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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

INVERSE PROBLEM OF SPECTRAL ANALYSIS FOR ONE CLASS OF DISCRETE DIRAC OPERATORS AND APPLICATIONS

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GENERAL CHARACTERISTICS OF WORK

Relevance and degree of development of the topic.

Stimulated by the classical problem moment, the spectral theory of second order difference operators dates back to the late XIX century. It turned out that there is a strong similarity between singular boundary value problems for second order difference equations and those for a Sturm-Liouville equation.

Similarities and differences between continuous and discrete boundary value problems have been treated in detail by F.Atkinson and Y.M.Berezanski. Note that the main difference here was that the second order difference equation

$$a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n$$

was a discrete analog of Sturm-Liouville equation in divergence form, i.e.

$$\left(p(x)y'\right)' + q(x)y = \lambda y .$$

Inverse problems have played a special role in further development of spectral theory. For now, a number of important results have been obtained for the inverse problems of spectral analysis for a Sturm-Liouville equation of the form $-y'' + q(x)y = \lambda y$. The most of these results belong to M.J.Ablowitz, X.Sigrun, Z.S.Agranovich, V.A.Marchenko, B.M.Levitan, V.A.Yurko.

Inverse problems of spectral analysis in different statements have been widely studied for a system of Dirac equations appearing in relativistic quantum theory, too. Such problems have been considered by M.G.Gasimov, B.M.Levitan, H.M.Huseynov and others.

Inverse problems of scattering for one-dimensional Schrodinger equation for different potentials have been treated by Z.S.Agranovich, V.A.Marchenко, B.M.Levitan, К.Chadan, P.Sabatier, V.S.Buslayev, V.Fomin, I.A.Anders, V.P.Kotlyarov, R.Newton, N.Y.Firsova and others.

Inverse problems of scattering for a system of Dirac equations have been studied by M.G.Gasimov, B.M.Levitan, H.M.Huseynov, I.S.Frolov and others.

The study of inverse problems of scattering for Sturm-Liouville equations and systems of Dirac equations was followed by the study of the same problems for discrete analogs of these equations and systems. Inverse spectral problems for a discrete Sturm-Liouville equation in different statements have been considered by Y.M.Berezanski, L.Fu, G.Hochstadt, S.V.Manakov, H.Flaschka, R.Z.Halilova, H.Sh.Huseynov, J.Bazargan, I.Yegorova, G.Teschl, Agil Kh.Khanmamedov and others. And the inverse problems of scattering in different statements for a system of discrete Dirac equations, or more precisely for the system of equations

$$\begin{cases} a_{1,n} y_{2,n+1} + b_{1,n} y_{1,n} + a_{2,n} y_{2,n} = \lambda y_{1,n}, \\ a_{1,n-1} y_{1,n-1} + +b_{2,n} y_{2,n} + a_{2,n} y_{1,n} = \lambda y_{2,n} \end{cases}$$

have been treated by H.M.Huseynov, G.A.Azimova, H.R.Mamedov, Agil Kh.Khanmamedov, Y.Aygar and M.Olgun.

One of the nonlinear equations integrated by means of the spectral theory of discrete Sturm-Liouville equation is a Langmuir chain of the form

$$\dot{a}_n = \frac{1}{2} a_n \left(a_{n-1}^2 - a_{n+1}^2\right), \; a_n = a_n(t) > 0, \; \cdot = \frac{d}{dt},$$

which is widely used in plasma physics and zoology. Cauchy problem for a Langmuir chain with different initial data has been considered by H.Flaschka, S.V.Manakov, Y.M. Berezanski, G.Teschl, H.M.Huseynov, Agil Kh.Khanmamedov, L.K.Asadova and others.

It should be noted that the case where the coefficients of the equation are step-like or the case where the scattering exists on only one side are of special interest in the inverse problems of scattering, both for continuous and discrete models. Inverse problems of scattering for Sturm-Liouville equation and its discrete analog in case where the coefficients on the left and right-hand sides converge to different limits have been considered by V.S.Buslayev, V.Fomin, I.A.Anders, V.P.Kotlyarov, Agil Kh.Khanmamedov, J.Bazargan, I.Yegorova, G.Teschl, I.Y.Yegorova and others. P.P.Kulish treated the inverse problem for Sturm-Liouville equation in case where the scattering exists on only one side. Similar problems for a discrete Sturm-Liouville equation have been studied by Agil Kh.Khanmamedov. However, this kind of inverse problems have not yet been considered neither for one-dimensional Dirac system, nor for its discrete analog.

Note that in case where the coefficients of the system of discrete Dirac equations ate step-like, some part of the continuous spectrum is a simple spectrum, while the other part is a double spectrum. Besides, new terms arise in the kernels of Marchenko-type main equations. And this creates extra difficulties for deriving expansion formulas with respect to the eigenfunctions and for treating main equations. On the other hand, serious difficulties in inverse problems also arise in case where the scattering exists on only one side. These difficulties are mainly due to the fact that the Yost solution and the Marchenko-type main equation are on only one side.

Thus, the study of inverse problem for a system of discrete Dirac equations in case where the scattering exists on only one side and in case where the coefficients are step-like is quite reasonable. The possibility of applying the last problem in nonlinear equations is also of special interest. This thesis is just dedicated to these problems.

Object and subject of research. Spectral analysis of discrete Dirac operators with step-like coefficients, direct and inverse problems of scattering, Cauchy problem for a Langmuir chain.

The goal and objectives of the study. The main purpose of this research is to study the direct and inverse problems of scattering for a system of discrete Dirac equations with step-like coefficients, to find the necessary and sufficient conditions for unique solvability of inverse problems, and to solve globally the Cauchy problem for a Langmuir chain with step-like initial condition by the method of inverse problem.

General technique of studies. We use the transform operator method for solving the inverse problems of scattering and the method of inverse spectral problem for integration of nonlinear equation. These methods earlier have been successfully used by V.A.Marchenko, B.M.Levitan, P.P.Kulish, V.S.Buslayev, V.Fomin, G.Teschl, Agil Kh.Khanmamedov and others to solve various inverse problems and to apply them in nonlinear chains. In this thesis, these methods are applied to the system of discrete Dirac equations with step-like coefficients and to the Cauchy problem for a Langmuir chain.

Main provisions of dissertation. The main points to be defended in this thesis are: the results concerning direct and inverse problems for discrete Dirac operator satisfying scattering condition on only one side by the transform operator method on the whole axis; the results concerning direct and inverse problems for discrete Dirac operator with step-like coefficients on the whole axis; the results concerning global solvability of the Cauchy problem for a Langmuir chain with step-like initial condition in the class of rapidly decreasing functions.

Scientific novelty. Scientific novelties of the research are:

- the study of direct problem of scattering for discrete Dirac operator satisfying scattering condition on only one side on the whole axis;

- the study of spectrum and resolvent of a discrete Dirac operator satisfying scattering condition on only one side, the expansion formula with respect to the eigenfunctions;

- the study of inverse problem of scattering for discrete Dirac operator satisfying scattering condition on only one side on the whole axis, necessary and sufficient conditions for the solution of inverse problem, solution algorithm for inverse problem;

- the study of direct problem of scattering for discrete Dirac operator with step-like coefficients on the whole axis;

- the study of spectrum and resolvent of discrete Dirac operator with step-like coefficients, expansion formulas with respect to the eigenfunctions;

- the study of inverse problem of scattering for discrete Dirac operator with step-like coefficients on the whole axis, finding Marchenko-type main equations, the proof of their unique solvability, necessary and sufficient conditions for the solution of inverse problem, solution algorithm for inverse problem;

- the study of Cauchy problem for a Langmuir chain with step-like initial condition, the proof of global solvability of this problem in the class of rapidly decreasing functions, formulas for finding rapidly decreasing solution by the method of inverse spectral problem.

Theoretical and practical value of the study. The results obtained in this thesis are theoretical. They can be used in the spectral theory of difference operators and in the integration of nonlinear equations.

Approbation and application. The main results of this work have been presented in the seminars of the Department of "Mathematical Analysis" of the Ganja State University (prof. A.M.Huseynov), in the seminars of the Department of "General Mathematics" of the Ganja State University (prof. F.H.Mamedov), in the seminar of the Department of "Nonharmonic Analysis" of the Institute of Mathematics and Mechanics of the ANAS (prof. B.T.Bilalov), in scientific conference on "Actual problems of theoretical and applied mathematics" dedicated to the 100th anniversary of academician M.L.Rasulov (Sheki, 2016), in international scientific conference on "Theoretical and applied problems of mathematics" (Sumgayit, 2017), in international scientific conference on "Contemporary problems of innovative technologies and applied mathematics in oil production" dedicated to the 90th anniversary of academician A.H.Mirzajanzadeh, in international scientific conference on "Contemporary problems of applied mathematics, computer science and mechanics" (Nalchik, Russia, 2020), and in international scientific conference on "Complex analysis, mathematical physics and nonlinear qquations" (Bannoe, Russia, 2020).

Personal contribution of the author. All the results obtained in the work are the personal contribution of the author.

Publications of the author. Eight research articles of the author have been published in the scientific periodicals recommended by the Higher Certifying Commission under the President of the Republic of Azerbaijan, two of them being indexed by Web of Science and SCOPUS and one having the impact factor 0.719. Short summaries of five articles by the author have been included in proceedings of four international (two of them abroad) and one national scientific conferences.

The name of the institution where the dissertation was completed. This thesis has been done in the Department of Mathematical Analysis of the Ganja State University.

Volume and structure of the dissertation (in signs, indicating the volume of each structural unit separately). The dissertation consists ~ 214458 characters (title page – 382, content ~ 1880 and introduction ~ 44556 знаков characters, I chapter ~ 86000 character, II chapter ~ 80000 character and conclusion – 1640 character) and the list of used literature consists of 100 items.

THE CONTENT OF THE DISSERTATION

This thesis consists of Introduction, two chapters and reference list. Introduction presents the justification for the relevance of the work, brief summary of previously known relevant results and information about the results obtained in this thesis.

Chapter 1 is dedicated to the study of direct and inverse problems of scattering for a system of discrete Dirac equations with scattering condition on only left-hand side.

In Section 1.1, the following system of discrete Dirac equations is considered:

$$\begin{cases} a_{1,n}y_{2,n+1} + b_{1,n}y_{1,n} + a_{2,n}y_{2,n} = \lambda y_{1,n}, \\ a_{1,n-1}y_{1,n-1} + +b_{2,n}y_{2,n} + a_{2,n}y_{1,n} = \lambda y_{2,n}, \ n \in Z, \end{cases} \quad (1)$$

where the coefficients $a_{1,n}, a_{2,n}, b_{1,n}, b_{2,n}$ take real values and satisfy the conditions

$$(-1)^{j-1} a_{j,n} > 0, n \in Z, \ a_{j,n} \to 0, b_{j,n} \to 0, n \to +\infty, \ j = 1,2, \quad (2)$$

$$\sum_{n<0} |n| \{ |a_{1,n} - 1| + |a_{2,n} + 1| + |b_{1,n}| + |b_{2,n}| \} < \infty, \ j = 1,2. \quad (3)$$

Also, the operator corresponding to the system (0.6) is introduced in this section. Denote by $\ell_2(-\infty,+\infty) \times \ell_2(-\infty,+\infty)$ a Hilbert space consisting of sequences $x = (x_{1,n}, x_{2,n})_{n=-\infty}^{+\infty}$ satisfying the condition $\sum_{n=-\infty}^{+\infty} \{ |x_{1,n}|^2 + |x_{2,n}|^2 \} < \infty$. Denote the operator generated by the left-hand side of the system of equations (1) in the space $\ell_2(-\infty,+\infty) \times \ell_2(-\infty,+\infty)$ by L, and the operator generated by this system in the space $\ell_2[1,+\infty) \times \ell_2[1,+\infty)$ with the boundary condition $y_{1,0} = 0$ by L_0. The spectrum $\sigma(L_0)$ of the operator L_0 consists of simple eigenvalues $\lambda_n, n = 1,2,...$ and the point $\lambda = 0$, with $\lambda_n \to 0, n \to \infty$.

Denote by $P_{j,n}(\lambda), Q_{j,n}(\lambda), \ j = 1,2$ the solutions of the system of equations (1) satisfying the initial conditions $P_{1,0}(\lambda) = Q_{2,1}(\lambda) = 0, \ P_{2,1}(\lambda) = 1, \ Q_{1,1}(\lambda) = \dfrac{1}{a_{2,1}}$.

Also, the Weyl function $m(\lambda)$ is introduced in Section 1.1, with $\psi_{j,n}(\lambda) = Q_{j,n}(\lambda) + m(\lambda) P_{j,n}(\lambda) \in \ell_1[1,\infty), \ j = 1,2$ for $\lambda \notin \sigma(L_0)$. The function $\psi_{j,n}(\lambda)$ is called a Weyl solution. The properties and the asymptotics at infinity of the Weyl solution are studied in this section.

In Section 1.2, the system (1) is considered for $\lambda = z + \frac{1}{z}, |z| \le 1$ and the Yost solution

$$f_{1,n}(z) = \alpha_1(n)(-z^2)^{-n}\left[-z^{-1} + \sum_{m=-\infty}^{-1}\left(K_{12}(n,m)(-z^2)^{-(m+1)} - K_{11}(n,m)z^{-1}(-z^2)^{-m}\right)\right],$$
$$f_{2,n}(z) = \alpha_2(n)(-z^2)^{-n}\left[1 + \sum_{m=-\infty}^{-1}\left(K_{22}(n,m)(-z^2)^{-m} - K_{21}(n,m)z^{-1}(-z^2)^{-m}\right)\right] \tag{4}$$

of this system is introduced. The following relationship formulas hold between the kernels of the solution and the coefficients of the system (1):

$$a_{1,n} = \frac{\alpha_1(n)}{\alpha_2(n+1)}, \quad a_{2,n} = -\frac{\alpha_2(n)}{\alpha_1(n)}, \tag{5}$$

$$b_{1,n} = -K_{12}(n.-1) - K_{21}(n+1,-1), \tag{6}$$
$$b_{2,n} = K_{12}(n.-1) + K_{21}(n,-1)$$

Also, the following forms of Weyl function and Weyl solution are used in this section:

$$M(z) = m\left(z + \frac{1}{z}\right), \ \Psi_{j,n}(z) = \psi_{j,n}\left(z + \frac{1}{z}\right). \tag{7}$$

It is proved that if $|z| = 1, z \ne \pm 1, z \ne z_n^{\pm 1}, n = 1,2,\ldots,$ then the relationship

$$\Psi_{j,n}(z) = a(z)\overline{f_{j,n}(z)} + \overline{a(z)}f_{j,n}(z), \ j = 1,2$$

between Weyl solution and Yost solution holds, where $\lambda_n = z_n + \frac{1}{z_n}, n = 1,2,\ldots$. Besides, the formula

$$a(z) = \frac{f_{2,1}(z) + a_{1,0}M(z)f_{1,0}(z)}{z - z^{-1}} \tag{8}$$

is obtained for the function $a(z)$. It is proved that the function $a(z)$ is analytic in the circle $|z| < 1$.

Theorem 1. *The function* $a(z)$ *can have only a finite number of zeros in the circle* $|z|<1$. *These zeros are simple and real.*

Denote by η_k, $0<\eta_k^2<1$, $k=1,2,..,P$ the zeros of the function $a(z)$. In this section, the normalizing numbers are defined by the formulas $M_k^{-2}=\sum_{n=-\infty}^{+\infty}\left\{\left|f_{1,n}(\eta_k)\right|^2+\left|f_{2,n}(\eta_k)\right|^2\right\}, k=1,\ldots,P$, and the absorption coefficient and the reflection coefficient are defined by the formulas $t(z)=\frac{1}{a(z)}, r(z)=\frac{\overline{a(z)}}{a(z)}$, respectively. The properties of absorption and reflection coefficients are studied.

The set of quantities $\{t(z);\eta_k, 0<\eta_k^2<1; M_k>0, k=1,...,P\}$ is called the scattering data for the system of equations (0.1). By inverse problem of scattering for the system of equations (1), we mean finding the coefficients of this system from the scattering data.

Section 1.3 is dedicated to the study of spectrum and resolvent of the operator L.

Theorem 2. *Let the conditions (2), (3) be satisfied. Then the continuous spectrum of the operator* L *generated by the system of equations (1) fills the interval* $[-2,2]$. *Besides, the operator* L *can have a finite number of simple real eigenvalues outside the interval* $[-2,2]$.

Also, the expression for the resolvent of the operator L is found in this section by means of Weyl and Yost solutions.

In Section 1.4, Marchenko-type main equations are derived. Let

$$F^{(1)}(n)=\sum_{k=1}^{P}M_k^2\begin{pmatrix}-\eta_k^{-1}\\-\eta_k^{-2}\end{pmatrix}\cdot\begin{pmatrix}-\eta_k^{-1} & -\eta_k^{-2}\end{pmatrix}\left(-\eta_k^2\right)^{-n}+$$

$$+\frac{1}{2\pi i}\int_{|z|=1}\frac{r(z)}{z}\begin{pmatrix}-z^{-1}\\-z^{-2}\end{pmatrix}\cdot\begin{pmatrix}-z^{-1} & -z^{-2}\end{pmatrix}\left(-z^2\right)^{-n}dz, \tag{9}$$

$$F^{(2)}(n)=\sum_{k=1}^{P} M_k^2 \begin{pmatrix} -\eta_k^{-1} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -\eta_k^{-1} & 1 \end{pmatrix} \left(-\eta_k^2\right)^{-n} + \\ + \frac{1}{2\pi i} \int_{|z|=1} \frac{r(z)}{z} \begin{pmatrix} -z^{-1} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -z^{-1} & 1 \end{pmatrix} \left(-z^2\right)^{-n} dz. \tag{10}$$

Theorem 3. *The kernels* $K_{ij}(n,m), i, j=1,2$ *and the quantities* $\alpha_j(n), j=1,2$ *appearing in (4) satisfy the following equations:*

$$\left(F_{11}^{(1)}(2n+m), F_{12}^{(1)}(2n+m)\right)+\left(K_{11}(n,m), K_{12}(n,m)\right)+ \\ + \sum_{k=-\infty}^{-1}\left(K_{11}(n,k), K_{12}(n,k)\right) F^{(1)}(2n+m+k)=0, n \le 0, m \le -1, \tag{11}$$

$$\left(F_{21}^{(2)}(2n+m), F_{22}^{(2)}(2n+m)\right)+\left(K_{21}(n,m), K_{22}(n,m)\right)+ \\ + \sum_{k=-\infty}^{-1}\left(K_{21}(n,k), K_{22}(n,k)\right) F^{(2)}(2n+m+k)=0, n \le 1, m \le -1, \tag{12}$$

$$\left(\alpha_1(n)\right)^{-2}=1+F_{11}^{(2)}(2n)+ \\ + \sum_{k=-\infty}^{-1}\left[K_{11}(n,k) F_{11}^{(2)}(2n+k)+K_{12}(n,k) F_{12}^{(1)}(2n+k)\right] \tag{13}$$

$$\left(\alpha_2(n)\right)^{-2}=1+F_{22}^{(2)}(2n)+ \\ + \sum_{k=-\infty}^{-1}\left[K_{21}(n,k) F_{21}^{(2)}(2n+k)+K_{22}(n,k) F_{22}^{(2)}(2n+k)\right] \tag{14}$$

Also, in this section the estimates

$$\left.\begin{aligned} \sum_{n=-\infty}^{-1}|n|\left|F_{12}^{(1)}(n-1)+F_{12}^{(1)}(n)\right|<\infty, \\ \sum_{n=-\infty}^{-1}|n|\left|F_{11}^{(1)}(n-1)+F_{11}^{(1)}(n)\right|<\infty. \end{aligned}\right\} \tag{15}$$

are obtained.

Section 1.5 deals with the existence and uniqueness of the solution of Marchenko-type main equations (11), (12).

Theorem 4. *Let the conditions (15) be satisfied. Then, for every fixed n, the equations (11), (12) have a solution in the space* $l_p(-\infty,-1]\times l_p(-\infty,-1](p=1,2)$ *and this solution is unique.*

Besides, the following theorem is proved in this section.

Theorem 5. *Let* $K_{ij}(n,m)$ *be the solutions of the equations (11), (12). Then the quantities* $\alpha_1^{-2}(n),\alpha_2^{-2}(n)$ *defined by the formulas (13), (14) are positive.*

Section 1.6 is dedicated to the inverse problem. First, the formula

$$M(z)=a(z)\overline{f_{2,1}(z)}+\overline{a(z)}f_{2,1}(z) \qquad (16)$$

is derived, which allows defining Weyl function in terms of scattering data. Then the following theorem for the solution of inverse problem is proved:

Theorem 6. *In order for the set of quantities* $\{t(z);\eta_k,0<\eta_k^2<1;M_k>0,k=1,...,P\}$ *to be a scattering data of the system of equations of the form (1) with the coefficients from the class (2) , (3), it is necessary and sufficient that the following conditions hold:*

1. the function $t(z)=\dfrac{1}{a(z)}$ *is continuous for* $|z|=1, z^2\neq\pm1$ *and the folloüing conditions are satisfied:*

$$t\left(\frac{1}{z}\right)=\overline{t(z)},\ t(z_n)=0,n=1,2,..$$

where $|z_n|=1,\ z_n\to i,n\to\infty$*. The function* $t(z)$ *is analytic in the circle* $|z|<1$ *except for the point* $z=0$ *and a finite number of poles* $z=\eta_k,0<\eta_k^2<1,k=1,..,P$*, and the function* $t(z)z\prod_{k=1}^{P}\dfrac{z-\eta_k}{1-\eta_k z}$ *is bounded in the circle* $|z|<1$*. The relation* $zt(z)\to d>0,z\to0$ *holds;*

2. the elements of the matrix $F^{(1)}(n)$ defined by the formula (9) satisfy the relations

$$\sum_{n=-\infty}^{-1} |n| \left| F_{12}^{(1)}(n-1) + F_{12}^{(1)}(n) \right| < \infty, \qquad \sum_{n=-\infty}^{-1} |n| \left| F_{11}^{(1)}(n-1) + F_{11}^{(1)}(n) \right| < \infty;$$

3. the function $m(\lambda)$ defined by the formulas (7),(16) is analytic on the whole complex plane except for simple poles $\lambda_n = z_n + \frac{1}{z_n}, n = 1,2,\ldots$ and takes real values on the real axis. Besides, the function $m(\lambda)$ must be a Weyl function of a self-adjoint operator of the form L_0 given on the semiaxis.

The inverse problem is solved as follows. If the scattering data are known, we construct the matrices $F^{(j)}(n)$ by the formulas (9), (10). Then we find the solutions $K_{ij}(n,m)$ from the main equations (11), (12). Further, we find the quantities $\alpha_1^{-2}(n), \alpha_2^{-2}(n)$ from the formulas (13), (14). For $n \le 0$, the coefficients $a_{j,n}, b_{j,n}, j = 1,2$ of the system (1) are found from the formulas (5), (6). Then, when $n \ge 1$, substituting $y_{2n-1} = y_{2,n}$, $y_{2n} = y_{1,n}$, $a_{2n} = a_{1,n}$, $a_{2n-1} = a_{2,n}$, $b_{2n} = b_{1,n}$, $b_{2n-1} = b_{2,n}$, we reduce the boundary value problem generated by the equation (1) and the boundary condition $y_{1,0} = 0$ to the boundary value problem

$$a_{n-1} y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, n \ge 1, \tag{17}$$

$$y_0 = 0. \tag{18}$$

Using the formulas (7), (16), we find the Weyl function $m(\lambda)$. This function is also the Weyl function of the problem (17), (18). For $n \ge 1$, by means of Weyl function we find the coefficients of the equation (17), and so the coefficients $a_{j,n}, b_{j,n}, j = 1,2$ of the system (1), using the method offered by Y.M.Berezanski or V.A.Yurko.

In Chapter 2, the system of discrete Dirac equations with step-like coefficients

$$\begin{cases} a_{1,n}y_{2,n+1} + a_{2,n}y_{2,n} = \lambda y_{1,n}, \\ a_{1,n-1}y_{1,n-1} + a_{2,n}y_{1,n} = \lambda y_{2,n}, \;\; n = 0, \pm 1, \pm 2, \ldots, \end{cases} \tag{19}$$

is considered, where the coefficients $a_{1,n}, a_{2,n}$ satisfy the conditions

$$\left. \begin{array}{l} a_{1,n} > 0, \; a_{2,n} < 0, \; n = 0, \pm 1, \pm 2, \ldots, \\ \sum\limits_{n \geq 1} |n| \left\{ |a_{1,n} - A| + |a_{2,n} + A| \right\} + \sum\limits_{n \leq -1} |n| \left\{ |a_{1,n} - 1| + |a_{2,n} + 1| \right\} < \infty \end{array} \right\} \tag{20}$$

with $0 < A < 1$ a given number.

In Section 2.1, Yost solutions of the system of equations (19) are introduced. Denote by Γ_1 and Γ_2 the complex λ-plane with the intervals $[-2A, 2A]$ and $[-2, 2]$ cut off, respectively. Consider the function

$$z_1 = z_1(\lambda) = -\frac{\lambda^2 - 2A^2}{2A^2} + \frac{\lambda}{2A}\sqrt{\lambda^2 - 4A^2}$$

in the plane Γ_1, where the analytic branch under the radical is chosen in such a way that $\sqrt{\lambda^2 - 4A^2} > 0$ for $\lambda > 2A$. Similarly, consider the function $z_2 = z_2(\lambda) = -\frac{\lambda^2 - 2}{2} + \frac{\lambda}{2}\sqrt{\lambda^2 - 4}$ in the plane Γ_2, where the analytic branch under the radical is chosen in such a way that $\sqrt{\lambda^2 - 4} > 0$ for $\lambda > 2$. Denote by $\partial\Gamma_j$ the boundary of the plane Γ_j, $j = 1,2$. Then, if the conditions (20) are satisfied, the system (19) has a solution which can be represented as

$$f_{j,n}(\lambda) = \alpha_j^+(n) \left(\frac{Az_1 - A}{\lambda} \right)^{j-2} z_1^n \left(1 + \sum_{m \geq 1} K_j^+(n,m) z_1^m \right), \; j = 1,2, \tag{21}$$

where $\alpha_j^+(n) > 0, K_j^+(n,m)$ are real and satisfy the following relations:

$$\left.\begin{aligned}&\frac{a_{1,n}}{A}=\frac{\alpha_2^+(n+1)}{\alpha_1^+(n)},\ \frac{a_{2,n}}{A}=-\frac{\alpha_1^+(n)}{\alpha_2^+(n)},\\&\frac{a_{1,n}^2-A^2}{A^2}=K_2^+(n,1)-K_1^+(n,1),\\&\frac{a_{2,n}^2-A^2}{A^2}=K_1^+(n-1,1)-K_2^+(n,1),\ n=0,\pm1,\ldots,\end{aligned}\right\}\qquad(22)$$

Similarly, if the conditions (20) are satisfied, then the system (19) has a solution which can be represented as

$$g_{j,n}(\lambda)=\alpha_j^-(n)\left(\frac{z_2^{-1}-1}{\lambda}\right)^{j-2}z_2^{-n}\left(1+\sum_{m\le-1}K_j^-(n,m)z_2^{-m}\right),\ j=1,2,\qquad(23)$$

where $\alpha_j^-(n)>0, K_j(n,m)$ are real and satisfy the following relations:

$$\left.\begin{aligned}&a_{1,n}=\frac{\alpha_1^-(n)}{\alpha_2^-(n+1)},\ a_{2,n}=-\frac{\alpha_2^-(n)}{\alpha_1^-(n)},\\&a_{1,n}^2-1=K_1^-(n+1,-1)-K_2^-(n+1,-1),\\&a_{2,n}^2-1=K_2^-(n+1,-1)-K_1^-(n,-1),\ n=0,\pm1,\ldots,\end{aligned}\right\},\qquad(24)$$

The following theorem is proved in this section:

Theorem 7. *$A^{2n+1}\alpha_1^+(n)\alpha_1^-(n)$ and $A^{2n}\alpha_2^+(n)\alpha_2^-(n)$ do not depend on n and are equal to each other.*

Section 2.2 deals with the direct problem of scattering. Relationship formulas

$$g_{j,n}(\lambda)=a_1(\lambda)\overline{f_{j,n}(\lambda)}+b_1(\lambda)f_{j,n}(\lambda),\ j=1,2,\lambda\in\partial\Gamma_1,\lambda^2\ne4A^2$$

$$f_{j,n}(\lambda)=a_2(\lambda)\overline{g_{j,n}(\lambda)}+b_2(\lambda)g_{j,n}(\lambda),\ j=1,2,\lambda\in\partial\Gamma_2,\lambda^2\ne4$$

are found between Yost solutions defined in Section 2.1. It is shown in this section that the functions $a_1(\lambda),a_2(\lambda)$ can be analytically extended to the plane Γ_2 and the function

$A^2\left(z_1^{-1}-z_1\right)a_1(\lambda)=\left(z_2^{-1}-z_2\right)a_2(\lambda)$ is continuous up to the boundary of Γ_2. The functions $a_1(\lambda), a_2(\lambda)$ can have a finite number of zeros $\lambda_k=\pm\mu_k, \mu_k>0,\ k=1,...,N$ on the plane Γ_2 which are symmetric with respect to the point $\lambda=0$. Let

$$\left(m_k^+\right)^{-2}=\sum_{n=-\infty}^{+\infty}\left\{f_{1,n}^2(\pm\mu_k)+f_{2,n}^2(\pm\mu_k)\right\}$$

$$\left(m_k^-\right)^{-2}=\sum_{n=-\infty}^{+\infty}\left\{g_{1,n}^2(\pm\mu_k)+g_{2,n}^2(\pm\mu_k)\right\}$$

Theorem 8. *The zeros of the function* $a_j(\lambda),\ j=1,2$ *are simple and the relations*

$$\dot a_1(\lambda)\frac{A^2\left(z_1-z_1^{-1}\right)}{\lambda}\Bigg|_{\lambda=\lambda_k}=C_k\left(m_k^+\right)^{-2}=C_k^{-1}\left(m_k^-\right)^{-2}, k=1,\dots,N$$

$$\dot a_2(\lambda)\frac{\left(z_2-z_2^{-1}\right)}{\lambda}\Bigg|_{\lambda=\lambda_k}=C_k\left(m_k^+\right)^{-2}=C_k^{-1}\left(m_k^-\right)^{-2}, k=1,\dots,N$$

hold at the points $\lambda_k=\pm\mu_k, \mu_k>0,\ k=1,...,N$, *where the points over the symbols denote a derivative with respect to* λ.

Then we define right and left reflection coefficients by the formulas $r^+(\lambda)=\dfrac{b_1(\lambda)}{a_1(\lambda)}, r^-(\lambda)=\dfrac{b_2(\lambda)}{a_2(\lambda)}$. The functions $r^+(\lambda), r^-(\lambda)$ are continuous on $\partial\Gamma_2$ except for maybe endpoints of it and satisfy the following relations:

$$r^+(\lambda-i0)=\overline{r^+(\lambda+i0)}=r^+(-\lambda-i0), -2A<\lambda<2A$$

$$r^-(\lambda-i0)=\overline{r^-(\lambda+i0)}=r^-(-\lambda-i0), -2<\lambda<2$$

$$1-\left|r^\pm(\lambda)\right|^2=\left|a_1(\lambda)\right|^{-2}\frac{A^2\left(z_1-z_1^{-1}\right)}{z_2-z_2^{-1}},\ \lambda\in\partial\Gamma_1,$$

$$1-\left|r^\pm(\lambda)\right|^2=\left|a_2(\lambda)\right|^{-2}\frac{z_2-z_2^{-1}}{A^2\left(z_1-z_1^{-1}\right)},\ \lambda\in\partial\Gamma_1$$

$$\left|r^{-}(\lambda)\right| = 1, \lambda \in \partial\Gamma_2 \setminus \partial\Gamma_1 .$$

The sets of quantities $\left\{r^{+}(\lambda),\ \lambda \in \partial\Gamma_1; \pm\mu_k; m_k^{+} > 0, k = 1,\ldots,N\right\}$ and $\left\{r^{-}(\lambda),\ \lambda \in \partial\Gamma_2; \pm\mu_k; m_k^{-} > 0, k = 1,\ldots,N\right\}$ are called right and left scattering data for the system of equations (19), respectively. By direct problem of scattering for the system of euqations (20), we mean a determination of right and left scattering data and the study of their properties.

In Section 2.3, we prove that the right scattering data are uniquely defined by the left scattering data.

Section 2.4 treats the spectrum and the resolvent of the operator L defined in the space $\ell_2(-\infty,+\infty)\times\ell_2(-\infty,+\infty)$ by the formulas

$$(Ly)_{1,n} = a_{1,n}y_{2,n+1} + a_{2,n}y_{2,n}, (Ly)_{2,n} = a_{1,n-1}y_{1,n-1} + a_{2,n}y_{1,n}, n \in Z .$$

It is shown that the continuous spectrum of L fills the interval $[-2,2]$ and can have a finite number of simple real eigenvalues outside $[-2,2]$. The eigenvalues of the operator L coincide with the zeros $\lambda_k = \pm\mu_k, \mu_k > 0,\ k = 1,\ldots, N$ of the function $a_2(\lambda)$. Also, expansion formulas with respect to the eigenfunctions corresponding to the continuous and discrete spectra of L are obtained in this section.

In Section 2.5, the inverse problem of scattering for the system of equations (20) is stated as follows: recover the coefficients of the system (20) from the left scattering data $\left\{r^{-}(\lambda),\ \lambda \in \partial\Gamma_2; \pm\mu_k; m_k^{-} > 0, k = 1,\ldots,N\right\}$. Marchenko-type main equations are derived in this section. Let

$$F_j^{+}(n) = \sum_{\lambda=\pm\nu_k} \left(m_k^{+}\right)^2 \left(\frac{Az_1 - A}{\lambda}\right)^{2-j} z_1^n +$$

$$+\frac{1}{2\pi i}\int_{\partial\Gamma_1} \frac{\lambda r^{+}(\lambda)}{A^2\left(z_1^{-1} - z_1\right)} \left(\frac{Az_1 - A}{\lambda}\right)^{2-j} z_1^n d\lambda +$$

$$+\frac{1}{2\pi i}\int_{\partial\Gamma_2^+\backslash\partial\Gamma_1^+}\frac{\lambda\left|a_2(\lambda)\right|^{-2}}{z_2^{-1}-z_2}\left(\frac{Az_1-A}{\lambda}\right)^{2-j}z_1^n d\lambda, \qquad (25)$$

$$F_j^-(n)=\sum_{\lambda=\pm\nu_k}\left(m_k^-\right)^2\left(\frac{z_2^{-1}-1}{\lambda}\right)^{2-j}z_2^{-n}+,$$

$$+\frac{1}{2\pi i}\int_{\partial\Gamma_2}\frac{\lambda r^-(\lambda)}{\left(z_2^{-1}-z_2\right)}\left(\frac{z_2^{-1}-1}{\lambda}\right)^{2-j}z_2^{-n}d\lambda \qquad (26)$$

where $\partial\Gamma_j^+$ denotes the upper boundary of $\partial\Gamma_j$.

Theorem 9. *For every fixed* $n, n=0,\pm1,\pm2,...,$ *the following relations are true for the kernels* $K_j^+(n,m), j=1,2$ *and the quantities* $\alpha_j^+(n), j=1,2$ *appearing in (21):*

$$K_j^+(n,m)+F_j^+(2n+m)+$$

$$+\sum_{r\geq1}K_j^+(n,r)F_j^+(2n+m+r)=0, m\geq1, j=1,2, \qquad (27)$$

$$\left(\alpha_j^+(n)\right)^{-2}=1+F_j^+(2n)+\sum_{r\geq1}K_j^+(n,r)F_j^+(2n+r) \qquad (28)$$

Theorem 10. *For every fixed* $n, n=0,\pm1,\pm2,...,$ *the following relations are true for the kernels* $K_j^-(n,m), j=1,2$ *and the quantities* $\alpha_j^-(n), j=1,2$ *appearing in (23):*

$$K_j^-(n,m)+F_j^-(2n+m)+,$$

$$+\sum_{r\leq-1}K_j^-(n,r)F_j(2n+m+r)=0, m\leq-1, j=1,2 \qquad (29)$$

$$\left(\alpha_j^-(n)\right)^{-2}=1+F_j^-(2n)+\sum_{r\leq-1}K_j^-(n,r)F_j^-(2n+r). \qquad (30)$$

The equations (27) and (30) are called Marchenko-type main equations.

Section 2.6 treats the Marchenko-type main equations. Also, the estimates

$$\sum_{n\geq,1}|n|\left|F_j^{+}(n+1)+F_j^{+}(n)\right|<\infty,\ j=1,2\ ,\tag{31}$$

$$\sum_{n\leq-1}|n|\left|F_j^{-}(n+1)+F_j^{-}(n)\right|<\infty,\ j=1,2\tag{32}$$

are obtained in this section.

Theorem 11. *Let the function* $r^{+}(\lambda)$ *be continuous on* $\partial\Gamma_1$ *and* $\left|r^{+}(\lambda)\right|<1, \lambda\in\partial\Gamma_1\setminus\{\pm 2A\}$. *If the conditions (31) are satisfied, then, for every fixed* n, *the equations (27) have a solution in the space* $l_p[1,+\infty)\times l_p[1,+\infty)(p=1,2)$ *and this solution is unique.*

Theorem 12. *Let the function* $r^{-}(\lambda)$ *be continuous on* $\partial\Gamma_2$ *and* $\left|r^{-}(\lambda)\right|<1, \lambda\in\partial\Gamma_2\setminus\{\pm 2\}$. *If the conditions (32) are satisfied, then, for every fixed* n, *the equations (29) have a solution in the space* $l_p(-\infty,-1]\times l_p(-\infty,-1](p=1,2)$ *and this solution is unique.*

Section 2.7 is dedicated to the solution of the inverse problem.

Theorem 13. *Let the function* $r^{+}(\lambda)$ *be continuous on* $\partial\Gamma_1$ *and* $\left|r^{+}(\lambda)\right|<1, \lambda\in\partial\Gamma_1\setminus\{\pm 2A\}$. *If the conditions (31) are satisfied, then, for every fixed* n, *the inequalities*

$$1+F_j^{+}(2n)+\sum_{r\geq1}K_j^{+}(n,r)F_j^{+}(2n+r)>0,\ j=1,2$$

hold, where $K_j^{-}(n,r)$ *is a solution of the equation (29).*

Similarly we prove the positivity of right-hand sides of the equalities (30). Also, a necessary and sufficient condition for the set of quantities $\left\{r^{-}(\lambda),\ \lambda\in\partial\Gamma_2;\pm\mu_k;m_k^{-}>0,k=1,...,N\right\}$ to be the left scattering data for the system of equations of the form (19), the coefficients of which belong to the class (20), is found in this section. And the inverse problem is solved using the following algorithm: given the scattering data $\left\{r^{-}(\lambda),\ \lambda\in\partial\Gamma_2;\pm\mu_k;m_k^{-}>0,k=1,...,N\right\}$, the quantities $F_j^{\pm}(n)$ are constructed by the formulas (25), (26). The solutions $K_j^{\pm}(n,m)$ are found from the main equations (27), (29).

Finally, the coefficients $a_{j,n},\ j=1,2$ are determined by one of the formulas (22) and (24).

In the last section of Chapter 2, the following Cauchy problem for a Langmuir chain is considered:

$$\dot{c}_n = c_n\left(c_{n-1}-c_{n+1}\right),\ \cdot = \frac{d}{dt},\ n=0,\pm1,\pm2,\ldots \tag{33}$$

$$c_n(0)=c_n^0,\ n\in Z, \tag{34}$$

where the sequence c_n^0 satisfies the condition

$$\sum_{n=-\infty}^{-1}|n|\left\{\left|c_n(0)-1\right|\right\}+\sum_{n=1}^{\infty}|n|\left\{\left|c_n(0)-A^2\right|\right\}<\infty. \tag{35}$$

We seek for the solution $c_n=c_n(t)$ of the problem (33), (34) such that for every $T>0$ the inequality

$$\left\|\sum_{n=-\infty}^{-1}\left(1+|n|\right)\left|c_n(t)-1\right|+\sum_{n=0}^{\infty}\left(1+|n|\right)\left|c_n(t)-A^2\right|\right\|_{C[0,T]}<\infty \tag{36}$$

holds. We call such solution a rapidly decreasing solution. First, the global solvability of this problem is proved in this section.

Theorem 14. *If the condition (35) holds, then the problem (33)-(34) has a solution in the class (36) and this solution is unique.*

Then, by means of the inverse problem of scattering for the system of equations (19), we find a solution of the problem (33)-(34). For this, we introduce the substitutions

$$c_{2n}=a_{1,n}^2,\ c_{2n-1}=a_{2,n}^2,\ a_{1,n}>0,\ a_{2,n}<0,\ n\in Z. \tag{37}$$

We also find the dynamics of scattering data of the system of equations of the form (19), the coefficients of which depend on the parameter t and satisfy the conditions (37).

Theorem 15. *Let the coefficients* $a_{1,n}=a_{1,n}(t), a_{2,n}=a_{2,n}(t)$ *of the system of equations (19) be defined by the formulas (37) and* $c_n=c_n(t)$ *be a solution of the problem (33)-(34) belonging to the* *class (36). Then, the variation of the left scattering data with respect to the time variable t is described by the following formulas:*

$$r^{-}(\lambda,t)=r^{-}(\lambda,0)\exp\left\{\left(z_2^{-1}-z_2\right)t\right\}$$
$$\mu_k(t)=\mu_k(0)=\mu_k, \qquad (38)$$
$$\left(m_k^{-}(t)\right)^{-2}=\left(m_k^{-}(0)\right)^{-2}\exp\left\{\left(z_2^{-1}(\mu_k)-z_2(\mu_k)\right)t\right\}, k=1,\ldots,N.$$

The formulas (37), (38) allow finding the rapidly decreasing solution of the problem (33)-(34). In fact, assuming $t=0$ in (37) and taking into account the condition (35), we construct the left scattering data $\left\{r^{-}(\lambda,0),\ \lambda\in\partial\Gamma_2;\pm\mu_k;m_k^{-}(0)>0,k=1,...,N\right\}$ for the system of equations (19) with the coefficients $a_{1,n}(0),a_{2,n}(0)$. Then we find the set of quantities $\left\{r^{-}(\lambda,t),\ \lambda\in\partial\Gamma_2;\pm\mu_k;m_k^{-}(t)>0,k=1,...,N\right\}$ using the formulas (38). Considering this set of quantities as the left scattering data, we solve the inverse problem and find the coefficients $a_{1,n}(t),a_{2,n}(t)$. Finally, we find the rapidly decreasing solution of the problem (33)-(34) by using the formulas (37).

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CONCLUSIONS

This thesis is dedicated to the inverse problems of scattering for the systems of discrete Dirac equations with step-like coefficients and their applications in nonlinear equations. Scientific novelties of the research are:

- the study of direct problem of scattering for discrete Dirac operator satisfying scattering condition on only one side on the whole axis;
- the study of spectrum and resolvent of a discrete Dirac operator satisfying scattering condition on only one side, the expansion formula with respect to the eigenfunctions;
- the study of inverse problem of scattering for discrete Dirac operator satisfying scattering condition on only one side on the whole axis, necessary and sufficient conditions for the solution of inverse problem, solution algorithm for inverse problem;
- the study of direct problem of scattering for discrete Dirac operator with step-like coefficients on the whole axis;
- the study of spectrum and resolvent of discrete Dirac operator with step-like coefficients, expansion formulas with respect to the eigenfunctions;
- the study of inverse problem of scattering for discrete Dirac operator with step-like coefficients on the whole axis, finding Marchenko-type main equations, the proof of their unique solvability, necessary and sufficient conditions for the solution of inverse problem, solution algorithm for inverse problem;
- the study of Cauchy problem for a Langmuir chain with step-like initial condition, the proof of global solvability of this problem in the class of rapidly decreasing functions, formulas for finding rapidly decreasing solution by the method of inverse spectral problem.

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