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ABSTRACT

of dissertation for the degree of Doctor of Philosophy

SOME BOUNDARY VALUE PROBLEMS FOR THREE-DIMENSIONAL BIANCHI INTEGRO-DIFFERENTIAL EQUATIONS AND THEIR APPLICATION TO OPTIMAL CONTROL

Specialty: 1203.01 – Computer Science

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GENERAL DESCRIPTION OF WORK

Relevance of the topic and degree of development. In the literature, there are many works on the construction of fundamental solutions for hyperbolic equations with dominant mixed derivatives with variable coefficients (or for pseudoparabolic equations). In this direction, the work of D. Colton, M. Kh. Shkhanukov and A. P. Soldatov shows that fundamental solutions can be defined as an analogue of the classical Riemann function, some classes of such equations with sufficiently smooth coefficients. However, the method of Riemann characteristics used in these works is a very limited method and, in general, does not allow such problems to be generalized to simple nonlocal problems, even to equations with constant coefficients. It should be especially noted that so far in the literature the Riemann function for various classes of equations has been constructed only for equations with sufficiently smooth coefficients. There are a number of articles that present analogues of the Riemann function for some special classes of equations with variable coefficients and dominant mixed derivatives $\frac{\partial^4}{\partial x^2 \partial y^2}$ and $\frac{\partial^3}{\partial x \partial y^2}$. From this point of view, we note the works of M.Kh. Shkhanukov and V.A. Vodakhova. R. Di. Vincenzo and A. Villani generalized the concept of the Riemann function for equations with variable coefficients and dominant shifted derivatives $\frac{\partial^3}{\partial x \partial y^2}$. Therefore, problems of correct solvability with nonsmooth variable coefficients associated with dominant mixed derivative hyperbolic equations are a very pressing issue related to the study and construction of fundamental solutions to boundary value problems. In this regard, the demonstrated work is devoted in this dissertation in the general case to the study of boundary value problems of a new type for hyperbolic equations with non-smooth coefficients (i.e. coefficients from Lp-type spaces) and with dominant mixed derivatives $\frac{\partial^3}{\partial x \partial y \partial z}$ and their development of a methodology for constructing a fundamental solution . Note that hyperbolic equations with dominant mixed derivatives $\frac{\partial^3}{\partial x \partial y \partial z}$ are mainly used in modeling vibration processes, and since vibration processes are widespread in nature, this equation has not only theoretical but also practical significance. The main researchers in this area are H.Bateman, M.K. Fage, V.I. Zhegalov, A.N. Mironov, L.B. Mironova, E.A. Utkina, O.M. Dzhokhadze, B. Midodashvili, O.A. Kosheeva, S.S. Akhiev, Sh.Sh. Yusubov, I.G. Mamedov and others. It should be noted that in the dissertation the Bianchi equation studied describes vibration processes, so it can be used in matters of seismic resistance of buildings during earthquakes. Since our country is located in a seismically active zone, the problems considered have not only theoretical but also practical significance, which demonstrates the relevance of the work from a theoretical and practical point of view.

The thesis is presented mainly on the basis of boundary value problems of a new type for three-dimensional integro-differential Bianchi equations, which are included in the class of third-order hyperbolic equations with three real characteristics. In addition, an important fundamental point is that the coefficients of the equation under consideration are functions that satisfy the conditions of summation and boundedness only with degree p, that is, the hyperbolic differential operator under consideration does not have a traditional conjugate differential operator. Therefore, the Riemann function for such an equation cannot be studied by the method of classical characteristics. This dissertation has developed a methodology that essentially uses modern methods of function theory and functional analysis to study such problems. Using this technique, a new concept of adjoint communication problem for boundary value problems is introduced. Such adjoint problems, unlike traditional adjoint problems defined by formal adjoint differential operators, by definition have the form of an integral equation and therefore make sense under fairly weak conditions imposed on the coefficients. With the help of such a conjugate problem, the concept of a fundamental solution is introduced and an integral representation of the solution to the corresponding boundary value problems is found.

The thesis mainly studies two types of boundary value problems for integro-differential equations of Bianchi type under certain conditions, such as integration over coefficients of degree p and boundedness. In contrast to the classics, boundary value problems posed in the arithmetic and geometric middle of the domain for integro-differential equations of Bianchi type, the concept of a generalized Riemann function is given and integral representations of solutions to the boundary value problem are found. This paper provides some generalizations of boundary value problems posed in the arithmetic and geometric middle of the domain, included in the class of correctly solvable problems. At the same time, optimality conditions were found in the form of maximum principle L.S. Pontryagin's by demonstrating the optimal control problem described by such boundary value problems.

Object and subject of study. The object of study of the dissertation is boundary value problems posed in the arithmetic and geometric middle of the domain for integro-differential equations of Bianchi type, and optimal control problems described by such boundary value problems. The subject of the research work is the construction of fundamental solutions to boundary value problems posed in the arithmetic and geometric middle of the domain, and finding integral representations of solutions for boundary value problems, as well as finding optimality conditions in the form of L.S. Pontryagin's maximum principle in the optimal control problem described by such boundary value problems.

Goals and objectives of the study. The goals and objectives of the dissertation are as follows:

- study of various classes of three-dimensional boundary value problems for 3D integro-differential equations with dominant third-order mixed derivatives of the Bianchi type and multidimensional local and nonlocal initial-boundary value problems in non-classical interpretations;

-for such equations from the class of hyperbolic equations with nonsmooth coefficients, mainly two types of posed boundary value problems are studied: in the arithmetic middle of the region and in the geometric middle of the region. Development of a variant of non-traditional increment methods in the study of optimal control problems described by such boundary value problems and their application to obtain new necessary first-order conditions in the form of the L.S. maximum principle L.S. Pontryagin's.

Research methods. The dissertation uses methods from the theory of partial differential equations, the theory of integral and integro-differential equations, the theory of boundary value problems, the theory of linear operator equations in Banach spaces, functional analysis and the theory of optimal controls, From the numerical solution algorithm in the form of the Picard method of successive approximations and the corresponding software.

Main results to be defended:

- Under sufficiently natural conditions in a non-classical formulation, the posed boundary value problem in the middle of the domain for the 3D Bianchi integro-differential equation is justified;
- The operator form of the posed boundary value problem in a non-classical formulation on the arithmetic middle of the domain is given; An equivalent integral equation for such a boundary value problem in a non-classical formulation was constructed and the correctness of the boundary value problem was proved;
- To study the posed boundary value problem in a non-classical formulation on the arithmetic middle of the domain for the 3D Bianchi integro-differential equation, after constructing the conjugate operator in integral form, a fundamental solution for the integral representation of the solution is constructed. Then boundary value problems posed at the arithmetic middle of the domain and some generalizations of them are given;

- The posed boundary value problem on the geometric middle of the domain for the 3D Bianchi integro-differential equation with non-smooth coefficients is substantiated and the equation under boundary conditions is reduced to a single operator equation;
- An equivalent integral equation was constructed for the posed boundary value problem in a non-classical formulation on the geometric middle of the domain and the correctness of the boundary value problem was proved;
- For an integral representation of the solution for the posed boundary value problem in a non-classical formulation, at the geometric middle of the domain, the conjugate operator is first constructed in integral form and then the fundamental solution is constructed. At the end, the posed boundary value problems on the geometric middle of the region and some of their generalizations are given;
- In the model case, for the problem of optimal control described by 3D Bianchi integro-differential equations, the necessary optimality conditions were obtained in the form of L.S. Pontryagin's maximum principle;
- Numerical solution algorithms have been developed for boundary value problems on the arithmetic and geometric middle of the domain in non-classical formulations.
- **Scientific novelty of the research:** The dissertation work obtained the main new scientific results presented below:
- In a non-classical formulation, the posed boundary value problem on the arithmetic middle of the domain for the 3D Bianchi integro-differential equation with non-smooth coefficients is justified;
- The posed non-classical boundary value problem on the arithmetic middle of the domain for the 3D Bianchi integro-differential equation with non-smooth coefficients is reduced to an equivalent integral equation and the correctness of such a boundary value problem is proven;
- To study the posed boundary value problem in a non-classical formulation on the arithmetic middle of the domain for the 3D

Bianchi integro-differential equation, after constructing the conjugate operator in integral form, a fundamental solution was constructed for the integral representation of the solution. Then boundary value problems posed at the arithmetic middle of the domain and some generalizations of them are given;

- The posed boundary value problem in a non-classical formulation on the geometric middle of the region is justified and the equation under the boundary conditions is reduced to a single operator equation;
- An equivalent integral equation was constructed for the posed boundary value problem on the geometric middle of the domain and the homeomorphism theorem was proved;
- For an integral representation of the solution for the posed boundary value problem in a non-classical formulation, at the geometric middle of the domain, the conjugate operator is first constructed in integral form and then the fundamental solution is constructed. At the end, some generalizations of the posed boundary value problems on the geometric middle of the region are given;
- For the problem of optimal control of the described boundary value problems on the geometric middle of the domain for a 3D Bianchi integro-differential equation with non-smooth coefficients, necessary and sufficient optimality conditions were obtained in the form of L.S. Pontryagin’s maximum principle;
- Numerical solution algorithms have been developed in the form of methods of successive Picard approximations for boundary value problems on the arithmetic and geometric middle of the domain in non-classical formulations.

Theoretical and practical importance of the reserch.

The results obtained in the dissertation are primarily theoretical in nature. However, this study has both theoretical and practical significance. So, since the Bianchi equation describes vibration processes and the problems are basically three-dimensional cuboids, such questions can also be applied to the seismic resistance of buildings during earthquakes.

Approbation of the work. The results of the dissertation work were presented at the 5th International Conference "Nonlocal boundary value problems and related problems of mathematical biology, informatics and physics", at the international conference "Modern problems and applied mathematics of innovative technologies in oil and gas products", dedicated to the 90th anniversary of Academician Azad Khalil oglu Mirzajanzade (Baku, 2018), at the Republican scientific conference "Current problems of mathematics and mechanics", dedicated to the 96th anniversary of the birth of the National Leader of the Azerbaijani people Heydar Aliyev (Baku, 2019),at the international scientific conference "Operators, functions and systems of mathematical physics" (Baku, 2019), at the International scientific conference "Modern problems of mathematics and mechanics", dedicated to the 60th anniversary of the Institute of Mathematics and Mechanics (Baku, 2019), at International scientific conference "Modern problems of mathematics and mechanics", dedicated to the 110th anniversary of the birth of academician Ibrahim Ibragimov (Baku, 2022), "Fundamental problems of mathematics and the use of intellectual technologies in education" at the II Republican scientific conference (Sumgait, 2022 .), at the III International scientific conference "Theoretical and Applied Problems of Mathematics" (Sumgait, 2023), at the International scientific conference "Modern problems of mathematics and mechanics", dedicated to the 100th anniversary of the National Leader Heydar Aliyev (Baku, 2023),at the International scientific and practical conference "Current problems of military art" dedicated to the 100th anniversary of the birth of National Leader Heydar Aliyev (Baku, 2023), at the V International conference "Problems of Cybernetics and Informatics" (PCI) (Baku, 2023), VII International scientific conference "Nonlocal boundary value problems and related problems of mathematical biology, informatics and physics" (Nalchik, 2023).

Authors personal contribution. All results and proposals obtained in the dissertation belong to the author. The formulation of the problem belongs to the scientific supervisor.

Authors publications. The main results obtained in the dissertation were published in 26 scientific works of the author. The list of works is given at the end of the abstract link.

The name of the institution where the dissertation work was performed. The dissertation work was completed at the Department of «Mathematical analysis and differential equations» of Sumgayit State University.

Total volume of the dissertation work indicating separately the volume of each structural units. Dissertation work (title page - 385 characters, table of contents - 2168 characters), introduction – 47090 characters, chapter I - 88849 characters, chapter II - 97928 characters, quantity, results, list of 138 references and from the attachments section. The total volume of the dissertation is 236420 characters (161 pages).

CONTENT OF THE DISSERTATION

The dissertation work consists of an introduction, two chapters, a conclusion, a list of references and a numerical solution algorithm. The first chapter consists of six and the second chapter consists of seven paragraphs. The introductory part of the work indicates the relevance of the topic and the degree of development, the goals and objectives of the research, research methods, the main conclusions of the defended, the scope of the research, the scientific novelty of the research, theoretical and practical value, and such tasks were highlighted. Finally, the results obtained in the thesis are briefly and concisely presented in the introduction.

Chapter I is devoted to boundary value problems in the middle of the domain for 3D Bianchi integro-differential equations with nonsmooth coefficients and the introduction of the new concept of the Riemann function.

In 1.1 the 3D Bianchi three-dimensional integro-differential equation is considered:

$$(V_{1,1,1}u)(x,y,z) \equiv u_{xyz}(x,y,z) + \sum_{\substack{i+j+k\leq 3 \\ i,j,k=0,1}} A_{i,j,k}(x,y,z) D_x^i D_y^j D_z^k u(x,y,z) +$$

$$+\int\limits_{\frac{x_0+x_1}{2}}^{x}\int\limits_{\frac{y_0+y_1}{2}}^{y}\int\limits_{\frac{z_0+z_1}{2}}^{z}\sum_{\substack{i+j+k<3\\ i,j,k=0,1}} K_{i,j,k}(\tau,\xi,\eta;x,y,z)D_x^iD_y^jD_z^k u(\tau,\xi,\eta)\times$$

$$\times d\tau d\xi d\eta=\varphi_{1,1,1}(x,y,z), \qquad (1)$$

Here $u=u(x,y,z)$ the required function is defined in the domain G; Functions $A_{i,j,k}(x,y,z)$ are given measurable functions in the domain $G=G_1\times G_2\times G_3$ and functions $K_{i,j,k}(\tau,\xi,\eta;x,y,z)$ are given measurable bounded functions in $G\times G$, where, $G_1=(x_0,x_1),\ G_2=(y_0,y_1),\ G_3=(z_0,z_1);\ \varphi_{1,1,1}(x,y,z)$ the given measurable function is in G.

Under these conditions for solving $u(x,y,z)$ equation (1), the desired function in the space of S.L. Sobolev $W_p^{(1,1,1)}(G)=$

$$=\{u\in L_p(G)/D_x^iD_y^jD_z^k u\in L_p(G);\ i,j,k=0,1\},\quad (\text{где},\quad 1\le p\le\infty)\quad .$$

Norm in space $W_p^{(1,1,1)}(G)$ is defined as follows: $\|u\|_{W_p^{(1,1,1)}(G)}=\sum\limits_{i,j,k=0}^{1}\left\|D_x^iD_y^jD_z^k u\right\|_{L_p(G)}$.

For equation (1), the stated conditions of the classical form in the middle of the domain are given in the following form

$$\begin{cases} u/_{x=\frac{x_0+x_1}{2}}=\Phi(y,z),\\ u/_{y=\frac{y_0+y_1}{2}}=\Psi(x,z),\\ u/_{\frac{z_0+z_1}{2}}=g(x,y), \end{cases} \qquad (2)$$

where, functions $\Phi(y,z),\Psi(x,z)$ and $g(x,y)$ given measurable functions in G. It is obvious that in conditions (2) functions Φ, Ψ, g $\Phi \in W_p^{(1,1)}(G_2 \times G_3), \Psi \in W_p^{(1,1)}(G_1 \times G_3), g \in W_p^{(1,1)}(G_1 \times G_2)$ in addition to these conditions, it must also satisfy the agreement conditions as follows:

$$\begin{cases} \Phi(\frac{y_0 + y_1}{2}, z) = \Psi(\frac{x_0 + x_1}{2}, z), \\ \Phi(y, \frac{z_0 + z_1}{2}) = g(\frac{x_0 + x_1}{2}, y), \\ \Psi(x, \frac{z_0 + z_1}{2}) = g(x, \frac{y_0 + y_1}{2}), \end{cases} \quad (3)$$

The problem is to find boundary conditions such that these boundary conditions do not contain additional information about the solution and do not require any additional conditions, such as agreement conditions (3). To do this, the following boundary conditions are considered:

$$\begin{cases} V_{0,0,0}u \equiv u(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}, \frac{z_0 + z_1}{2}) = \varphi_{0,0,0} \\ (V_{1,0,0}u)(x) \equiv u_x(x, \frac{y_0 + y_1}{2}, \frac{z_0 + z_1}{2}) = \varphi_{1,0,0}(x), \\ (V_{0,1,0}u)(y) \equiv u_y(\frac{x_0 + x_1}{2}, y, \frac{z_0 + z_1}{2}) = \varphi_{0,1,0}(y), \\ (V_{0,0,1}u)(z) \equiv u_z(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}, z) = \varphi_{0,0,1}(z), \\ (V_{1,1,0}u)(x, y) \equiv u_{xy}(x, y, \frac{z_0 + z_1}{2}) = \varphi_{1,1,0}(x, y), \\ (V_{0,1,1}u)(y, z) \equiv u_{yz}(\frac{x_0 + x_1}{2}, y, z) = \varphi_{0,1,1}(y, z), \\ (V_{1,0,1}u)(x, z) \equiv u_{xz}(x, \frac{y_0 + y_1}{2}, z) = \varphi_{1,0,1}(x, z), \end{cases} \quad (4)$$

where, $\varphi_{0,0,0} \in R$ is a given number and $\varphi_{i,j,k}$ the given functions satisfy the following condition:

$\varphi_{1,0,0}(x) \in L_p(G_1), \varphi_{0,1,0}(y) \in L_p(G_2), \varphi_{0,0,1}(z) \in L_p(G_3),$

$\varphi_{1,1,0}(x,y) \in L_p(G_1 \times G_2), \varphi_{0,1,1}(y,z) \in L_p(G_2 \times G_3), \varphi_{1,0,1}(x,z) \in L_p(G_1 \times G_3).$

In this section we prove that the classical boundary value problem (1), (2) in the middle of the domain is, generally speaking, equivalent to a problem of the form (1), (4).

In 1.2, the operator form of problem (1), (4) is specified:

$$Vu = \varphi, \tag{5}$$

where, V the vector operator and is defined using the equality

$V = (V_{0,0,0}, V_{1,0,0}, V_{0,1,0}, V_{0,0,1}, V_{1,1,0}, V_{0,1,1}, V_{1,0,1}, V_{1,1,1}) : W_p^{(1,1,1)}(G) \to \mathrm{E}_p^{(1,1,1)}$

and φ the given vector element is in the form

$\varphi = (\varphi_{0,0,0}, \varphi_{1,0,0}, \varphi_{0,1,0}, \varphi_{0,0,1}, \varphi_{1,1,0}, \varphi_{0,1,1}, \varphi_{1,0,1}, \varphi_{1,1,1})$

from spaces

$\mathrm{E}_p^{(1,1,1)} \equiv R \times L_p(x_0, x_1) \times L_p(y_0, y_1) \times L_p(z_0, z_1) \times L_p(G_1 \times G_2) \times$
$\times L_p(G_2 \times G_3) \times L_p(G_1 \times G_3) \times L_p(G).$

In 1.3, problem (1), (4) is reduced to an equivalent integral equation using the integral representation of functions $u \in W_p^{(1,1,1)}(G)$. A homeomorphism theorem is proved for the existence of a bounded inverse operator using the constructed equivalent integral equation.

In 1.4, schemes for constructing the conjugate operator in integral form are specified. Using equalities $f(Vu) = (V^* f)(u)$ and also from the general form of the linear functional on $W_p^{(1,1,1)}(G)$ it turns out that the operator V has a conjugate operator in the following form:

$V^* = (\omega_{0,0,0}, \omega_{1,0,0}, \omega_{0,1,0}, \omega_{0,0,1}, \omega_{1,1,0}, \omega_{0,1,1}, \omega_{1,0,1}, \omega_{1,1,1}) : \mathrm{E}_q^{(1,1,1)} \to \mathrm{E}_q^{(1,1,1)}$

where, the operators $\omega_{i,j,k}$ are defined using specifically integral equalities.

In 1.5, a fundamental solution will be constructed for problem (1), (4). So, first, taking an arbitrary point $(x,y,z)\in\overline{G}$, the following system is considered:

$$\begin{cases}\omega_{0,0,0}f=1,\\(\omega_{1,0,0}f)(\alpha)=\theta(x-\alpha),\alpha\in(x_0,x_1),\\(\omega_{0,1,0}f)(\beta)=\theta(y-\beta),\beta\in(y_0,y_1),\\(\omega_{0,0,1}f)(\gamma)=\theta(z-\gamma),\gamma\in(z_0,z_1),\\(\omega_{1,1,0}f)(\alpha,\beta)=\theta(x-\alpha)\theta(y-\beta),(\alpha,\beta)\in G_1\times G_2,\\(\omega_{0,1,1}f)(\beta,\gamma)=\theta(y-\beta)\theta(z-\gamma),(\beta,\gamma)\in G_2\times G_3,\\(\omega_{1,0,1}f)(\alpha,\gamma)=\theta(x-\alpha)\theta(z-\gamma),(\alpha,\gamma)\in G_1\times G_3,\\(\omega_{1,1,1}f)(\alpha,\beta,\gamma)=\theta(x-\alpha)\theta(y-\beta)(z-\gamma),(\alpha,\beta,\gamma)\in G,\end{cases}\tag{6}$$

here, $\theta(z)$ Heaviside function on R, that is $\theta(z)=\begin{cases}1, z>0\\0, z\le 0\end{cases}$.

Definition 1. If for any given point $(x,y,z)\in\overline{G}$ system (6) has at least one solution

$$f(x,y,z)=(f_{0,0,0}(x,y,z),\ f_{1,0,0}(\alpha;x,y,z),$$
$$f_{0,1,0}(\beta;x,y,z),\ f_{0,0,1}(\gamma;x,y,z),\ f_{1,1,0}(\alpha,\beta;x,y,z),$$
$$f_{0,1,1}(\beta,\gamma;x,y,z), f_{1,0,1}(\alpha,\gamma;x,y,z), f_{1,1,1}(\alpha,\beta,\gamma;x,y,z))\in \mathrm{E}_q^{(1,1,1)},$$

then we will call this solution a fundamental solution (θ-fundamental) for problem (1), (4).

In this section we prove the following theorem:

Theorem 1. Problem (1), (4) has a unique fundamental solution $f(x,y,z)$. Wherein,solution $u\in W_p^{(1,1,1)}(G)$ of the problem (1), (4) using the θ-fundamental solution can be represented in the following form:

$$u(x,y,z)=\varphi_{0,0,0}f_{0,0,0}(x,y,z)+\int\limits_{\frac{x_0+x_1}{2}}^{x_1}\varphi_{1,0,0}(\alpha)f_{1,0,0}(\alpha;x,y,z)d\alpha+$$

$$+\int\limits_{\frac{y_0+y_1}{2}}^{y_1}\varphi_{0,1,0}(\beta)f_{0,1,0}(\beta;x,y,z)d\beta+\int\limits_{\frac{z_0+z_1}{2}}^{z_1}\varphi_{0,0,1}(\gamma)f_{0,0,1}(\gamma;x,y,z)d\gamma+$$

$$+\int\limits_{\frac{x_0+x_1}{2}}^{x_1}\int\limits_{\frac{y_0+y_1}{2}}^{y_1}\varphi_{1,1,0}(\alpha,\beta)f_{1,1,0}(\alpha,\beta;x,y,z)d\alpha d\beta+$$

$$+\int\limits_{\frac{y_0+y_1}{2}}^{y_1}\int\limits_{\frac{z_0+z_1}{2}}^{z_1}\varphi_{0,1,1}(\beta,\gamma)f_{0,1,1}(\beta,\gamma;x,y,z)d\beta d\gamma+$$

$$+\int\limits_{\frac{x_0+x_1}{2}}^{x_1}\int\limits_{\frac{z_0+z_1}{2}}^{z_1}\varphi_{1,0,1}(\alpha,\gamma)f_{1,0,1}(\alpha,\gamma;x,y,z)d\alpha d\gamma+$$

$$+\int\limits_{\frac{x_0+x_1}{2}}^{x_1}\int\limits_{\frac{y_0+y_1}{2}}^{y_1}\int\limits_{\frac{z_0+z_1}{2}}^{z_1}\varphi_{1,1,1}(\alpha,\beta,\gamma)f_{1,1,1}(\alpha,\beta,\gamma;x,y,z)d\alpha d\beta d\gamma.$$

In 1.6 the stated boundary value problems in the arithmetic middle of the domain and some generalizations of them are given.

Chapter II is devoted to various classes of local and nonlocal boundary value problems for 3D Bianchi integro-differential equations with nonsmooth coefficients and their application to optimal control problems.

In 2.1, a boundary value problem defined in a non-classical formulation in the geometric middle of the domain is studied and the 3D Bianchi integro-differential equation is considered as follows:

$$(V_{1,1,1}u)(x,y,z)\equiv u_{xyz}(x,y,z)+\sum_{\substack{i+j+k\leq 3\\ i,j,k=\overline{0,1}}}A_{i,j,k}(x,y,z)D_x^iD_y^jD_z^ku(x,y,z)+$$

$$+\int\limits_{\sqrt{x_0x_1}}^{x}\int\limits_{\sqrt{y_0y_1}}^{y}\int\limits_{\sqrt{z_0z_1}}^{z}\sum_{\substack{i+j+k\leq 3\\ i,j,k=\overline{0,1}}}K_{i,j,k}(\tau,\xi,\eta;x,y,z)D_x^iD_y^jD_z^ku(\tau,\xi,\eta)\times$$

$$\times d\tau d\xi d\eta = \varphi_{1,1,1}(x,y,z), \qquad (7)$$

Here $u = u(x,y,z)$ the required function is defined in the domain G; Functions $A_{i,j,k}(x,y,z)$ are given measurable functions in the domain $G = G_1 \times G_2 \times G_3$ and functions $K_{i,j,k}(\tau,\xi,\eta;x,y,z)$ are given measurable bounded functions in $G \times G$, where, $G_1 = (x_0,x_1)$, $x_0 \ge 0$, $G_2 = (y_0,y_1), y_0 \ge 0$, $G_3 = (z_0,z_1)$, $z_0 \ge 0$; $\varphi_{1,1,1}(x,y,z)$ the given measurable function is in G.

Under these conditions for solving $u(x,y,z)$ equation (7), the desired function in the space of S.L. Sobolev $W_p^{(1,1,1)}(G) =$

$= \{u \in L_p(G) / D_x^i D_y^j D_z^k u \in L_p(G);\ i,j,k = 0,1\}$, (где, $1 \le p \le \infty$) .

Norm in space $W_p^{(1,1,1)}(G)$ is defined as follows:

$$\|u\|_{W_p^{(1,1,1)}(G)} = \sum_{i,j,k=0}^{1} \left\| D_x^i D_y^j D_z^k u \right\|_{L_p(G)} .$$

For equation (7), the stated conditions of the classical form in the geometric middle of the domain are given in the following form

$$\begin{cases} u /_{x=\sqrt{x_0 x_1}} = \Phi(y,z), \\ u /_{y=\sqrt{y_0 y_1}} = \Psi(x,z), \\ u /_{z=\sqrt{z_0 z_1}} = g(x,y), \end{cases} \qquad (8)$$

where, functions $\Phi(y,z), \Psi(x,z)$ and $g(x,y)$ given measurable functions in G. It is obvious that in conditions (8) functions Φ, Ψ, g $\Phi \in W_p^{(1,1)}(G_2 \times G_3), \Psi \in W_p^{(1,1)}(G_1 \times G_3), g \in W_p^{(1,1)}(G_1 \times G_2)$ in addition to these conditions, it must also satisfy the agreement conditions as follows:

$$\begin{cases} \Phi(\sqrt{y_0 y_1}, z) = \Psi(\sqrt{x_0 x_1}, z), \\ \Phi(y\ , \sqrt{z_0 z_1}) = g(\sqrt{x_0 x_1}, y), \\ \Psi(x\ , \sqrt{z_0 z_1}) = g(x, \sqrt{y_0 y_1}), \end{cases} \qquad (9)$$

The problem is to find boundary conditions such that these boundary conditions do not contain additional information about the solution and do not require any additional conditions, such as agreement conditions (9). To do this, the following boundary conditions are considered:

$$\begin{cases} V_{0,0,0}u \equiv u(\sqrt{x_0x_1}, \sqrt{y_0y_1}, \sqrt{z_0z_1}) = \varphi_{0,0,0} \\ (V_{1,0,0}u)(x) \equiv u_x(x, \sqrt{y_0y_1}, \sqrt{z_0z_1}) = \varphi_{1,0,0}(x), \\ (V_{0,1,0}u)(y) \equiv u_y(\sqrt{x_0x_1}, y, \sqrt{z_0z_1}) = \varphi_{0,1,0}(y), \\ (V_{0,0,1}u)(z) \equiv u_z(\sqrt{x_0x_1}, \sqrt{y_0y_1}, z) = \varphi_{0,0,1}(z), \\ (V_{1,1,0}u)(x,y) \equiv u_{xy}(x, y, \sqrt{z_0z_1}) = \varphi_{1,1,0}(x,y), \\ (V_{0,1,1}u)(y,z) \equiv u_{yz}(\sqrt{x_0x_1}, y, z) = \varphi_{0,1,1}(y,z), \\ (V_{1,0,1}u)(x,z) \equiv u_{xz}(x, \sqrt{y_0y_1}, z) = \varphi_{1,0,1}(x,z), \end{cases} \tag{10}$$

where, $\varphi_{0,0,0} \in R$ is a given number and $\varphi_{i,j,k}$ the given functions satisfy the following condition:

$$\varphi_{1,0,0}(x) \in L_p(G_1), \varphi_{0,1,0}(y) \in L_p(G_2), \varphi_{0,0,1}(z) \in L_p(G_3),$$
$$\varphi_{1,1,0}(x,y) \in L_p(G_1 \times G_2), \varphi_{0,1,1}(y,z) \in L_p(G_2 \times G_3), \varphi_{1,0,1}(x,z) \in L_p(G_1 \times G_3).$$

In this section we prove that the classical boundary value problem in the geometric middle of the domain (7), (8), generally speaking, is equivalent to a problem of the form (7), (10). As you can see, problem (7), (10) is more natural in its formulation than problem (7), (8). This is due to the fact that in the formulation of problem (7), (10) the right-hand side of the boundary conditions does not require any agreement conditions. Therefore, problem (7), (10) can be considered as a problem of a new type, posed in the geometric middle of the domain.

In 2.2 the operator form of this boundary value problem in the geometric middle of the domain is given.

In 2.3, an equivalent integral equation will be constructed for a boundary value problem posed in the geometric middle of the region. Using the constructed integral equation, we prove the theorem on the correct solvability of the boundary value problem (7)-(10) posed in the geometric middle of the domain (the homeomorphism theorem).

2.4. is devoted the construction of the conjugate operator in integral form.

2.5. is devoted to the construction of a fundamental solution for a boundary value problem posed in the geometric middle of the domain, and, as in 1.5, a system of the form (6) is considered, and a is determined fundamental solution of problem (7), (10).

In this section we prove the following theorem:

Theorem2. Problem (7), (10) has a unique fundamental solution $f(x, y, z)$. In this case, the solution of the problem (7), (10) can be represented similarly to Theorem 1 using the θ-fundamental solution.

In 2.6, the posed boundary value problems in the geometric middle of the domain and some of their generalizations are considered.

2.7. is devoted to obtaining the necessary optimality conditions for optimal control problems described by the 3D Bianchi integro-differential equation with non-smooth coefficients.

It is assumed that the controlled process is described by the equation

$$(V_{1,1,1}u)(x,y,z) \equiv u_{xyz}(x,y,z) + \sum_{\substack{i+j+k\leq 3 \\ i,j,k=\overline{0,1}}} A_{i,j,k}(x,y,z) D_x^i D_y^j D_z^k u(x,y,z) +$$

$$+ \int\limits_{\sqrt{x_0 x_1}}^{x} \int\limits_{\sqrt{y_0 y_1}}^{y} \int\limits_{\sqrt{z_0 z_1}}^{z} \sum_{\substack{i+j+k\leq 3 \\ i,j,k=\overline{0,1}}} K_{i,j,k}(\tau,\xi,\eta; x,y,z) D_x^i D_y^j D_z^k u(\tau,\xi,\eta) d\tau d\xi d\eta =$$

$$= \varphi(x,y,z,v(x,y,z)), \tag{11}$$

$(x, y, z) \in G = (x_0, x_1) \times (y_0, y_1) \times (z_0, z_1)$ under the conditions of the geometric middle of the domain in the following form:

$$\begin{cases} V_{0,0,0}u \equiv u(\sqrt{x_0x_1}, \sqrt{y_0y_1}, \sqrt{z_0z_1}) = \varphi_{0,0,0} \\ (V_{1,0,0}u)(x) \equiv u_x(x, \sqrt{y_0y_1}, \sqrt{z_0z_1}) = \varphi_{1,0,0}(x) \\ (V_{0,1,0}u)(y) \equiv u_y(\sqrt{x_0x_1}, y, \sqrt{z_0z_1}) = \varphi_{0,1,0}(y) \\ (V_{0,0,1}u)(z) \equiv u_z(\sqrt{x_0x_1}, \sqrt{y_0y_1}, z) = \varphi_{0,0,1}(z) \\ (V_{1,1,0}u)(x,y) \equiv u_{xy}(x, y, \sqrt{z_0z_1}) = \varphi_{1,1,0}(x,y) \\ (V_{0,1,1}u)(y,z) \equiv u_{yz}(\sqrt{x_0x_1}, y, z) = \varphi_{0,1,1}(y,z) \\ (V_{1,0,1}u)(x,z) \equiv u_{xz}(x, \sqrt{y_0y_1}, z) = \varphi_{1,0,1}(x,z) \end{cases} \tag{12}$$

Here $A_{i,j,k}(x,y,z)$, $K_{i,j,k}(\tau,\xi,\eta;x,y,z)$ and $\varphi_{i,j,k}$ – given non-smooth functions; $\varphi(x,y,z,v)$ given function in $G \times R^r$, satisfies the Carathéodory condition and for any positive $\delta > 0$ there is such a function $\varphi_\delta^0(x,y,z) \in L_p(G)$, almost everywhere for everyone $(x,y,z) \in G$ $|\varphi(x,y,z,v(x,y,z))| \le \varphi_\delta^0(x,y,z)$ and for everyone $v \in R^r$, $\|v\| = \sum_{i=1}^{r} |v_i| \le \delta$; $v(x,y,z) = (v_1(x,y,z), \cdots, \ v_r(x,y,z))$ r - dimensional control vector function. It is assumed that $v(x,y,z) =$ $= (v_1(x,y,z), \cdots, v_r(x,y,z))$ the vector function is measurable and bounded in G and takes its meaning at almost all points $(x,y,z) \in G$ from any given set $U \subset R^r$. Then we will call this vector function admissible control. Let us denote the set of all admissible controls by U_∂ .

In this section we consider the optimal control problem as follows: find such an admissible control $v(x,y,z)$ from U_∂, which gives the smallest value to the following multipoint functional for solving problem (11)-(12):

$$S(v) = \sum_{k=1}^{N} \Big[a_k^{(1,0,0)} u(x_k^{(1)}, y_1, z_1) + a_k^{(0,1,0)} u(x_1, y_k^{(1)}, z_1) + \\ + a_k^{(0,0,1)} u(x_1, y_1, z_k^{(1)}) \Big], \tag{13}$$

where, $(x_k^{(1)}, y_k^{(1)}, z_k^{(1)}) \in \overline{G}$ - given points; $a_k^{(1,0,0)}$, $a_k^{(0,1,0)}$, $a_k^{(0,0,1)} \in R$ - given numbers.

To obtain the necessary and sufficient optimality condition, the increment of the functional (13) is first calculated. Using integral representations of functions from the space $W_p^{(1,1,1)}(G)$, the increment of the functional is found in integral form:

$$\Delta S(v) = \int\limits_{\sqrt{x_0 x_1}}^{x_1} \int\limits_{\sqrt{y_0 y_1}}^{y_1} \int\limits_{\sqrt{z_0 z_1}}^{z_1} \left[R(x,y,z) + (\omega_{1,1,1} f)(x,y,z) \right] \Delta u_{xyz}(x,y,z)\,dxdydz -$$

$$- \int\limits_{\sqrt{x_0 x_1}}^{x_1} \int\limits_{\sqrt{y_0 y_1}}^{y_1} \int\limits_{\sqrt{z_0 z_1}}^{z_1} f_{1,1,1}(x,y,z) \Delta\varphi(x,y,z)\,dxdydz.$$

Then for the optimal control problem (11)-(13) the conjugate equation will be constructed:

$$(\omega_{1,1,1} f)(x,y,z) + R(x,y,z) = 0, \quad (x,y,z) \in G, \qquad (14)$$

where, $(\omega_{1,1,1} f)(x,y,z)$ the second type is the three-dimensional Volter integral operator.

In the considered optimal control problem (11)-(13), since the control function $v(x,y,z)$ enters the right side of the equation in a nonlinear form, considering the needle-shaped variation of the admissible control, we find that

$$S(v_\varepsilon) - S(v) == -\iiint\limits_{G_\varepsilon} f_{1,1,1}(x,y,z)[\varphi(x,y,z,v(x,y,z) +$$

$$+ \Delta v_\varepsilon(x,y,z)) - \varphi(x,y,z,v(x,y,z))]dxdydz =$$

$$= -\iiint\limits_{G_\varepsilon} f_{1,1,1}(x,y,z)[\varphi(x,y,z,\hat{v}) - \varphi(x,y,z,v(x,y,z))]dxdydz. \qquad (15)$$

Thus, since the optimal control problem (11)-(13) is linear, from (15) it follows:

Theorem 3. Let us assume that the adjoint equation (14) is solved $f_{1,1,1}(x,y,z) \in L_q(G)$. Then, for any admissible controls $v(x,y,z)$, a necessary and sufficient condition for optimality for almost all $(x,y,z) \in G$ is the fulfillment of the maximum condition:

$$\max_{\hat{v}\in U_\partial} H(x,y,z,\ f_{1,1,1}(x,y,z),\hat{v}) = H(x,y,z,\ f_{1,1,1}(x,y,z), v(x,y,z)),$$

where, $H(x,y,z,\ f_{1,1,1},v) = f_{1,1,1}\cdot\varphi(x,y,z,v)$ -Hamilton-Pontryagin function.

Note that the adjoint equation (14) introduced here for the optimal control problem (11)-(13) is more natural than the classical adjoint problem. Because when constructing this conjugate equation, no smoothness conditions were imposed on the coefficients of the equation. This is one of the biggest advantages of the conjugate task.

In this section, in order not to increase the volume of the dissertation, an optimal control problem is demonstrated, described only by boundary conditions specified in the geometric middle of the domain. It should be noted that from the optimal control problem described by the boundary value problem posed in the geometric middle of the domain, in the particular case, $x_0 = 0,\ y_0 = 0,\ z_0 = 0$ we obtain an optimal control problem described by Goursat conditions.Similarly, one can consider the optimal control problem described by the boundary value problem posed at the arithmetic middle of the domain, which is presented in this published article.

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CONCLUSION

The dissertation work is devoted to boundary value problems posed in the arithmetic and geometric middle of the domain for integro-differential equations of Bianchi type, as well as optimal control problems described by such boundary value problems.

The dissertation work obtained new scientific results presented below:

- In a non-classical formulation, the posed boundary value problem on the arithmetic middle of the domain for the 3D Bianchi integro-differential equation with non-smooth coefficients is justified;
- The posed non-classical boundary value problem on the arithmetic middle of the domain for the 3D Bianchi integro-differential equation with non-smooth coefficients is reduced to an equivalent integral equation and the correctness of such a boundary value problem is proven;
- To study the posed boundary value problem in a non-classical formulation on the arithmetic middle of the domain for the 3D Bianchi integro-differential equation, after constructing the conjugate operator in integral form, a fundamental solution was constructed for the integral representation of the solution. Then boundary value problems posed at the arithmetic middle of the domain and some generalizations of them are given;
- The posed boundary value problem in a non-classical formulation on the geometric middle of the region is justified and the equation under the boundary conditions is reduced to a single operator equation;
- An equivalent integral equation was constructed for the posed boundary value problem on the geometric middle of the domain and the homeomorphism theorem was proved;
- For an integral representation of the solution for the posed boundary value problem in a non-classical formulation, at the geometric middle of the domain, the conjugate operator is first constructed in integral form and then the fundamental solution is constructed. At the end, some generalizations of the posed

boundary value problems on the geometric middle of the region are given;

- For the problem of optimal control of the described boundary value problems on the geometric middle of the domain for a 3D Bianchi integro-differential equation with nonsmooth coefficients, necessary and sufficient optimality conditions are obtained in the form of the maximum principle;
- Numerical solution algorithms have been developed for boundary value problems on the arithmetic and geometric middle of the domain in non-classical formulations.

The main results of the dissertation were published in the following works:

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