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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

SOLVABILITY AND SPECTRAL PROPERTIES OF SOME OPERATOR-DIFFERENTIAL EQUATIONS

Specialty: 1211.01 –Differential equations
Field of science: Mathematics

Applicant: **Elvin Sakhavat Rzayev**

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The work was performed in the department of "Nonharmonic analysis" of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan

Scientific supervisor:

Honored Scientist, doctor of mathematical sciences, professor
Araz Rafig Aliyev

Official opponents:

doctor of physical-mathematical sciences, professor
Nazim Bakhish Kerimov

candidate of physical-mathematical sciences, associate professor
Elvin Ibrahim Azizbayov

doctor of philosophy in mathematics
Elchin Jamal Ibadov

Dissertation council ED 1.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics the Ministry of Science and Education of the Republic of Azerbaijan.

Chairman of the Dissertation council:

corresponding member of ANAS,
doctor of physical-mathematical sciences, professor
_______________ **Misir Jumail Mardanov**

Scientific secretary of the Dissertation council:

candidate of physical-mathematical sciences
_______________ **Abdurrahim Farman Guliyev**

Chairman of scientific seminar:

doctor of physical-mathematical sciences, professor
_______________ **Nizameddin Shirin Isgenderov**

GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the topic.

One of the research methods for partial differential equations is the use of theory of operator-differential equations. In the middle of the past century, E.Hille, T.Kato, K.Iosida, S.Agmon, L.Nirenberg and Azerbaijani mathematician Z.I.Khalilov have studied the solvability of the Cauchy problem, behavior of the solution and the problem of stability for the first and second order differential equations with operator coefficient in Banach spaces. Since these works are the first in this field, the authors can be considered the founders of the theory of operator-differential equations.

Further, boundary value problems for second and higher order operator-differential equations have been studied by many foreign and Azerbaijani mathematicians and fundamental results have been obtained. Among these studies special interest has been shown to solvability problems. The articles of M.G.Gasymov, Yu.A. Dubinsky, S.S.Mirzoyev, Q.V. Radzievsky, V.K. Romanko, A.A. Shkalikov, N.I.Yurchuk, V.V.Vlasov and the book of S.Yakubov and Ya.Yakubov should be noted among the works on solvability. Among these works, M.G.Gasymov's[1,2,3] papers have special scientific weight. The reason for this is that the author related the well-posedness and unique solvability of boundary value problems for the considered higher order non-typical operator-differential equations with the completeness or multiple completeness of a part of eigen and adjoined vectors of an appropriate polynomial operator pencil.

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[1] Гасымов, М.Г. К теории полиномиальных операторных пучков // - Москва: Доклады АН СССР, - 1971. т. 199, № 4, - с. 747-750.

[2] Гасымов, М.Г. О кратной полноте части собственных и присоединенных векторов полиномиальных операторных пучков // - Иреван: Известия АН Арм.ССР, серия математика, - 1971. т. 6, № 2-3, - с. 131-147.

[3] Гасымов, М.Г. О разрешимости краевых задач для одного класса операторно-дифференциальных уравнений // - Москва: Доклады АН СССР, - 1977. т. 235, № 3, - с. 505-508.

The fundamental theory on multiple completeness of eigen and adjoined vectors of a class of polynomial operator pencils in M.V.Keldysh's[4] work (the brief content of this work was published by M.V.Keldysh in 1951) was the basis for M.G.Gasymov's papers.

The listed works differed with their effeciency and led to some studies. Some part of the studies were led to estimating the norms of intermediate derivative operators involved in operator-differential equations and as a result it was revealed that the solvability conditions found for such type equations are easily verifiable in practice. S.S.Mirzoyev first calculated the exact value of the norms of intermediate derivative operators for vector functions and obtained unimprovable sufficient conditions for solvability of boundary value problems for a wide class of elliptic and quasi-elliptic type operator-differential equations. S.S.Mirzoyev's results have found continuous development in A.R.Aliyev's works devoted to solvability of operator-differential equations with discontinuous coefficients.

Some non-classical problems of mathematical physics are reduced to problems with an operator in boundary conditions. Note that well-posed and unique solvability and Fredholmness of boundary value problems for second and third order operator-differential equations with an operator in the boundary conditions have been widely studied on a finite interval and on the half-axis. But these studies far from being complete. The works devoted to such kinds of problems for fourth order operator-differential equations are relatively few. As an example we can note the book of S.Yakubov and Y.Yakubov[5], the papers of V.S.Aliyev[6], B.A.Aliyev and

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[4] Келдыш, М.В. О полноте собственных функций некоторых классов несамосопряженных линейных операторов // - Москва: Успехи математических наук, - 1971. т. 26, № 4, - с. 15–41.

[5] Yakubov, S. Differential-Operator Equations: Ordinary and Partial Differential Equations / Yakubov, S., Yakubov, Y. - Chapman Hall/CRC Monogr. Surv. Pure Appl. Math., - vol. 103. Chapman & Hall, Boca Raton, - 2000. - 541 p.

[6] Aliyev, V.S. On normal solvability of boundary-value problems for elliptic type fourth order operator-differential equations // - Baku: Transactions of National Academy of Sciences of Azerbaijan. Series of Physical-Technical and Mathematical Sciences, - 2004. vol. 24, no. 7, - p. 9-16.

Y.Yakubov[7]. The represented dissertation work has been devoted to unique and well-posed solvability of boundary value problems for fourth-order operator-differential equations with operators in the boundary conditions. From the point of view of application, it should be underlined that fourth order operator-differential equations studied in the work are of great interest in theory of bending of thin and thick elastic plates. These problems of mechanics help to study the completeness of elementary solutions of equations. It should be noted that the completeness of elementary solutions in some solution spaces were previously shown in the papers of I.I. Vorovich, V.Ye.Kovalchuk, Yu.A.Ustinov, V.I.Yudovich, M.B.Orazov, A.A.Shkalikov, S.S.Mirzoyev, A.R.Aliyev and their followers. This dissertation work also touches these problems. And in its turn, this leads to proving the multiple completeness of a part of eigen and adjoined vectors of the appropriate polynomial operator pencil.

In the last 15 years, A.R. Aliyev and his followers have studied fourth-order differential equations with repeated characteristics encountered in the problems of stability of plates made of plastic material, found sufficient conditions for solvability expressed by the operator coefficients of the equations and studied some related spectral problems.

Object and subject of the study. The object and subject of the dissertation work are various boundary value problems for a class of elliptic type fourth order operator-differential equations on a half-axis with an operator coefficient in the boundary conditions.

Goals and objectives of the study. The main goal of the study is to find regular solvability conditions for various boundary value problems for a class of elliptic type fourth order operator-differential equations on a half-axis with an operator coefficient in the boundary conditions, to estimate the norms of intermediate derivative operators in Sobolev type vector function spaces, determine their relation with

[7] Алиев, Б.А., Якубов, Я. Фредгольмовость краевых задач для эллиптического дифференциально-операторного уравнения четвертого порядка с операторными граничными условиями // - Минск: Дифференциальные уравнения, - 2014. т. 50, № 2, - с. 210-216.

regular solvability conditions and study some spectral problems related to these equations.

Research methods. In the dissertation work, the methods of theory of differential equations in vector function spaces were used, the methods of Fourier transformation were attracted. Along with these the sections of functional analysis such as the theory of linear operators in Hilbert space, the theory of semi-groups of operators and methods of the theory of generalized functions have been widely used.

Main theses to be defended. The following theses are defended:

1. Finding regular solvability conditions for various boundary value problems for a class of elliptic type fourth order operator-differential equations on a half-axis with an operator coefficient in the boundary conditions.

2. Constructing explicit representation of regular solution of the principal part of the elliptic type fourth order operator-differential equation with an operator coefficient in the boundary conditions.

3. Estimating the norms of intermediate derivative operators in Sobolev type vector functions spaces and defining their relation with regular solvability conditions of the studied boundary value problems.

4. Finding conditions that determine the completeness of elementary solutions for a class of elliptic type fourth order homogeneous operator-differential equations on a half-axis.

Scientific novelty of the study. The following scientific novelties have been obtained:

1. Regular solvability conditions for various boundary value problems on a half-axis for a class of elliptic type fourth order operator-differential equations with an operator coefficient in the boundary conditions have been obtained.

2. Explicit representation of regular solution of the principal part of the elliptic type fourth order operator-differential equation with an operator coefficient in the boundary conditions has been constructed.

3. The norms of intermediate derivative operators in Sobolev type vector functions spaces have been estimated.

4. The relation between the estimation of the norms of intermediate derivative operators and regular solvability conditions of the studied boundary value problems have been determined.

5. The conditions that determine the completeness of elementary solutions for a class of elliptic-type fourth order homogeneous operator-differential equations in the regular solutions space within operator coefficient various non-homogeneous boundary conditions on a half-axis have been found.

Theoretical and practical importance of the study. The dissertation work is mainly of theoretical character. The obtained results allow to study new classes of boundary value problems for fourth order elliptic type partial differential equations and appropriate spectral problems. Furthermore, the results obtained in the dissertation work can be used in the problems of elasticity theory, for example in the problems of bending of thin and thick elastic plates.

Approbation and application. The results of the dissertation work were reported in the seminar of "Functional Analysis" of the Institute of Mathematics and Mechanics of Ministry of Science and Education (prof. Hamidulla Aslanov), of "Differential Equations" (prof. Akper Aliyev), in the seminar of the department “Applied Mathematics” of Baku State University (prof. Hamzaga Orujov), of "Differential and Integral Equations" (prof. Nizameddin Iskenderov), in the seminar of the department of "General and Applied Mathematics" of Azerbaijan State Oil and Industry University (prof. Araz Aliyev). The results were discussed also with prof. Sabir Mirzoyev. Furthermore, the results of the dissertation work were reported at the following scientific conferences: International scientific conference "Actual Problems of Mathematics and Informatics" devoted to the 90-th anniversary of National Leader Heydar Aliyev (Baku, May 29-31, 2013), at the International scientific conference "Spectral Theory of Differential Operators" devoted to 75-th anniversary of acad. M.G.Gasymov held at the Institute of Mathematics and Mechanics of ANAS (Baku, December 8-10, 2014), at the VII International Youth scientific-practical conference "Mathematical Modeling of Processes and Systems" (Sterlitamak, Russian Federation, December 7-9, 2017), at the International scientific seminar "Spectral Theory and Its Applications" dedicated to the 80-th anniversary of academician M.G.Gasymov at the Institute of Mathematics and Mechanics of ANAS (Baku, June 7-8, 2019), at the International conference "Modern Problems of Mathematics and Mechanics" dedicated to the 100-th anniversary of the National Leader Heydar Aliyev (April 26-28, 2023), "Spectral Theory of Operators and Issues Related to Them" dedicated to the 75th anniversary of the birth of the honored scientist of Russia, professor Y.T.Sultanayev at the International scientific-practical conference (Ufa, Russian Federation, October 26-27, 2023).

Author's personal contribution. All the results obtained belong to the author.

Author's publications. The author's 5 works have been published in the publishing houses recommended by the Higher Certification Commission of the Republic of Azerbaijan (one of them in SCIE list of Web of Science Core Collection, one in ESCI list of Web of Science Core Collection), 1 conference proceedings and 4 theses (in total 10 works). The list of references is at the end of the author's thesis.

The name of the organization where the work has been performed.

The work was performed in the department of "Nonharmonic analysis" of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

Total volume of the dissertation work in signs indicating separately the volume of each structural unit.

Total volume of the work is 177169 signs (title page – 354 signs, contents – 2769 signs, introductions – 44000 signs, chapter I – 66600 signs, chapter II – 35000 signs, chapter III – 27400 signs, conclusion – 1046). The list of references consists of 65 names.

The author expresses his deep gratitude to his supervisor prof. Araz Aliyev for his regular attention to the dissertation work.

MAIN CONTENT OF THE DISSERTATION WORK

The work consists of introduction, three chapters and a list of references.

In the introduction of the work we substantinate the rationale of the topic, show the development degree, objective and subjective of the topic, give scientific novelty, underline theoretical and practical importance and give information on the approbation of the work.

Assume that H denotes separable Hilbert space $(x, y \in H)$ with a scalar product (x, y), the operator A is a positive definite self-adjoint operator in H i.e. $A = A^* \geq cE,\ c > 0$, here E is a unit operator.

As is known, the domain of the operator A^α $(\alpha \geq 0)$ will be Hilbert space $(x, y \in D(A^\alpha))$ with respect to the scalar product $(x, y)_{H_\alpha} = (A^\alpha x, A^\alpha y)$ i.e. $H_\alpha = D(A^\alpha)$, $(x, y)_{H_\alpha} = (A^\alpha x, A^\alpha y)$, $x, y \in H_\alpha$. Accept that for $\alpha = 0$ $H_0 = H$.

By $L_2(\mathbb{R}_+; H)$ we denote a set of all vector functions $f(t)$ with the values from H , almost everywhere defined on $\mathbb{R}_+ = (0, +\infty)$ and square integrable in $\mathbb{R}_+$. This set is a Hilbert space with the scalar product

$$(f, g)_{L_2(\mathbb{R}_+; H)} = \int_0^{+\infty} (f(t), g(t))dt$$

and the norm

$$\|f\|_{L_2(\mathbb{R}_+; H)} = \left(\int_0^{+\infty} \|f(t)\|_H^2\, dt\right)^{1/2} < +\infty.$$

Now according to J.-L.Lions and E.Majenes[8] book we introduce the linear space

$$W_2^4(\mathbb{R}_+; H) = \left\{u(t):\ A^4 u(t) \in L_2(\mathbb{R}_+; H), u^{(4)}(t) \in L_2(\mathbb{R}_+; H)\right\}.$$

This space is a complete Hilbert space with respect to the scalar product

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[8] Лионс, Ж.-Л. Неоднородные граничные задачи и их приложения / Ж.-Л.Лионс, Э.Мадженес – Москва: Мир, - 1971. – 371 с.

$$(u,v)_{W_2^4(\mathbb{R}_+;\,H)} = \left(u^{(4)}, v^{(4)}\right)_{L_2(\mathbb{R}_+;\,H)} +$$
$$+(A^4u, A^4v)_{L_2(\mathbb{R}_+;\,H)},\ \ u(t), v(t) \in W_2^4(\mathbb{R}_+;H).$$

The norm in the space $W_2^4(\mathbb{R}_+;\ H)$ is determined as

$$\|u\|_{W_2^4(\mathbb{R}_+;H)} = \left(\|A^4u\|^2_{L_2(\mathbb{R}_+;H)} + \left\|u^{(4)}\right\|^2_{L_2(\mathbb{R}_+;H)}\right)^{1/2}.$$

Here and in the sequel, these derivatives are understood as generalized derivatives.

Under the denotation $L(X,Y)$ we will understand a set of linear bounded operators acting from one Hilbert space X to another Hilbert space Y. The denotation $\sigma(\cdot)$ means the spectrum of the operator $(\cdot)$.

In the separable Hilbert space H we consider the following elliptic type fourth order operator-differential equation:

$$u^{(4)}(t) + A^4u(t) + \sum_{j=1}^{4} A_j u^{(4-j)}(t) = f(t),\ \ t \in \mathbb{R}_+, \qquad (1)$$

here $A = A^* \geq cE,\ c > 0,\ A_j,\ j = 1,2,3,4$, are linear and generally speaking unbounded operators, $f(t) \in L_2(\mathbb{R}_+;H),\ u(t) \in W_2^4(\mathbb{R}_+;H)$.

Definition 1. If the vector-function $u(t) \in W_2^4(\mathbb{R}_+;H)$ satisfies equation (1) almost everywhere in $\mathbb{R}_+$, then $u(t)$ is said to be a regular solution of equation (1).

Equation (1) is studied separately with each of the following boundary conditions:

$$u(0) = 0,\ \ u'(0) = Tu''(0); \qquad (2)$$
$$u(0) = 0,\ \ u''(0) = Ku'(0); \qquad (3)$$
$$u'(0) = Tu''(0),\ \ u'''(0) = 0; \qquad (4)$$
$$u''(0) = Ku'(0),\ \ u'''(0) = 0, \qquad (5)$$

here $T \in L\left(H_{3/2}, H_{5/2}\right),\ K \in L\left(H_{5/2}, H_{3/2}\right)$.

Definition 2. If for arbitrary $f(t) \in L_2(\mathbb{R}_+;H)$ the equation (1) has a regular solution $u(t)$ satisfying the boundary conditions of (2) in the sense of the convergence

$$\lim_{t\to 0}\|u(t)\|_{H_{7/2}} = 0,\ \ \lim_{t\to 0}\|u'(t) - Tu''(t)\|_{H_{5/2}} = 0$$

and the inequality

$$\|u\|_{W_2^4(\mathbb{R}_+;H)} \leq const\|f\|_{L_2(\mathbb{R}_+;H)}$$

is valid, then boundary value problem $(1), (2)$ is said to be regularly solvable.

Definition 3. If for arbitrary $f(t) \in L_2(\mathbb{R}_+; H)$ the equation (1) has a regular solution $u(t)$ satisfying the boundary conditions of (3) in the sense of convergence

$$\lim_{t\to 0}\|u(t)\|_{H_{7/2}} = 0, \ \lim_{t\to 0}\|u''(t) - Ku'(t)\|_{H_{3/2}} = 0$$

and the inequality

$$\|u\|_{W_2^4(\mathbb{R}_+;H)} \leq const\|f\|_{L_2(\mathbb{R}_+;H)}$$

is valid then the boundary value problem $(1), (3)$ is said to be regularly solvable.

Regular solvability definitions of boundary value problems $(1), (4)$ and $(1), (5)$ are given in the same way.

Our goal in chapter I is to show regular solvability of boundary value problems $(1), (2)$; $(1), (3)$; $(1), (4)$ and $(1), (5)$ for the case $A_j = 0, \ j = 1, 2, 3, 4,$ imposing definite conditions on the operator coefficients.

By $P_{0,T}^{\{0\}}, P_{0,K}^{\{0\}}, P_{0,T}^{\{3\}}$ and $P_{0,K}^{\{3\}}$ we denote the operator acting from the spaces $W_{2,T}^4(\mathbb{R}_+; H; \{0\})$, $W_{2,K}^4(\mathbb{R}_+; H; \{0\})$, $W_{2,T}^4(\mathbb{R}_+; H; \{3\})$ and $W_{2,K}^4(\mathbb{R}_+; H; \{3\})$ respectively to the space $L_2(\mathbb{R}_+; H)$ by the following rules:

$$P_{0,T}^{\{0\}}u(t) = u^{(4)}(t) + A^4u(t), \ u(t) \in W_{2,T}^4(\mathbb{R}_+; H; \{0\}),$$
$$P_{0,K}^{\{0\}}u(t) = u^{(4)}(t) + A^4u(t), \ u(t) \in W_{2,K}^4(\mathbb{R}_+; H; \{0\}),$$
$$P_{0,T}^{\{3\}}u(t) = u^{(4)}(t) + A^4u(t), \ u(t) \in W_{2,T}^4(\mathbb{R}_+; H; \{3\}),$$
$$P_{0,K}^{\{3\}}u(t) = u^{(4)}(t) + A^4u(t), \ u(t) \in W_{2,K}^4(\mathbb{R}_+; H; \{3\}),$$

here

$$W_{2,T}^4(\mathbb{R}_+; H; \{0\}) = \{u(t): \ u(t) \in W_2^4(\mathbb{R}_+; H), \ u(0) = 0, u'(0) = \\ = Tu''(0)\},$$
$$W_{2,K}^4(\mathbb{R}_+; H; \{0\}) = \{u(t): \ u(t) \in W_2^4(\mathbb{R}_+; H), \ u(0) = 0, u''(0) = \\ = Ku'(0)\},$$
$$W_{2,T}^4(\mathbb{R}_+; H; \{3\}) = \{u(t): \ u(t) \in W_2^4(\mathbb{R}_+; H), \ u'(0) = Tu''(0), \\ u'''(0) = 0\},$$

$$W_{2,K}^{4}(\mathbb{R}_+;H;\{3\}) = \{u(t) : u(t) \in W_2^4(\mathbb{R}_+;H),\ u''(0) = Ku'(0),\ u'''(0) = 0\}.$$

The following statement is valid for the operator $P_{0,T}^{\{0\}}$.

Theorem 1. *Assume that* $C = A^{5/2}TA^{-3/2}$ *and* $-\frac{1}{\sqrt{2}} \notin \sigma(C)$. *Then for arbitrary* $f(t) \in L_2(\mathbb{R}_+;H)$ *the equation* $P_{0,T}^{\{0\}}u(t) = f(t)$ *has a unique regular solution and the representation of this solution is as follows:*

$$u(t) = u_0(t) + \frac{1}{4(\omega_1 - \omega_2)} e^{\omega_1 tA} A^{-\frac{7}{2}}\left(E + \sqrt{2}C\right)^{-1} A^{\frac{5}{2}} \times$$
$$\times \left[-\frac{\omega_4}{\omega_3} \int_0^{+\infty} (e^{-\omega_4 As} - e^{-\omega_3 As})\left(A^{-2}f(s)\right) ds + \right.$$
$$+T \int_0^{+\infty} (\omega_2 e^{-\omega_4 As} + \omega_1 e^{-\omega_3 As})\left(A^{-1}f(s)\right) ds -$$
$$\left. -\omega_2{}^2 TA^2 \int_0^{+\infty} (\omega_4 e^{-\omega_4 As} + \omega_3 e^{-\omega_3 As})\left(A^{-3}f(s)\right) ds \right] -$$
$$-\frac{1}{4} e^{\omega_2 tA} \left[\frac{1}{\omega_1 - \omega_2} A^{-\frac{7}{2}}\left(E + \sqrt{2}C\right)^{-1} A^{\frac{5}{2}} \times \right.$$
$$\times \left(-\frac{\omega_4}{\omega_3} \int_0^{+\infty} (e^{-\omega_4 As} - e^{-\omega_3 As})\left(A^{-2}f(s)\right) ds + \right.$$
$$+T \int_0^{+\infty} (\omega_2 e^{-\omega_4 As} + \omega_1 e^{-\omega_3 As})\left(A^{-1}f(s)\right) ds -$$
$$\left. -\omega_2{}^2 TA^2 \int_0^{+\infty} (\omega_4 e^{-\omega_4 As} + \omega_3 e^{-\omega_3 As})\left(A^{-3}f(s)\right) ds \right) +$$
$$\left. + \int_0^{+\infty} (\omega_4 e^{-\omega_4 As} + \omega_3 e^{-\omega_3 As})\left(A^{-3}f(s)\right) ds \right],$$

where

$$u_0(t)=\int_0^{+\infty} G(t,s)f(s)\,ds,$$

$$G(t,s)=\frac{1}{4}\begin{cases}-\left(\dfrac{e^{\omega_2 A(t-s)}}{\omega_1}+\dfrac{e^{\omega_1 A(t-s)}}{\omega_2}\right)A^{-3}, & for\;\; t-s>0\,,\\ \left(\dfrac{e^{\omega_4 A(t-s)}}{\omega_3}+\dfrac{e^{\omega_3 A(t-s)}}{\omega_4}\right)A^{-3}, & for\; t-s<0\,,\end{cases}$$

$$\omega_1=-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,\;\; \omega_2=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i,\;\; \omega_3=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,$$

$$\omega_4=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i.$$

The following statement is valid for the operator $P_{0,K}^{\{0\}}$ as well.

Theorem 2. *Assume that $B=A^{3/2}KA^{-5/2}$ and $-\sqrt{2}\notin\sigma(B)$. Then for arbitrary $f(t)\in L_2(\mathbb{R}_+;H)$ the equation $P_{0,K}^{\{0\}}u(t)=f(t)$ has a unique regular solution and the representation of this solution is as follows:*

$$u(t)=u_0(t)+\frac{1}{4({\omega_2}^2-{\omega_1}^2)}e^{\omega_1 tA}A^{-\frac{7}{2}}\left(E+\frac{1}{\sqrt{2}}B\right)^{-1}A^{\frac{3}{2}}\times$$

$$\times\left[\omega_2 KA\int_0^{+\infty}\left(\frac{e^{-\omega_4 As}}{\omega_3}+\frac{e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-3}f(s)\right)ds-\right.$$

$$-K\int_0^{+\infty}\left(\frac{\omega_4 e^{-\omega_4 As}}{\omega_3}+\frac{\omega_3 e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-2}f(s)\right)ds+$$

$$+\int_0^{+\infty}\left(\frac{{\omega_4}^2 e^{-\omega_4 As}}{\omega_3}+\frac{{\omega_3}^2 e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-1}f(s)\right)ds-$$

$$\left.-{\omega_2}^2A^2\int_0^{+\infty}\left(\frac{e^{-\omega_4 As}}{\omega_3}+\frac{e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-3}f(s)\right)ds\right]-$$

$$-\frac{1}{4}e^{\omega_2 tA}\left[\frac{1}{{\omega_2}^2-{\omega_1}^2}A^{-\frac{7}{2}}\left(E+\frac{1}{\sqrt{2}}B\right)^{-1}A^{\frac{3}{2}}\times\right.$$

$$\times\left(\omega_2 KA\int_0^{+\infty}\left(\frac{e^{-\omega_4 As}}{\omega_3}+\frac{e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-3}f(s)\right)ds-\right.$$

$$-K\int_0^{+\infty}\left(\frac{\omega_4 e^{-\omega_4 As}}{\omega_3}+\frac{\omega_3 e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-2}f(s)\right)ds+$$

$$+\int_0^{+\infty}\left(\frac{{\omega_4}^2 e^{-\omega_4 As}}{\omega_3}+\frac{{\omega_3}^2 e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-1}f(s)\right)ds-$$

$$\left.-{\omega_2}^2A^2\int_0^{+\infty}\left(\frac{e^{-\omega_4 As}}{\omega_3}+\frac{e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-3}f(s)\right)ds\right)+$$

$$\left.+\int_0^{+\infty}\left(\frac{e^{-\omega_4 As}}{\omega_3}+\frac{e^{-\omega_3 As}}{\omega_4}\right)\left(A^{-3}f(s)\right)ds\right].$$

The appropriate statements are valid for the operator $P_{0,T}^{\{3\}}$ and $P_{0,K}^{\{3\}}$ as well.

Our goal in chapter II is to show regular solvability of the considered boundary value problems $(1),(2)$; $(1),(3)$; $(1),(4)$ and $(1),(5)$ in the case $A_j\neq 0,\ j=1,2,3,4,$ imposing certain conditions on the operator coefficients. Note that in this chapter when determining solvability condition the existence of relation with finding the norms of intermediate derivative operators is shown and they are estimated in Sobolev type spaces.

Theorem 3. *Assume that* $C=A^{5/2}TA^{-3/2}$, $ReC\geq 0$, $A_jA^{-j}\in L(H,H)$, $j=1,2,3,4,$ *and the inequality* $\sum_{j=1}^{4}c_j\left\|A_jA^{-j}\right\|_{H\to H}<1$ *is satisfied. Here* $c_1=1$, $c_2=\frac{1}{2}$, $c_3=\frac{1}{\sqrt{2}}$, $c_4=1$. *Then for each* $f(t)\in L_2(\mathbb{R}_+;H)$ *the boundary value problem (1), (2) has a unique regular solution.*

Theorem 4. *Assume that* $B = A^{3/2}KA^{-5/2}$, $ReB \geq 0$, $A_jA^{-j} \in L(H,H)$, $j = 1,2,3,4$, *and the inequality* $\sum_{j=1}^{4} c_j \left\| A_jA^{-j} \right\|_{H\to H} < 1$ *is satisfied. Here* $c_1 = 1$, $c_2 = \frac{1}{2}$, $c_3 = \frac{1}{\sqrt{2}}$, $c_4 = 1$. *Then for each* $f(t) \in L_2(\mathbb{R}_+;H)$ *the boundary value problem* $(1),(3)$ *has a unique regular solution.*

Theorem 5. *Assume that* $C = A^{5/2}TA^{-3/2}$, $ReC \geq 0$, $A_jA^{-j} \in L(H,H)$, $j = 1,2,3,4$, *and the inequality* $\sum_{j=1}^{4} c_j \left\| A_jA^{-j} \right\|_{H\to H} < 1$ *is satisfied. Here* $c_1 = \frac{1}{\sqrt{2}}$, $c_2 = \frac{1}{2}$, $c_3 = c_4 = 1$. *Then for each* $f(t) \in L_2(\mathbb{R}_+;H)$ *the boundary value problem* $(1),(4)$ *has a unique regular solution.*

Theorem 6. *Assume that* $B = A^{3/2}KA^{-5/2}$, $ReB \geq 0$, $A_jA^{-j} \in L(H,H), j = 1,2,3,4$, *and the inequality* $\sum_{j=1}^{4} c_j \left\| A_jA^{-j} \right\|_{H\to H} < 1$ *is satisfied. Here* $c_1 = \frac{1}{\sqrt{2}}$, $c_2 = \frac{1}{2}$, $c_3 = c_4 = 1$. *Then for each* $f(t) \in L_2(\mathbb{R}_+;H)$ *the boundary value problem* $(1),(5)$ *has a unique regular solution.*

The numbers c_j, $j = 1,2,3,4$, involved in theorems 3, 4, 5, 6 are the numbers obtained from the estimation of the norms of the intermediate derivative operators

$$A^j \frac{d^{4-j}}{dt^{4-j}} : W_{2,T}^4(\mathbb{R}_+;H;\{0\}) \to L_2(\mathbb{R}_+;H),$$

$$A^j \frac{d^{4-j}}{dt^{4-j}} : W_{2,K}^4(\mathbb{R}_+;H;\{0\}) \to L_2(\mathbb{R}_+;H),$$

$$A^j \frac{d^{4-j}}{dt^{4-j}} : W_{2,T}^4(\mathbb{R}_+;H;\{3\}) \to L_2(\mathbb{R}_+;H),$$

$$A^j \frac{d^{4-j}}{dt^{4-j}} : W_{2,K}^4(\mathbb{R}_+;H;\{3\}) \to L_2(\mathbb{R}_+;H), \quad j = 1,2,3,4,$$

with respect to the norms $\left\| P_{0,T}^{\{0\}} u \right\|_{L_2(\mathbb{R}_+;H)}$, $\left\| P_{0,K}^{\{0\}} u \right\|_{L_2(\mathbb{R}_+;H)}$, $\left\| P_{0,T}^{\{3\}} u \right\|_{L_2(\mathbb{R}_+;H)}$, $\left\| P_{0,K}^{\{3\}} u \right\|_{L_2(\mathbb{R}_+;H)}$.

In chapter III we consider homogeneous case of equation (1) in separable Hilbert space H:

$$u^{(4)}(t)+A^4u(t)+\sum_{j=1}^{4}A_ju^{(4-j)}(t)=0,\ \ t\in\mathbb{R}_+, \tag{6}$$

here $A=A^*\geq cE,\ c>0,\ A_j,\ j=1,2,3,4,$ are linear, generally speaking unbounded operators, $u(t)\in W_2^4(\mathbb{R}_+;H)$.

Equation (6) are studied separately for each of the following boundary conditions and according to the previous chapters definitions of regular solution and regular solvability of boundary value problem are given:

$$u(0)=\varphi,\ \ u'(0)-Tu''(0)=\psi, \tag{7}$$
$$\varphi\in H_{7/2},\psi\in H_{5/2},T\in L\left(H_{3/2},H_{5/2}\right);$$

$$u(0)=\varphi,\ \ u''(0)-Ku'(0)=\psi, \tag{8}$$
$$\varphi\in H_{7/2},\psi\in H_{3/2},K\in L\left(H_{5/2},H_{3/2}\right);$$

$$u'(0)-Tu''(0)=\psi,\ \ u'''(0)=\varphi, \tag{9}$$
$$\psi\in H_{5/2},\varphi\in H_{1/2},T\in L\left(H_{3/2},H_{5/2}\right);$$

$$u''(0)-Ku'(0)=\psi,\ \ u'''(0)=\varphi, \tag{10}$$
$$\psi\in H_{3/2},\varphi\in H_{1/2},K\in L\left(H_{5/2},H_{3/2}\right).$$

Theorem 7. *Assume that the conditions of theorem 5 are satisfied. Then for each* $\varphi\in H_{7/2}$, $\psi\in H_{5/2}$ *the boundary value problem* $(6),(7)$ *has a unique regular solution.*

Theorem 8. *Assume that the condsitions of theorem 6 are satisfied. Then for each* $\varphi\in H_{7/2}$, $\psi\in H_{3/2}$ *the boundary value problem* $(6),(8)$ *has a unique regular solution.*

The appropriate statements are valid for the boundary value problems $(6),(9)$ and $(6),(10)$ as well.

Next, in this chapter we consider the following elliptic type fourth order polynomial operator pencil

$$P(\lambda)=\lambda^4E+A^4+\sum_{j=1}^{4}\lambda^{4-j}A_j \tag{11}$$

corresponding to equation (6) in a separable Hilbert space H. Operator coefficients of the operator pencil (11) satisfy the same conditions as the operator coefficient of equation (6).

Denote by $\sigma_\infty(H)$ the set of completely continuous operators acting in space H. As we know[9], for $Q \in \sigma_\infty(H)$ the operator $(Q^*Q)^{1/2}$ is a completely continuous self-adjoint operator and its eigenvalues are called s-numbers of the operator Q.

Numbering the non-zero s-numbers of the operator Q in the descending order taking into account their multiplicity we introduce, the denotation

$$\sigma_p = \left\{Q : Q \in \sigma_\infty(H), \sum_{j=1}^{\infty} s_j^p(Q) < \infty\right\}, 0 < p < \infty.$$

It is clear that σ_p are Neimann-Shatten classes.

If the equation $P(\lambda_0)x = 0$ has a non-zero (nontrivial) solution x_0, then λ_0 is called an eigenvalue of the operator pencil $P(\lambda)$, while x_0 is called an eigenvector corresponding to the eigenvalue λ_0.

Note that we require the operator coefficients of the pencil (11) to fulfill the following conditions:

$$A = A^* \geq cE, c > 0,\ A^{-1} \in \sigma_\infty(H);$$

$$A_j A^{-j} \in L(H,H), j = 1, 2,3,4,\ \ (E + A_4 A^{-4})^{-1} \in L(H,H).$$

Under these conditions the results of M.V.Keldysh's[4] works follow that the spectrum of the operator pencil $P(\lambda)$ is discrete. This means that exception of the set of isolated eigenvalues $\{\lambda_n\}$, which can only have limit point at infinity, all $\lambda \in \mathbb{C}$ have the resolvent $P^{-1}(\lambda)$.

If the number λ_0 is an eigenvalue of the operator pencil $P(\lambda)$ while x_0 is one of the eigenvectors corresponding to the eigenvalue λ_0 and the elements of the system $x_1, x_2, \ldots, x_m$ satisfy the equalities

$$\sum_{k=0}^{4} \frac{1}{k!} \frac{d^k P(\lambda)}{d\lambda^k}\bigg|_{\lambda=\lambda_0} x_{p-k} = 0, \qquad p = 0, 1, 2, \ldots, m \quad (12)$$

$$(x_{-1} = x_{-2} = x_{-3} = x_{-4} = 0),$$

then the system $x_1, x_2, \ldots, x_m$ is called a chain of adjoined vectors to the eigenvector x_0.

[9] Горбачук, В.И. Граничные задачи для дифференциально-операторных уравнений / В.И.Горбачук, М.Л.Горбачук – Киев: Наукова думка, - 1984. – 284 с.

Assume that λ_n are eigenvalues of the operator pencil (11), $x_{0,n}, x_{1,n}, \dots, x_{m,n}$ is a system of eigen and adjoined vectors responding to the eigenvalues λ_n. Equalities (12) yield that the functions

$$u_{h,n}(t) = e^{\lambda_n t}\left(x_{h,n} + \frac{t}{1!}x_{h-1,n} + \dots + \frac{t^h}{h!}x_{0,n}\right), h = \overline{0, m} \quad (13)$$

satisfy the equation (6). These functions are called elementary solutions of the equation (6). It is clear that for $Re\lambda_n < 0$ these solutions are decreasing solutions and are belong to the space $W_2^4(\mathbb{R}_+; H)$.

It is clear from M.V.Keldysh's[4] work that to any eigenvalue λ_n we can adjoin the system of canonical eigen and adjoined vectors of the operator pencil $P(\lambda)$.

If in the boundary conditions $(7) - (10)$ we accept $T = 0$, $K = 0$, then we can write these conditions in the general form as follows :

$$u^{(l_i)}(0) = \xi_{l_i}, \qquad i = 0, 1. \quad (14)$$

Here $\xi_{l_i} \in H_{7/2 - l_i}$, $i = 0, 1$, l_i are whole numbers, so they satisfy the conditions $0 \leq l_0 < l_1 \leq 3$ and $l_0 + l_1 \neq 3$.

We can easily show that by substitution method we can reduce the boundary value problem (6), (14) to the boundary value problem

$$v^{(4)}(t) + A^4 v(t) + \sum_{j=1}^{4} A_j v^{(4-j)}(t) = g(t), \ \ t \in \mathbb{R}_+, \quad (15)$$

$$v^{(l_i)}(0) = 0, \qquad i = 0, 1, \quad (16)$$

For the boundary value problem $(15), (16)$ the validity of the statement from S.S.Mirzoyev's[10,11] works is known.

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10 Мирзоев, С.С. Условия корректной разрешимости краевых задач для операторно-дифференциальных уравнений // - Москва: Доклады АН СССР, - 1983. т. 273, № 2, - с. 292-295.

11 Мирзоев, С.С. Вопросы теории разрешимости краевых задач для операторно-дифференциальных уравнений в гильбертовом пространстве и связанные с ними спектральные задачи: / Диссертация на соискание доктора физико-математических наук. / - Баку, 1994. - 229 с.

Theorem 9. *Assume that* $A_j A^{-j} \in L(H,H),\ j = 1,2,3,4,$ *and the inequality* $\sum_{j=1}^{4} c_j \left\| A_j A^{-j} \right\|_H < 1$ *is satisfied. Here*

$$c_j = \sup_{0 \neq v \in \overset{0}{W}{}_2^4(\mathbb{R}_+;H;\{l_i\})} \left\| A^j v^{(4-j)} \right\|_{L_2(\mathbb{R}_+;H)} \times$$

$$\times \left\| v^{(4)}(t) + A^4 v(t) \right\|_{L_2(\mathbb{R}_+;H)}^{-1}, j = \overline{1,4},$$

$$\overset{0}{W}{}_2^4(\mathbb{R}_+;H;\{l_i\}) =$$

$$= \left\{ v(t) \colon v(t) \in W_2^4(\mathbb{R}_+;H), v^{(l_i)}(0) = 0, i = 0,1 \right\}.$$

Then for each $g(t) \in L_2(\mathbb{R}_+;H)$ *the boundary value problem* $(15), (16)$ *has a unique regular solution.*

Wide information on calculation of the numbers c_j in theorem 9 can be found in S.S.Mirzoyev's[10,11,12] works.

For $Re\lambda_n < 0$ by means of the solution (13) we determine the vector

$$\tilde{x}_{h,n}^{(l_0,l_1)} = \left\{ x_{h,n}{}^{(l_0)}, x_{h,n}{}^{(l_1)} \right\} \in \widetilde{H}_{l_0,l_1} \equiv H_{7/_2 - l_0} \oplus H_{7/_2 - l_1}$$

here $x_{h,n}^{(l_i)} \equiv \frac{d^{l_i}}{dt^{l_i}} u_{h,n}(t) \Big|_{t=0},\ i = 0,1,\ h = 0,1,\dots,m.$

We call the system $\left\{ \tilde{x}_{h,n}^{(l_0,l_1)} \right\}$ a derivative chain of eigen and adjoined vectors of the operator pencil $P(\lambda)$ generated by the boundary value problem (6), (14).

Note that in S.S.Mirzoyev's[11] works it was proved that within the conditions of theorem 9, for $(E + A_4 A^{-4})^{-1} \in L(H,H)$ subject to one of the conditions $A^{-1} \in \sigma_p, 0 < p \leq 1$ or $A^{-1} \in \sigma_p,\ 0 < p < \infty,\ A_j A^{-j} \in \sigma_\infty(H),\ j = 1,2,3,4,$ each of the systems $\left\{ \tilde{x}_{h,n}^{(l_0,l_1)} \right\}$ is complete in the appropriate space $\widetilde{H}_{l_0,l_1}$.

Lets determine the following derivative chains:

$$\left\{ \bar{x}_{h,n}^{(0,1,2)} \right\}, \qquad \bar{x}_{h,n}^{(0,1,2)} = \left\{ x_{h,n}^{(0)},\ x_{h,n}^{(1,2)} \right\} \in \widetilde{H}_{0,1} \equiv H_{7/_2} \oplus H_{5/_2},$$

$$x_{h,n}^{(0)} = u_{h,n}(0), \qquad x_{h,n}^{(1,2)} \equiv x_{h,n}^{(1)} - T x_{h,n}^{(2)} = u_{h,n}{}'(0) - T u_{h,n}{}''(0);$$

[12] Mirzoyev, S.S. On the norms of operators of intermediate derivatives // - Baku: Transactions of National Academy of Sciences of Azerbaijan. Series of Physical-Technical and Mathematical Sciences, - 2003. vol. 23, no. 1, - p. 157-164.

$$\left\{\bar{x}_{h,n}^{(0,2,1)}\right\}, \qquad \bar{x}_{h,n}^{(0,2,1)} = \left\{x_{h,n}^{(0)},\, x_{h,n}^{(2,1)}\right\} \in \widetilde{H}_{0,2} \equiv H_{7/_2} \oplus H_{3/_2},$$
$$x_{h,n}^{(0)} = u_{h,n}(0), \qquad x_{h,n}^{(2,1)} \equiv x_{h,n}^{(2)} - Kx_{h,n}^{(1)} = u_{h,n}{}''(0) - Ku_{h,n}{}'(0);$$
$$\left\{\bar{x}_{h,n}^{(1,2,3)}\right\}, \qquad \bar{x}_{h,n}^{(1,2,3)} = \left\{x_{h,n}^{(1,2)},\, x_{h,n}^{(3)}\right\} \in \widetilde{H}_{1,3} \equiv H_{5/_2} \oplus H_{1/_2},$$
$$x_{h,n}^{(1,2)} = x_{h,n}^{(1)} - Tx_{h,n}^{(2)} = u_{h,n}{}'(0) - Tu_{h,n}{}''(0),\; x_{h,n}^{(3)} = u_{h,n}{}'''(0);$$
$$\left\{\bar{x}_{h,n}^{(2,1,3)}\right\}, \qquad \bar{x}_{h,n}^{(2,1,3)} = \left\{x_{h,n}^{(2,1)},\, x_{h,n}^{(3)}\right\} \in \widetilde{H}_{2,3} \equiv H_{3/_2} \oplus H_{1/_2},$$
$$x_{h,n}^{(2,1)} = x_{h,n}^{(2)} - Kx_{h,n}^{(1)} = u_{h,n}{}''(0) - Ku_{h,n}{}'(0),\; x_{h,n}^{(3)} = u_{h,n}{}'''(0).$$

It is clear that the derivative chains $\left\{\bar{x}_{h,n}^{(0,1,2)}\right\}$, $\left\{\bar{x}_{h,n}^{(0,2,1)}\right\}$, $\left\{\bar{x}_{h,n}^{(1,2,3)}\right\}$ and $\left\{\bar{x}_{h,n}^{(2,1,3)}\right\}$ respectively correspond to the boundary value problems (6), (7); (6), (8); (6), (9) and (6), (10). It is known that[13], for proving double completeness of these chains in the trace spaces it sufficies to show their equivalence to the appropriate system $\left\{\tilde{x}_{h,n}^{(l_0,l_1)}\right\}$. Since this procedure is similar for each of the chains, in this work we do it for example, for the chain $\left\{\bar{x}_{h,n}^{(0,2,1)}\right\}$ i.e. we show the equivalence of the systems $\left\{\tilde{x}_{h,n}^{(0,2)}\right\}$ and $\left\{\bar{x}_{h,n}^{(0,2,1)}\right\}$ in the space $H_{7/_2} \oplus H_{3/_2}$. The noted considerations mentioned above allow to state the main theorem of third chapter.

In theorem 8 for each $\varphi \in H_{7/_2}$, $\psi \in H_{3/_2}$ sufficient conditions providing the existence of a unique solution of the boundary value problem (6), (8) from the space $W_2^4(\mathbb{R}_+;H)$ were determined. By $W^{(0,2,1)}(P)$ we denote the set of these solutions. According to the theorems on intermediate derivatives and traces the set[8] $W^{(0,2,1)}(P)$ is a closed subspace of the $W_2^4(\mathbb{R}_+;H)$ space.

The following statement is valid.

Theorem 10. *Assume that the conditions of theorem 8 are satisfied. Furthermore, if alongside with the condition*

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[13] Мильман, В.Д. Геометрическая теория пространств Банаха. Часть. I. Теория базисных и минимальных систем // - Москва: Успехи математических наук, - 1970. т. 25, № 3, - с. 113-174.

$(E + A_4 A^{-4})^{-1} \in L(H, H)$ *one of the following conditions*
$A^{-1} \in \sigma_p, 0 < p \leq 1,$ *or*
$A^{-1} \in \sigma_p, 0 < p < \infty,\ A_j A^{-j} \in \sigma_\infty(H), j = 1, 2, 3, 4,$
is satisfied, the system of decreasing elementary solutions of the boundary value problem (6), (8) *is complete in the space* $W^{(0,2,1)}(P)$.

Note that similar theorems can be valid for boundary value problems (6), (7); (6), (9) and (6), (10) as well.

CONCLUSION

The dissertation work has been devoted to regular solvability of various boundary value problems on a half-axis for a class of elliptic type fourth order operator-differential equations with operator coefficient boundary conditions.

The main results of the dissertation work are the following:

1. Sufficient regular solvability conditions of various boundary value problems on a half-axis for a class of elliptic type fourth order operator-differential equations with operator coefficient boundary conditions were found.
2. Explicit representation of the regular solvability of the principal part of the elliptic type fourth order operator-differential equation with operator coefficient boundary conditions was structured.
3. The norms of intermediate derivative operators in Sobolev type vector functions spaces have been estimated.
4. The relation between the estimation of the norms of intermediate derivative operators and the regular solvability conditions of the considered boundary value problems was determined.
5. On a half-axis within various non-homogeneous boundary conditions with operator coefficient that determine the conditions of the completeness of the elementary solutions for a class of elliptic type fourth order homogeneous operator-differential equations in the space of regular solutions were found.

The results of the dissertation work have been published in the following works:

1. *Rzayev, E.S.* On a boundary value problem for a fourth order differential equation with operator coefficients // Abstracts of the International Conference on Actual Problems of Mathematics and Informatics dedicated to the 90-th anniversary of Haydar Aliyev, - Baku: -29-31 May, – 2013, – p. 92-93.
2. *Рзаев, Э.С.* Об одной краевой задаче для дифференциального уравнения четвертого порядка с операторными коэффициентами // - Baku: Journal of Qafqaz University, Series of Mathematics and Computer Science, - 2013. vol. 1, no. 2, - pp. 168-172.
3. Aliev, A.R., *Rzayev, E.S.* Solvability of boundary value problem for elliptic operator-differential equations of fourth order with operator boundary conditions // - Baku: Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan, - 2014. vol. 40, Special Issue, - p. 13-22. (**Web of Science Core Collection, ESCI**)
4. Al-Aidarous, E.S. Fourth order elliptic operator-differential equations with unbounded operator boundary conditions in the Sobolev-type spaces / Eman Al-Aidarous, Araz Aliev, *Elvin Rzayev* [et al.] // - United Kingdom: Boundary Value Problems, – 2015. vol. 2015, no. 191, – p. 1-14. (**Web of Science Core Collection, SCIE**)
5. Алиев, А.Р., *Рзаев, Э.С.* О разрешимости краевой задачи с ограниченным оператором в краевых условиях для одного класса эллиптических операторно-дифференциальных уравнений // Материалы VII Международной молодежной научно-практической конференции «Математическое моделирование процессов и систем», Часть 1, – Уфа: – 7–9 декабря, – 2017, – с. 64-69.
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anniversary of the academician M.G. Gasymov, - Baku: -7-8 June, - 2019, – p. 33-35.
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8. Aliev, A.R., *Rzayev, E.S.* On solvability of boundary-value problem for fourth-order elliptic equation with operator coefficients // Proceedings of the International Conference on “Modern Problems of Mathematics and Mechanics” dedicated to the 100-th anniversary of the National Leader Heydar Aliyev, - Baku: -26-28 April, – 2023, – p. 58-60.
9. *Rzayev, E.S.* On the completeness of a system of decreasing elementary solutions for one class of fourth-order operator-differential equations on the semi-axis // - Baku: Baku Mathematical Journal, - 2023. vol. 2, no. 2, - p. 199-207.
10. Алиев, А.Р., *Рзаев, Э.С.* О полноте производных цепочек полиномиального операторного пучка четвертого порядка // Материалы Международной научно-практической конференции «Спектральная теория операторов и смежные вопросы», посвященной 75-летию проф. Я. Т. Султанаева, – Уфа: – 26–27 октября, – 2023, – с. 6.

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Address: AZ 1141, Baku city, B.Vahabzade str, 9.

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