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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

HOMOLOGIES OF FUZZY MODULES

Specialty: 1201.01 – Algebra

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GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the topic. In studying some problems arising in social sciences, economy, engineering, medical diagnostics and in many other fields of science, the classic methods of mathematics are not effective enough. In recent years, there have been established various non-traditional theories in connection with the solution of these problems.

The foundation of non-classic theories was laid by Lotfi-Zadeh. In 1965 he built theory of fuzzy sets and with this on the one hand he gave a theory of multi-valued logic, and on the other hand, this theory had a great importance in solving a number of applied problems. Theory of fuzzy sets is applied on almost all fields of mathematics as algebra, geometry, functional analysis, etc.

In 1968 Chang has applied fuzzy sets to topology. After that, a lot of studies in this field have been conducted. These studies mainly are related to the general topology. Many of these results were given in Ying-Ming's book. Since in fuzzy topological spaces, the topology has no fuzziness, Shostak first have a new definition of a fuzzy topological space. This topology itself is a fuzzy set satisfying some conditions.

In 1971, Rosenfeld has applied fuzzy set in algebra, gave fuzzy groups and conducted some applications. Then, fuzzy structures were included in a ring, modules, algebra, etc. and some studies in this direction were conducted.

The generalization of theory of fuzzy set, the theory of intuitionistic fuzzy sets was introduced by Atanassov. Then the generalization of intuitionistic fuzzy sets, theory of neutrosophic sets, was given by Smarandache and some studies in this field were conducted. Intuitionistic fuzzy and neutrosophic sets have found their applications in algebra, topology. The last years, the studies were carried out in algebra on soft G -modules.

Theory of soft sets containing the properties of fuzzy, intuitionistic fuzzy, neutrosophic sets was built in 1999 by Molotsov. Maji and Roy had great cervices in application of these sets. Then fuzzy and soft structures were combined and fuzzy soft sets were established. Given the soft groups, in 2007, soft sets began to be applied in algebra. Afterwards soft rings, soft module were given and some of their properties were studied. As a continuation of this, fuzzy and intuitionistic fuzzy sets structure and soft sets structure were combined and fuzzy soft groups, rings, modules and other structures in algebra were given and some studied related to them were conducted.

The last years, intuitionistic fuzzy structures with the action of one group in modules were introduced and some studies on this field were conducted. Application of neutrosophic sets and neutrosophic soft modules in algebra was also given.

Unlike algebra, soft sets were applied in topology only in 2011. Then intensive research on this field were carried out. It should be noted that the results concerning mainly the general topology were obtained in fuzzy sets. But a powerful apparatus as the methods of algebraic topology was not widely used in these studies.

The problem of closure is one of the necessary problems with respect to algebraic operations in new categories structured in some fields of mathematics. Since direct and inverse limits contain all algebraic operations in itself, the closure problem in these categories can be solved by showing the existence of direct and inverse limits.

As can be seen, algebra and topology are widely used in studying fuzzy sets. Therefore, the works done in this field are urgent studies with an applied importance in future.

Object and subject of the study. Fuzzy modules with effects under the group and fuzzy topology on algebraic structures.

Goal and objectives of the study. Research of some algebraic problems in fuzzy structures.

Research methods. In the work, the methods of modern algebra, homological algebra and algebraic topology are applied.

The main clauses to be defended.

1. Fuzzy soft G-modules, intuitionistic fuzzy soft G-modules categories were built and closure problem of algebraic categories of these categories with respect to algebraic operations were studied.
2. Giving the notion of exact sequence in intuitionistic fuzzy G-modules categories, some exact sequences were built.
3. Constructing homological modules in intuitionistic fuzzy G-modules categories, it was proved that the axioms of homological theory is satisfied.
4. The category of neutrosophic G-modules being the extension of intuitionistic fuzzy G-modules was built.
5. The notion of neutrosophic soft modules being the extension of neutrosophic modules, was introduced and the closure problem in this category of modules was studied. The existence of inverse limit in the category of neutrosophic soft modules was proved.
6. Fuzzy, intuitionistic fuzzy (Shostak) topology in soft sets was introduced and studies related to the base, continuity in the newly obtained space was conducted.

Scientific novelty of the study. Giving an action of one group in fuzzy modules, a new category is constructed and the properties of this category is studied. The results obtained here allow to build theory of descriptions of fuzzy categories. One of the important issues in the newly obtained category is the problem of closure with respect to algebraic operations. In the category of neutrosophic modules, the closure problem is solved completely. Applying fuzzy sets to soft sets, a bridge is built between algebra and topology. So, a category containing ordinary topological spaces and soft topological spaces was built.

Theoretical and practical importance of the study. The dissertation work is mainly of theoretical character. In the work, the categories of fuzzy soft G-modules, intuitionistic fuzzy soft G-modules, neutrosophic G-modules, neutrosophic soft modules were constructed. The results obtained here enable to construct theory of descriptions of fuzzy groups. The importance of descriptions theory in mathematics is obvious. Fuzzy structures arose from the need of practice and we hope that these studies will be widely used in solving practical problems.

Approbation and application. The results of the dissertation work were reported in the Republican conference “Actual problems of Mathematics and Mechanics” dedicated to the 100-th anniversary of corr.-member of ANAS, famous scientist and outstanding mathematician Goshgar Teymur oglu Ahmedov (Baku, 2017), in the XII Republican Scientific conference of doctoral students and young researches devoted to the 100-th anniversary of the People Republic of Azerbaijan (Baku, 2018), in the Republican Scientific conference “Actual problems of mathematics and mechanics” devoted to 95-th anniversary of the National leader of Azerbaijan Haydar Aliyev (Baku, 2018), in the Scientific conference «8-th International Eurasian conference on mathematical sciences and applications» (Baku, 2019) and in the Scientific conference “IX İnternational conference of the Georgian mathematical union» held in Georgia (Batumi, 2018).

The name of the organization where the work was executed. The work was executed in the chair of “Algebra and geometry” of “Mechanics –Mathematics” department of Baku State University.

Applicant’s personal contribution. All new scientific novelties and results belong to the applicant.

Published scientific works. The main results of the dissertation work were published in applicants 10 scientific works, 2 of which are in the journals included in the WOS and 2 in the journals included in the Scopus database. 2 of the published articles are single-authored. In addition, the results of the dissertation were reported at 5 international and national scientific conferences, and these reports were reflected in the relevant conference materials list in the form of theses.

Total volume of the dissertation work indicating the volume of structural section of the dissertation separately in signs. The dissertation work consists of introduction, three chapters, results, a list of references of 129 titles. Total volume of the work –

223356 signs (title page – 328 signs, table of contents – 1422 signs, introduction - 38725 signs, chapter I - 76.000 signs, chapter II - 56000 signs, chapter III - 50000 signs).

THE BRIEF CONTENT OF THE DISSERTATION

In the introduction we justity the rationale of the work, give brief information on the study of the topic, briefly show the main results and give information on approbation of the work.

In subchapter 1 chapter I we give some operations on fuzzy soft G-modules and build a category.

Definition 1. Let K be a ring, M be a left (or right) modules on K, G be a group. Let the action of G group on M module be given, i.e. the function $\mu : G \times M \to M$ satisfying the following conditions be given.

1) $\mu(1_G, m) = m, \quad \forall m \in M$ (1_G G is a unit element of the group)
2) $\mu(g_1 g_2, m) = \mu(g_1, \mu(g_2, m))$
3) $\mu(g, k_1 m_1 + k_2 m_2) = k_1 \mu(g, m_1) + k_2 \mu(g, m_2)$

In this case the module M is said to be a G-module.

Definition 2. Let (F, A) be a fuzzy soft set on M. If for $\forall a \in A$ the fuzzy set $F(a) : M \to [0,1]$ satisfies the following conditions:

a) $F(a)(ax + by) \geq F(a)(x) \wedge F(a)(y) \quad \forall a, b \in K,\ x, y \in M$

b) $F(a)(g \cdot m) \geq F(a)(m)$

then the pair (F, A) is said to be a fuzzy soft G-module on M.

Theorem 1. If (F, A) (H, B) be two fuzzy soft G-modules on M, then their intersection $(F, A) \cap (H, B)$ is also a fuzzy soft G- module on M.

Theorem 2. Let (F, A), (H, B) be two fuzzy softs G-modules on M. If $A \cap B = \varnothing$ their union $(F, A) \cup (H, B)$ is a fuzzy soft G-module on M.

Theorem 3. Let (F,A), (H,B) be two fuzzy softs G-modules on M. Then $(F,A)\wedge(H,B)$ is also a fuzzy soft G-module on M.

Theorem 4. Let (F,A) on M, (H,B) on N be two fuzzy soft G-modules, then $(F,A)\times(H,B)$ is a fuzzy soft G-module on $M\times N$

Definition 3. Let (F,A), (H,B) be two fuzzy soft G-modules on M, their sum $(F,A)+(H,B)=(G,C)$ is defined as follows; $G_c(x)=\underset{x=a+b}{\vee}(F_c(a)\wedge H_c(b))$ for $\forall c\in C$ and here $C=A\bigcap B$.

Theorem 5. Let (F,A), (H,B) be two fuzzy soft G-modules on M, then their sum $(F,A)+(H,B)$ is also a fuzzy soft G-module on M.

Definition 4. Let (F,A), (H,B) be two fuzzy soft G-modules on M, their product $(F,A)\cdot(H,B)=(G,C)$ is defined as follows: $G_c(x)=\underset{x=\sum(a_i+b_i)}{\vee}\left\{\underset{i}{\wedge}(F_c(a_i)\wedge H_c(b_i))\right\}$ for $\forall c\in C$ and here $C=A\bigcap B$.

Theorem 6. The product of two fuzzy soft G-modules (F,A), (H,B) on M is also a fuzzy soft G-module on M.

Definition 5. Let M be a G-module and N be a submodule of M. If the submodule N is invariant under the action of the group G, i.e. for $\forall g\in G$ and $n\in N$, $g\cdot n\in N$ then the submodule N is said to be G-submodule.

Let (F,A) be a fuzzy soft G - module on M, the fuzzy soft set $F_{a/N}$ for $\forall a\in A$ be defined as $F_a/N : N\rightarrow[0,1]$.

Theorem 7. Let (F,A) be a fuzzy soft G-module on M, then $F_{a/N}$ is a fuzzy soft G - module on N.

Theorem 8. Let (F,A) be a fuzzy soft G-module on M, N be G-submodule of M, then $(\tilde{F},A)$, is a fuzzy soft G-module on the M/N factor module.

In subchapter 2 of chapter 1 we consider intuitionistic fuzzy soft G-modules that are the extensions of fuzzy soft G-modules and prove similar theorems. In subchapter 3 of chapter 1 we consider a sequence of intuitionistic fuzzy soft G-modules.

Assume that (F,A) is an intuitionistic fuzzy soft G-module on M. Let (G,B) be an intuitionistic fuzzy soft G-module on N. $f:M\to N$, is a homomorphism of G-module, $\varphi: A\to B$ is the mapping of sets.

Definition 6. If for each $a\in A$ $f:\left(M,F_a,F^a\right)\to\left(N,G_{\varphi(a)},G^{\varphi(a)}\right)$ is a homomorphism of intuitionistic fuzzy of G-modules, then the pair $(f,\varphi):(F,A)\to(G,B)$ is called homomorphism of intuitionistic fuzzy soft G-modules.

Assume that $(f,\varphi):(F,A)\to(G,B)$ is a homomorphism of an intuitionistic fuzzy soft G-modules, $\ker f < M$ be a kernel of f. We define the structure of intuitionistic fuzzy soft G-modules on the $\ker f$ G-submodules as follows, for $\forall a\in A$,

$$\overline{F}(a)=\left(\overline{F}_a,\overline{F}^a\right),\ \overline{F}_a=F_a\Big/_{Kerf},\overline{F}^a=F^a\Big/_{Kerf}.$$

Let $\operatorname{Im} f < N$ be an image of f. In the same way we show the structure of intuitionistic fuzzy soft G-modules on the $\operatorname{Im} f$ G-submodule, for $\forall b\in B$

$$\overline{G}(b)=\left(\overline{G}_b,\overline{G}^b\right),\ \overline{G}_b=G_b\Big/_{\operatorname{Im} f},\ \overline{G}^b=G^b\Big/_{\operatorname{Im} f}.$$

Theorem 9. Let M and N be G-modules, f be a homomorphism of G-modules. If (G,B) is an intuitionistic fuzzy G-module on N, then $\left(f^{-1}(G),B\right)$, M is an intuitionistic fuzzy soft G-module on M.

Theorem 10. If $\{(F_i, A_i)\}_{i\in I}$ is a family of intuitionistic fuzzy soft G-modules on $\{M_i\}_{i\in I}$, then $\oplus_{i\in I}(F_i, A_i)$ is an intuitionistic fuzzy soft G - modules on $\{M_i\}_{i\in I}$.

Definition 7. The support of the intuitionistic fuzzy soft set for an arbitrary intuitionistic fuzzy soft set (F, A) on the set X is denoted as (F^*, A) and is determined as $F^*(a) = \{x \in X, F_a(x) > 0, F^a(x) < 1\}$. It is clear that (F^*, A) is a soft set on X.

Theorem 11. a) Assume that (F, A) is an intuitionistic fuzzy soft G - module. Then (F^*, A) is a soft G -submodule of M.

b) For the soft G -modules $(F, A),(G, B)$ of M we obtain $((F, A)+(G, B))^* = (F^*, A)+(G^*, B)$.

c) $((F, A)\cap(G, B))^* = (F^*, A)\cap(G^*, B)$.

Definition 8. For $\forall a \in A$ and $\forall x \in M$, we define the two intuitionistic fuzzy soft sets $(\tilde{\Phi}, A)$ and $(\tilde{M}, A)$ on M as follows:

$$\tilde{\Phi}(a)(x) = \begin{Bmatrix} (1,0), x = 0 \\ (0,1), x \neq 0 \end{Bmatrix}; \tilde{M}(a)(x) = (1,0).$$

Then $(\tilde{\Phi}, A)$, $(\tilde{M}, A)$ intuitionistic fuzzy soft sets, according to fuzzy soft modules are called 0 and 1 fuzzy soft modules.

Definition 9. Assume that, $(F, A),(G, B)$ are intuitionistic fuzzy soft G -modules. If the condition $(F, A)\cap(G, B) = (\tilde{\Phi}, A\cap B)$ is satisfied, then the sum $(F, A)+(G, B)$ is called a direct sum of (F, A) and (G, B) and is written as $(F, A)\oplus(G, B)$.

Theorem 12. Assume that $(F, A),(G, B),(H, C)$ are soft G - modules of M such that $(F, A) = (G, B)\oplus(H, C)$, then $(F^*, A) = (G^*, B)\oplus(H^*, C)$.

Assume that,

$$\ldots \xrightarrow{f_{i-1}} M_{i-1} \xrightarrow{f_i} M_i \xrightarrow{f_{i+1}} M_{i+1} \xrightarrow{f_{i+2}} \ldots \qquad (1)$$

is a sequence of G -modules and G -module homomorphisms.

Definition 10. Let $M_i, i \in Z$ be G-modules, (F_i, A) be intuitionistic fuzzy soft G-modules on M_i. If for $\forall a \in A$

$$\ldots \to (M_{i-1}, F_{i-1}(a)) \to (M_i, F_i(a)) \to (M_{i+1}, F_{i+1}(a)) \to \ldots$$

the sequence of intuitionistic fuzzy G-modules is exact, then the sequence

$$\begin{aligned}&\ldots \xrightarrow{(f_{i-1},1_A)} (F_{i-1}, A) \xrightarrow{(f_i,1_A)} (F_i, A) \xrightarrow{(f_{i+1},1_A)} \\ &\to (F_{i+1}, A) \xrightarrow{(f_{i+2},1_A)} \ldots\end{aligned} \qquad (2)$$

of intuitionistic fuzzy soft G-modules is called an exact sequence of intuitionistic fuzzy soft G-modules.

Theorem 13. Assume that for the direct sum $(F, A) \oplus (G, B)$ of intuitionistic fuzzy soft G-modules $(F, A), (G, B)$, $(F^*, A) + (G^*, B)$ is a direct sum of the soft G-modules. Then the sequence

$$0 \longrightarrow (F, A) \xrightarrow{(i,1_A)} (F, A) \oplus (G, A) \xrightarrow{(\pi,1_A)} (G, A) \longrightarrow 0$$

is exact.

Theorem 14. Assume that $M \xrightarrow{f} N \xrightarrow{g} P$, is an exact sequence of G-modules on the N module and (F, A), (G, A), (H, A) are intuitionistic fuzzy soft G-modules on M, N and P respectively. Then the sequence $(F, A) \xrightarrow{(f,1_A)} (G, A) \xrightarrow{(g,1_A)} (H, A)$ of intuitionistic fuzzy soft G-modules is exact in (G, A), if and only if the sequence $F^*(a) \xrightarrow{f'} G^*(a) \xrightarrow{g'} H^*(a)$ of G-modules for all $a \in A$ is exact in $G^*(a)$, here f' and g', f and g are contractions of $F^*(a)$ and $G^*(a)$, respectively.

In subchapter 4 of chapter 1 we construct homological modules of intuitionistic fuzzy G-modules.

Let G be a group, M be G-module. By (M, μ, v) we denote intuitionistic fuzzy G-module.

Definition 11. If $\partial_{n-1} \circ \partial_n = 0$ is satisfied for the sequence

$$\{(M_n, \mu_n, v_n), \partial_n : (M_n, \mu_n, v_n) \to (M_{n-1}, \mu_{n-1}, v_{n-1})\}_{n \in Z} \quad (3)$$

of intuitionistic fuzzy G-modules, this sequence is said to be a chain complex of intuitionistic fuzzy G-modules.
Assume that (3) and

$$\left\{\left(M_n^{'},\mu_n^{'},\nu_n^{'}\right),\partial_n^{'}:\left(M_n^{'},\mu_n^{'},\nu_n^{'}\right)\to\left(M_{n-1}^{'},\mu_{n-1}^{'},\nu_{n-1}^{'}\right)\right\}_{n\in Z} \qquad (4)$$

is two chain complexes of intuitionistic fuzzy G-modules on $\{M_n\}$, $\{M_n^{'}\}$, respectively.

Definition 12. Assume that $\left\{\varphi_n,\psi_n:(M_n,\mu_n,\nu_n)\to\left(M_n^{'},\mu_n^{'},\nu_n^{'}\right)\right\}$ are morphisms of chain complexes of intuitionistic fuzzy G-modules and $D=\left\{D_n:(M_n,\mu_n,\nu_n)\to\left(M_{n+1}^{'},\mu_{n+1}^{'},\nu_{n+1}^{'}\right)\right\}$ is a family of homomorphisms of intuitionistic fuzzy G-modules. If the equality $\varphi_n-\psi_n=D_{n-1}\circ\partial_n+\partial_{n+1}^{'}\circ D_n$ is satisfied, then the family of homomorphisms of the modules $D=\left\{D_n:M_n\to M_{n+1}^{'}\right\}_{n\in Z}$ is called a chain homotopy, $\{\varphi_n\}$, $\{\psi_n\}$ are called chain homotopic mappings and are denoted as $\{\varphi_n\}\sim\{\psi_n\}$.

Theorem 15. The chain homotopy relation is an equivalent relation and is invariant with respect to superposition.

Definition 13. The intuitionistic fuzzy G-module $H_n(C)=\left(Ker\partial_n/\operatorname{Im}\partial_{n+1},\tilde{\mu}_n,\tilde{\nu}_n\right)$ is said to be n-dimensional homological module of intuitionistic fuzzy G chain complex.

If $\left\{\varphi_n:(M_n,\mu_n,\nu_n)\to\left(M_n^{'},\mu_n^{'},\nu_n^{'}\right)\right\}$ are morphism of chain complexes of intuitionistic fuzzy G-modules, for $\forall[x]\in H_n(C)$ we define the homomorphism $\varphi_{n*}:H_n(C)\to H_n\left(C^{'}\right)$ as $\varphi_{n*}[x]=[\varphi_n(x)]$.

Theorem 16. The opposition $C\to H_n(C)$ is a functor going from the category of chain complexes of intuitionistic fuzzy G-modules to the category of intuitionistic fuzzy G-modules.

Theorem 17. The homological functor of chain complexes of intuitionistic fuzzy G-modules is invariant with respect to chain homotopy.

So, if $\{\varphi_n\} \sim \{\psi_n\} : \{(M_n, \mu_n, v_n), \partial_n\} \to \{(M'_n, \mu'_n, v'_n), \partial'_n\}$, then $\varphi_{n*} = \psi_{n*} = H(C) \to H_n(C')$.

Theorem 18. If

$$0 \to C' \xrightarrow{\varphi} C \xrightarrow{\psi} C'' \to 0 \tag{5}$$

is a splitted short exact sequence of chain complexess of intuitionistic fuzzy G-modules, then the sequence

$$\begin{aligned} \ldots \longleftarrow H_{n-1}(C') \xleftarrow{\partial_{n*}} H_n(C'') \longleftarrow H_n(C) \longleftarrow \\ \longleftarrow H_n(C') \longleftarrow \ldots \end{aligned} \tag{6}$$

of homological modules of intutive fuzzy G-modules is exact.

In subchapter 5 of chapter 1, the results of subchapter 1 and 2 is taken to the neutrosophic G-modules.

In subchapter 1 of chapter II neutrosophic soft modules are given and a category is built.

Definition 14. Assume that $(\tilde{F}, A)$ is a neutrosophic soft set on M. If for $\forall a \in A$ $\tilde{F}(a) = (T_a, I_a, F_a)$ is a neutrosophic submodule of M, then the pair $(\tilde{F}, A)$ on M is called a neutrosophic soft module and is denoted as $\tilde{F}_a$.

Definition 15. Assume that $(\tilde{F}^1, A)$ and $(\tilde{F}^2, B)$ are two neutrosophic soft modules on M and N, respectively and $f : M \to N$ is a homomorphism of the modules, $g : A \to B$ is a mapping of the sets. İf the following conditions,

$$f\left(T^1_{(a)}\right) = \tilde{F}^2(g(a)) = T^2_{g(a)},$$

$$f\left(I^1_{(a)}\right) = \tilde{F}^2(g(a)) = I^2_{g(a)},$$

$$f\left(F^1_{(a)}\right) = \tilde{F}^2(g(a)) = F^2_{g(a)},$$

are satisfied, the pair $(f, g) : (\tilde{F}^1, A) \to (\tilde{F}^2, B)$ is said to be a neutrosophic soft homomophisms of neutrosophic soft modules.

It is clear that for $\forall a \in A \;\; f:\left(M, \widetilde{F}^1_{(a)}\right) \to \left(N, \widetilde{F}^2_{g(a)}\right)$ are neutrosophic homomorphisms of neutrosophic modules.
Neutrosophic soft modules and their homomorphisms form one category.
Theorem 19. Assume that $\left(\widetilde{F}^1, A\right)$ and $\left(\widetilde{F}^2, B\right)$ are two neutrosophic soft modules on M. Then their intersection $\left(\widetilde{F}^1, A\right) \cap \left(\widetilde{F}^2, B\right)$ is a neutrosophic soft module on M.
Theorem 20. Assume that $\left(\widetilde{F}^1, A\right)$ and $\left(\widetilde{F}^2, B\right)$ are two neutrosophic soft modules on M. Then $\left(\widetilde{F}^1, A\right) \wedge \left(\widetilde{F}^2, B\right)$ is a neutrosophic soft module on M.
Theorem 21. Assume that $\left(\widetilde{F}^1, A\right)$ and $\left(\widetilde{F}^2, B\right)$ are two neutrosophic soft modules on M. If $A \cap B = \varnothing$, then $\left(\widetilde{F}^1, A\right) \cup \left(\widetilde{F}^2, B\right)$ is a neutrosophic soft module on M.
Theorem 22. If $\left\{\left(\widetilde{F}_i, A_i\right)\right\}_{i\in I}$, is a family of neutrosophic soft modules on $\{M_i\}_{i\in I}$ then $\prod\limits_{i\in I}\left(\widetilde{F}_i, A_i\right)$ is a neutrosophic soft module on $\prod\limits_{i\in I} M_i$
.
Theorem 23. If $\left\{\left(\widetilde{F}_i, A_i\right)\right\}_{i\in I}$, is a family of neutrosophic soft modules on a family of modules $\{M_i\}_{i\in I}$ then $\bigoplus\limits_{i\in I}\left(\widetilde{F}_i, A_i\right)$, is a neutrosiphic soft module on $\bigoplus\limits_{i\in I} M_i$.
In subchapter 2 of chapter II a closure problem in the category of neutrosopic modules is solved.
Theorem 24. The category of neutrosophic modules has zero objects, sums, products, kernels and cokernels.
Denote the category of neutrosophic modules by NM.
Definition 16. The functor $D:\Lambda^{op} \to NM$ where Λ is a directed set (is considered as a category) is called an inverse system of neutrosophic modules, the limit of D is called a limit of the inverse system.

Assume that

$$\left(\underline{M},\underline{T},\underline{I},\underline{F}\right)= \\ =\begin{pmatrix} \left\{\left(M_\alpha,T_\alpha,I_\alpha,F_\alpha\right)\right\}_{\alpha\in\wedge}, \\ \left\{p_\alpha^{\alpha'}:\left(M_{\alpha'},T_{\alpha'},I_{\alpha'},F_{\alpha'}\right)\to\left(M_\alpha,T_\alpha,I_\alpha,F_\alpha\right)\right\}_{\alpha\prec\alpha'} \end{pmatrix} \qquad (7)$$

is an inverse system of neutrosophic modules.

Let $A=\left\{\pi_\alpha:\prod_{\alpha\in\wedge}M_\alpha\to M_\alpha\right\}_{\alpha\in\wedge}$ be a family of projections and $\left(\prod_{\alpha\in\wedge}M_\alpha,T_A,I_A,F_A\right)$ be a direct product of neutrosophic modules. Then the family $\left(\lim\limits_{\leftarrow}M_\alpha,T_A\middle|\lim\limits_{\leftarrow}M_\alpha,I_A\middle|\lim\limits_{\leftarrow}M_\alpha,F_A\middle|\lim\limits_{\leftarrow}M_\alpha\right)$ is a neutrosophic module.

Theorem 25. All inverse systems in the form of (7) has a limit in the category *NM* and this limit equals the neutrosophic module $\left(\lim\limits_{\leftarrow}M_\alpha,T_A\middle|\lim\limits_{\leftarrow}M_\alpha,I_A\middle|\lim\limits_{\leftarrow}M_\alpha,F_A\middle|\lim\limits_{\leftarrow}M_\alpha\right)$.

Let us consider the inverse system (7) and give the homomorphism of the modules $d:\prod_\alpha M_\alpha\to\prod_\alpha M_\alpha$ by the formula

$$d\left(\{x_\alpha\}\right)=\left\{x_\alpha-p_\alpha^{\alpha'}\left(x_{\alpha'}\right)\right\}_{\alpha\prec\alpha'}.$$

Definition 17. $\lim\limits_{\leftarrow}{}^{(1)}M_\alpha,(T_A)^\pi,(I_A)^\pi,(F_A)^\pi,$ is called the "first derivative functor" of the inverse system of neutrosophic modules given in (7) and is denoted as $\lim\limits_{\leftarrow}{}^{(1)}$.

Proposal 1. $\lim\limits_{\leftarrow}{}^{(1)}$ is a functor.

Lemma 1. $\lim\limits_{\leftarrow}(M_\alpha, T_\alpha, I_\alpha, F_\alpha) = \ker \overline{d}$ and

$\lim\limits_{\leftarrow}{}^{(1)}(M_\alpha, T_\alpha, I_\alpha, F_\alpha) = co\ker \overline{d}$.

Theorem 26. Assume that,

$$(M_1, T_1, I_1, F_1) \xleftarrow{p_1^2} (M_2, T_2, I_2, F_2) \xleftarrow{p_2^3} \ldots$$

is an inverse sequence of the neutrosophic modules. For each infinite subsequences of this sequence $\lim\limits_{\leftarrow}{}^{(1)}$ is unchangable.

Theorem 27. If for $\forall \{x_n''\} \in \ker \overline{d}$ the condition $\lim\limits_{n\to\infty} T_n''(x_n'') = 0$, $\lim\limits_{n\to\infty} I_n''(x_n'') = 0$ and $\lim\limits_{n\to\infty} F_n(x_n'') = 1$ is satisfied, then for the following short exact sequence of the inverse system of neutrosophic modules,

$$\begin{array}{ccccccccc} & & \vdots & & \vdots & & \vdots & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & (M_2', T_2', I_2', F_2') & \to & (M_2, T_2, I_2, F_2) & \to & (M_2'', T_2'', I_2'', F_2'') & \to & 0 \\ & & \vdots & & \vdots & & \vdots & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & (M_1', T_1', I_1', F_1') & \to & (M_1, T_1, I_1, F_1) & \to & (M_1'', T_1'', I_1'', F_1'') & \to & 0 \end{array}$$

the sequence

$$0 \to \varprojlim (M_n', T_n', I_n', F_n') \to \varprojlim (M_n, T_n, I_n, F_n) \to$$
$$\to \varprojlim (M_n'', T_n'', I_n'', F_n'') \to \varprojlim{}^{(1)} (M_n', T_n', I_n', F_n') \to$$
$$\to \varprojlim{}^{(1)} (M_n, T_n, I_n, F_n) \to \varprojlim{}^{(1)} (M_n'', T_n'', I_n'', F_n'') \to 0$$

is exact.

In subchapter 3 of chapter II we study the closure problem of in the category of neutrosophic soft modules.

In subchapter 1 of chapter III we consider fuzzy topology in soft sets.

Let E be a set of parameters, X be a universal set. By $SS(X,E)$ we indicate the family of all soft sets X .

Definition 18. A mapping $\tau : SS(X,E) \to [0,1]$ is called a gradation of openness of soft sets on X if it satisfies the following conditions:

a) $\tau(\Phi) = \tau(\tilde{X}) = 1,$

b) $\tau((F,E) \tilde{\cap} (G,E)) \geq \tau(F,E) \wedge \tau(G,E),$
$\forall (F,E),(G,E) \in SS(X,E),$

c) $\forall \{(F_i,E)\}_{i\in\Delta}$ for the family $\tau\left(\bigcup_{i\in\Delta}(F_i,E)\right) \geq \bigwedge_{i\in\Delta} \tau(F_i,E)$

The triple (X,E,τ) is called a fuzzy topological space of soft sets and denoted by FTS .

Definition 19. A mapping $v : SS(X,E) \to [0,1]$ is called a gradation of closedness of soft sets on X if it satisfies the following conditions:

a) $v(\Phi) = v(\tilde{X}) = 1,$

b) $v((F,E) \tilde{\cup} (G,E)) \geq v(F,E) \wedge v(G,E),$
$\forall (F,E),(G,E) \in SS(X,E)$

c) for the family $\forall \{(F_i,E)\}_{i\in\Delta}$, $v\left(\bigcap_{i\in\Delta}(F_i,E)\right) \geq \bigwedge_{i\in\Delta} v(F_i,E),$

The triple (X,E,v) is called a fuzzy cotopological space of soft sets and denoted by $FCTS$.

Theorem 28. Let (X,E,τ) be a fuzzy topological space. Then for each $\forall r \in (0,1]$, $\tau_r = \{(F,E) \in SS(X,E) | \tau(F,E) \geq r\}$, is descending family of soft topologies of soft sets on X .

Theorem 29. Let $\{\sigma_r\}_{r\in(0,1]}$ be a descending family of soft topologies on X , then $\tau(F,E) = \vee\{r | (F,E) \in \sigma_r\}$ is gradation of openness. Also, $\tau_r = \sigma_r$.

Definition 20. Let (X,E,τ) be a fuzzy topological space.

a) If β satisfies the following conditions,

$\tau(F,E)=\bigvee_{\underset{i\in\Delta}{\cup}(G_i,E)=(F,E)}\bigwedge_{i\in\Delta}\beta(G_i,E)$, for $\forall(F,E)\in SS(X,E)$, then $\beta:SS(X,E)\to[0,1]$ is called a base of τ.

Theorem 30. If $\beta:SS(X,E)\to[0,1]$ satisfies the following conditions,

a) $\beta(\Phi)=\beta(\tilde{X})=1$;

b) $\beta((F,E)\tilde{\cap}(G,E))\geq\beta(F,E)\wedge\beta(G,E)$, $\forall(F,E),(G,E)\in SS(X,E)$, then $\tau_\beta(F,E)=\bigvee_{\underset{j\in J}{\cup}(G_i,E)=(F,E)}\bigwedge_{j\in J}\beta(G_j,E)$, is a gradation of openness and β is a base of τ_β.

Theorem 31. Let (X,E,τ) be a FTS and $Y\subset X$. Define mapping $\tau_Y:SS(Y,E)\to[0,1]$ by the rule $\tau_Y(F,E)=\vee\{\tau(G,E):(F,E)=(G,E)\tilde{\cap}\tilde{Y},(G,E)\in SS(X,E)\}$. Then τ_Y is a gradation of openness on Y and $\tau_Y((G,E)\tilde{\cap}\tilde{Y})\geq\tau(G,E)$.

Definition 21. Let (X,E,τ), (Y,E',σ) be two fuzzy soft topological spaces and $(f,\varphi):(X,E,\tau)\to(Y,E',\sigma)$ be a mapping. Then (f,φ) is called a continuous mapping at the soft point $x_e\in(X,E)$ if for each arbitrary soft set $(f,\varphi)(x_e)=(f(x))_{\varphi(e)}\in(G,E')\in SS(Y,E')$, there exists $(F,E)\in SS(X,E)$ such that $x_e\in(X,E)$, $\tau(F,E)\geq\sigma(G,E')$ and $(f,\varphi)(F,E)\subset(G,E')$. If (f,φ) is a continuous mapping for each soft point, then (f,φ) is a continuous mapping.

Theorem 32. Then $(f,\varphi):(X,E,\tau)\to(Y,E',\sigma)$ is a continuous mapping if and only if $\tau((f,\varphi)^{-1}(G,E'))\geq\sigma(G,E')$ is satisfied, $\forall(G,E')\in SS(Y,E')$.

Theorem 33. Then $(f,\varphi):(X,E,\tau)\to(Y,E',\sigma)$ is a continuous mapping if and only if $(f_r,\varphi_r):(X,E,\tau_r)\to(Y,E',\sigma_r)$ is a continuous mapping on soft bitopological space for each $r\in(0,1]$.

Theorem 34. Let (X,E,τ), (Y,E',σ) be two FTSs and β be a base of σ on Y. Then $(f,\varphi):(X,E,\tau)\to(Y,E',\sigma)$ is a continuous mapping if and only if $\beta(G,E')\le\tau\left((f,\varphi)^{-1}(G,E')\right)$ for $\forall(G,E')\in SS(Y,E')$.

So we can give the notion of factor space of fuzzy soft topological spaces.

Let $\{(X_\lambda,E_\lambda,\tau_\lambda)\}_{\lambda\in\Lambda}$ be a family of fuzzy soft topological spaces and for $\forall\lambda\neq\lambda'$ the conditions $X_\lambda\cap X_{\lambda'}=\varnothing,\ E_\lambda\cap E_{\lambda'}=\varnothing$ be satisfied. By $\tilde{X}$ we show connection of all soft points related to these spaces and let $E=\bigcup\limits_{\lambda\in\Lambda}E_\lambda$. Then the $(\tilde{X},E)$ is a family of soft sets on $X=\bigcup\limits_{\lambda\in\Lambda}X_\lambda$ with parameters E and if for soft point $x_e\in(\tilde{X},E)$, if $x\in X_\lambda$ then $e\in E_\lambda$. If $e\in E_\lambda$ then $x\in X_\lambda$ is satisfied. For arbitrary $(F,E)\in(\tilde{X},E)$, $(F,E)_\lambda=\{F(e)\cap X_\lambda\}_{e\in E}$.

Theorem 35. Let $\{(X_\lambda,E_\lambda,\tau_\lambda)\}_{\lambda\in\Lambda}$ be a family of FTSs, different X'_λ s be disjoint. Then τ which is defined as follows

$$\tau(F,E)=\bigwedge_{\lambda\in\Lambda}\tau_\lambda\left((F,E)_\lambda\right),\forall(F,E)\in(\tilde{X},E)$$

is gradation of openness on X .

Definition 22. The fuzzy topological space (X,E,τ) is called the direct sum of $\{(X_\lambda,E_\lambda,\tau_\lambda)\}_{\lambda\in\Lambda}$, denoted by $(X,E,\tau)=\bigoplus\limits_{\lambda\in\Lambda}(X_\lambda,E_\lambda,\tau_\lambda)$.

İt is clear that since $i_\lambda : X_\lambda \to X = \underset{\lambda\in\Lambda}{\cup} X_\lambda$ and $j_\lambda : E_\lambda \to E = \underset{\lambda\in\Lambda}{\cup} E_\lambda$ are embedding mappings for all $\lambda \in \Lambda$, the mapping $(i_\lambda, j_\lambda) : (X_\lambda, E_\lambda, \tau_\lambda) \to (X, E, \tau)$ is a continuous mapping.

Theorem 36. Let $\{(X_\lambda, E_\lambda, \tau_\lambda)\}_{\lambda\in\Lambda}$ be a family of FTSs, $X = \prod_{\lambda\in\Lambda} X_\lambda$ be a set, $E = \prod_{\lambda\in\Lambda} E_\lambda$ be parameter set and for each $\lambda \in \Lambda$, $p_\lambda : X \to X_\lambda$ and $q_\lambda : E \to E_\lambda$ be two projections maps. Define $\beta : SS(Y, E) \to [0,1]$ as follows:

$$\beta(G,E) = \vee\left\{ \overset{n}{\underset{j=1}{\wedge}} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right) \middle| (F,E) = \overset{n}{\underset{j=1}{\cap}} \left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right) \right\}.$$

Then β is a base on FTS and for each $\lambda \in \Lambda$, $(p_\lambda, q_\lambda) : (X, E, \tau_\beta) \to (X_\lambda, E_\lambda, \tau_\lambda)$ are continuous maps.

The results obtained in fuzzy soft topological spaces for intuitionistic soft topological spaces are generalized and proved in subchapter 2 of chapter III.

Definition 23. A mappings $(\tau, \tau^*) : SS(X,E) \to [0,1]$ is called an intuitionistic fuzzy topology on X if the following conditions hold:

a) $\tau(F,E) + \tau^*(F,E) \le 1;\ \forall (F,E) \in SS(X,E)$

b) $\tau(\Phi) = \tau(\tilde{X}) = 1,\ \tau^*(\Phi) = \tau^*(\tilde{X}) = 0$

c) $\tau((F,E) \tilde{\cap} (G,E)) \ge \tau(F,E) \wedge \tau(G,E)$,

$\tau^*((F,E) \tilde{\cap} (G,E)) \le \tau^*(F,E) \vee \tau^*(G,E)$,

$\forall (F,E), (G,E) \in SS(X,E)$

ç) $\tau\left(\underset{i\in\Delta}{\cup}(F_i, E)\right) \ge \underset{i\in\Delta}{\wedge} \tau(F_i, E)$, $\tau^*\left(\underset{i\in\Delta}{\cup}(F_i, E)\right) \le \underset{i\in\Delta}{\vee} \tau^*(F_i, E)$ for the family $\forall \{(F_i, E)\}_{i\in\Delta}$, The quadruple $\left(X, E, \tau, \tau^{*}\right)$ is called intuitionistic fuzzy topological spaces of soft sets.The intuitionistic fuzzy topological space $\left(X, E, \tau, \tau^{*}\right)$ is denoted by $IFTS$.

CONCLUSION

The main results of the dissertation work are the followings:

- The categories of fuzzy soft G-modules, intuitionistic fuzzy soft G-modules were built and the closure problem of these algebraic categories with respect to algebraic operations were studied.
- The notion of exact sequence in the category of intuitionistic fuzzy soft G-modules was given and exact sequence was constructed.
- Homological modules in the category of intuitionistic fuzzy G-modules were constructed, and it was proved that the axioms of homological theory are satisfied.
- Category of neutrosophic G-modules being the extension of intuitionistic fuzzy G-modules was constructed.
- The notion of neutrosophic soft modules being the extension of neutrosophic modules, was introduced and the closure problem in this category of modules was studied. The existence of the inverse limit in the category of neutrosophic soft modules was proved.
- Fuzzy, intuitionistic fuzzy (Shostak) topology in soft sets was introduced and some researches related to the basis, continuity in the newly obtained topological space was conducted.

The basic results of the dissertation work are in the following works:

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