REPUBLIC OF AZERBAIJAN

On the rights of manuscript

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

MATHEMATICAL MODELING AND METHODS FOR STUDYING NONSTATIONARY WAVE PROCESSES IN RHEOLOGICAL MEDIA

Specialty:	1203.01 –Computer sciences
Field of science:	Mathematics

Applicant: **Ulviyya Sangan Aliyeva**

Baku - 2025

The dissertation work was completed at the Department of "Mathematical Analysis and Differential Equations" of Sumgayit State University.

Scientific advisor: Doctor of Mathematical Sciences, prof.
Nabi Tapdig Gurbanov

Official opponents: Doctor of Physical and Mathematical Sciences,pof.
Adalat Karim Karimov

Doctor of Mathematical Sciences, Assoc. Prof.
Naila Ismayil Valiyeva

Candidate of Physical and Mathematical Sciences, Assoc. Prof.
Mubariz Bilal Rasulov

ED 1.19 Dissertation Council operating under the Institute of Management Systems of the Ministry of Science and Education of the Republic of Azerbaijan of the Higher Attestation Commission under the President of the Republic of Azerbaijan

Chairman of the Dissertation Council: Doctor of Mathematical Sciences, Professor
Knyaz Shiraslan Mammadov

Scientific Secretary of the Dissertation Council: Doctor of Philosophy in Mathematics,
Nigar Ogtay Shukurova

Chairman of the Scientific Seminar: Doctor of Mathematical Sciences, Associate Professor
Anar Beybala Rahimov

GENERAL CHARACTERISTICS OF THE WORK

Relevance and level of development of the topic. In modern scientific research and technical sciences, various mathematical models are used depending on the nature of the process and reflect the most essential features. Mathematical models are effective methods for studying mechanical systems when determining the main indicators and provide an opportunity to briefly describe complex technical systems using mathematical symbols, and also allow you to determine the interaction of various parameters. The creation of accurate mathematical models is associated with great difficulties.

Analyzing this process, we encounter two different problems. The first problem is in describing the differential equation of the characterizing system, choosing the equation of state and determining the initial-boundary conditions. The second problem arises in solving these problems and physically interpreting the results obtained.

The development of modern technology has caused the intensive use of polymer, composite and other materials with rheological properties, which is inextricably linked with the creation of new materials with pre-set various physical and mechanical properties. The emergence of such materials is widely used in new technology, which contributes to the creation of new structures operating under various conditions. Therefore, taking into account the dependence of the rheological properties of these materials on the types of loading becomes relevant, ignoring which can lead to loss of bearing capacity and destruction of structures.

The study of the physical and mechanical properties of these materials and the analysis of their application in industrial structures and engineering have shown the need to use mathematical methods in strength and stability calculations for the corresponding structures. At present, there are many well-developed methods for solving static and quasi-static problems. It should be noted that the greatest interest, from the point of view of application, is presented by dynamic problems, solving which it is possible to answer questions about the stability and strength of structural elements, building structures, etc., operating under conditions of intensively changing impacts.

In this area, a huge number of practical problems have been studied to meet technical requirements, where the rheological properties of materials were described by elementary models of Maxwell, Voigt, standard linear body, etc.

The absence of a single method suitable for arbitrary hereditary kernels for solving dynamic problems is the reason for the unsolved nature of many important practical problems.

This circumstance predetermines the relevance of the topic of the dissertation work, aimed at the development and analysis of mathematical models that allow the construction of solution methods, corresponding software, numerical calculation and optimization of structural elements, assessment of their strength and reliability during operation under the influence of intensive loads.

Object and subject of the research. The main object of the dissertation is mathematical modeling and development of a method for solving non-stationary wave problems and problems of oscillations of a system with rheological properties, as well as a study of the influence of various boundary and initial conditions and layering of the medium on wave motion with arbitrary nuclei.

The aim of the work is to construct a more accurate mathematical model and develop a method for solving the problem of propagation of non-stationary waves in solids, taking into account the rheological properties of the material of the medium with arbitrary difference kernels.

To achieve this goal, the following tasks were formulated and studied:

1. The general mathematical model has been improved and a numerical method for studying the propagation of non-stationary wave processes in rheological media with arbitrary rheology has been proposed.

2. Mathematical modeling and numerical solution of vibrations of mechanical systems taking into account the rheological properties of the medium material are investigated

3. Modeling and algorithms for studying the vibrations of a pipe made of composite material and an analysis of the influence of frequency on the error of solutions are formulated

4. A mathematical model and numerical analysis of wave processes in layered rheological media under various initial-boundary conditions have been developed

Research methods. To solve the tasks set, theories of partial differential equations, theories of integral and integro-differential equations, methods of integral transformations, separation of variables, the method of successive approximations and numerical methods were used.

The main provisions submitted for defense:

1. Construction of a more accurate mathematical model of dynamic wave processes in a medium with rheological properties and methods for their solutions were investigated

2. Mathematical modeling and algorithms for studying solutions to the equations of oscillations of mechanical systems with rheological properties under various initial-boundary conditions

3. Mathematical modeling and implementation of solutions to the problem of vibrations of a multilayer reinforced pipe taking into account the rheological properties of the material and numerical analysis of the error of the solutions.

4. Modeling and numerical analysis of wave processes in layered rheological media under different initial-boundary conditions and analysis of the influence of parameters on the wave field.

Scientific novelty of the research. The following results were obtained in the dissertation:

1. A more accurate mathematical model has been improved and a methodology for studying the propagation of dynamic non-stationary waves in rheological media for arbitrary rheological functions has been developed

2. Mathematical models and algorithms for studying the solution of the equation of oscillations of relaxing systems under various initial-boundary conditions are constructed

3. Modeling and implementation of solutions to the problem of vibrations of reinforced multilayer pipes and numerical calculation of the error of solutions for a specific core

4. Mathematical modeling of the reaction of layered media with rheological properties under dynamic loading and numerical analysis of wave processes.

The validity and reliability of the scientific provisions of the conclusions and results presented in the dissertation are determined by the correctness of the mathematical formulation of the problems and methods for solving them using the laws of mathematics and mechanics, as well as a comparison of the results obtained with existing solutions.

Theoretical and practical significance of the research. The dissertation work is theoretical in nature with practical application, in which new results are obtained. Scientific value is determined by the correctness of the physical and mathematical formulation of the problems under consideration and the methods for solving them using the fundamental laws of mechanics and mathematics, and by comparing the research results with existing known solutions.

The practical value of the dissertation is determined by a wide range of technical applications of the problems under consideration and the use of polymer, composite and other materials with rheological properties in modern technology. The results of the work can be used to solve a set of applied problems in engineering calculations for the strength, durability and operational reliability of various types of structures made of polymer and composite materials under the influence of intensively changing dynamic loads.

Testing and application. The results obtained in the dissertation were reported and discussed at various international and national conferences and seminars:

- at the XII International Conference "Fundamental and Applied Problems of Mathematics", dedicated to the 85th anniversary of Professor Alischev. September 19-22, 2017, Makhachkala, Russia, pp.96-98

- XII International conference on Appled mathematics and mexanics in Aerospace industry. 24-31 May, 2018, Aluchta, Crimea, Russia, (NPNG-2018), p.406-409
- at the XII Republican Conference of Young Researchers and Doctoral Students. ASPU, November 22-23, 2018, Baku
- at the XIII International Conference on Applied Mathematics and Mechanics in Aerospace Industry (AMMIA - 2020) September 6-13, Alushta, Crimea, Russia, 2020, pp.6-13.
- at the X IV International Conference dedicated to the 90th anniversary of Dagestan State University, September 16-19, 2021, Makhachkala, Russia, pp.16-19
- at the X IV International Scientific Nadirov Readings dedicated to the 90th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan Nadirov N.K., February 25, 2022, Atyrau, Kazakhstan, pp.17-23
- at the III All-Russian Conference with International Participation February 7-9, 2022, Makhachkala, Russia.
- 8th International conference on control and optimization with Industrial Applications. 24-26 August, 2022, Baku, Azerbaijan.
- at the II Republican Conference. Fundamental Problems of Mathematics and Application of Intelligent Technologies in Education, December 15-16, 2022, Sumgait, Azerbaijan.
- 5th International Conference on Problems of Cybernetics and Informatics, PCI 28-30 August 2023, Baku ·
- PLMO-2024 3rd international Conferehce on problems o logistics, management and operation in the east-west transport corridor. Baku.
- The 9th International Conference on Control and Optimization with Industrial Applications, 27-29 August 2024, Istanbul.

The dissertation was presented at a meeting of the scientific seminar of the Research Institute of Applied Mathematics at Baku State University.

The dissertation was repeatedly presented and discussed at the seminar of the Department of Mathematical Analysis and Differential Equations of Sumgait State University.

Name of the institution where the dissertation work was completed. The dissertation work was completed at the Department of Mathematical Analysis and Differential Equations of Sumgait State University.

Author's publications. The main results of the dissertation work are published in 24 works by the author, the list of which is given at the end of the dissertation abstract.

Structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, a summary of the results, a bibliography, and appendices. The bibliography includes 106 references. The work includes 15 tables, 27 figures, and three appendices. The total length of the dissertation is 160 pages.

Summary of the dissertation

The introduction of the dissertation defines the purpose and relevance of the issue under consideration, provides an analytical review and analysis of works related to the problem considered in the dissertation, and also provides a brief summary of the work for all chapters.

The first chapter is devoted to the mathematical modeling of the behavior of structural structures composed of rheological materials under external non-stationary influences and the construction of effective methods based on existing approximate numerical methods for solving theoretical and practical problems.

The first paragraph of the first chapter is devoted to the mathematical modeling of non-stationary wave processes in rheological media and the study of solutions for arbitrary hereditary functions describing the physical properties of the material of the medium.

Mathematically, the problem is reduced to solving a system of integro-differential equations, where the relationship between voltage σ_{ij} and the deformation is given in the form:

$$s_{ij}(t) = \int_0^t \Gamma(t-\tau)\, e_{ij}(\tau)d\tau; \sigma_{ij}(t) = \sigma\delta_{ij} + s_{ij(t)}; s_{ij}\delta_{ij} = 0$$

$$\sigma(t) = \int_0^t \Gamma_1(t-\tau)\, \theta(\tau)d\tau; \varepsilon_{ij}(t) = \frac{1}{3}\theta(t)\delta_{ij} + e_{ij}(t); e_{ij}\delta_{ij} = 0 \quad (1)$$

where $\delta_{ij} = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases}$ is the unit Kronecker tensor, $\sigma$and $\theta$are the spherical stress and volumetric strain tensor, respectively, $s_{ij}$and $e_{ij}$are the components of the stress and strain deviators, and the functions $\Gamma(t)$and $\Gamma_1(t)$are the kernels of shear and volumetric relaxation, respectively.

The essence of this formula is that in a small element of the environment, stress $s_{ij}(t)$and $\sigma(t)$On the interval, the deformations $0 < \tau < t < \infty$and , respectively, $\theta(t)$are uniquely determined $e_{ij}(t)$. It is assumed that the state of the medium is invariant with respect to the origin of time. It is known that the equations of motion of a continuous medium have the form:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \rho F_i + \frac{\partial \sigma_{ij}}{\partial x_j} (i, j = 1,2,3) \quad (2)$$

Here $\rho$is the density of the medium, $F_i$is the projection of the external mass force, $u_i$is the displacement

Note that to determine the function $\bar{u}(x,t)$from equation (2), boundary and initial conditions are required.

The boundary conditions can be as follows:

$$\sigma_{ij}\ell_i\Big|_{s_1} = F_{i0} \quad (3)$$

or $$u_i|_{s_2} = u_{i0}$$

The initial conditions can be as follows:

$$u(x,t) = f_1(x)$$

$$\frac{\partial u(x,t)}{\partial t} = f_2(x) \text{ при } t = 0. (4)$$

Thus, the solution of non-stationary problems is reduced to solving equation (2) under conditions (3) and (4).

Theorem. If functions $f_1(x)$And $f_2(x)$differentiable functions satisfying zero boundary conditions, then problem (2), (3) and (4) has a solution determined by the formula

$$u(x,t)=\sum_{n=1}^{\infty}T_n(t)V_n(x)$$

Theorem. If equation (2) has a solution satisfying conditions (3) and (4), then it is unique.

The second section of the first chapter examines the modeling and analysis of the solution to the equation of forced transverse oscillations of a beam in a sphere without resistance. Mathematically, the problem reduces to solving an integro-differential equation.

$$EJ[\frac{\partial^4 w(x,t)}{\partial x^4}+\int_0^t \Gamma(t-\tau)\frac{\partial^4 w(x,\tau)}{\partial x^4}d\tau]+\rho F\frac{\partial^2 w(x,t)}{\partial t^2}=q(x,t), \quad (5)$$

where $w(x,t)$is the transverse displacement, $\Gamma(t)=-\frac{dR(t)}{dt}$, $R(t)$is the relaxation function, $\rho$is the density of the material, $F$is the cross-section of the beam, $q(x,t)$is the transverse force, $J$is the moment of inertia of the cross-section of the beam relative to the neutral axis at the initial

$$w(x,t)=u_0;\frac{\partial w(x,t)}{\partial t}=v_0 \text{ at } t=0(6)$$

and boundary conditions

$$w(x,t)=0;\frac{\partial^2 w(x,t)}{\partial x^2}=0 \qquad \text{at} \qquad x=0(7)$$

$$w(x,t)=0;\frac{\partial^2 w(x,t)}{\partial x^2}=0 \qquad \text{at} \qquad x=\ell \quad (8)$$

This means that both ends of the beam are pinned. Using the separation of variables method and applying the Bubnov -Galerkin procedures, we obtain two independent equations:

differential equation

$$u_k^{IV}(x)-\lambda_k^4 u_k(x)=0 \qquad (9)$$

with the corresponding boundary conditions and the integro-differential equation

$$z_k''(t)+\varepsilon\omega_k^2 z_k(t)-\varepsilon\omega_k^2\int_0^t \Gamma(t-\tau)z_k(\tau)\,d\tau=\omega_k^2 q_k(t)(10)$$

with initial conditions

$$z_k(t)=u_{0k};z_k'(t)=V_{0k} \text{ at } t=0(k=1,2,3,\dots). (11)$$

Here $\omega_k^2 = \frac{EJ}{\rho F}\lambda_k^4, \lambda_k$- forms the spectrum of fundamental numbers of the problem, ε- is some small parameter, $0 < \varepsilon << 1$:

$$q_k(t) = \frac{1}{\omega_k^2 \rho F}\int_0^\ell q(x,t)u_k(x)dx.$$

By solving equation (9), we find the eigenfunctions $\{u_k(x)\}$and eigenvalues $\lambda_k$of this boundary value problem.

Using the integral Laplace transform, the image of the solution of the integro-differential equation (10), taking into account conditions (11), is obtained in the following form:

$$\bar{z}_k(p) = \frac{pu_{0k}+v_{0k}}{p^2+\omega_k^2-\varepsilon\omega_k^2\bar{\Gamma}(p)} + \frac{\bar{q}_k(p)}{p^2+\omega_k^2-\varepsilon\omega_k^2\bar{\Gamma}(p)} \quad (12)$$

Where p– the Laplace transform parameter, $\bar{z}_k(p)$and$\bar{\Gamma}(p)$ Laplace images of the functions of the same name $z_k(t)$and$\Gamma(t)$ respectively. Here, the first term corresponds to free oscillation, and the second term to forced oscillation. Omitting the indices for simplicity of notation, we represent the first term as a series:

$$\bar{\phi}_k(p) = \frac{pu_0+v_0}{p^2+\omega^2}\sum_{m=0}^{\infty}\left(\frac{\varepsilon\omega^2\bar{\Gamma}(p)}{p^2+\omega^2}\right)^m \quad (13)$$

Theorem. If the condition is satisfied

$$\left|\frac{\varepsilon\omega^2\bar{\Gamma}(p)}{p^2+\omega^2}\right| < 1,$$

then the series (13) is a convergent series for all time values.

Then, after some transformations, we represent the solution (13) in the form of an absolutely convergent series:

$$\bar{\phi}(p) = \frac{pu_0+v_0}{\bar{\alpha}(p)}\left[1 + \varepsilon\omega^2\frac{\bar{\beta}(p)}{\bar{\alpha}(p)} + \varepsilon^2\omega^4\frac{\bar{\beta}^2(p)}{\bar{\alpha}^2(p)} + \dots\right] \quad (14)$$

Where

$$\bar{\alpha}(p) = (p + \frac{1}{2}\varepsilon\omega\Gamma_s)^2 + \omega^2(1 - \frac{1}{2}\varepsilon\Gamma_c)^2$$

$$\bar{\beta}(p) = \bar{\Gamma}(p) + \frac{p}{\omega}\Gamma_s + \Gamma_c + \frac{1}{4}\varepsilon(\Gamma_s^2 + \Gamma_c^2)$$

The original of the first member has the form:

$$\phi_1(t) = exp(-\frac{1}{2}\varepsilon\omega\Gamma_s t)[u_0 \cos\omega(1 - \frac{1}{2}\varepsilon\Gamma_c)t +$$

$$+\frac{v_0-\frac{1}{2}\varepsilon\omega\Gamma_s}{\omega(1-\frac{1}{2}\varepsilon\Gamma_C)}\sin\omega(1 - \frac{1}{2}\varepsilon\Gamma_c)t] \quad (15)$$

This is the solution to equation (10) obtained by the averaging method. The second approximation of the solution is defined as follows:

$$\phi_2(t) = \varepsilon\omega^2\phi_1(t) * L^{-1}\left\{\frac{\bar{\beta}(p)}{\bar{\alpha}(p)}\right\}$$

Where

$$\psi(t) = L^{-1}\left\{\frac{\bar{\beta}(p)}{\bar{\alpha}(p)}\right\} =$$

$$= \Gamma(t) * exp(-\frac{1}{2}\varepsilon\omega\Gamma_s t)\cdot\frac{1}{\omega\left(1-\frac{1}{2}\varepsilon\Gamma_c\right)} sin\,\omega\,(1-\frac{1}{2}\varepsilon\Gamma_c) +$$

$$+A\,exp(-\tfrac{1}{2}\varepsilon\omega\Gamma_s t)\,sin[\,\omega(1-\tfrac{1}{2}\varepsilon\Gamma_c)t+\theta], \qquad (16)$$

And $$\theta = arctg\frac{\omega(1-\frac{1}{2}\varepsilon\Gamma_c)}{d-\frac{1}{2}\varepsilon\omega\Gamma_s}; A = \frac{\Gamma_s}{\omega}\sqrt{1+\frac{(d-\frac{1}{2}\varepsilon\omega\Gamma_s)^2}{\omega^2(1-\frac{1}{2}\varepsilon\Gamma_c)^2}}$$

$$d = \frac{\Gamma_c}{\Gamma_s}\omega + \frac{\varepsilon\omega}{4\Gamma_s}(\Gamma_s^2+\Gamma_c^2).$$

The subsequent approximation of the solution of free oscillation is determined in a similar manner, which is not difficult.

The original of the second term of equation (12) is found by convolution of functions$\phi_1(t)$ And $\psi(t)$.

In the third section of this chapter, a mathematical model is constructed for the transverse vibrations of a long cylinder secured by an elastic shell when subjected to an external load that is periodic in time.

If we neglect the influence of shear deformation and rotational inertia on deflection, we obtain the equation of oscillation in the form[1]:

$$\frac{\partial^2 u(z,t)}{\partial t^2} + \frac{E_0 J_0 + 3G_0 J_{ц}}{m}\frac{\partial^4 u(z,t)}{\partial z^4} = \frac{3G_0 J_{ц}}{m}\int_0^t R(t-\tau)\frac{\partial^4 u(z,t)}{\partial z^4}d\tau (17),$$

Where m– mass, E_0- elastic modulus, G_0- shear modulus, $J_0$and $J_{ц}$-constants.

Equation (17) is solved under the following initial and boundary conditions:

$$u(z,t) = f(z); \frac{\partial u(z,t)}{\partial t} = \psi(z) \text{ at } t = 0 (18)$$

$$u(z,t) = 0; \frac{\partial^2 u(z,t)}{\partial z^2} = 0 \text{ at } z = 0 \text{and } z = l (19)$$

This means that the ends of the cylinder are hinged.

Problem (17) - (19) is solved using the method described in the second section of this chapter. The solution was investigated for $q(t) = q_0 H(t)$. It was found that $\phi_k(t)$changes abruptly from zero to $\phi_k(0)$:

$$\phi_k(0) = \frac{q_0}{m}\sqrt{\frac{2}{\ell}} \cdot \frac{\ell}{\pi k} sin\frac{\pi k}{2}, (20)$$

where the maximum deflection at the point $z = \frac{\ell}{2}$will be as follows:

$$u_{max} = \frac{2q_0}{\pi^4}\frac{\ell^4}{E_0 J_0 + 2G_0 J_y}(21)$$

In the fourth paragraph A numerical solution method based on the Euler- Kromer method is proposed for an integro-differential convolution equation describing the nonstationary oscillations of relaxing systems. A software implementation of the method in Python was developed , and numerical studies were conducted, demonstrating the method's effectiveness and finding an error estimate. The results of the numerical experiment are $\lambda = 5,10,30,50,100$presented in tables (tab.1, fiq.1,2,3.).

Statement of the problem

The equation required is to be solved:

$$\frac{d^2U}{dt^2} + \lambda^2 U(t) - \epsilon\lambda^2 \int_0^t k(t-S)U(S)\, dS = f(t)$$

Where :

- Core,$k(t) = \varepsilon t^{\alpha-1} e^{-\beta t}$
- External force $f(t) = \sigma_0 \cdot H(t)$,
- Initial conditions: $U(0) = 0, U'(0) = 11$,
- Parameters : $\epsilon = 0.1, \alpha = 0.074, \beta = 0.05, \lambda = 60.$

Beginning of the algorithm.

Step 1: Transformation into an ODE system

1. 1. Introduction of variables

Let's enter the speed $V(t) = \frac{dU}{dt}$.

Then:

$$\frac{dU}{dt} = V(t)$$

$$\frac{dV}{dt} = f(t) - \lambda^2 U(t) + \epsilon\lambda^2 \int_0^t k(t-S)U(S)\ dS$$

1.2 . Initial conditions

$$U(0) = 0,$$
$$V(0) = 1.$$

Step 2: Time Discretization

2.1. Grid construction

Time is divided into$N$ steps by step $\Delta t = 0.001$:

$$t_i = i\Delta t, i = 0,1,\dots,N.$$

2.2. Integral approximation

The integral is represented as a sum over subintervals :

$$\int_0^{t_i} k(t_i - S)U(S)\ dS \approx \sum_{j=0}^{i-1} \int_{t_j}^{t_{j+1}} k(t_i - S)U(S)\ dS.$$

Step 3: Linear approximation$U(S)$

3.1 Piecewise linear model

At each interval $[t_j, t_{j+1}]$:

$$U(S) \approx U_j + \frac{U_{j+1} - U_j}{\Delta t}(S - t_j).$$

3.2. Expansion of the integral

The integral is decomposed into two terms:

$$\int_{t_j}^{t_{j+1}} k(t_i - S)U(S)\ dS = \epsilon U_j \int_{t_j}^{t_{j+1}} (t_i - S)^{\alpha-1} e^{-\beta(t_i - S)} dS$$

$$+\epsilon \frac{U_{j+1} - U_j}{\Delta t} \int_{t_j}^{t_{j+1}} (S - t_j)(t_i - S)^{\alpha-1} e^{-\beta(t_i - S)} dS$$

Step 4: Analytical evaluation of integrals

4.1 CalculationA_j

Replacement $x = t_i - S$, $dx = -dS$:

$$A_j = \epsilon U_j \int_{t_i - t_{j+1}}^{t_i - t_j} x^{\alpha-1} e^{-\beta x} dx$$
$$= \epsilon U_j \beta^{-\alpha} \left[\Gamma\left(\alpha, \beta(t_i - t_{j+1})\right) - \Gamma\left(\alpha, \beta(t_i - t_j)\right)\right]$$

4.2. CalculationB_j

Replacement$x = t_i - S$

$$B_j = \epsilon \frac{U_{j+1} - U_j}{\Delta t} \left((t_i - t_j) \int x^{\alpha-1} e^{-\beta x} dx - \int x^{\alpha} e^{-\beta x} dx\right)$$

$$= \epsilon \frac{U_{j+1} - U_j}{\Delta t} \left((t_i - t_j) \beta^{-\alpha} \Delta\Gamma(\alpha) - \beta^{-(\alpha+1)} \Delta\Gamma(\alpha + 1) \right)$$

Step 5: Numerical scheme

5.1. Euler- Kromer method

Speed and position update:

$$V_i = V_{i-1} + \left[\sigma_0 - \lambda^2 U_{i-1} + \epsilon\lambda^2 \sum_{j=0}^{i-1} (A_j + B_j) \right] \Delta t,$$

$$U_i = U_{i-1} + V_i \Delta t.$$

5.2 Initial conditions

$$U_0 = 0,$$

$$V_0 = 1.$$

Step 6: Implementation in Python

6.1 Code structure

- Import libraries: numpy , scipy.special , matplotlib .
- Precalculation of constants : $\Gamma(\alpha)$, β^{α}.
- Nested loops for summing the integral.

6.2 Key functions

- gammainc (a, x): Regularized incomplete gamma function.
- gamma (a): The full gamma function.

Step 7: Solution validation

7.1 Case $\epsilon = 0$

Analytical solution:

$$U(t) = \frac{\sigma_0}{\lambda^2} (1 - \cos(\lambda t)) + \frac{1}{\lambda} \sin(\lambda t)$$

7.2. Energy Control

Total energy:

$$E(t) = \frac{1}{2} V^2 + \frac{\lambda^2}{2} U^2 - \sigma_0 U$$

8. Conclusion

- The integro-differential equation is transformed into a system of ODEs.
- The integral term is calculated using incomplete gamma functions.
- A robust numerical method has been implemented.

Tab. 1: Numerical results of $U\ (\ t\ )$ for different values *of* λ indicated in the indices

T	*U* 5	At *10*	At *15*	At *30*	*At* 50	*U* 100
0.77	0.214220284	0.007379163	0.013333183	0.009705256	0.002491845	0.000408589
0.771	0.213554902	0.007929265	0.012903353	0.009820981	0.002411287	0.000420534
0.772	0.212888579	0.008479753	0.012474411	0.009931698	0.002328667	0.000431158
0.773	0.212221325	0.009030591	0.012046423	0.010037352	0.002244159	0.000440388
0.774	0.211553151	0.009581741	0.011619453	0.010137894	0.002157939	0.000448161
0.775	0.210884068	0.010133166	0.011193565	0.010233276	0.002070187	0.00045443
0.776	0.210214088	0.01068483	0.010768824	0.010323456	0.001981083	0.000459159
0.777	0.20954322	0.011236695	0.010345294	0.010408395	0.001890809	0.000462325
0.778	0.208871478	0.011788725	0.009923037	0.010488058	0.00179955	0.000463918
0.779	0.208198871	0.012340884	0.009502118	0.010562412	0.001707488	0.000463942
0.78	0.207525411	0.012893134	0.009082598	0.010631429	0.001614808	0.000462414
0.781	0.206851108	0.013445439	0.00866454	0.010695085	0.001521696	0.000459362
0.782	0.206175974	0.013997763	0.008248007	0.01075336	0.001428335	0.000454827

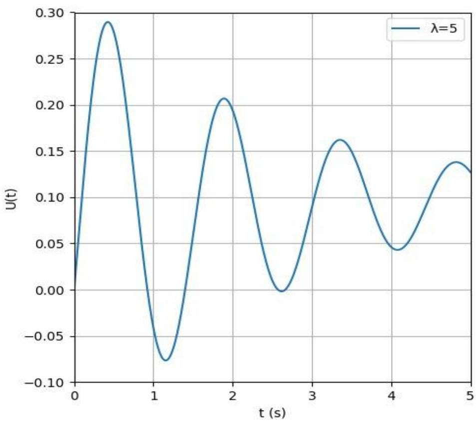

Fig. 1. Graph of the function $U(t)$ for λ=5

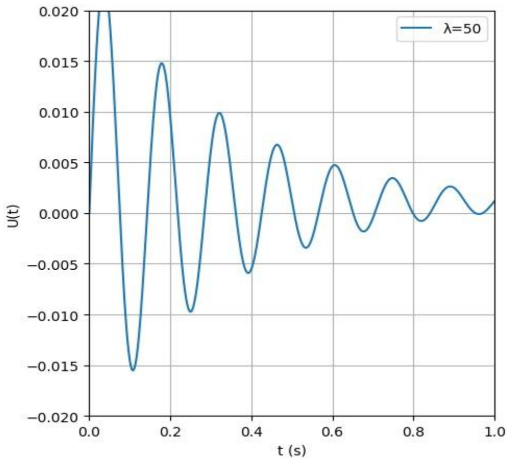

Fig. 2. Graph of the function $U(t)$at λ=50

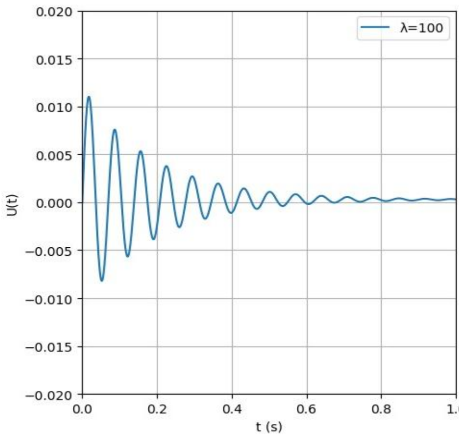

Fig. 3. Graph of the function $U(t)$at λ=100

We note that when implementing a solution justified by the choice of the Euler- Cromer method , the stability of this method was demonstrated and the global and local discretization errors were determined. As a result, it was found that the integral term characterizing the rheological properties of the medium acts as a damping factor and exponentially suppresses the oscillation amplitudes.

The second chapter is devoted to mathematical modeling and deformation research in composite and reinforced materials, which are relevant today due to the widespread use of composite materials in various fields of engineering.

As is known, when studying composite materials, three different but related problems arise.

The first problem is calculating the bond strength between the components at their interfaces, which accounts for the overall strength of the reinforced material. The second problem is the combined deformation of the various elements, which is a complex mechanical process. The third problem is establishing the relationship between the reinforcing material and the binder polymer to ensure optimal properties of the composite as a whole.

The first paragraph establishes the principles of interaction of composite elements, on the basis of which a mathematical model of the vibration of thick-walled multilayer reinforced pipes is constructed, taking into account the rheological properties of the material.

Then the study of oscillations is reduced to solving the equation:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = q_n^2 \frac{\partial^2 u(x,t)}{\partial x^2} + \varepsilon \int_0^t \Gamma(t-\tau) \frac{\partial^3 u(x,\tau)}{\partial x^2 \partial \tau} d\tau + F(x,t) \quad (22)$$

Here $q_n = \frac{a_n}{\gamma_{cp}}$; $\varepsilon\Gamma(t) = \frac{1}{\gamma_{cp}}[a_n'\omega(t) + a_n'' g_{\beta k}(t)]$

$$F(x,t) = \frac{1}{\gamma_{cp}}\left\{b_n \frac{\partial q_a}{\partial x} + c_n \frac{\partial q_b}{\partial x} \sum_{k=1}^{n} \int_0^t g_{\beta k}(t-\tau)\left[b_n' \frac{\partial q_a}{\partial x} + b_n'' \frac{\partial q_b}{\partial x}\right]\right\},$$

Where $g_{\beta k}$- Ilyushin function of coherent creep, β_k- design parameter, $a_n, a_n', a_n'', b_n, b_n'$ and b_n''- constants depending on the pipe dimensions.

Thus, the problem of longitudinal vibration of a multilayer pipe is reduced to solving equation (22) under boundary and initial conditions, which we take in the following form:

$$u(x,t) = 0 \ ; \frac{\partial u(x,t)}{\partial x} = 0 \ \text{at} \ x = 0; \ x = \ell (23)$$

$$u(x,t) = \phi_1 \ ; \frac{\partial u(x,t)}{\partial t} = \phi_2 \text{at} \ t = 0. (24)$$

Using the methodology described in the first chapter, problem (22)- (24) is solved.

In the second paragraph of the second chapter, a model is constructed for studying the radial vibrations of a reinforced pipe with a rheological binder.

Let's assume that the outer and inner layers of a pipe are composed of elastic fibers, with a layer of viscoelastic material between them. Let's assume that the pipe is subject to external and internal pressure, and that the longitudinal deformation is independent of the coordinate r. In this case, the change in shear stress in the radial and longitudinal directions can be neglected. We assume that the pipe material is incompressible, and the deformations are axisymmetric. Then, from the equation of motion of the binder layer in the radial direction, taking into account the results of the first section of this chapter, we obtain the integro-differential equation:

$$V''(t) + \lambda^2 V(t) = \varepsilon\lambda^2 \int_0^t \omega(t-\tau)\, dv(\tau), (25)$$

where is the $\varepsilon > 0$small parameter, $\omega(t)$- depends on the relaxation kernel of the connected creep function, $\lambda$and ε- depend on the structure of the reinforced pipe, $V(t) = \frac{u(t)}{r}$- is the desired function. Thus, the solution to the problem is reduced to solving the integro-differential equation (25) with the initial condition

$$V(t) = V_0; \frac{dV(t)}{dt} = V' \text{ at } t = 0.$$

the solution to this equation is obtained from equations (15) and (16) by replacing $V_0 = U_0, V' = V_0$and $\Gamma(t)$with $\omega(t)$;

The third paragraph of this chapter dedicated to the development of an algorithm and software for the numerical solution of the integro-differential equation of oscillation in the MATLAB system (fiq.4,5).

A fundamental result was obtained that the influence of the second term on the solution depends on the oscillation frequency, and at low frequencies this influence is insignificant and increases with increasing frequency. The amplitude of the second term of the series $z_2(t)$ at some time values it is 10-25% of the amplitude of the first term of the series $z_1(t)$ and the amplitudes of all terms decrease exponentially, and the phases are shifted.

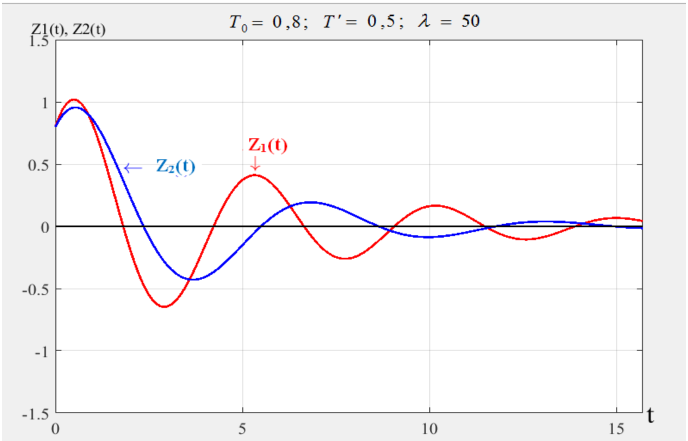

Fig. 4. Graphs of the functions of the first and second approximations $Z_1(t)$and $Z_1(t)$at λ=50

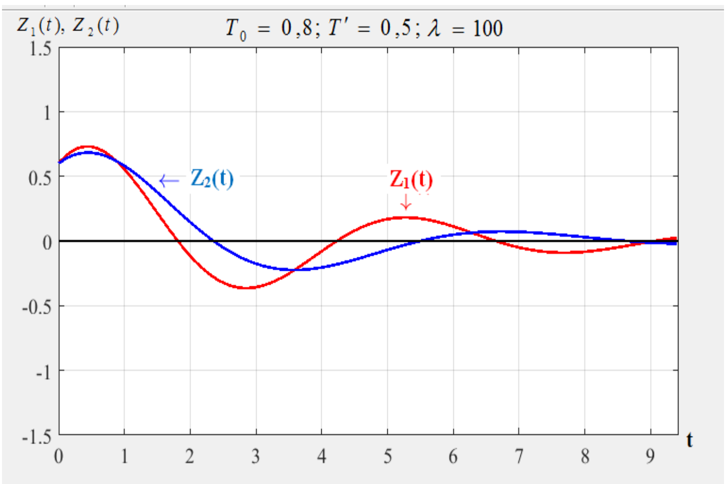

Fig. 5. Graphs of the functions of the first and second approximations $Z_1(t)$and $Z_1(t)$at λ=100

In the third chapter, mathematical models and an algorithm for studying wave processes in layered rheological media are developed.

It should be noted that in this case a system of refracted and reflected waves arises, which form a complex pattern depending on the layering of the material medium and the arrangement of layers with different physical and mechanical properties.

The first section of this chapter attempted to outline the key stages of studying the propagation of transient waves in a layered cylinder using calculations for arbitrary hereditary kernels. Therefore, refined mathematical models for the problem of propagation of dynamic transient shear waves in layered cylinders were created, and a methodology for solving these problems was developed to optimize structural control.

The solution is investigated for the case where the layers of a cylindrical shell are composed of materials with different mechanical properties. The resulting solutions are examined for the case where the standard linear solid model describes the properties of the material. It is shown that in this case, the Laplace image of the kernel is determined by the formula[1]

$$\bar{K}(p) = \frac{\chi}{p+\tau^{-1}} \quad (28)$$

and the resulting solution can be applied to practical calculations for sufficiently small values of z/c. To calculate displacements, the formula can be used

$$\phi(z,t) = (t - \sqrt{\frac{a}{b}}\frac{z}{c}) + \frac{1}{\pi}\int_a^b \frac{1}{s^2} e^{st} \sin(\frac{zs}{c}\sqrt{\frac{a-s}{b-s}}) ds \quad (29)$$

Where $a = \chi + \tau^{-1}; b = \frac{1}{\tau}; c^2 = \frac{\mu}{\rho}$, $\chi$and $\tau$are determined by experiment.

In the second paragraph of the third chapter The above models are applied **to** the study of the propagation of non-stationary plane waves in layered rheological half-spaces.

---

[1] Ilyasov, M.Kh. Unsteady viscoelastic waves / M.Kh. Ilyasov. –Baku: Azerbaijan.Hava Yollary , –2011. –330 p.

It is known that some seismological problems, as well as problems related to the seismic resistance of structures, are reduced to the study of wave processes in layered media, taking into account the rheological properties and heterogeneity of the medium.

This section considers similar problems. Let there be a layer of thickness between two half-spaces in a rectangular H coordinate system $OXYZ$. The first half-space occupies the region $x < 0, -\infty < y, z < +\infty$, this layer occupies the region $0 < x < H, -\infty < y, z < +\infty$,
and the second half-space is occupied by the region $x > H, -\infty < y, z < +\infty$and all regions are isotropic, homogeneous and viscoelastic with different parameters.

Let in the environment along the axisOX a non-stationary wave propagates $t = 0$from infinity and at a given moment in time it falls on the boundary plane of the layer $x = 0$.

Determining the wave state in each medium at subsequent moments in time is mathematically reduced to solving a system of equations

$$\frac{\partial \sigma_i(x,t)}{\partial x} = \rho_i \frac{\partial^2 u_\tau(x,t)}{\partial t^2} (i = 1,2,3) \tag{30}$$

We assume that the system is $t < 0$at rest, so the initial conditions will be in the form:

$$u_i(x,t) = \frac{\partial u_i(x,t)}{\partial t} = 0 \text{ at } t = 0 \tag{31}$$

We accept contact and boundary conditions in the form:

$$u_1(x,t) = u_2(x,t); \sigma_1(x,t) = \sigma_2(x,t) \text{ at } x = 0 \tag{32}$$

$$u_2(x,t) = u_3(x,t); \sigma_2(x,t) = \sigma_3(x,t) \text{ at } x = H \tag{33}$$

$$\sigma_1(x,t) = \sigma_0 H(t) \text{ at } x = 0; u_3(x,t) \to 0 \text{at} x \to \infty \tag{34}$$

where $\sigma_i(x,t)$are the stresses, $u_i(x,t)$are the displacements in the direction of the axis OX, $\rho_i$are the densities of the materials in the environment, $H$is the layer thickness, $\sigma_0 = const$, $H(t)$is the unit Heaviside function. We take the constitutive relation as follows:

$$\sigma_i(x,t) = \int_0^t [R_i^{(1)}(t-\tau) + \frac{2}{3} R_i(t-\tau)]\, d\left(\frac{\partial u(x,t)}{\partial x}\right) \tag{35}$$

where $R_i^{(1)}(t)$and $R_i(t)$– functions of volume and shear relaxation. Here, for , $i = 1$all relations refer to the first half-space, for $i = 2$, to the layer, and for $i = 3$, to the second half-space.

From this it is clear that mathematically the problem is reduced to solving the system of equations (30) - (35).

The solution of the problem in Laplace images using the method described in the first paragraph of this chapter, when Poisson's ratio ν is constant, is obtained in the following form:

$$u_1(x,t) = \sigma_0 \left[\phi(z_1,t) - D_1 \sum_{n=0}^{\infty} (-1)^n A^n \phi(z_2,t) \right]$$

$$u_2(x,t) = \sigma_0 \left[2D_2 \sum_{n=0}^{\infty} (-1)^n A^n \phi(z_3,t) + D_3 \sum_{n=0}^{\infty} (-1)^n A^n \phi(z_4,t) \right]$$

$$u_3(x,t) = 4D_4 \sigma_0 \sum_{n=0}^{\infty} (-1)^n A^n \phi(z_5,t). \quad (36)$$

Of interest here is the definition of the wave field in a half-space $x > H$ for all moments of time $t > 0$.

Note that in solution (36) the first term corresponds to the solution of the elastic problem. In this case

$$u_3(x,t) = \begin{cases} 4D_4 \sigma_0 \sum_{n=0}^{\infty} |A^n| H(t - z_5); \text{ при } t - z_5 > 0 \\ 0; \qquad\qquad\qquad\qquad \text{при } t - z_5 < 0 \end{cases} \quad (37)$$

The remaining terms in solution (36), except for the first, arise due to the rheology of the medium and the displacements $u_3(x,t)$vanish at $t < 0$. This dependence completely determines the wave field under step action at the boundary $x = 0$.

The third section of the third chapter is devoted to the numerical solution of the problem of transient wave propagation in rheological layered media, based on the fourth-order Runge-Kutta method and implemented using model examples. The calculation results are presented in tables, and graphical representations are constructed using the following data (fiq.6):

$$K(t) = \varepsilon t^{\alpha-1} \exp(-\beta t)$$

$$\alpha = (0{,}02 - 0{,}56) \quad \beta = \{0{,}05; 0{,}075; \ 0{,}1\}$$

$$\varepsilon = (0{,}091 - 0{,}99) \quad \lambda = \{1; 10; 50; 100\}$$

For this core, a block diagram and a program for implementing the solution to the problem were constructed, and corresponding computer images were obtained.

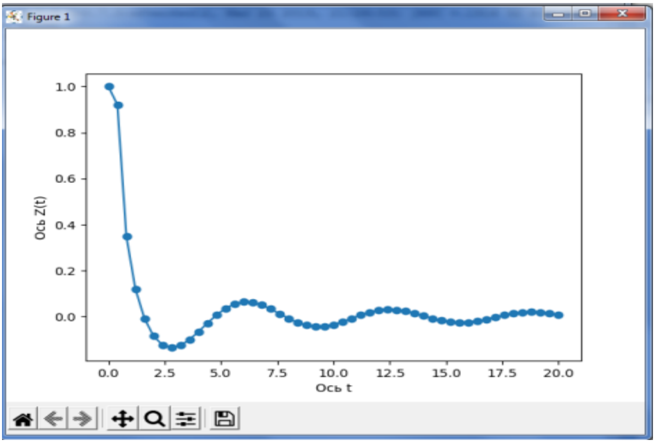

Fig. 6. Graph of the function $Z(t)$

It should be noted that the numerical solution of such problems is associated with the problem of adequately describing wave processes, which is difficult to implement in the case of multilayer barriers due to the appearance of reflection and refraction of waves from contact surfaces.

The calculation results showed that the amplitudes of waves reflected from the layer boundaries decrease over time, which indicates a decrease in the intensity of subsequent reflections.

MAIN RESULTS OF THE DISSERTATION WORK

1. More accurate mathematical models of the propagation of dynamic non-stationary wave processes in a medium with rheological properties under random external influences have been developed, and methods for solving the specified class of problems for arbitrary relaxation functions have been constructed on the basis of methods of operational calculus and successive approximations [3]; in dissertation 1.1. [12]; in dissertation 1.2.

2. A mathematical model was constructed, an algorithm and software were developed for the numerical solution of an equation describing the oscillation of mechanical systems with rheological

properties under different initial conditions, a solution method based on the Euler- Cromer method and implemented in the Python program was proposed. The influence of the rheological properties of the material on the oscillation amplitudes was investigated [21]; in the dissertation 1.2. [14]; in the dissertation 1.3. [23]; in the dissertation 1.4.

3. Mathematical models and computational algorithms based on successive approximations for solving the problem of oscillations of a multilayer-reinforced pipe taking into account the coupled effect of the layers, and software for numerical solution in the Matlab system were developed. It was found that the influence of the second approximation on the solution depends on the oscillation frequency, and this increases with increasing frequency and, for some time values, amounts to 10-15% of the amplitude of the first term. The amplitudes of all terms decrease exponentially, and the phases are shifted [8]; in the dissertation 2.1. [20]; in the dissertation 2.3.

4. Mathematical models and algorithms based on the Runge-Kutta method were constructed for the study and analysis of wave processes in rheological multilayer media. The influence of viscosity and layering of the medium on the wave field was studied and it was shown that the amplitude of waves reflected from the layer boundaries decreases over time [18]; in dissertation 3.1. [15]; in dissertation 3.3.

5. An algorithm and software for the numerical solution of the problem have been developed. The developed models and solution methods can be used to solve applied problems in engineering calculations of the strength and reliability of structural elements made of materials with rheological properties.

The main results of the dissertation were published in the following scientific papers

1. Alieva , U.S. Forced transverse vibrations of a long viscoelastic cylinder fastened to an elastic shell // –Sumgait : Sumgait State University , Scientific News , series: Natural and technical sciences , –2022. cild 22, No. 3, –pp. 12-16 . https://www.ssu-scientificnews.edu.az/pdf/T22-3.pdf

2. Alieva, U.S. Dynamic torsion waves in two-layer viscoelastic cylinders // –Bakı: Az.TU, Elmi əsərlər, Fundamental elmlər, No. 1, –2021 . –pp.54-58. https://drive.google.com/file/d/1Y7zPDvis0I6ZR8UJIqus5TiV9yvbnexp/view
3. Alieva , U.S. Study of dynamic waves in a viscoelastic medium taking into account the dependence of the relaxation function and density on coordinates // Ministry of Science and Higher Education of the Russian Federation Dagestan State University Fundamental and applied problems of mathematics and computer science Proceedings of the XIV international conference dedicated to the 90th anniversary of the Dagestan state University , –Makhachkala : –16-19 September , –20 21 . –p. 35-38.
4. Alieva , U.S. Study of dynamic waves in physically nonlinear media taking into account rheology // Azərbaycan Dövlət Pedaqoji Universieti Doktorantların və Gənc tədqiqatcıların XXII Respublika Elmi Konfransının Materialları, –Bakı: –22-23 November, –2018 . cild I , –p. 17-19 .
5. Alieva , U. S. Study of non -stationary plane waves in layered spaces // International scientific conference "Theoretical and Applied Problems of Mathematics", dedicated to the 100th anniversary of the birth of national leader Heydar Aliyev, organized jointly by Sumgait State University and the Institute of Mathematics and Mechanics . –Sumgait: April 25-26, –2023. - pp. 36-39.
6. Aliyeva , U.S. Study of the solution of the equation of longitudinal-transverse oscillation of rods taking into account the viscosity of the medium // –Sumgait : Sumgait State University , Scientific News , series: Natural and technical sciences , –2021. Vol. 21, No. 1, –pp. 9-12. https://www.ssu-scientificnews.edu.az/pdf/T21-1.pdf
7. Alieva , U.S. Study of the stability of a compressed rod in a viscoelastic medium // Proceedings of the XIII International Conference on Applied Mathematics and Mechanics in the Aerospace Industry (AMMAI-2020) , Alushta, –Crimea : –September 6-13 , –2020. –p.117-1 18 .

http://www.npnj.ru/files/npnj2020_web.pdf
8. Alieva , U. S. Oscillations of a reinforced viscoelastic pipe // II Republican Conference Fundamental Problems of Mathematics and Application of Intelligent Technologies in Education. –Sumgait : –December 15-16 , –2022. No. 10, –pp. 65-68 . https://www.ssu-conferenceproceedings.edu.az/pdf/riyaziyyat2022.pdf .
9. Babadzhanova , V.G., Aliyeva , U.S. Study of wave processes in a thick-walled pipe under internal pressure. –Sumgait : Sumgait State University , Scientific News , series: Natural and technical sciences , –2019. Vol. 19, No. 4, –pp. 14-18. https://www.ssu-scientificnews.edu.az/pdf/T19-4.pdf .
10. Kurbanov, N. T . , Alieva, U.S. A study of dynamic waves in a half-space taking into account the inhomogeneity of the medium // Fundamental and applied problems of mathematics and informatics. Proceedings of the XII International Conference dedicated to the 85th anniversary of Professor M.G. Alishaev . Ministry of Education and Science of the Russian Federation, Dagestan State University. –Makhachkala: –September 19-22, –2017. –pp. 96-98. https://elibrary.ru/item.asp?id=30470763
11. Kurbanov, N.T. , Alieva, U.S. Study of dynamic waves of inhomogeneous half-spaces taking into account rheology // –Crimea: Proceedings of the XII international conference on applied mathematics and mechanics in aerospace industry, NPNJ Alushta, –May 24-31, –2018. –pp. 406-409. https://elibrary.ru/item.asp?id=35651544
12. Kurbanov, N.T., Alieva, U.S. Study of dynamic stability of viscoelastic rods / / –Kharkov : Vysnik NTU , –2012. –pp. 86-90. Higher Attestation Commission of Ukraine. https://repository.kpi.kharkov.ua/items/d7bdf9d4-04c6-4ed4-8f63-326dc092f432
13. Kurbanov, N.T., Alieva, U.S. Study of bending of a deformable rod taking into account creep properties // –Sumgait : Sumgait

State University , Scientific News , series: Natural and technical sciences , –2018 . Volume 18, No. 3, –pp. 14-19 .

14. Kurbanov , N.T., Alieva , U.S. Study of vibrations of viscoelastic cylindrical shells at low viscosity // –Kharkov : Vysnik NTU, –2010. pp . 93-100 –. Higher Attestation Commission of Ukraine. file:///C:/Users/CompTel/Downloads/49516D0%A2%D0%B5%D0%BA%D1%81%D1%82%20%D1%81%D1%82%D0%B0%D1%82%D1%82%D1%96-99163-1-10-20150913.pdf.
15. Kurbanov , N.T., Babadzhanova , V.G., Alieva , U.S. Study of unsteady waves in inhomogeneous rheological media with low viscosity // –Sumgait : Sumgait State University , Scientific News , series: Natural and technical sciences , –2021 . Volume 21, No. 3, –pp. 4-9.
16. Kurbanov , N.T., Alieva , US. Study of unsteady shear waves in a two-layer viscoelastic cylinder with low viscosity // Ministry of Science and Higher Education of the Russian Federation, Dagestan State University. " Actual problems of mathematics and information technology ." Proceedings of the III All -Russian conference with international participation. –Makhachkala : –February 7-9, –2022. pp. 106-109 –. http://science.dgu.ru/NewsItem.aspx?id=14066
17. Kurbanov , N.T., Babadzhanova , V.G., Aliyeva , U.S. On one method for solving the integro-differential equation of oscillations of a relaxing system // Ministry of Education and Science of the Republic of Kazakhstan. Collection of materials of the XIV international scientific Nadirov readings "A striking example of the continuity of scientific traditions and loyalty to the profession". –Atyrau : –February 25 , –2022. –pp. 17-23. https://cloud.mail.ru/public/q995/zE4rHmjPn
18. Kurbanov N.T., Gadieva S.S., Alieva U.S. Dynamic torsion of a viscoelastic cylinder. Vest.DSU , Natural Sciences series. Issue 1, Volume 37, pp. 18-23, Makhachkala, 2022 (Higher School of Economics of Russia).

19. Kurbanov , N.T., Gadieva , S.S., Alieva , U.S. Study of dynamic torsion of a viscoelastic cylindrical shell under load distributed over the lateral surface /, N.T.Kurbanov , S.S. Gadieva , U.S. Alieva // The scientific heritage , –Budapest : –2021. vol.1 , no.74, –p.21-25 , RINTS. http://www.scientific-heritage.com/wp-content/uploads/2021/09/The-scientific-heritage-No-74-74-2021-Vol-1.pdf
20. Kurbanov , NT, Babajanova , VG, Aliyeva, US An investigation of the radial vibration of the reinforced pipe with a viscoelastic binder // The 8th International Conference on Control and Optimization with Industrial Applications, –Baku : –August 24-26, –2022 –p.201-203. http://www.coia-conf.org/upload/editor/files/COIA2022_V1.pdf
21. Kurbanov NT, Babajanova VG, Aliyeva US On a method for solving an equation of vibrations of a viscous-elastic beam. Seminar of Ilia Vekua Institute of Applied Mathematics Reports, vol 48, p.28-35, 2022, Ibane Javakhishvili , Tbilisi State University, Tbilisi. http://www.viam.science.tsu.ge/enlarged/2022/ .
22. Kurbanov NT, Babajanova VG, Aliyeva US About one design scheme for solving the problem of vibration of reinforced viscoelastic pipe/ 5th International Conference on Problems of Cybernetics and Informatics/ PCI-2023/ Baku 28-30 August, 2023. Kod 195003/ SCOPUS
23. Kurbanov NT, Babajanova VG, Aliyeva US, Agamalıyeva K. Constructing Approximate Solution to the Problem of Vibrations of Viscoelastic Systems . PLMO-2024 3rd international Conferehce on problems o logistics, management and operation in the east-west transport corridor. Baku May 15-17, 2024. https://plmo.cyber.az/2024/papers/00.html
24. Kurbanov NT, Babajanova VG, Aliyeva US, Agamalıyeva K. Mathematical modeling and analysis of the Solution of Vibrations of Rods with rheological features. The 9th

International Conference on Control and Optimization with Industrial Applications, Istanbul.27-29 August , 2024 .
file:///C:/Users/User/Downloads/BA_COIA24.pdf

The dissertation defense will take place on October 24, 2025 at 15:^{00} at a meeting of the dissertation council E D 1.19, operating on the basis of the Institute of Control Systems of the Ministry of Science and Education of the Republic of Azerbaijan.

Vahabzade street , 68

The dissertation is available in the library of the Institute of Systems of Management of the Ministry of Science and Education of the Republic of Azerbaijan.

Electronic versions of the dissertation and its abstract are available on the official website of the Institute of Control Systems of the Ministry of Science and Education of the Republic of Azerbaijan (http://www.isi.az) .

The abstract was sent to the appropriate addresses on September 23, 2025.

Signed for printing: 22.09. 2025

Paper format: A5

Volume: 37938

Circulation : 20