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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

THE WAYS OF SOLUTION OF ALGEBRAIC PROBLEMS IN THE TEACHING OF INFORMATICS IN HIGH EDUCATION INSTITUTIONS

Speciality: 5801.01 – "Theory and methodology of education and training (methodology of teaching informatics)"

Field of science: Pedagogy

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Baku – 2026

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GENERAL DESCRIPTION OF THE RESEARCH

Relevance and the degree of research of the topic. The basic consept of the development of Education system of Azerbaijan in the period of independence was defined by all azerbaijanian's national Lieder Haydar Aliyev. Modern education system agreed with international priorities and mirroring modern ideas not only declared by Haydar Aliyev, but was performed in real actions due his advices. In 15 June of 1999 was accepted the decree of the President of Republic with the number 168 named "Reform Program for Azerbaijan's System of education", in which the main aim of education reforms' Program was defined as a defence of acquired potentials and its development in high, special, bachelor degree, preschool and school levels of education.

It was arisen a new "Low of Azerbaijan Republic on Education", as a result of performed works in the direction of development Azerbaijan's system of education. In that low, the basic principles of education in world's system of education as humanity, democrace, equality, nationality, quality, and integration and liberalism, in major. Just two last notions (integration and liberalism) could not be realized, if was not be applied information technologies before them.

For active integration into the World's education standards was an important document accepted in 30 October of 2006 Dcabinet of Ministers, with the number 233, named "General Consept of Education of Azerbaijan Republic (National Curriculum)". In the light of Free society, occurrence of a new teacher-student relations, passing from dogmatic mental action to analytic intellectual thinking in education were an important characteristics of a new concept.

One of basic questions standing in front of high education of Azerbaijan is a preparation of high level pedagogical staff, having abilities in modern education technologies. An important condition for successful organization of education process is a support for high level education. Nowadays, studying of psychological and pedagogical owns of electronic education (briefly, e-education or e-learning) is a necessary education technology admitting these

demands [1]. Low acts making necessary an application of e-education in Azerbaijan were included into the low of Azerbaijan Republic on Education (clauses 12.1.3, 13.1.3, 13.2, 13.3). In the decree of the President of Azerbaijan dated 24 October of 2013 named "Governmental Strategy on development of education in Azerbaijan Republic" [33] the quality of education is declared as a haed strategic pririty. Admiting of modern demands of students in qualified education, in perfect humanism, in being of useful patriot sitizen is an important mission.

In accordance with Low main demands for the permance of this obligation are: reachability of information recourses, security of effectiveness and perfectness of scientific information; applicability of modern information technologies; existence of saticfactory number of references given to use of learners in the library hails and satisfactory number of working terminals; organization of computer laboratories with possibility of Internet connection; completion of pedagogical personnel; maintenance of scientific libraries with electronic funds for references and so on.

Continuously expansioning intrusion of information and communication technologies into our life is caused arising of more effective methods of teaching of various subjects in all stages of education. Interactive education method being applied by using of computers made education prosess more effective [4-6]. Use of computers was firstly applied in subject geometry, in which the level of visualization is more appreciated. Construction of sections of space figures helps in the development of space imagination. Arising of more perfect generations of computers, increasing of the speed of calculation makes their increasing use in mathematics and other natural sciences, for solving of various questions of these branches. Today is possible solving of various questions, which were seemed impossible being solved using computers, applying them. Modern science includs areas which is not possible to act without computers.

The use of computers in education prosess stood demand of modern period. The computer technologies has exclusive advantages in studying of various algebraic structures. There are prepared new simple methods of solution by using of computers, for problems

which were solved using complicated methods or were not solvable before. As a result of continues development, there were introduced methods of electron education.

Object and subject of the research. The main object of investigation of dissertation is a study of teaching of high algebra brunch by electronic tools. There are specific methods of electronic teaching. The subject of dissertation is a study of some high algebra questions using e-education methods of teaching.

The aim and objectives of the research. Despite that the investigation deep the psychological and pedagogical features of e-education was not properly investigated. By this reason, in modern period of the teaching of algebraic questions the main problem is studing of question on psychological and pedagogical feature of e-education.

The main aim of investigations for the dissertation consists in learning of basics of theoretical and practical aspects of optimization of education in the field of high algebra by organizers and high cvalifed performers of e-learning in universities.

E-learning (education), as a teaching technology, is arosed in 1990. Firstly elements of e-learning came into the world in some universities and private institutes as a partial application of new teaching technology and these abilities could be realized only by universities with developed economy. In first periods this new technology was offered as a compact discs in application for books. In modern life e-education came into the world in the beginning of 2000th years. Arising of activeinspire with tools panel being able to connect to Internet icreased possibilites of e-education. In the difference from compact discs, now it was possible in every stage of studing, to get prepared materials touch in various themes. In developed universities they began apply new electronic education technologies. Later the number of learners getting an education by electronic ways of teaching begin to increase. It begin to set up a new generation of online learners. Such an education gave possibility to some category of persons to get education, do not leaving of the place of living and working.

Gradually, e-learning tools began active intrusion into the education. Despite that, many investigations in this field preferred to talk about positive features of e-learning. As a basic reason for introducing and development of the concept on e-learning it is possible to indicate construction of its theoretical topics and investigation of personal behavior, social and cultural aspects.

In international network, investigations gradually begin to take place, stimulating learning of pedagogical problems taking into the mind interests of peoples and teaching organizations. Despite that in e-learning there is not an immediate connection between learner and learning, e-learning has many possibilities consisting in existing of wide information recourses. Besides, learning of pedagogical and psichological aspects of a new form of education begins be very actual. As a result of such investigations gradually e-learning begins be more perfect.

Research methods. The main methods of investigations used in the dissertation are observation, research, archive-investigation, the study of student questionnaire, a quiz performed between students and so on. The methodological fundamentals of investigation are based on basic principles: information technology must be systematic, be inactive, and must stimulate personal activity.

The main provisions for defense:

1. The use of prepared program packages (traditional, or online) increases usefulness of the teaching, excludes the loss of time and makes the lesson be interactive and useful.

2. The effectiveness of e-learning depends on trustity of questions which must be solved by using of e-learning.

3. There is need in the use of a number of questions of high algebra by modern online program packages. Arising in the high algebra new teaching methodologies and electronic technologies with investigation demands the preparation of new staff personals, and this is one of questions waiting for their solution.

Scientific novelty of research. The scientific novelty investigation is connected with clearing of the use of e-learning in teaching of high algebra in high scools with the problem waiting for its solution and valid defining of psichological and pedogogical

features of the teaching. In modern period sucsessful application of e-learning connected with preparation of valid methodologies for various systems and subsystems using oline teaching technologies. One of basic circumstances influncing to the quality of e-learning is a use of online program packages or packages prepared by students or teachers.

Theoretical and practical usefulness of investigation. One of functional abilities of educational systems is a preparation of students for social life. In traditional educational technologies this is bought into the life by supervision of teachers. But in e-education learners are in the contact with virtual world and are disjoined from social life. By this reason there arise the problem on agreement between social life and virtual educational action performed by educational personal. To eliminate such difficulties, the useful way may be finding of appropriate agreement between traditional education system and e-learning. In modern period many of question waiting for their solutions demands cooperativ use ofb several branches of the science. In the dissertation the main question being solved is clearing of specific questions of creation of e-learning methods in the teaching of high algebra and defining ways garanteeing its practical use.

Approbation and application. Results of the dissertation were talked about in the seminar acting at the faculty of mathematics and informatics of Ganja State University devoted to the modern problems of Mathematics. These results were reported about at various international conferences. Some of these conferences are indicated below.

1. 8th Internaional Conference on Control and Optimization with industrial applications, 2022, 24-26 August, Baku, Azerbaijan.
2. International scientific-technical conference "New technologies in modern industry" held in 29-30 April 2010. Tbilisi.
3. International conference "Mathematical theories, problems of their application and education", Ganja, 23-25 September 2011.
4. International Conference "Modern Problems Of Mathematics And Mechanics" devoted 60th Anniversary of The

Institute Of Mathematics And Mechanics. 23- 25 October, 2019 , Baku.

The name of organization in which the dissertation is completed. The work was performed at the Department of Informatics and Algebra of Ganja State University.

The total volume of the dissertation with reference to the volume of the structural parts of the dissertation. The dissertation consists of an Introduction, 2 chapters, a Conclusion and Bibliography. The Introduction comprises 6 pages, 11706 characters; Chapter I comprises 65 pages, 106331 characters , Chapter II comprises 65 pages and 80025 characters, the Conclusion comprises 2 pages, 2238 characters. The total volume of the dissertation is 200300 characters, excluding the list of References and Appendices.

BASIC CONTENT OF DISSERTATION

The Introduction section of the dissertation justifies the relevance of the research topic, determines the degree of its development in the known literature, indicates the goals and objectives of the research, reveals its scientific novelty, and provides information on the theoretical and practical effectiveness of the research results and the approval of the work.

Chapter I of the research paper is entitled **"Methodological Problems of E-Learning in Universities"** and consists of five paragraphs. In the first paragraph it is defined the notion of e-learning or e-education and investigated its pedagogical aspects. The e-learning is sa new event in the education prosess. The main feature of this novelty consists of excluding of educational prosess from complete supervision of pedagogical staff and posessing of same part of their time in connection with virtual world. It is natural that this new situation causes born of new branches of education pedagogics and investigation of its correlation with initial pedagogics connected with substitution of it by new pedagogics [25-31, 35-36, 38, 40, 43, 45, 54, 55, 60-67, 70-79].

In the second paragraph psychological aspects of e-learning is studied. In this section influence of geiving of e-learning technologies

some freedom to the learners is studied. There were discussed the role played in e-learning for categories of psychology such notions as recognition, attention, understanding, memory [4-24].

Some of basic questions of dissertation is a question on studying of pedagogical and psychological aspects of e-learning in teaching of the subject line of high algebra and ways of its application. For being impressiv of e-learning it required firstly existence of wide possibilities of teaching. These possibilities concerns firstly to existence of psychological support for students which causes suitable conditions for education. Suitable circumstances make possible to open personal features of students more convexly. One of mostly difficulties of e-education is an absence of immediate connections between learners and the teachers. In this process the visual control for complete development is weak (especially in lectures). As a result it is possible non revealing of such inappropriate situations as scattering of attention, weakining of responsibility with respect to lessons which can caus weekining of recognition.

The third paragraph is characterized with studying of methodological aspects of teaching of high algebra by using electronic technologies. Here the content of the subject high algebra, studying of structure and the analyse of the program are given.

In the forth paragraph it given a comprehensive information about the content and structure are given. Modern course of high algebra is a subject covered during three semesters (as a subject of professional training), and by complemented selected subjects. Selection of the material and planning of hours partially depends on teachers. The subjects Algebra 1, 2, 3 means giving of minimal system of knowledge. In selected subjects the themes of additional sections of high algebra are taking place. In modern period, the online application programs are enriching by new technologies by covering high algebra and other disciplines. Most of these programs are useful for online and free learning. Here, the role of teachers basically, consists of being directing and guiding force. To perform this role, it required teachers deep knowledge in the field of that application programs and tools connected with them.

The course of high algebra is very large and if we take into attention the fact that with every section is connected the Internet sreds as Sage, Magma in which there are great number of complete materials, then it stood clear that the time of taking of requred materials has great importance for effectiveness of the teaching. Here great importance has defining of valid proportion between the use of teaching materials prepared by teacher and materials which taken from the internet. In the dissertation great attention is given to questions which are important from the e-learning is the discussed the question on use of online programs not only in the teaching, but also in the preparation of course and free work projects.

In the subject Algebra-1 teaching in Ganja State University (also in ASPU) the content is given in subject program. This program is as follows:

1. Sets and relations. 2. Algebras. 3. Number structures. Ring of integral numbers, field ofv rational numbers, field of real numbers. Field of complex numbers: axiomatic definition of complex numbers, operations over them, invers of complex numbers. Various interpretations of complex numbers: algebraic, geometric, trigonometric, exponential and matrx views. Roots of unit. On roots of unit over any field. Power and root of complex number. Muavr formula. Various groups set up by roots of unit. Primitive roots.

4. Matrices and determinants. 5. System of linear equations. 6. Linear spaces.

The basic notions considered here are: linear combination, linear independence. Sum of vectors, multiplication of vectors to the number. n-dimensional (arithmatic) linear spaces. The rank of vector systems. Dimension and basis of linear spaces. Isomorphism of equal dimensional linear spaces. Passing from one basis to other. Linear subspaces, their sum and direct sum. Intersection of linear sub spaces, theorem on their dimensions. The space of solutions of homogenious system of linear spaces. Connection between the set of solution and corresponding homogenious system of linear equations. Linear subspaces. Fundamental system of solutions.

The subject program of the course Algebra-2 (I course II semester) overlaps following themes: 1. Linear operators. 2. Euclid spaces. 3. Basic algebraic structures. 4. Integral numbers.

The last topic covers following themes closely connected with secondary school. Theorem on division with reminder. Theory of division. GCD and LCD and their properties. Euclid's algorithm. Prime and composit numbers. Expansion into the prime factors. Systematic fractions. Continuous fractions. Arithmetic functions. Euler function. Congruences and their properties. Residues and their properties. Residues and their properties. Rings oh prime ideals. Rings of residue classes as a factor-ring. Theorems of Ferma and Euler. Congruence's of first degree and systems of linear congruence's. Quadratic residues. Legendre's symbol. Criterian of Euler. Congruences of high degree. Congruences of prime moduli. Exponent with respect to module and primitive roots.

The subject Algebra -3 (II course III semester) includes the following sections:

1. Polynomials of one variables. 2. Polynomials of several variables. 3. Symmetric polynomials. 4. Algebraic extension of the field.

The basic notions connected with these topics are: Rezultant. Exclusion of variable from the system of equations. Basic theorem of Algebra. Viet's formulae. Closeness of Field of Complex numbers.

The fifth paragraph of Chapter I discusses the content and structure of the subject programs on Higher Algebra. It is noted here that the content and structure of the subject are determined by the subject program. The modern Higher Algebra course is a supplementary course covered by the so-called main (compulsory) course and elective subjects in three semesters. The selection of material and the planning of the hours allocated to it also depend on the teacher. The subjects of Algebra 1, 2, 3 involve the provision of a minimal system of knowledge in algebra. Elective subjects include additional sections of Algebra and topics important for a number of applications. In modern times, online application programs are rich in new technologies that cover almost all of the most important issues of algebra and a number of other areas related to it. Most of

these programs are suitable for online training and also for free learning and use. The role of the teacher here is mainly to be a guiding and guiding force in obtaining the system of skills related to the course. To fulfill this role adequately, it is important for teachers to have in-depth knowledge of these applications and the tools associated with them.

The algebra course is quite extensive, and if we take into account that there is quite a lot of material on each section of this subject in Internet environments such as Sage and Magma, then the question of what materials related to this course to study in a limited time frame is important for the effectiveness of the training. Here, it is important to have the right balance between the use of online programs and the teaching of teaching material prepared by and under the guidance of a teacher. In the dissertation work, a large place is given to the use of online programs on topics important from the point of view of e-learning, as well as the study of research material that online programs can be useful not only in teaching, but also in the preparation of final, course and independent works.

The second chapter of the research work is called **"Methodology of teaching some sections of higher algebra using e-learning methods"** and consists of five paragraphs.

In the this chapter the espessially important sections of high algebra from the point of a view e-education such as system of linear equations, normal forms of matrices, solving of algebraic equations, repesentation of numbers by continuous fractions, theory of polynomials, programming of some problems of number theory of geometric charakter, construction of graphs of functions. For students it is given the ways of solution of practically useful problems [2-3, 48-53, 57, 68-69].

The first paragraph examines the problems encountered in online teaching of the topic of Systems of Linear Algebraic Equations. Duaring solution of practical problems it is possible getting of more complicated systems of linear equations. In those systems the number of variables and equations may be large numbers.

Definition 2.1.1. If all free coefficients of a system of linear equations are equal to zero then such a system is called a homogeneous system of linear equations. Other systems are called non-homogenous. The system got from non-homogenous by substituting all of free members by zero, is called the homogenous system corresponding to the given one.

Definition 2.1.2. The system which has non-empty set of solutions is called a joint system of equations, and the system which has not solutions is called non-joint system of equations.

These definitions and notions are best known from the theory of the system of linear equations. In simplest cases solution may be found by graphic methods. This is useful to bring the theory of linear equations obviousness and this method clears a geometric meaning of solutions. After it is considered the graphic solutions of system of linear equations by online Sage programs.

Continuous fractions has great importance in several questions of Algebra and Number Theory.

The second paragraph provides information about continued fractions, which are one of the forms of writing real numbers and have wide applications in many areas of mathematics, and the teaching of problems related to them.

Here one gives some information on expansion of numbers into continuous fractions and computer methods.

Definition 1. Saying finite continuous fraction one means a fraction having a view

$$a_0 + \cfrac{b_1}{a_1 + \cfrac{b_2}{a_2 + \ddots \cfrac{}{\ddots + \cfrac{b_n}{a_n}}}}$$

in which the numbers $a_0,a_1,...,a_n$, $b_1,b_2,...,b_n$ – *are integral numbers.*

In particular case, when $b_1=b_2=...=b_n=1$, $a_i \geq 1$ and $a_n > 1$, then such a fraction is called chain fractions. So, the continuous fraction is a fraction of a view

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{a_n}}},$$

in which the numbers $a_0,a_1,...,a_n$ are natural umbers, and called the elements of continuous fractions. Finite continuous fractions symbolically is denoted as $[a_0,a_1,...,a_n]$.

Continuous fractions has several properties.

1. Every positive rational number can be represented as continuous fraction.

2. There exists only one continuous fractions equal to given rational number.

There is second and more general method of getting for representation of rational numbers in the view of continuous fractions.

Suppose now a real number $\alpha_0 = \alpha$ be given. Denote by a_0 the integral part of this number. Then we have

$$\alpha = \alpha_0 = a_0 + \alpha_1', 0 \le \alpha_1' < 1;$$

If $\alpha_1' > 0$ then we have $\alpha_1 = 1/\alpha_1' > 1$. Again, denoting by a_1 the integral part of the number α_1, we can write

$$\alpha_1 = a_1 + \alpha_2', 0 \le \alpha_2' < 1.$$

Continuing process, after of $n+1$ steps we obtain

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{\alpha_{n+1}}}};$$

if here the number α_{n+1} still will not an integral number, then is possible to continue the process. For rational number α, always is possible find a natural number n such that α_{n+1} is an integral number.

Definition 2.2.2. For $0 \le s \le n$ the fraction $\delta_s = [a_0, a_1, ..., a_s]$ is called s -th close fraction.

To begin studing of properties of close fractions, let us consider two sequence of integral numbers, set up by incomplete quotients: $P_0, P_1, ..., P_n$ and $Q_0, Q_1, ..., Q_n$. This sequence is defined by following recurrent formulae:

$$\left.\begin{array}{l} P_s = a_s P_{s-1} + P_{s-2}, \\ Q_s = a_s Q_{s-1} + Q_{s-2}. \end{array}\right\},$$

in which $P_0 = a_0, P_1 = a_0 a_1 + 1, Q_0 = 1, Q_1 = a_1$, when $s \ge 2$.

In Sage online program there is a prosedure having possibilities for expansion of number into continuous fractions. This prosedure easily can be online used by students and teachers. It is not difficult to expand a number into continuous fractions, and take demanded number of incomplete quotients. The rational number allows expansion into finite, quadratic irrationalities allows expansion inti periodic infinite, but other real numbers allow non-periodic continuous fractions.

Quadratic irrationaities can be expanded into periodic continuous fractions. Other irrationalities allows expansion into non-periodic continuous fractions. There is a conjecture on continuous fractions consisting in the statement that in the expansion of algebraic numbers by continuous fractions all of incomplete quotients are bounded. Among investigations of this direction the role of computer modelling is large. So, as a result of analysis of statistic data, the conjecture formulated above may be incorrect. Analysis of data shows that in expansions of algebraic irrationalities week increasing of incomplete quotients was observed. Science every computer evaluations can give only finite number of incomplete quotients, then the complete solution of the problem remains open.

In this section a program in pascal has been constructed for approximate evaluation of real roots of cubic polynomials with real coefficients (these roots always are existing) [2, 3, 9, 57]. It is obviouse that the root can be evaluated by online program Sage, but in regard to evaluating process, for developing responsibilities of

students to solve put problems it requred in exesizes to give him problems connected with constructin of algorithms and programs and organize theit performance.

The third paragraph discusses the methodology for teaching matrices and operations on them through programming languages.

The theme of canonic forms of matrices is one of basic sections of the theory of matrices. Such forms makes possible to study the construction of linear operators connected with them. By this reason the question plays an importante role. At the same time, Smit canonic forms is importante in many practic questions. But with grooth of sizes evaluation of Smith canonic forms admits great difficulties. Since evalution takes very big time and intellectual potential, in a bounded frames of the time this work can not be performed. In other hand to understand the content of theoretical material by students is importante, because they must understand some questions connected with technical performance, and understand some points which does not tuched in theoretical part. In first stage is useful considering matrices of small dimensions, without using online systems. This allows to understand theoretical content of the teaching material.

In the sequal stages in applications is possible consideration of computer methods for studying Smith canonic forms for matrices of larger order and questions connected with finding of Smith canonic forms and also pass to solution of other questions (defining of transformations leading to the Smith canonic forms and so on).

Finding of Smith canonic forms is possible to perform in Euclid rings, then this question is basicly prefarable to consider on the ring of integral numbers or on the ring of polynomials. Solving of the problem is based on the theorem on division with reminder (Euclid algoritm). By this reason the algoritm of solution, schematically, is possible to give by applying several times Euclid's algorithm and elementary transformations. So, introducing appropriate parameters is possible give an algorithm of solution and construct the programs in some language.

But for e-learning more appropriate way is a use of prepared befor programs which allows avoid the loss of the time and be

garanteed from mistakes. In conditions of e-learning this can be got directly from Internet, addressing to free teaching and applicable program. For solving of this problem it can be used Sage mathematical program. But other sreds (like Mathematic, Magmit) can be used also Maple, MathCad and so on) can be used for solution of this problem also. Here giving freedom to students also may be useful. Student can freely select the sred in accordance with own responsibilities and knowledge. Very important feature is full solution of the problem.

Disjoining of real roots of the polynomial is such a distribution of the roots into disjoint intervals by such way that every interval contains only one its root. For that there are various ways. From theoretical and practice point of a view most effective method is the method of Sturm. Befor introducing of this method let us define the number of sign changing in the finite sequence of distinct from zero real numbers. Consider following sequence of real numbers:

$$4, -2, 1, -4, 1, 3, -2, -7, 6 \qquad (1)$$

In this sequence signs are distributed by following way:

$$+, -, +, -, +, +, -, -, + . \qquad (2)$$

Moving along the system (2) from left to rihght standing of vatious signs consequentli is observed six times:

$$(+, -), (-,+), (+, -), (-,+), (+,-), (-, +) .$$

In this case one says that in the sequence (1) there are 6 sign changes. It is clear that the pair $(+, +)$ does not contain sign changes.

Suppose that we are given with some polynomial $f(x)$ with real coefficients having not repeated roots. With every such polynomial one associates some system of polynomials named Sturm's system.

In questions connected to polynomials, such polynomials with unteqral coefficients are considered that at a given poitnt their value is sufficiently small. The solution of these problems are succesfully solved by using of LLL algorithm, by the help of computers. There is an algorithm in modern Sage program allowing solve it.

The main content of the algorithm is consists of folloing. Let in the space R^n some basis $b_2, ..., b_n$ be given. As it is known, by using

of the orthogonalization process of Gram-Smith a new orthogonal basis $b^*_1, b^*_2, ..., b^*_n$ can be constructed. This process can be performed as follows. By an induction, ve put

$$b^*_i = b_i - \sum_{j<i} \mu_{i,j} b^*_j, b^*_1 = b_1;$$

here the coefficient $\mu_{i,j}$ are defined by following way:

$$\mu_{i,j} = \frac{b_i \cdot b^*_j}{b^*_j \cdot b^*_j}.$$

It is sould be noted that by these formulae the construction of the orthogonal basis can be constructed using the program Sage. Using Sage we can orthogonalize a given basis.

Defintion. If the L□R^n is a subgroup satisfying the condition RL=R^n , then L is called a lattice. Some basis $b_1, b_2, ..., b_n$ is called reduced, if for every pair of indecies in Gram-Smith process is possible take

$$|\mu_{i,j}| \le \frac{1}{2} \qquad (2.1.1)$$

Such that at $i \ge 2$ the following inequality holds

$$|b^*_i| \ge \left(\frac{3}{4} - \mu^2_{i,i-1}\right) |b^*_{i-1}|^2. \qquad (2.1.2)$$

The main consecuence for applications is as follows.

Theorem 2.5.1. Let in R^n some lattice L and ins reduced LLL basis $b_1, b_2, ..., b_n$ be given. Then following relations are satisfied:

1. $d(L) \le \prod_{i=1}^{n} |b_i| \le 2^{n(n-1)/4} d(L)$;
2. For everi $1 \le j \le i \le n$ the following inequality is satisfied:

$|b_j| \le 2^{(i-1).2} |b^*_i|$;

$|b_1| \le 2^{(n-1)./4} d(L)^{1/n}$

and for arbitrary non-zero vector $x \in L$ the inequality below is satisfied:

$|b_1| \le 2^{(n-1)./2} |x|$.

3. For every independent system of vectors $x_1,...,x_s \in L$ and any index $1 \le j \le s$ following ineqality holds

$$\left|b_j\right| \le 2^{(n-1)./2} \max(\left|x_1\right|,...,\left|x_s\right|).$$

4. If $\left|b_1\right| \le 2^{(n-1)./4} d(L)^{1/n}$ and vector $x \in L$ are non-zero then the inequality below holds:

$$\left|b_1\right| \le 2^{(n-1)./2} \left|x\right|.$$

5. For arbitrary independent system of vectors $x_1,...,x_s \in L$ and index $1 \le j \le s$ we have

$$\left|b_j\right| \le 2^{(n-1)./2} \max(\left|x_1\right|,...,\left|x_s\right|).$$

The fourth paragraph of the second chapter discusses the methodology for teaching the problem of the number of integers in various domains in the Python language [48, 50, 56, 69]. In high algebra and number theory in various domains $\Omega \subset \mathbf{R}^2$ of the plain is studied the number of lattica points, that is, the points with integral coordinates. On the base of geometric reasoning it is possible to state that the number of such points approximately equals to the area of corresponding domain.

In some articles by using probabilistic approaches authors considering to the same question from other points of a view investigated the number of lattice points in circles with shifted center. They show that the relation of the difference between the number of lattice points and the area of the circle in modulus to the square root of the radius of the circle tends to infinity, as radius tends to infinity, tends to absolutely continuous function of the center of a circle in Lebesgue sense, that is

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{N_\alpha(R) - \pi R^2}{\sqrt{R}} dR = f(\alpha);$$

here $N_\alpha(R)$ means the number of lattice points in the circle with the center $\alpha \in [0,1]^2$ and the radius $\sqrt{R}$, and $f(\alpha)$ is a function absolutely continuous in uniqe square.

In [56] it was shown that for the reminder term the estimation $R = o(r^{1/2}(\log r)^{1/2})$ is fair at some shift of the center of the circle. Since the direction and value of the shift are not known, then we addressed to the solution of the question by using of computers. We had dissected the unite square into small squares by little steps. In the unite quadrate we take some lattice with small steps. The center of initial circle shiftes to some lattice point. Suppose that we are diven with a point $(u,v) \in [0,1]^2$. Denote by $N(u,v;r)$ the number of lattice points in the circle with center (u,v) and radius r. The relativ error is studied by taking various values for radius, and used special program in Python. Analogical problems are considered in balls.

In algebra and number theory is studied one of geometric questions is a question on lattice points in balls. In such problems it is given some formulae for the number of lattice points depending on someprramters. Denote by $r(n)$ the number of representation the number n as asumof three natural numbers as below:

$$n = x^2+y^2+z^2;$$

here x; y; z are integral numbers.

Then the function T(N) is possible give by the equality:

$$T(N) = \sum_{n \le N} r(n) .$$

This sum expresses the number of lattice points in the sphere $x^2 + y^2 + z^2 = N$. From geometric resoning is clear that the number T(N) approximately equals to the volume of the ball included in the sphere. To estimate the error, let us denote

$$R = R(r) = T(r^2) - (4/3)\pi r^3 .$$

Estimation for modulus of this difference, at large values of the radius is known as a sphere problem. From the result of Gauss on the number of lattice points in circles follows that $R << r^2$.

In 1935 İ.M.Vinoqradov using the method of trigonometris sums obtained:

$$R << r^{1.4+\varepsilon};$$

where $\varepsilon > 0$ is positive number. This relation later was improved by him and Chen as follows:

$$R << r^{4/3+\varepsilon}.$$

Later these estimates were improved by other mathematicians as follows:

$R << r^{29/22+\varepsilon}$ (Çamizo F . and İvanes H. (1995)),

$R << r^{21/16+\varepsilon}$ (Heath-Braun (1997)),

$R << r^{17/14+\varepsilon}$ (Arxipova L. G. (2008)).

In 1926 Szego G. showed that $R = \Omega(r\sqrt{\log r})$, and in 2000 Tsang K. M. had showen the estimate

$$R = \Omega_{\pm}(r\sqrt{\log r}).$$

There is a conjecture stating that

$$R << r^{1+\varepsilon}.$$

Besides with these results, some mathematicians have investigated the problem from other points of a view. These authors began to study the number of lattice points in shifted balls like the question on lattice points in shifted circles. They shifted the ball's center from the origin to the point $\alpha \in [0,1)^3$ and study the number of lattice points. Here the main attention is paid to expressin the number of lattice points as a function of center $\alpha \in [0,1)^3$. Here we, using computer methods, show that during changing of the position of the center $(\theta,\eta,\xi) \in [0,1)^3$ with some steps dispersion of the number of lattice points with the center at that point and the radius r from the volume of the ball is an order of radius. Denote by $N(\theta,\eta,\xi)$ the number of lattice points in the ball with the center $(\theta,\eta,\xi) \in [0,1)^3$ and radius r By special program in Python we investigate a relative error

$$\frac{\left|N(\theta,\eta,\xi) - (4.3)\pi r^3\right|}{r}.$$

The Sage program has many wide possibilities of graphic imaging. By moving the cursor, is possible to watch the image in any aspect angle, move, rotate and so on. It possible to draw graphics of some elementary functions, and in dissertation are provided some

characteristic examples. Here the graphic solution for several non-standard equations.

The last paragraph presents the conduct of the pedagogical experiment and the analysis of the results.

Result of the dissertation are validate by pedagogical experiment. This experiment was performed in 2020-2021 years at II course of mathematical-informatic speciality. In two semigroups in laboratory trainings for the subject “languages of programming -1” were carried out e-learning lessons and online SAGE lessons. In same groups lessons were carried out by traditional nethods. (teachers were Sabina Babayeva and Lala Rahimova). With that aim there were carried out lessons with title:

1. Construction of approximative solution algorithm for cubic equations.
2. Aproximative solution of cubic eequation (by using of Pascal language)
3. Construction of an algorithm for representation of real number by continuous fractions
4. Expansion of real numbers into continuous fractions

The experiment was spent in three stages: defining, studying and controlling stages.

The stage of defining. In the stage the main purpose was clearing of pedagogical situation. İn semigroups of which was planned an experiment were organized meetings with students and teachers, gathered data, were formulated beginning statements. In other semigroups lessons of teachers were litened trough the subject “languages of programming -1”. In this stage with teachers and students was spent a quiz. During quiz to them made appeal with following questions:

Questions addressed to teachers:

1) Are you satisfied with knowledges of students though the subject “languages of programming -1”?

2) Are you satisfied with the content of themes of the subject “languages of programming -1”?

3) Which difficulties are you met during teaching of the theme “solution of equation” and how they can be avoided?

4) Which languages of programming you count available during teaching of the theme of continuous fraction?

Questions addressed to students:

1) Which ways you know for the solution of third degree equation from the subject algebra-3?

2) What must you know for the construction of third degree equations?

3) What continuous fraction is?

4) By what tools one can expand the real numbers into continuous fractions?

5) Which possibilities of computer technics can be used for solution of problems?

Answers of teachers and students have been gathered. By gazering of personal datas it was defined that 72% of teachers and students prefer use of computers and programming in teaching content line.

During analyzing results of quiz it is cleared that the results of survey approximately equal in all semigroups. As a result of analysis it is formed such a mind that the content of searched theme in traditional programs is not suitable for demands of students. By this reason to the theme required a new approach. So, in considered question to theoretic knowledge and practic contenc must be given priority. So, giving attention to theoretic knowledge is useful application of supremacy of Pascal language. It is required the construction of algorithm for the problem and program in Pascal language and performed. For checking of the results may be performed online by using of system SAGE. It is must be noted that the word SAGEQ means in English: System for Algebraic and Geometry Experimentation. As it seen from the name, this sysyem exactly is intended for question of arithmetic, algebra and geometry. The system of creation written in Python and was intended for creative using by users. This means that every person having sufficient level of knowledge in Python can enrich the system by own procedures using internet. System is open to every user in internet and was not monopolized by any corporation and organization. By this reason its use is completely free and without

fee. But many of analogical programs are not open for all users. The user not only can offer own proposals, but the full use of them is possible after some payment. From this point of a view for the online education SAGE is very useful and was several years applied by the author as an experiment.

By applying of this methodology, it required to prepare works of creative and scientific character for demonstration of its effectiveness in studying of informatics and other subjects (for example, algebra). In the teaching process independent action of students must be stimulated.

Jointly with students, there were performed repeating exercises in Pascal and it was preferable them to teaching by using of planned methodology. This process was performed during lessons. By this way the theoretical knowledge of students was supported. As a result, it was constructed the algorithm for solving cubic equations with complex coefficients by using of Cardan's formulae.

Second stage was a stage of learning experiment. Here the basic goal was a realization of offered teaching supplement. For reaching of this hint there were defined control semigroups. The experimental were the semigroups in which the competitor was teaching. For initial help, it was observed the solution of cubic equations by using of online Sage program. All of teachers made familiar with setting and aim of experiment.

After of construction of the algorithm of solution of cubic equations with complex coefficients, preparation of the Pascal program for its solution. During preparation of the program following two modulus were offered to students semigroups: 1) construction of sub-programs for defining of signs of auxiliary quantities t and s; 2) Calculation of auxiliary cubic roots. In addition, independent exercises for application of these program were offered to students and the results were analyzed in additional discussions. In control semigroups the teachers only were being informed with the materials of the experiment.

The percent of quality in control semigroups was 47,8%.

The percent of quality in experimental semigroups was 62,5%. Durin pedagogical experiment collected materials of the experiment

verified in experimental and control semigroups and performed their comparison. After of assessment it was observed that the results in experimental semigroups are better.

The results of the pedagogical experiment can be generalized as follows:

- İn experimantal semigroups experimental indicators of the subject programming ganguages-1 were better than in control semigroups and ended with the growth of interests of students.
- The materials being preparated by teachers were in accor - dance with the level of the knowledge of students, and the teaching methods of teachers were in the sphere of interests of students, caused increasing of their interests to theoretical questions, and growth of their intellectual abilities.

In the **Conclusion** section of the dissertation, the conclusions reached during the research are summarized as follows:

1. The study of the history of the emergence of e-learning technologies and the difficulties they face, as well as the unprecedented pandemic situation, shows that e-learning is the demand of the time. For many reasons, e-learning should be developed forward, overcoming all difficulties, and effective methods of a new form of teaching should be developed. For this reason, research on e-learning technologies is relevant.

2. The dissertation work has determined that teaching algebra using e-learning methods requires creative work on the part of the teacher. At a time when all areas of life are rapidly covered by ICT, one of the important issues is the preparation of personnel who are deeply aware of these technologies and at the same time able to use them in the relevant field of science. These two important aspects of teaching algebra require the organization of learning activities in such a way that the teaching of the basic knowledge of algebra as a subject and the use of new technologies complement each other and ensure the effectiveness of training.

3. At the same time, e-learning should increase students' enthusiasm for scientific research and ensure the correct determination of the advantages and scope of use of new technologies in this direction.

4. It is of particular importance that some issues are taught online under the guidance of a teacher in a specially equipped laboratory using the Internet.

5. Due to a number of its mentioned features, it is important to give preference to the SAGE system in terms of stimulating scientific research.

6. It is advisable to consider some scientific research issues in order to stimulate students' involvement in scientific activity.

7. Building the right model for solving the problem and building a program for its solution requires teachers to have deep theoretical knowledge and to form this knowledge in students.

The content, key propositions, and results of the research are reflected in the following articles published by the applicant:

1. Асланова Н.Ш. Методика преподавания темы «кубические уравнения» в университетах средствами электронного обучения // – Черкасск: Вестник Черкассаого Национального Университета имени Б.Хмельницкого: серия педагогические науки, – 2021. №3, – с.89-99.
2. Асланова Н.Ш. Преподавание темы «кубические уравнения» в университетах средствами электронного обучения // XIV Международная научная конференция «фундаментальные и прикладные проблемы математики и информатики», приуроченная к 90-летию Дагестанского государственного университета. – Махачкала: – 16-19 сентября, – 2021, – с.57-59.
3. Aslanova N.Sh. On Calculation of Elements of Continued Fractions for Cubic Algebraic Numbers // – Baku: Journal of Contemporary Applied Mathematics, – 2020. V.10, № 1, – p.10-13.
4. Aslanova, N. Sh., Jabbarov, I.Sh. On the Number of Lattice Points in the Shif-ted Circles // – Baku: Azerbaijan Journal of Mathematics, – 2020. V.10, № 2, – p.175-190.
5. Aslanova, N.Ş. Cəbri tənliklər mövzusunda Şturm sisteminin e-təlim üçün uyğun olan məsələləri // – Bakı: Pedaqoji

Universitetin Xəbərləri: Riyaziyyat və təbiət elmləri seriyası, – 2021. C.69, №4, – s.18-30.
6. Aslanova N.Ş. Cəbr fənninin e-təlim vasitəsi ilə tədrisinin metodoloji problemləri // – Gəncə: GDU. Elmi xəbərlər. Fundamental, humanitar və təbiət elmləri seriyası, – 2022. №3, – s.150-155.
7. Aslanova N.Ş. Elektron təlimin psixoloji aspektləri // – Bakı: BSU. Humanitar elmlərin öyrənilməsinin aktual problemləri. Ali məktəblərarası elmi məqalələr məcmuəsi, – 2022. №4, – s.226-230.
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9. Jabbarov I.Sh. Aslanova N.Sh. On Optimization Method in the Lattice Points Error Term Problem with Numerical Calculations // Baku: Proceedings of the 8th International Conference on Control and Optimization with industrial applications, – 24-26 August, – 2022, v.2, – p.252-254.

The defense will be held on 18 September 2026, at 14^{00} at the meeting of the Dissertation council FD 2.15 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at Azerbaijan State Pedagogical University.

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Dissertation is accessible at the Azerbaijan State Pedagogical University Library.

Electronic version of the abstract is available on the official website of the of the Azerbaijan State Pedagogical University.

Abstract was sent to the required addresses on 17 August 2026.

Signed for print: 25.06.2026
Paper format: 60×84$^{1/16}$
Volume: 37627 characters
Number of hard copies: 20