rho^{(2)} = 0.01$
$\nu^{(1)} = \nu^{(2)} = 0.25$
h/R
0.0
0.1
0.2
0.3
0.4
0.5

Fig. 11

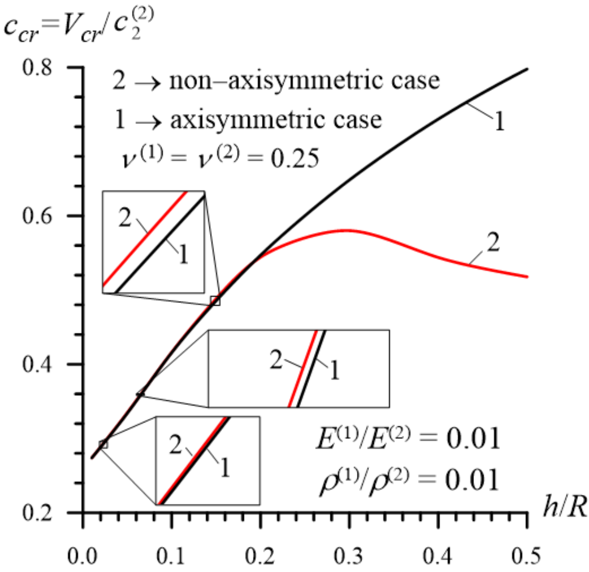

$c_{cr}=V_{cr}/c_2^{(2)}$
2 → non–axisymmetric case
1 → axisymmetric case
$\nu^{(1)} = \nu^{(2)} = 0.25$
$E^{(1)}/E^{(2)} = 0.01$
$\rho^{(1)}/\rho^{(2)} = 0.01$
h/R
0.8
0.6
0.4
0.2
0.0
0.1
0.2
0.3
0.4
0.5
1
2

Fig. 12

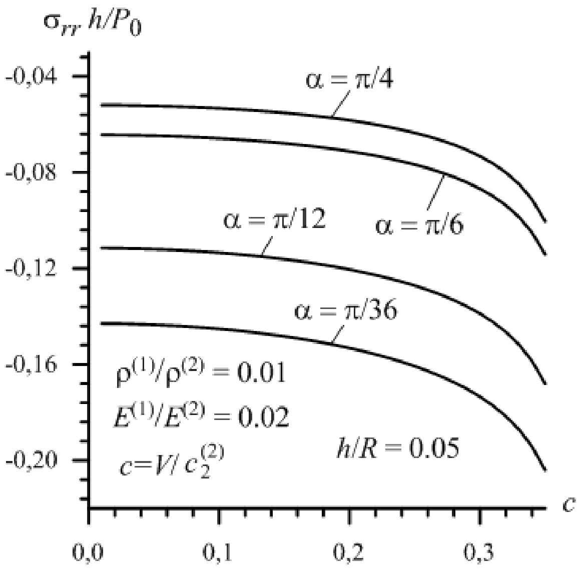

$\sigma_{rr}\, h/P_0$
-0,04
-0,08
-0,12
-0,16
-0,20
$\alpha = \pi/4$
$\alpha = \pi/6$
$\alpha = \pi/12$
$\alpha = \pi/36$
$\rho^{(1)}/\rho^{(2)} = 0.01$
$E^{(1)}/E^{(2)} = 0.02$
$c=V/c_2^{(2)}$
$h/R = 0.05$
c
0,0
0,1
0,2
0,3

Fig. 13

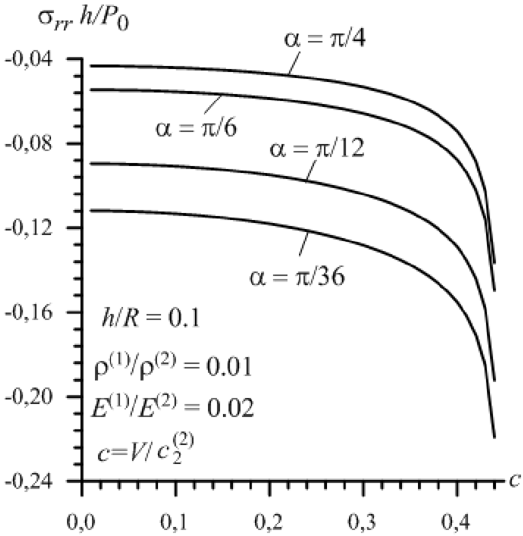


Fig. 14

In Chapter 3 the problems of dynamics of moving and oscillating-moving loads acting on the inner surface of a two-layer hollow cylinder were studied. Not only axisymmetric, but also non-axisymmetric problems are considered. In the axisymmetric case, we study the dynamics of a ring load moving with a constant velocity and acting on the inner surface of a two-layer hollow cylinder. In the three-dimensional non-axisymmetric case, we study not only the dynamics of a non-axisymmetric moving load, but also the forced oscillation under the action of non-axisymmetric loads that are harmonically changing over time, acting on the inner surface of a two-layer cylinder.

When formulating problems related to the axisymmetric case (Fig. 15), we use the equation and the relation (1) – (3), boundary conditions (4) for $\omega = 0$, and contact conditions (5). The thickness of the inner layer is denoted by h_2, and the thickness of the outer layer - by h_1. Therefore, in the boundary conditions (4), h_2 is written instead of h. In addition, the following boundary conditions are added to these conditions satisfied on the outer surface:

$$\sigma_{rr}^{(1)}\Big|_{r=R+h_1} = 0, \quad \sigma_{rz}^{(1)}\Big|_{r=R+h_1} = 0, \tag{36}$$

When formulating the problem related to the three-dimensional axisymmetric case (Fig. 16), the equation and relation (18), (19), boundary conditions (20) with the replacement of h by h_2, and contact conditions (18) are used. The following boundary conditions are added these conditions, satisfied on the outer surface of the hollow cylinder:

$$\sigma_{rr}^{(1)}\Big|_{r=R+h_1} = 0, \quad \sigma_{r\theta}^{(1)}\Big|_{r=R+h_1} = 0 \quad \sigma_{rz}^{(1)}\Big|_{r=R+h_1} = 0, \tag{37}$$

Solutions are obtained in the same way as the above for axisymmetric and non-axisymmetric problems. In this case, the solution and the expression with the upper index (1) are rewritten for the outer layer of the cylinder with the introduction of corresponding additional terms with new unknown constants. Based on the obtained analytical expressions of the Fourier transforms of the desired values, numerical results are obtained on the critical velocity and distribution of interface stresses acting on the interface of the layer media.

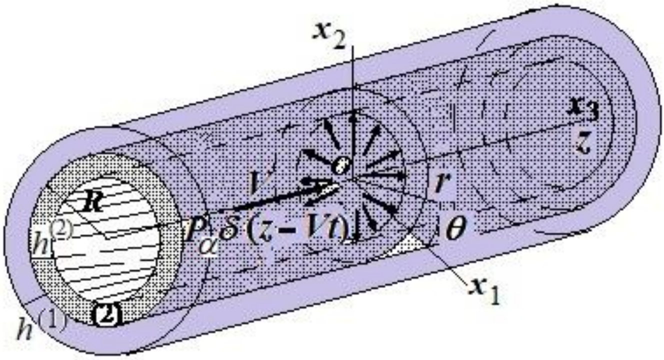


Fig. 15

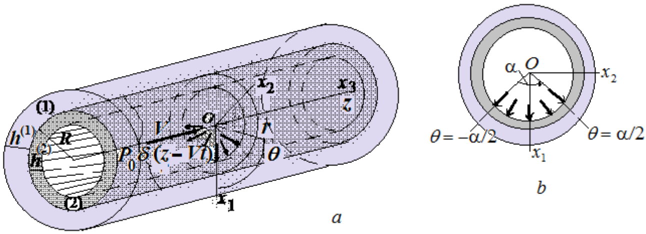


Fig. 16

Consider fragments of numerical results given in Tables 3 and 4, which show the effect of the ratio h_1/R on the value of critical velocities obtained for different values of the ratio h_2/R in the case of axisymmetric loading.

We also consider fragments of numerical results in the axisymmetric case related to the interface normal stress, which are shown in Fig.17. These results were obtained for $E^{(1)}/E^{(2)} = 0.35$, $\nu^{(1)} = \nu^{(2)} = 0.25$, $\rho^{(1)}/\rho^{(2)} = 0.1$, $c_2^{(1)}/c_2^{(2)} = \sqrt{3} \approx 1.8708$ at different values of the ratio h_1/R in cases $h_2/R = 0.1$ (a), 0.3 (b) и 0.5 (c). These results are obtained for the non-axisymmetric case as weel.

Table 3.

The value of the dimensionless critical velocity $V_{\text{кр}}/c_2^{(2)}$ obtained for a bi-layered hollow cylinder for various values of h_1/R and h_2/R for $\rho^{(1)}/\rho^{(2)} = 0.01$, $\nu^{(1)} = \nu^{(2)} = 0.25$ and $E^{(1)}/E^{(2)} = 0.05$ in cases when $F = 0$ (upper value) and $F = \infty$ (lover value)

h_2/R	h_1/R						
	0.1	0.3	0.5	1.0	2.5	5.5	∞
0.5	0.8375	0.9160	0.9350	0.9355	0.9355	0.9355	0.9355
	0.7903	0.8133	0.8559	0.8807	0.8809	0.8809	0.8809
0.3	0.7064	0.8369	0.8864	0.8881	0.8881	0.8881	0.8881
	0.6396	0.6930	0.7729	0.8027	0.8028	0.8028	0.8028
0.1	0.5439	0.8159	0.8437	0.8437	0.8437	0.8437	0.8437
	0.4177	0.6592	0.7310	0.7311	0.7311	0.7311	0.7311

Table 4.

The value of the dimensionless critical velocity $V_{кр}/c_2^{(2)}$ obtained for bi-layered hollow cylinder for various values of h_1/R and h_2/R for $\rho^{(1)}/\rho^{(2)} = 0.1$, $\nu^{(1)} = \nu^{(2)} = 0.25$ and $E^{(1)}/E^{(2)} = 0.35$ in cases when $F = 0$ (upper value) and $F = \infty$ (lover value)							
h_2/R	h_1/R						
	0.1	0.3	0.5	1.0	2.5	5.5	∞
0.5	0.8003 / 0.7917	0.8173 / 0.7961	0.8238 / 0.8037	0.8260 / 0.8099	0.8261 / 0.8101	0.8261 / 0.8101	0.8261 / 0.8101
0.3	0.6501 / 0.6374	0.6801 / 0.6472	0.6930 / 0.6624	0.6977 / 0.6735	0.6977 / 0.6738	0.6977 / 0.6738	0.6977 / 0.6738
0.1	0.4223 / 0.3869	0.5112 / 0.4502	0.5279 / 0.4867	0.5291 / 0.4900	0.5291 / 0.4900	0.5291 / 0.4900	0.5291 / 0.4900

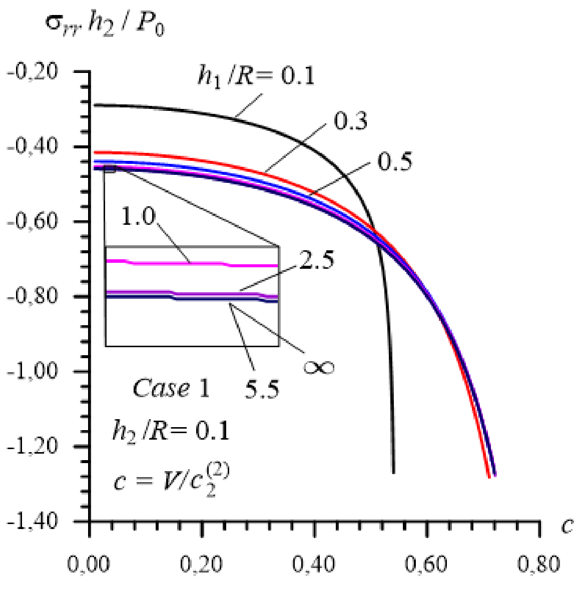


Fig. 17a

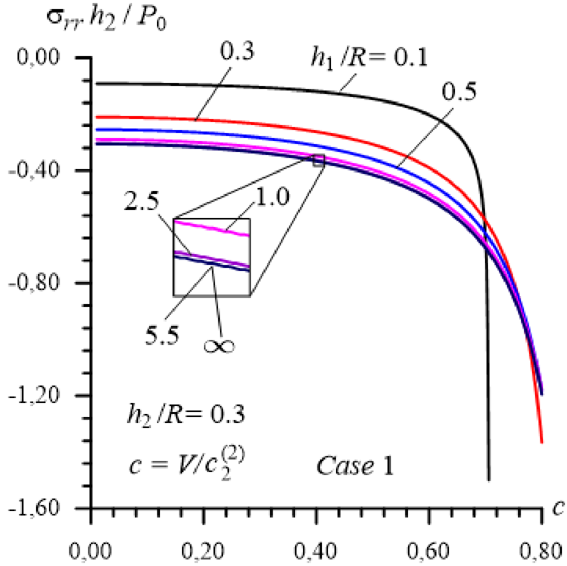


Fig. 17b

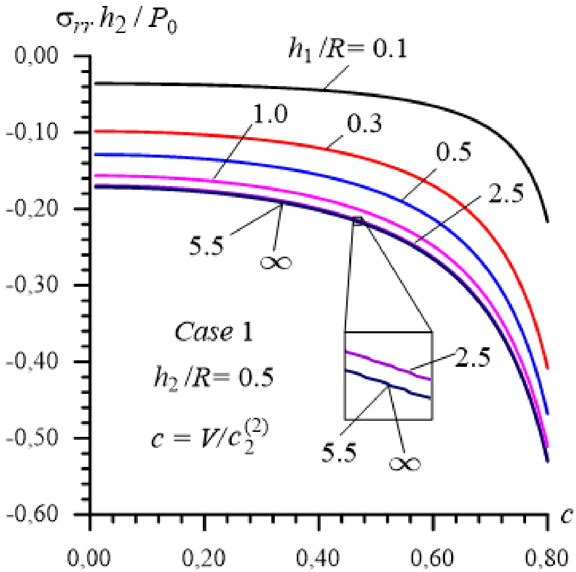


Fig. 17c

The results obtained in the work and conclusions made on the basis of relevant analyses are summarized in conclusion.

This concludes the presentation of the content of the dissertation.

Main results and conclusions

I. Thus, in this dissertation, based on the piecewise homogeneous body model with the use of exact three-dimensional equations of elastodynamics and TLTEWIS, a theory is developed on the dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium, including:

- formulation of problems and development of analytical and numerical methods for solving the corresponding problems of dynamic stress-strain state;
- investigation of various classes of two-dimensional (axisymmetric) and three-dimensional (non-axisymmetric) problems on determining the influence of the speed and vibration of a moving load on the distribution of stresses acting on the interface between the cylinder materials and the surrounding medium;
- determination of critical velocities of moving and oscillating moving loads and the influence of problem parameters on the value of these critical velocities;
- detection of «gyroscopic effects» (or «Coriolis acceleration effect») on the value of critical velocities and on the distribution of interface stresses;
- study of forced vibration of the «hollow cylinder + environment» system;
- study of the influence of homogeneous initial stresses on the value of critical velocities and interface stresses;
- determination of the effect of non-axisymmetricity of moving and oscillating-moving loads on the value of critical velocities.

II. The following specific results were obtained:

The dynamics of a moving ring load acting on the inner surface of a hollow cylinder surrounded by an elastic medium was studied in the framework of a piecewise homogeneous body model using exact equations and relations of elastodynamics corresponding to the axisymmetric case. It is assumed that on the interface between the hollow cylinder and surrounded medium there is a «shearial-spring» type of non-ideal contact with the parameter F ($0 \leq F \leq \infty$).

Numerical results on the critical velocity and stress distribution acting on the interface between a hollow cylinder and the surrounding medium are presented and discussed.

The following specific results were obtained:

– the value of the critical velocity significantly depends on the parameter F, which characterizes the degree of imperfection of the contact forces, and the specified imperfection of the contact forces leads to a decrease in the values of the critical speed;
– the value of the critical velocity decreases with an increase in the external radius R of the cross section of a hollow cylinder at its constant thickness and approaches the corresponding critical velocities obtained for a system consisting of a covering layer and a half-plane;
– an increase in the elastic modulus of the hollow cylinder material leads to a decrease in the dimensionless value of the critical velocity $V_{cr}/c_2^{(2)}$, where $c_2^{(2)}$ is the shear wave velocity in the hollow cylinder material, V_{cr} is the critical velocity of the moving load;
– the absolute values of normal and tangential stresses acting on the interface between the media of the hollow cylinder and the surrounding elastic medium monotonically increase with increasing velocity of the moving load;
– a decrease in the elastic modulus of the environment leads to a decrease in the absolute values of the above stresses;
– the absolute values of interface stresses and displacements increase with the growth of the external radius R of the cross-section of a hollow cylinder at a fixed thickness of this cylinder.

2. The influence of homogeneous initial stresses in the system «hollow cylinder+surrounding infinite medium» on the value of the critical velocity of a moving ring load acting on the inner surface of

a hollow cylinder surrounded by an infinite medium was studied in the framework of a piecewise homogeneous body model using exact equations and relations of the three-dimensional linearized theory of elastic waves in bodies with initial stresses. It is assumed, that the initial stresses in the body appear as a result of the action of uniformly distributed normal tensile (or compressive) forces in the direction of the cylinder axis. Numerical results on the effect of initial stresses on the values of the critical velocity and on the values of interface stresses are presented and discussed. Based on these results, the following main conclusions are made:

– the initial stretching (compression) of the hollow cylinder and the environment leads to an increase (decrease) in the critical velocity values;
– the value of the above influence increases with a decrease in the ratio h/R, where h is the thickness of the hollow cylinder, and R is the outer radius of the cross-section of the hollow cylinder. Moreover, the critical velocity values decrease with decreasing ratio h/R;

3. The axisymmetric problem of forced harmonic oscillation of a system consisting of a hollow cylinder and the surrounding medium under the action of harmonic-time-varying annular forces uniformly distributed in the circumferential direction and pointwise distributed relative to the cylinder axis, acting on the inner surface of the cylinder was studied. It is assumed that non-ideal contact forces of the «shearial-spring» type are performed on the interface of media. Numerical results on the frequency response of interface voltages are presented and discussed. Based on these results, the following main conclusions are made:

– the frequency characteristics of interface stresses are non-monotonic, i.e. there are such values of the vibration frequency of external forces (we denote these frequencies by Ω^* and call them «resonant» frequencies) at which the absolute values of stresses become maximum (these maximum values of stresses are called «resonant» values). Therefore, the nature of the forced vibration of the system «hollow cylinder + infinite environment» is similar to the forced vibration of the system «spring+mass+damper»;

– «resonant» frequencies and «resonant» stress values decrease with increasing elastic modulus of the cylinder material.

4. It is investigated axisymmetric dynamics oscillating-moving ring load acting inside of a hollow cylinder surrounded by an infinite medium, and this study focuses on the influence of frequency of vibration of moving load at critical velocity. Also, the influence of this frequency on the distribution of interface stresses depending on the coordinates and on the speed of the moving load is studied in detail. Numerical results are presented and discussed that illustrate the above-mentioned influence of the vibration frequency of moving forces on the critical velocity and on the distribution of interface stresses. Based on these results, the following main conclusions are made:

– if the moving ring load does not vibrate, then this load has only one critical speed and its value decreases with decreasing ratio h/R;
– when the driving load vibrates, several critical velocities appear, some of which are greater and some of which are less than the critical velocity obtained in the absence of load vibration;
– the appearance and non-appearance of critical velocities at relatively large values of the vibration frequency of the moving load also depends on the mechanical properties of the selected material pairs of the system under consideration and on the ratio h/R;
– the critical velocities obtained for all the considered vibration frequencies of the moving load decreases with a decrease in the ratio h/R;
– the frequency response of interface stresses depends not only on the speed of the moving load, but also significantly depends on the ratio of the mechanical properties of the selected material pairs for the system under consideration.

5. A three-dimensional non-axisymmetric problem on the dynamics of a moving load acting on the inner surface of a hollow cylinder surrounded by an infinite elastic medium is investigated using exact three-dimensional equations and relations of

elastodynamics. It is assumed that the moving load is point-located relative to the axial coordinate and is non-axisymmetrically distributed along the inner circumference of the cross-section of the hollow cylinder in the limit of the central angle α and through the constant vertical component of the total load. A method for solving these types of problems is developed, which is based on the transition to a moving coordinate system, using the Fourier transform and the Fourier representation of transformations of the desired quantities through Fourier series relative to the angular coordinate. It is possible to obtain analytical expressions for the coefficients of the Fourier series and thus determine the Fourier transforms of the desired values. The originals of the required values are determined numerically. Numerical results on the critical velocity and the distribution of interface stresses are presented and analyzed. Based on these results, the following main conclusions are formulated:

- the non-axisymmetricity of the moving load can significantly reduce the value of the critical speed and the magnitude of this decrease depends significantly on the ratio h/R (where h is the thickness of the cylinder, R is the outer radius of the cylinder cross section) and on the ratio $E^{(1)}/E^{(2)}$ where, $E^{(1)}$ $E^{(2)}$) is the modulus of elasticity of the environment material (cylinder);
- for relatively small values of the ratio h/R and $E^{(1)}/E^{(2)}$ for $0.01 \leq h/R \leq 0.5$ the non-axisymmetricity of the moving load does not affect the minimum critical velocity and this critical velocity coincides with the critical velocity obtained in the corresponding axisymmetric case;
- the value of critical speeds does not depend on the angle α;
- among the interface stresses in a quantitative sense, the radial normal stress plays a dominant role;
- the nature of the distribution of the interface normal stress as a function of the circumferential coordinate θ depends on the value of the angle α, etc.

6. A three-dimensional non-axisymmetric problem of forced vibration of a system consisting of a hollow cylinder and an infinite medium surrounding is investigated. It is assumed that the internal

surface of the cylinder is affected by forces that are harmonically variable in time and point-wise located relative to the cylinder axis, which are non-axisymmetrically distributed along the inner circumference of the cylinder cross-section. The solution method described in the previous paragraph was also used for this study. Numerical results on the frequency characteristics of interface stresses and the distribution of these stresses on the interface surface are presented and discussed. Based on these results, the following main conclusions are formulated:

- in the case when the elastic modulus of the cylinder material $E^{(2)}$ significantly greater than the elastic modulus of the ambient material $E^{(1)}$) (for example, when $E^{(1)}/E^{(2)} \leq 0.5$), the frequency response of the normal stress is complex and the corresponding graphs have intervals of «rise-descent», which are caused by the reflection of waves from the interface of media;
- when the elastic modulus of the cylinder material is slightly greater than the elastic modulus of the surrounding material (for example, when $E^{(1)}/E^{(2)} \leq 0.8$), the frequency response of the normal stress is non-monotonic;
- in the case when the materials of the system under consideration are the same (i.e. $E^{(1)}/E^{(2)} \leq 0.8$), the frequency response of the normal stress changes very smoothly and has a pronounced non-monotonic character, which implies that there is a frequency value at which the voltage gets its absolute maximum value;
- the above non-monotonicity of frequency characteristics also occurs when the elastic modulus of the surrounding material is greater than the elastic modulus of the cylinder material (i.e., $E^{(1)}/E^{(2)} = 1$), and so on.

7. The dynamics of a non-axisymmetrically distributed oscillating-moving load acting on the inner surface of a hollow cylinder is studied using exact three-dimensional equations and relations of elastodynamics. The distribution of the oscillating-moving force was assumed as in the previous paragraph, and the methods specified in paragraphs 5 and 6 are used to solve the corresponding boundary value problems. Numerical results on

influence of the so-called «gyroscopic effect» on the critical velocity and on the distribution of normal and shear stresses acting on the interface between the cylinder materials and the surrounding medium are presented and discussed. Based on these results, the following main conclusions are made:

– To determine the critical speed, it is sufficient to use the zero and first terms in the Fourier series expansion of the Fourier transforms of the desired quantities. In this case, the critical velocity defined through the zero term coincides with the critical velocity related to the corresponding axisymmetric cases, and the critical velocities defined through the first term are the critical velocities corresponding to the non-axisymmetric case;
– Due to the «gyroscopic effect», several, and in many cases two critical speeds may appear, the first (second) of which is less than (greater than) the critical speed obtained in the absence of vibration;
– The first (second) critical speed decreases (increases) with increasing frequency of the vibro-moving load;
– The distribution of the interface normal stress depends on the frequency of the oscillating load not only in quantitative but also in qualitative terms.

8. The axisymmetric problem of the dynamics of a moving ring load acting on the inner surface of a hollow two-layer circular cylinder is studied assumed that between the layers of the cylinder there are non-ideal contact relations of the «shearial-spring» type with the parameter F $(0 \leq F \leq \infty)$. Numerical results on the critical velocity and the distribution of interface stresses are obtained, according to which the following main conclusions are made:

– the value of the critical velocity significantly depends on the entire thickness of a two-layer hollow cylinder and increases with increasing thickness;
– an increase in the thickness of the outer layer of the cylinder causes an increase in the values of the critical velocity and these values approach the corresponding value of this velocity obtained for the case when a hollow cylinder is surrounded with

an infinite medium;

– an increase in the values of the parameter F leads to a monotonous decrease in the critical speed;
– in cases where the value of the load speed is sufficiently «far» from the corresponding critical speed, the value of the interface normal stress increases with the thickness of the outer layer of the cylinder. However, in cases where the speed of the load is sufficiently «close» to the corresponding critical velocities, the absolute value of the specified normal stress increases with a decrease in the thickness of the outer layer of the cylinder, etc.

9. Using three-dimensional exact equations and relations of elastodynamics, the dynamics of a moving force that is non-axisymmetrically distributed in the circumferential direction and point-centered in the axial direction acting on the inner surface of a two-layer hollow cylinder is studied. Numerical results related to the critical velocity and interface normal voltage are presented. This study focuses on the influence of the h_1/h_2 ratio (where h_1 is the thickness of the outer layer, h_2 is the thickness of the inner layer) on the critical velocity value, as well as the value of the interface normal voltage. Based on these studies, the following main conclusions are made:

In cases $V/c_2^{(2)} < 1$ (where V is the speed of the load, $c_2^{(2)}$ is the velocity of the shear wave in the material of the inner layer) two critical velocities are obtained: the velocity of the lower value is called the minimum, and the velocity of the higher value is called the maximum;

For relatively small values of the ratio h_2/R (where R is the outer radius of the cross-section of the inner layer of the cylinder), for example, for $h_2/R = 0.1,$ 0.2 and 0.3, the minimum critical velocities correspond to the critical velocities obtained in the axisymmetric case;

For relatively large values of the ratio $h_2/R,$ for example, for $h_2/R = 0.5,$ minimum critical velocities correspond to the non-axisymmetric case, and with an increase in the ratio h_1/h_2, these critical velocities approach the corresponding critical velocities obtained for the system «hollow cylinder + surrounding infinite medium»;

In all the considered cases, as the ratio h_1/h_2 increases, the values of critical velocities increase and approach the corresponding critical velocities obtained for the system «hollow cylinder + surrounding infinite medium»;

As the h_1/h_2 ratio increases, the absolute values of the interface normal stress increase and approach the corresponding values obtained in the case when a hollow cylinder is surrounded by an infinite medium.

Three-dimensional forced vibration of a two-layer hollow cylinder caused by non-axisymmetrically distributed in the circumferential direction and point-centered in the axial direction of harmonic time-varying forces was also studied. Numerical studies are performed on the frequency response of the interface normal stress in the case when the material of the inner layer is made of steel, and the material of the outer layer is made of aluminum. The obtained numerical results allow us to draw the following main conclusions:

In the case when the materials of the layers are the same, due to the reflection of waves from the outer surface of a two-layer hollow cylinder, the frequency response is complex;

In the case where material layers are different, the above frequency response is further complicated by not only the reflection waves from the outer surface of a hollow cylinder, and also due to wave reflection from the surface of the partition environments of the materials of layers;

In addition to the above, the complexity of the frequency response obtained in the case under consideration may also be caused by the occurrence of resonant cases, etc.

III. All the considered problems on the dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium of infinite and finite thickness in the radial direction are solved for the first time in the framework of a piecewise homogeneous body model using exact three-dimensional equations of elastodynamics and TLTEWIS.

IV. The obtained results can be described as the solution to major scientific and technical objectives, creating techniques based on the model of piecewise-homogeneous bodies with attraction of exact three-dimensional equations of elastodynamics and TLTEWIS to determine the critical velocity of moving and oscillating-moving objects in underground transport constructions and double-layer gun barrel.

At the same time, the main scientific results can be considered as a new scientific direction, which consists in developing the theory of dynamics of a moving and oscillating-moving load acting on the inner surface of a hollow cylinder surrounded by an elastic medium based on the model of a piecewise homogeneous body with the use of exact three-dimensional equations of elastodynamics and TLTEWIS.

V. The practical significance of the results of this work is to create a theoretical basis for predicting and preventing catastrophic events caused by moving and oscillating objects in underground transport structures and in multi-layer gun barrels.

The main results of the dissertation were published in the following works:

1. Mehdiyev, M.A., Akbarov, S.D. On the forced vibration of the bi-material elastic system consisting of the hollow cylinder and surrounding elastic medium // VII International Joint Conference of Georgian Mathematical Union & Georgian Mechanical Union. Continuum Mechanics and Related Problems of Analysis dedicated to 125-th birthday anniversary of academician N.Muskhelishvili, − Batumi (Georgia): 5-9 September,−2016, −p. 153-154.
2. Akbarov, S.D., Mehdiyev, M.A. Forced vibration of the elastic system consisting of the hollow cylinder and surrounding elastic medium under perfect and imperfect contact// −Seoul: Structural Engineering & Mechanics, −2017, 62, № 1,−p. 113-123.
3. Akbarov, S.D., Mehdiyev, M.A. On the influence of the initial stresses on the critical velocity of the moving ring load acting in the interior of the hollow cylinder surrounded by an infinite elastic medium// The 2017 World Congress on Advances in

Structural Engineering & Mechanics (ASEM17). Volume of Abstracts, - Ilsan (Seoul), Korea: 28 August-1 September, - 2017, - p. 47.

4. Akbarov, S.D., Mehdiyev, M.A. On the influence of the initial stresses on the critical velocity of the moving ring load acting in the interior of the hollow cylinder surrounded by an infinite elastic medium// In proceeding of the «World Congress on Advances in Structural Engineering & Mechanics (ASEM17)», - Ilsan (Seoul), Korea: 28 August-1 September, - 2017, - 24p.

5. Akbarov, S.D., Mehdiyev, M.A. The interface stress field in the elastic system consisting of the hollow cylinder and surrounding elastic medium under 3D non-axisymmetric forced vibration//−USA: Computers, Materials & Continua,−2018, 54, № 1, −p. 61-81.

6. Akbarov, S.D., Mehdiyev, M.A. Mathematical modeling and theoretical study on the tunnel+soil system // 1st International Conference on Innovations in Natural Science & Engineering (ICINSE 2018), Volume of Abstracts, − Turkish Republic of Northern Cyprus: 3-6 January, − 2018, − p. 5-6.

7. Ozisik, M., Mehdiyev, M.A., Akbarov, S.D. The influence of the imperfectness of contact conditions on the critical velocity of the moving load acting in the interior of the cylinder surrounded with elastic medium// −USA: Computers, Materials & Continua,−2018, 54, № 2, −p. 103-136.

8. Akbarov, S.D., Mehdiyev, M.A. Influence of initial stresses on the critical velocity of the moving load acting in the interior of the hollow cylinder surrounded by an infinite elastic medium//−Seoul: Structural Engineering & Mechanics,−2018, 66, № 1, −p. 45-59.

9. Mehdiyev, M.A., Akbarov, S.D. Parametric resonance of the system consisting the circular hollow cylinder and surrounding elastic medium under action in the interior// Proceedings of the 6th International Conference on Control and Optimization with Industrial Applications Engineering (COIA-2018), Volume I, − Baku: 11-13 July− 2018, −p. 270-272.

10. Akbarov, S.D., Mehdiyev, M.A., Ozisik, M. Three-

dimensional dynamics of the moving load acting on the interior of the hollow cylinder surrounded by the elastic medium//−Seoul: Structural Engineering & Mechanics, −2018, 67, № 2, −p. 185-206.

11. Akbarov, S.D., Mehdiyev, M.A. Dynamics of the system consisting of the hollow cylinder and surrounding infinite elastic medium under acting an oscillating moving ring load on the interior of the cylinder// Coupled Systems Mechanics,−2018, 7, № 5, −p. 525-554.

12. Mehdiyev, M.A. 3D non-axisymmetric dynamic interface shear stress field in the elastic system consisting of the hollow cylinder and surrounding elastic medium// − Baku: Transactions of NAS of Azerbaijan, Series of Physical-Technical & Mathematical Sciences, Issue Mechanics, −2018, 38, № 7, −p. 43-55.

13. Mehdiyev, M.A. On the parametric resonance cases of the system consisting of the circular cylinder and surrounding elastic medium under action in the interior of the cylinder time-harmonic oscillating moving load// − Baku: Caspian Journal of Applied Mathematics, Ecology & Economics, −2018, 6, № 2, −p. 90-98.

14. Akbarov, S.D., Mehdiyev, M.A. 3D Dynamics of the oscillating-moving ring load acting in the interior of the hollow cylinder surrounded with elastic medium// −Seoul: Structural Engineering & Mechanics,−2019, 71, № 6, −p. 713-738.

15. Mehdiyev, M.A., Akbarov, S.D. The «gyroscopic effect» on interface stresses in a «hollow cylinder + surrounding medium» system under action of the oscillating-moving ring load in the interior of the cylinder// 21.Ulusal Mekanik Kongresi, − Niğde, Ömer Halisdemir Üniversitrsi (Turkey), 02-06 Eylül,− 2019, −p. 368-371.

16. Mehdiyev, M.A. On the 3D dynamic normal stress field on the interface of the bi-layered hollow cylinder under action a moving load in the interior of that// − Baku: Caspian Journal of Applied Mathematics, Ecology & Economics, −2019, 7, № 1, −p. 56-64.

17. Mehdiyev, M.A. On the critical velocity of the moving load acting in the interior of the bi-layered hollow cylinder in the 3D

dynamic state//– Baku: Transactions of NAS of Azerbaijan, Series of Physical-Technical & Mathematical Sciences, Issue Mechanics, –2020, 40, № 7, –p. 20-27.

18. Mehdiyev, M.A. On the frequency response of the interface normal stress in the bi-layered hollow cylinder in the 3D dynamic state//–Baku: Transactions of NAS of Azerbaijan, Series of Physical-Technical & Mathematical Sciences, Issue Mechanics, –2020, 40, № 8, –p. 14-21.

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