REPUBLIC OF AZERBAIJAN

On the rights of the manuscript

ABSTRACT

of the dissertation for the degree of Doctor of Science

THE DEVELOPMENT AND APPLICATIONS OF NUMERICAL METHODS OF SOLUTION TO CONTROL AND BOUNDARY-VALUE PROBLEMS BY EVOLUTION SYSTEMS WITH NON-ACCURATE GIVEN INITIAL CONDITIONS

Specialty: 1203.01 – Computer sciences

Field of science: Mathematics

Applicant: **Yegana Ramiz Ashrafova**

Baku – 2021

The work was performed at the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Scientific consultant: corresponding member of ANAS, doctor of physical and math. sciences, prof.

Kamil Rajab Aida-zade

Official opponents:

doctor of physical and mathematical sciences, prof.
Elimkhan Nadir Mahmudov

doctor of physical and mathematical sciences, prof.
Andrey Leonid Karchevsky

doctor of physical and mathematical sciences, prof.
Hamlet Farman Quliyev

doctor of physical and mathematical sciences
Yusif Soltan Qasimov

Dissertation council ED 1.19 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Chairman of the Dissertation council:

Dr. Sc. (Phys.-Math.), prof.
Qalina Yurievna Mehdieva

Scientific secretary of the Dissertation council:

Cand. Sc. (Phys.-Math.), ass. prof.
Elkhan Nariman Sabziev

Chairman of the scientific seminar: corr. member of ANAS, Dr. Sc. (Phys.-Math.), prof.

Vagif Rza Ibrahimov

GENERAL DESCRIPTION OF WORK

The relevance and elaboration degree of the topic. Mathematical models of many processes in biology, ecology, chemistry, petro-chemistry, pipeline transport of gas and liquid are the systems with parameters distributed in time and space, are described by partial differential equations. Classical mathematical statements of problems for calculating the dynamics of these processes require setting initial and boundary conditions corresponding to the type of differential equations. Setting of the form of boundary conditions and controlling them for one-dimensional non-stationary objects is associated with the possibility of taking measurements at the boundary points of the object and determining changes in its state at these points. Modern means of measuring and computer technology are quite good at solving this problem. It is much more complicated with the measurement, and even more so with the direct control of the state of all points of an object distributed in the phase space. Therefore, setting the initial state of an object required by classical statements of mathematical physics problems is practically impossible, and, therefore, it is necessary to study the state of the processes occurring in it and the dynamics of their change, by using only information about boundary conditions without knowing of initial conditions. Such problems are called boundary propagation problems. Their study was started by A.N. Tikhonov and continued in the works, Belyaev V.D., Bokalo M., Vafodorova G.O., Drija M., Kirilich V.M., Guseinov R.V., Lavrenyuk S.P. ., Lorenzi A., Lupton T., Moiseev E.I., Allwright J.S. But the research and development of numerical methods for solving these problems has received less attention.

The practical interest in the study of these problems is due to the fact that with a sufficiently long functioning of any evolutionary process due to the presence in it of the external forces, particularly friction (dissipation) forces inherent in any real physical system, the influence of the initial conditions on its course weakens with time and the most effective process control is determined by boundary controls in the absence of other control actions.

The main research objects of this dissertation are optimization problems, optimal control, as well as computational and inverse problems described by large systems of difference and differential equations with ordinary and partial derivatives regarding long-term processes with inaccurate given initial and nonlocal (unseparated) boundary conditions. Such problems, as a rule, arise in the mathematical modeling of complex objects using decomposition methods with respect to spatial and / or temporal variable. In particular, these kinds of problems are encountered in problems associated with the control of oil and gas transportation processes through pipeline networks of complex structure. Approximations of the problems under consideration lead in the general case to two-point problems with respect to systems of ordinary differential or discrete equations of large dimensions of a block structure. But the direct use of the sweep methods for boundary conditions proposed in the works of A.A. Abramov, V.M. Abdullaev, S.K. Godunov, E.S. Nikolaev, V.S. Ryaben'ky, A.A. Samarsky without taking into account the specifics is not effective, because taking into account the specifics of the structure of the conditions of the problems, as a rule, can significantly accelerate their solution. Note that many mathematical models of complex processes encountered in practice are obtained using their decomposition into simpler subobjects with previously known mathematical models or into subobjects for which it is not difficult to construct them. The subject of research is decomposition, which can be carried out according to spatial and / or temporal variables, and the decomposition of a complex object is carried out so that the intermediate (internal) states of the different subobjects do not affect each other and they are associated with arbitrary, but a small number of other subobjects.

The approach to calculate the state of such objects proposed in this dissertation is based on the idea of sweeping boundary conditions, while in the resulting formulas for sweeping the values of the initial or final vector of variables of a block is used only the matrix of the subsystem of this block. In contrast to the approaches proposed in the works of Abdullaev V.M., Voevodin A.F., Godunov S.K., Ryaben'ky

V.S. taking into account the specifics of the systems under consideration, firstly, the proposed formulas do not use the inverse of matrix, secondly, these formulas allow unambiguous block-by-block sweep of unseparated conditions between blocks, and thirdly, due to the previous feature, the process of sweeping of conditions as a whole is one stage. The proposed approach develops the works of Akimov E.N., Bikov A.N., Yanenko N.N., which developed special parallelization algorithms for calculating the state of such objects, based mainly on dividing blocks into separate subunits, the total number of which is determined by the number of processors (cores) of the used computer system.

The trunk pipeline of a complex looped structure with long extent can be considered as a complex system. The fluid flow process in a pipeline network is described by a system of partial differential equations of hyperbolic type with unseparated boundary and inaccurate given initial conditions. For numerical solution, the methods of lines, grids and special schemes of sweep methods are used which are different from those proposed by Abdullaev V.M., Voevodin A.F., Godunov S.K., Nikolaev E.S., Ryaben'ki V.S., Samarskii A.A.. Note that the developed schemes of the sweep methods can be parallelized, similarly to the schemes proposed in the works of Akimov E.N., Bykov A.N., Buzalo N.S., Voevodin A.F., Ryaben'ky V.S., Shugrin S.M., Yanenko N.N., and others.

In general, the works of Aliev F.A., Asanov AT, Bitsadze A.V., Valle-Poussen, Valikov E.A., Dzhumabaev D.S., Zapsarbaev L.K., Ilyin V. A., Ionkin N.I., Kabdrakova S.S., Kamynin L.I., Nakhushev A.M., Moiseev E.I., Pikone M., Steklov V.A., Tamarkin Ya.D. and etc. played an important role in the study of problems with nonlocal conditions.

The first investigations of the unsteady fluid motion in pipes were carried out by Gromeki I.S., Zhukovsky N.E., Leibenzon M.S., Charny I.A. In the works of Aida-zade K.R., Verigin A.N., Gamzaev Kh.M., Guseinzade M.A., Kadymov Ya.B., Kublanovsky L.B., Nezamaev N.A., Musaev V.G., Pashaev A.M., Smirnov M.E., Yufin V. A., Adamkowski A., Herty M., Kim Ch., Nouri-Borujerdi A., Seaid

M., Seok WH, Töppel M., Wichowski R., and other authors various methods for calculating transients in linear and complex (branched and unbranched) pipelines were applied taking into account resistance and inertia forces.

Starting from the 60-70s of the XX century, scientists such as Butkovsky A.G., Vasiliev F.P., Egorov A.I., Krabs V., Kurzhansky M.A., Lions J.-L., Potapov M.M., Russell D.L., Razgulina A. V., Sirin M., Sirazetdinov T.K., Chernousko F.L. were engaged in the study of problems of optimal control of processes described by systems of a hyperbolic type. Today, there is a lot of work in the field of optimal control. Among these works, one can distinguish the works of such scientists as Aida-zade K.R., Aliev F.A., Akhiev S.S., Aschepkov L.T., Vasiliev O.O., Velichenko V.V., Gabasov R., Gasanov K.G., Evtushenko Yu.G., Ibiev F.T., Iskenderov A.D., Kirillova F.M., Kuliev G.F., Mahmudov E.N., Mardanov M.J., Mansimov K.B., Melikov T.K., Mizukami K., Moiseev N.N., Mutallimov M.M., Polyak B.T., Tagiev R.K., Fedorenko R.P., Sharifov Y.A., Yagubov M.G. and etc.

The works of many scientists, like Vabishchevich P.N., Denisov A.M., Ivanchov I.M., Kabanikhin S.I., Kamynin V.L., Karchevsky A.L., Kozhanov A.I., Lavrentiev M. M., Otelbaev M., Pulkina L.S., Romanov V.G., Sabitov KB, Cannon JR, Lesnic D., Yang FL and others, and in our republic - Aida-zade K.R., Akhundov A.Ya., Bahyshev Sh.M., Gasanov A., Gasimov Yu.S., Denisov A.M., Iskenderov A.D., Iskenderov N.Sh., Ismailov M.I., Megraliev Y.T., Namazov G.K., Ragimov A.B., Tagiev R.K., Yagubov G.Ya. and others are devoted to the study of various aspects of inverse problems with respect to processes described by partial differential equations.

If we take into account the high modern level of development and use of computational, information and measuring technologies in the systems for the main transportation of hydrocarbon raw materials, then the use of numerical methods, algorithms, and solution programs developed in the dissertation in problems of controlling the modes of transportation of hydrocarbon raw materials is of practical interest.

A key criterion for the effectiveness of evaluating automated pipeline control systems is the ability to quickly detect leaks. The range of these tools is very wide: from the methods used in everyday life to aerospace monitoring. A significant number of works exist in this area: Kutukov S.E., Balogun HA, Burn S., Chis T., Cobacho R., Colombo AF, Dhammika De S., Donavan M., De Silva D., Geiger G., Hyun K., Junseok H., Karney BW, Lin HY, Liu JH, Lee G., Lee P., Mashford J., Ming Liu, Oyedeko KF, Su HG, Stewart B., Zhou Zhi-Jie et al. Colombo A.F., Lee P., and Karney B.W. did a review of studies on leak detection methods in the unsteady mode of fluid motion.

Despite numerous scientific advances, research on obtaining adequate mathematical models, numerical methods of calculation and optimal control that would satisfy the modern requirements of practice are relevant and are actively continuing.

Object and subject of research.

The objects of research of this dissertation work are boundary value problems and the corresponding inverse problems and problems of optimal control of evolutionary systems of large dimension, block structure and with non-accurate given initial conditions. The subject of the research is approaches based on the idea of transferring boundary conditions for calculating the state of large systems of block structure and methods for solving problems of optimal control of such objects.

The main goal and tasks of research.

The main goal of the dissertation is to develop and apply effective numerical methods for solving boundary value problems and control problems for evolutionary systems with non-accurate given initial conditions. In accordance with this, the following tasks were set and solved.

1. To develop the numerical schemes for solving the large systems of ordinary differential (ODE) and discrete equations with block structure and connected only by boundary values.
2. To investigate the solution to the problem of optimal control of a large ODE system of a block structure with blocks connected only

by boundary conditions and to develop numerical solution methods.

3. To study the impact duration of the initial conditions on the processes described by equations of parabolic and hyperbolic types, depending on the values of the parameters involved in the statement of the problem.
4. To investigate the formulation and methods for solving the problems of optimal control of evolutionary processes under inaccurate given initial conditions.
5. To develop the numerical schemes for computation the state of wave processes occurring at objects of a complex structure.
6. To investigate the formulation and develop numerical methods for the solution to inverse problems on determination the locations and powers of concentrated sources at objects with distributed parameters of complex structure.
7. To develop the software for solving the considered problems applying modern computer technologies.
8. To apply the obtained results to solving computational, optimization and inverse problems on determination the locations and powers of concentrated sources on the example of unsteady flow in hydraulic networks of complex structure.

The main research methods are:
theories and numerical methods for solving differential equations with ordinary and partial derivatives with nonlocal initial-boundary conditions and corresponding problems of optimal control and inverse problems, finite-dimensional optimization, modern information technologies and programming tools.

The main provisions to be defended:

1. the solution to large linear systems of ODE and discrete equations of block structure with non-local conditions;
2. the solution to problems of optimal control of large ODE systems of block structure with blocks which are associated by boundary conditions;

3. analysis of the time duration of the influence of initial conditions on long-term evolutionary processes depending on the process parameters;
4. the solution to problems of optimal control of evolutionary processes under inaccurate given initial conditions;
5. the state calculation, the solution to optimal control problems and identification places and powers of sources for wave processes;
6. development of software for solving the considered problems with the use of modern computer technologies;
7. application of the obtained results for solving calculation, optimization and inverse problems on the example of hydraulic networks of complex structure.

The scientific novelty of the dissertation is as follows:

1. The numerical methods have been developed for the solution to the systems of linear differential and discrete equations of large dimension and a block structure, the subsystems of which are connected only by unseparated boundary conditions.
2. The numerical algorithm for numerical solution to optimal control problem for a large ODE system of a block structure with blocks connected only by boundary conditions is proposed.
3. For evolutionary processes the duration time of influence of the initial conditions on the state of the process depending on various parameters involved in the formulation of the problem was studied.
4. The statements of optimal control problems for evolutionary processes with inaccurate given initial conditions are presented and a numerical algorithm for their solution is proposed.
5. Numerical schemes have been developed for calculating the state of processes described by a system of differential equations of a hyperbolic type of large dimension with unseparated boundary conditions.
6. The statement is studied and a numerical algorithm for solving the problem optimal control of a process described by a system of differential equations of hyperbolic type of large dimension with

unseparated boundary conditions and inaccurate given initial conditions is proposed.

7. Formulations, formulas and algorithms for solving inverse problems to determine the locations and powers of sources for wave processes are proposed.

Theoretical and practical value of the study. The results obtained in the work have both theoretical and applied value.

The theoretical significance of the work lies in the fact that the class of problems of optimal control of evolutionary processes described by equations with partial derivatives of parabolic and hyperbolic types under inaccurate given initial conditions is investigated. The systems of differential equations considered in the work have a block structure, a large dimension, and the independent subsystems of differential equations which are interconnected only by nonlocal boundary conditions. The calculation formulas necessary for the solution to the problems of optimal control of dynamic systems under inaccurate given initial conditions are obtained and justified.

The practical value of the dissertation consists in the fact that the results of studies carried out in the dissertation are important for controlling long-term processes, in particular, in the exploitation of oil and gas fields, pipeline transportation of liquid, gas, monitoring of the ecological state of regions and others.

The developed mathematical methods and algorithms, for instance, can be included in the systems of dispatching operational control of the modes of transportation of oil and gas in pipeline systems.

These methods will allow the dispatching personnel of the transport organization to carry out effective operational control, forecasting and managing hydrocarbon transport through complex pipeline systems.

Approbation of the work. The main results of the work were reported at the following international conferences:

Intern. Conf. "Control and Optimization with Industrial Applications" - COIA-, 2013, 2015,2018 (Baku, Borovets(Bulgaria)); Intern. Conf. “Problems of Cybernetics and Informatics” PCI-2010, 2012 (Baku); 24th Mini Euro Conf. “Continuous Optimization and Information-Based Technologies in the Financial Sector, (MEC EurOPT), 2010

(Turkey, Izmir), Int. Russian-Bulgarian Symposium "Equations of a mixed type and related problems of analysis and computer science", Russia, (Nalchik-Khabez 2010), (Russian-Kazakh Symposium-Nalchik 2011), (Russian-Abkhazian Symposium-Elbrus, 2012); IV Int. conf. “Nonlocal boundary value problems and related problems of mathematical biology, computer science, and physics” (Nalchik-Terskol, 2013, 2012); (Russian-Kazakh Symposium - Nalchik, 2014); Int. conf. “Actual problems of mathematics, computer science, mechanics and control theory” (Kazakhstan, Almaty, 2009); Int. conf. “Actual problems of modern mathematics, computer science and mechanics-II” (Kazakhstan, Almaty, 2011); The 4th Congress of the Turkic World Mathematical Society (TWMS) (Baku, 2011); Intern. Conf. "Optimation Methods and Applications", OPTIMA-2012 (Costa Da Caparica, Portugal, 2012), OPTIMA-2013, 2014, 2015, 2017, 2018, 2019 (Petrovac, Montenegro); Fifth Int. conf. "Mathematics, its applications and mathematical education" (MPMO-2014) Russia, Irkutsk, 2014), VI Int. conf. (MPMO-2017) Ulan-Ude, 2017; Int. scientific and practical. conf. “Innovative technologies in the oil and gas industry” Russia Stavropol, 2015,2018), Int. conf. “Applied Mathematics and Fundamental Computer Science” Omsk, 2016, 2017, 2018, 2019; Int. scientific conference "Computer science and applied mathematics" dedicated. 25th anniversary of Independence of the Republic. Kazakhstan and the 25th anniversary of the Institute of Information and Computational Technologies, Kazakhstan, Almaty, 2016; V All-Russian scientific and practical. conf. “Mathematical modeling of processes and systems”, Bashkiria, 2016; Int. Conf. “Actual problems of mathematics and mechanics”, dedicated to the 80th anniversary of Honored Scientist, Y. J. Mamedov 2010, 85th anniversary of 2015, Baku; "Oil-gas, oil refining and petrochemicals", dedicated. The 90th anniversary of the AGNA, 2010; Int. conf. “Actual problems of mathematics and computer science”, dedicated to the 90th birthday of Heydar Aliyev (Baku, 2013); Int. conf. "Non-Newtonian systems in the oil and gas industry", ded. Acad. 85th Anniversary OH. Mirzajanzade (Baku, 2013). I Int. scientific conf. “The role of the multidisciplinary approach in solving urgent problems of fundamental and applied sciences” Baku, 2014, Intern. Workshop on "Non-Harmonic Analysis and Differential Operators" 2016, Baku, Azerbaijan; XI Int. Chetaev scientific conf. “Analytical mechanics, stability and control”, Kazan, 2017; Int. conf. “Actual problems of applied mathematics and physics” Kabardino-Balkarian Republic, Elbrus region, 2017; IFAC Conferences & Symposia: TECIS-2018, Baku-2018.

- **at scientific seminars** of the Institute of Control Systems, Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan, Research Institute of Applied Mathematics at Baku State University, Department of Applied Mathematics of the Azerbaijan State Oil Academy (Oil and Industrial University), Department of “Equations of Mathematical Physics” at Baku State University, Faculty of Computer Sciences at Ege University and Dokuz Eilul University in Izmir, Turkey.

Application of the work.

The main scientific results of the dissertation were used in the works carried out within the framework of the following two projects supported by the Science Development Fund under the President of the Republic of Azerbaijan:

- “Mathematical modeling and optimization of complex systems, software development and numerical methods for solving these problems, and their application” (Grant No. EIF-2010-1 (1) -40/11 - responsible project executor), (2011-2012);
- “Numerical solution of boundary, coefficient-inverse, and optimization problems with respect to differential equations with nonlocal boundary equations with given and undefined initial conditions” (Grant - No. EİF / GAM-2-2013-2 (8) - 25/06/1 project Manager) (2014-2015).

In 2014, 2015, 2018, 2019, the results associated with calculations and optimal control of evolutional processes in the absence of accurate initial conditions and dynamic objects of large dimension and block structure were included in the Most Important Scientific Results of National Academy of Sciences of Azerbaijan.

Publications. On the topic of the dissertation author published 51 scientific papers, of which 26 are articles, 20 of which were published in foreign countries, including 13 of them are in Scopus, 8 are in Web of Science Core Collection and 25 papers were published in materials and abstracts of international conferences.

Institution where the dissertation work was executed. The dissertation was performed at the Institute of Control Systems of the National Academy of Sciences of Azerbaijan.

Structure and volume of the dissertation. The dissertation consists of an introduction, 5 chapters, a list of references from 224 titles and Appendices. The total volume of the dissertation is 336 pages of typewritten text; the main volume is 275 (550000 signs) pages, including 27 tables and 27 figures. In particular, the first chapter is of 170 000, the second is of 92 000, the third is of 80 000, the fourth is of 28 000, and the fifth is of 84 000 signs.

CONTENT OF THE DISSERTATION

The first chapter, consisting of eight sections, is devoted to the development of numerical methods for solving boundary value problems described by systems of ordinary differential equations and systems of linear discrete equations of block structure and large dimension.

In 1.1, a comparative analysis of the results of studies of the solution of large dynamical systems with nonlocal conditions is given**.**

In 1.2, a solution to a system of large dimension and block structure consisting of L independent subsystems of linear non-autonomous differential equations is studied:

$$\dot{u}^k(x) = A^k(x)u^k(x) + B^k(x), \quad x \in [0, l^k], \qquad (1)$$

$$u^k(\cdot) \in R^{n_k}, \quad k = 1, ..., L.$$

Here $A^k(x),\ B^k(x) -$ are known continuous $n_k -$dimensional respectively square matrix and vector functions, and $A^k(x) \neq const,\ x \in (0, l^k)$; unknown $n_k -$dimensional vector functions

$u^k(x)-$are continuously differentiable at $x\in[0,l^k]$; $l^k>0-$ are given; $k=1,...,L.$ Subsystems (1) are interconnected only by boundary conditions:

$$\sum_{j=1}^{L} G^{sj^*} u^j(0) + \sum_{j=1}^{L} Q^{sj^*} u^j(l^j) = r^s, \quad s=1,...,n, \tag{2}$$

$$G^{sj} = (g_1^{sj},...,g_{n_j}^{sj})^*, \quad Q^{sj} = (q_1^{sj},...,q_{n_j}^{sj})^*,$$

which can be written in matrix form:

$$Gu(0)+Qu(l)=r, \tag{3}$$

where $G,Q-$ are given $n\times n$ dimension square matrices, $n=\sum_{k=1}^{L} n_k$, moreover, the rank of the extended matrix (G,Q) is: $rang(G,Q)=n$; $r=(r^1,...,r^n)^*-$ are given n- dimensional vector, $^*-$ is transpose sign.

Problem (1), (2) is characterized by the following specific features: 1) the subsystems of differential equations of system (1) are mutually independent, 2) the solutions $u^k(x)$, $k=1,...,L,$ of subsystems connect non-separable boundary conditions characterized by weakly but arbitrarily filled connection matrices G,Q; 3) a large number (L) of subsystems, and therefore, a large order n of system (1) as a whole; 4) in real problems, there is a relation $n>>n_k$, $k=1,...,L$. For a numerical solution, a scheme of the sweep method of boundary conditions was developed and justified, which allow to carry out the transfer of conditions in blocks, and not for the entire system at the same time. This significantly increases the efficiency of solving the problem in comparison with the use of known schemes of the sweep method.

Let in S-th condition from (2) the first nonzero coefficient of the left values of the solutions of all subsystems, for $x=0$ is G^{sk}, i.e., $G^{sk}\neq 0_{n_k}$.

Definition 1.1. We'll say that n_k-dimensional vector-function $\alpha^{sk}(x)$ and a function $\beta^{sk}(x)$ are as follows

$$\alpha^{sk}(0) = G^{sk}, \quad \beta^{sk}(0) = r^s, \tag{4}$$

carry out the transfer of the left boundary value of the solution of the k-th subsystem (1) in the s-th condition from (2) to the right, if for an arbitrary solution $u^k(x)$ of k-th subsystem (1) for all $x \in [0, l^k]$ the next equality holds

$$\alpha^{sk^*}(x) u^k(x) + \left[\sum_{j=k+1}^{L} G^{sj^*} u^j(0) + \sum_{j=1}^{L} Q^{sj^*} u^j(l^j) \right] = \beta^{sk}(x). \tag{5}$$

It is clear that, the condition (5), taking into account (4), coincides with the s-th condition of (2) at $x = 0$. Functions $\alpha^{sk}(x)$, $\beta^{sk}(x)$ we'll call sweep coefficients. Substituting the values of function $\alpha^{sk}(x)$, $\beta^{sk}(x)$ at $x = l^k$ in (5), we obtain the equality equivalent to the s-th condition from (2)

$$\sum_{j=k+1}^{L} G^{sj^*} u^j(0) + \sum_{j=1}^{L} \tilde{Q}^{sj^*} u^j(l^j) = \tilde{r}^s,$$

where designated $\tilde{Q}^{sk} = Q^{sk} + \alpha^{sk}(l^k)$, $\tilde{r}^s = \beta^{sk}(l^k)$, $\tilde{Q}^{sj} = Q^{sj}$, $j = 1, ..., L,\ j \neq k$. Sweep functions $\alpha^{sk}(x)$, $\beta^{sk}(x)$, used to sweeping the boundary values of the solutions of the subsystems involved in the boundary conditions (3), from one end to the other are not unique. In particular, their constructive construction was proposed in the following theorem.

Theorem 1.1. If $G^{sk} \neq 0_{n_k}$ and n_k- dimensional vector function $\alpha^{sk}(x)$ and a function $\beta^{sk}(x)$ at $x \in (0, l^k]$ are the solution to the following Cauchy problems:

$$\dot{\alpha}^{sk}(x) = -A^{k^*}(x) \alpha^{sk}(x), \qquad \alpha^{sk}(0) = G^{sk}, \tag{6}$$
$$\dot{\beta}^{sk}(x) = B^{k^*}(x) \alpha^{sk}(x), \qquad \beta^{sk}(0) = r^s,$$

then these functions are sweep coefficients for the left boundary value of an arbitrary solution of the k-th subsystem (1) in the s-th condition, and, therefore, satisfy condition (5) at $x \in [0, l^k]$.

It is important to note that, as follows from Definition 1.1 and Theorem 1.1, the sweeping of the left value of the solution of the k-th subsystem in the s-th condition is carried out only if the corresponding coefficient in this condition is nonzero.

Similar to the right sweep, the left sweep method is given.

Let in the s-th condition of (2) the first nonzero coefficient for the right values of the solutions of all subsystems are Q^{sk}, i.e., $Q^{sk} \neq 0_{n_k}$.

Definition 1.2. We'll say that n_k- dimensional vector function $\alpha^{sk}(x)$ and function $\beta^{sk}(x)$ are as follows

$$\alpha^{sk}(l^k) = Q^{sk}, \quad \beta^{sk}(l^k) = r^s, \tag{7}$$

carry out the transfer of the right boundary value of the solution of the k-th subsystem (1) in the s-th condition from (2) to the right, if for an arbitrary solution $u^k(x)$ of k-th subsystem (1) for all $x \in [0, l^k]$ the next equality holds:

$$\alpha^{sk^*}(x) u^k(x) + \left[\sum_{j=1}^{L} G^{sj^*} u^j(0) + \sum_{j=k+1}^{L} Q^{sj^*} u^j(l^j) \right] = \beta^{sk}(x). \tag{8}$$

It is clear that, the condition (8), taking into account (7), coincides with the s-th condition of (2) at $x = l^k$. If such sweep functions are known, then equality (8) at $x = 0$ takes the form:

$$\sum_{j=1}^{L} \tilde{G}^{sj^*} u^j(0) + \sum_{j=k+1}^{L} Q^{sj^*} u^j(l^j) = \tilde{r}^s, \tag{9}$$

where $\tilde{G}^{sk} = G^{sk} + \alpha^{sk}(0),\ \tilde{G}^{sj} = G^{sj},\ \tilde{r}^s = \beta^{sj}(0),\ \ j = 1, \ldots, L,\ j \neq k$. The condition (9), which is equivalent to s-th condition (2), involve n_k-dimensional variable, defined at the right end, one less than before sweeping.

Similar to Theorem 1.1, the following theorem holds.

Theorem 1.2. If $Q^{sk} \neq 0_{n_k}$ and n_k- dimensional vector function $\alpha^{sk}(x)$ and function $\beta^{sk}(x)$ at $x \in [0, l^k]$ are the solution to the following Cauchy tasks:

$$\dot{\alpha}^{sk}(x) = -A^{k^*}(x)\alpha^{sk}(x), \quad \alpha^{sk}(l^k) = Q^{sk},$$

$$\dot{\beta}^{sk}(x) = B^{k^*}(x)\alpha^{sk}(x), \quad \beta^{sk}(l^k) = r^s, \tag{10}$$

then these functions are sweep coefficients for the right boundary value of an arbitrary solution of $k-$th subsystem (1) in the s-th condition (2), and therefore they satisfy the sweep condition (8).

Note that the advantage of the sweep method, requiring the solution of auxiliary vector Cauchy problems over the known approach using the fundamental matrix, requiring a solution to the matrix Cauchy problem of dimension ($n \times n$) is that in real problems, from the existing n conditions, as a rule, only a few can be nonlocal, the rest are given at one end.

The advantage of the proposed approach in comparison with the direct use of sweep methods in general is obvious, because here, the transfer is carried out only with respect to those variables whose coefficients are non-zero in the boundary conditions, while the transfer is carried out using only the subsystem of differential equations in which the transferred variable is involved.

To the mathematical formulation (1), (2) is reduced, for example, the problem of calculating artificial dynamic neural networks with a complex structure and large dimension, the problem of calculating unsteady processes of fluid, gas flow in pipeline networks of a complex structure, etc.

In 1.3, a numerical approach for the solution to a system of ordinary second-order differential equations with associated only nonseparated boundary conditions is studied. For example, the application of the direct method to the calculation of oscillatory processes and heat transfer processes in multilink coupled structures described by second-order hyperbolic and parabolic equations leads to this problem.

The specificity of the considered system of equations is that the equations themselves are assumed to be independent of each other, i.e. the functions involved in them are connected only by boundary conditions. The mathematical statement of the problem under consideration is presented in the general case as a two-point problem with respect to a system of ordinary differential equations of large dimension. For a numerical solution of the problem, generally speaking, it is possible to use various known schemes of the sweep method. For an effective numerical solution of the problem, an approach is proposed based on the idea of sweeping conditions, but essentially using the specific features of the problem. This allowed the transfer of the boundary condition for each equation of the system separately, independently of other equations of the system. As a result, at the end, it is necessary to solve an algebraic system of equations of the same order as the original system of differential equations. The essential thing is that the proposed approach is easily parallelized. The obtained formulas and solution schemes are given. The illustrative test problem considered in the dissertation is obtained as a result of applying the direct method for calculating fluid flow modes using an example of a fragment of a complex pipeline transport network.

In 1.4, a decomposition approach for the solution to the linear discrete block-diagonal systems with unseparated between blocks boundary conditions is proposed. Mathematical models of many large objects of complex structure are characterized by the following features:

1) a large number of sub-objects L;
2) a large dimension of the state vector of sub-objects or a long duration of functioning n_k, $k=1,...,L$;
3) weak and arbitrary relations between sub-objects, i.e. weak and arbitrary filled matrices т.е. слабой и произвольной заполненностью матриц G, Q.

Features 1), 2) for real objects lead to the fact that the order of the algebraic system can exceed several thousand and tens of thousands. Feature 3) leads to nonseparated boundary conditions, which necessitates the use of methods for transferring boundary conditions.

Consider a blocky tridiagonal system of equations:

$$A_i^k y_{i+1}^k - C_i^k y_i^k + B_i^k y_{i-1}^k = -F_i^k, \quad i=1,...,n_k-1, \quad k=1,...,L. \qquad (11)$$

Here $y^k = \left(y_0^k, ..., y_{n_k}^k\right)^* - (n_k + 1)$ -dimensional vector, that determines the state of the $k-$th process (subsystem); A_i^k, B_i^k, C_i^k and $F_i^k -$ are given numbers; n_k – the duration of $k-$th process, $k = 1, ..., L$.

We introduce the notation

$$n = \sum_{k=1}^{L} n_k - L, \quad m = 2L, \quad M = n + m,$$

$$(y_0, y_1) = ((y_0^1, ..., y_0^L)^*, (y_1^1, ..., y_1^L)^*) \in R^m, \qquad r = \left(r^1, ..., r^m\right)^*,$$

$$(y_{n-1}, y_n) = ((y_{n_1-1}^1, ..., y_{n_L-1}^L)^*, (y_{n_1}^1, ..., y_{n_L}^L)^*) \in R^m.$$

Here $M -$ is the number of unknowns in system (11), consisting of L subsystems (blocks) with n total number of equations, y_0, y_1 and y_n, y_{n-1} – are accordingly, the state of all subprocesses at the initial and final (individual for each subprocess) time instants.

Subprocesses under consideration are interconnected by means of m initial and final states in the form of nonseparated boundary conditions between blocks, given in the form:

$$\sum_{j=1}^{L} \left[g_l^{sj} y_{n_j}^j + q_l^{sj} y_{n_j-1}^j\right] + \sum_{j=1}^{L} \left[g_0^{sj} y_1^j + q_0^{sj} y_0^j\right] = r^s, \quad s = \overline{1, m}. \qquad (12)$$

Taking into account the specific structure of the system, formulas are obtained for transferring the values of the initial and final variables in the boundary conditions, which is carried out block by block independently of each other. The result is an algebraic system, the dimension of which is determined by the number of subsystems, and only the values of the initial or final variables of all subsystems are unknown.

Definition 1.5 We say that the variables α_i^{sk}, β_i^{sk}, γ_i^{sk}, $i = 1, ..., n_k$, carry out the sweeping to the right the left value of the solution of $k-$th subsystem (11) in $s-$th condition, if for any solution of $k-$th subsystem (11) the equalities hold

$$\alpha_i^{sk} y_i^k + \beta_i^{sk} y_{i+1}^k + \sum_{j=k+1}^{L} g_0^{sj} y_1^j + \sum_{j=k+1}^{L} q_0^{sj} y_0^j + \sum_{j=1}^{L} g_l^{sj} y_{n_j}^j + \sum_{j=1}^{L} q_l^{sj} y_{n_j-1}^j = \gamma_i^{sk}, \quad (13)$$

$$\alpha_0^{sk} = q_0^{sk}, \quad \beta_0^{sk} = g_0^{sk}, \quad \gamma_0^{sk} = r^s. \tag{14}$$

Theorem 1.6 If g_0^{sk} and q_0^{sk} at the same time are not equal to zero, then the parameters α_i^{sk}, $\beta_i^{sk}, \gamma_i^{sk}$, $i = 1,\dots,n_k$, determined from the recurrence relations

$$\begin{aligned}
&\alpha_i^{sk} = \beta_{i-1}^{sk} + \alpha_{i-1}^{sk}\frac{C_i^k}{B_i^k}, \ \alpha_0^{sk} = q_0^{sk}, \\
&\beta_i^{sk} = (\beta_{i-1}^{sk} - \alpha_i^{sk})\frac{A_i^\nu}{C_i^\nu}, \beta_0^{sk} = g_0^{sk}, \\
&\gamma_i^{sk} = \gamma_{i-1}^{sk} + (\alpha_i^{sk} - \beta_{i-1}^{sk})\frac{F_i^k}{C_i^k}, \gamma_0^{s1} = r^s, \quad i = 1,\dots,n_k, \\
&\gamma_0^{sk+1} = \gamma_{n_k}^{sk},
\end{aligned} \tag{15}$$

are the sweep coefficients of the left value in the $s-$th condition (12) with respect to the solution of $k-$th subsystem of equations of system (11). The formulas are derived in a similar way and a theorem is proved for sweeping to the left.

In 1.5 the solution of linear discrete two-step block-diagonal systems of equations of large dimension with subsystems connected only by boundary conditions between blocks is studied. Taking into account the specific structure of the system, the formulas are obtained for transferring the values of the initial and final variables in the boundary conditions, which is carried out block by block independently of each other. The result is an algebraic system whose dimension is determined by the number of blocks, and only the values of the initial or final variables of all blocks are unknown.

In 1.6 the numerical solution of a system of independent hyperbolic equations related only by unseparated boundary conditions is studied. Approximating the system of differential equations, initial and boundary conditions, is obtained a system of tridiagonal algebraic equations, for the solution of which the sweep method given in 1.4 is used.

In 1.7 the numerical solution of the optimal control problem for a complex object described by a large-dimensional ODE system of a block structure with blocks connected only by boundary conditions is studied. The necessary optimality conditions are obtained in which the adjoint problem has the same specificity as the direct problem. To solve the optimal control problem, it is proposed to apply first-order numerical optimization methods using functional gradient formulas involved in the necessary optimality conditions. To solve direct and conjugate initial-boundary value problems having a block structure and unseparayed nonlocal boundary conditions with a weakly filled Jacobi matrix, were used the special schemes of the sweep method proposed above, which take into account the specifics of systems of differential equations and boundary conditions that allow the transfer of boundary conditions for each block separately.

In 1.8 the results of numerical experiments carried out on examples of solving test problems corresponding to the problems considered in the first chapter are presented.

The second chapter, consisting of five sections, is devoted to studying the length of duration time of the influence of the initial conditions on the state of the evolutionary process using numerical methods on the example of specific boundary value problems and the corresponding problems of optimal control of evolutionary processes with inaccurate given initial conditions.

In 2.1, the solution of boundary value problems with respect to evolutionary processes described by parabolic equations under inaccurate given initial conditions is studied. The initial-boundary-value problem with respect to the heating process of a rod limited in length is considered:

$$u_t = au_{xx} + f(x,t), \quad t_0 < t \le T, \quad 0 < x < l, \tag{16}$$

$$u(0,t) = \mu_1(t), \quad u(l,t) = \mu_2(t), \quad 0 \le t \le T. \tag{17}$$

It is assumed that the initial state of the process is not known exactly, but piecewise continuous functions

$$u(x,t_0) = u_0(x) = u_0(x;\gamma), \qquad x \in [0,l], \tag{18}$$

which could be initial for the process, depend on the $r-$dimensional vector of parameters $\gamma \in R^r$ whose values belong some given admissible set D:

$$D = \{\gamma \in R^r : \underline{\gamma_i} \le \gamma_i \le \bar{\gamma}_i,\ i = 1,...,r\}\,. \qquad (19)$$

In this case, it is assumed $\rho_D(\gamma)$-the density function of the distribution of the parameters γ of the initial state on the set D is given. Here $\underline{\gamma_i}, \bar{\gamma}_i -$are given numbers; $a > 0-$is a coefficient of thermal diffusivity; $u(x,t)-$ is the temperature of the rod at a point at a point $x \in [0,l]$ in time $t \in [t_0, T]$; $f(x,t)$, $\mu_1(t)$, $\mu_2(t)$ are given continuous functions.

The classical solution to problem (16)–(18) is assumed continuous everywhere in a closed domain, in particular, requiring matching of the initial and boundary conditions. The continuity condition for the initial functions is very restrictive for practical applications. For considered in (18) initial functions with parameters from D, the fulfillment of conjugation conditions (initial conditions with boundary ones) is not considered.

Indeed, if we consider the simplest problem of cooling a uniformly heated rod at zero temperature at the edges and $u_0(x) \ne 0$, then the solution to the problem is continuous in a closed domain should be discontinuous at points $(0,t_0)$ and (l,t_0). This example shows that the condition of continuity of the initial value and the conditions of its conjugation with boundary values exclude practically important cases from consideration. Using the function that is the fundamental solution of the heat equation constructed by A.N. Tikhonov[1], we can uniquely determine the solution to the problem for the case when the initial function $u_0(x,\gamma)$ belongs to the class of piecewise continuous functions. It is clear that for all initial functions with parameters from

[1] Tikhonov, A.N., Samarskii, A.A. Equations of mathematical physics. M. “Nauka”, 1977, 736p.

D, the conditions for the existence and uniqueness of a solution to the corresponding initial-boundary-value problem are satisfied.

It follows from the form and properties of the fundamental solution that, if the moment of interest to us is sufficiently far from the initial one, then over time the influence of the initial conditions on the temperature distribution along the rod decreases and the temperature of the rod is practically determined only by the values of the boundary conditions.

In 2.2, the well-known fact is confirmed by numerical experiments:

at sufficiently large values of the duration time of the process, the influence of the initial conditions on its current state decreases. A qualitative analysis of the duration of the influence of the initial conditions depending on the values of the parameters and functions involved in the mathematical model of the process under study, as well as the range of variation of the possible values of the initial and boundary conditions, is carried out.

It was established by numerical experiments that the current state of the process mainly depends on the boundary conditions and the existing capacities of internal or external sources after a certain moment of time T, and little depends on the set of possible initial states with parameters from D.

In 2.3 An analytical representation of the solution of the initial-boundary-value problem is obtained that describes, according to I.A. Charny[2], unsteady fluid flow in a linear section of the pipeline:

$$\begin{cases} -\dfrac{\partial P(x,t)}{\partial x} = \dfrac{\rho}{S}\dfrac{\partial Q(x,t)}{\partial t} + 2a\dfrac{\rho}{S}Q(x,t), \\ -\dfrac{\partial P(x,t)}{\partial t} = c^2\dfrac{\rho}{S}\dfrac{\partial Q(x,t)}{\partial x}, \qquad (x,t)\in\Omega=(0,l)\times(0,T]. \end{cases} \tag{20}$$

Here $P(x,t),\ Q(x,t)$ is the pressure and flow rate of the fluid in the pipeline at a point $x\in(0,l)$ in time $t>0$; l is the length of the considered linear part of the pipeline; $a=16\nu/d^2$, d is the internal

---

[2] Charny, I.A., Unsteady motion of a real fluid in pipes, 2nd -ed. M.: Nedra, – 1975. –296 p.

diameter of the pipeline, S is the cross-sectional area of the pipeline section, ρ is the density of the fluid, ν is the kinematic viscosity coefficient, which for a droplet fluid we can assume that, it is independent of pressure, and therefore $a = const$; c is the speed of sound in the tested fluid; T is the given duration of the study of the modes of operation of the process.

Due to the very complex form of the obtained analytical representation of the solution, it is impossible to conduct any qualitative analysis of its behavior, to investigate the dependence of the duration of the influence of the initial conditions on the factors and parameters involved in the formulation, even in the case of only one linear section, not to mention case of the complex loopback structure of the pipeline network.

In 2.4 numerical methods have been used to analyze the duration time of the influence of the initial conditions on the unsteady mode of fluid flow in the linear section of the pipeline under given boundary conditions (modes) and the dependence of this duration on the values of the initial conditions themselves, on the geometric dimensions of the pipeline, the properties of the pumped liquid, and also the boundary modes. The duration of the influence of the initial conditions on the process almost weakly depends on the values of the initial modes (conditions) themselves. With an increase in the friction coefficient, the duration of the influence of the initial conditions increases and the growth rate of the influence time is the same with growth of a.

Identified patterns are important in the calculations, forecasting, optimization of transient modes that arise for various technical and technological reasons during the transportation of hydrocarbon raw materials through trunk pipelines.

In 2.5 is studied the formulations of the problems of optimal control of the boundary conditions of evolutionary processes that have been functioning for a long time, in connection with which there is no accurate information about their initial state (initial conditions). In this regard, only controlling boundary conditions play an important role.

In 2.5.1 the problems of optimal control of processes described by differential equations of the parabolic type are studied in the absence of accurate information about the initial values of the state of the process. The process of thermal conductivity (16) is considered as an illustration.

A numerical solution of the problem is carried out in which, by controlling the temperatures $\mu_1(t),\ \mu_2(t),\ t \in (0,T]$ that determine the boundary conditions, at a given moment $T > 0$, the temperature distribution in the rod must be brought as close as possible to a given distribution $U(x),\ \ 0 \le x \le l$, i.e. the functional defined, for example, in the form:

$$J(\mu) = \int_D I(\mu,\gamma)\rho_D(\gamma)d\gamma \to \min, \tag{21}$$

$$I(\mu,\gamma) = \int_0^l (u(x,T;\mu,\gamma) - U(x))^2 dx + \alpha \left\|\mu(t)\right\|^2_{L_2[0,T]}$$

get the minimum value. Here $u(x,T;\mu,\gamma)$ is the solution to the problem (16) – (19) for given parameter $\gamma \in D$ in the initial condition $u(x,0) = \varphi(x;\gamma)$ and for admissible control $\mu(t) \in L_2[0,T]$. Based on the numerical experiments, an analysis is made of the dependence of the behavior of the obtained optimal boundary conditions on the change in the set of possible initial conditions.

In 2.5.2 the problem of optimal control of processes described by differential equations of a hyperbolic type is studied in the absence of accurate information on the initial values of the state of the process. The wave process is considered as an illustration.

For the corresponding optimal control problems considered in 2.5.1 and 2.5.2, analytical formulas for the gradients of the corresponding target functionals are obtained. These formulas allow us to use effective numerical methods of first-order optimization for solving problems. The obtained results can be used in studies related to the control of many long-term processes with distributed parameters, described by systems of partial differential equations.

In 2.5.3 the results of numerical experiments carried out on examples of solving model problems are presented.

The third chapter, consisting of five sections, is devoted to the modeling and calculation of transients that occur during the transportation of the fluid in pipeline networks of complex structure.

In 3.1 is carried out a comparative analysis of the results of studies on numerical modeling of the modes of unsteady fluid flow in pipelines.

In 3.2 is considered the formulation of the problem of computation of the transient modes in oil pipelines of complex loopback structure. The fluid flow in each linear section according to I.A. Charny can be adequately described by a system of two first-order partial differential equations of hyperbolic type. There are nonseparated boundary conditions at the connection points of the sections, determined by an analog of the first Kirchhoff law.

An unsteady fluid flow regime is considered for a fairly general case of a complex loopback hydraulic network containing M sections and N nodes. We introduce the following notations: $I-$is the set of nodes; $J=\{(k,s):k,s\in I\}-$is the set of sections; $I_k^+ -$ the set of node numbers from which sections lead to $k-$th node; $I_k^- -$ the set of node numbers to which sections lead from $k-$th node; $I_k = I_k^+ \bigcup I_k^- -$the set of nodes, adjacent to $k-$th node.

The unsteady isothermal flow of a droplet liquid under a laminar regime with a constant density ρ on the (k,s)-th linear section of the oil pipeline network with pipelines of a diameter d^{ks} and length l^{ks} can be adequately described by the following linearized by Charny I.A. hyperbolic system of differential equations:

$$\begin{cases} -\dfrac{\partial P^{ks}(x,t)}{\partial x} = \dfrac{\rho}{S^{ks}}\dfrac{\partial Q^{ks}(x,t)}{\partial t} + 2a^{ks}\dfrac{\rho}{S^{ks}}Q^{ks}(x,t), \\ -\dfrac{\partial P^{ks}(x,t)}{\partial t} = c^2\dfrac{\rho}{S^{ks}}\dfrac{\partial Q^{ks}(x,t)}{\partial x}, x\in(0,l^{ks}), t\in(0,T], \end{cases} \quad s\in I_k^+,\ \ k\in I. \quad (22)$$

Here: $t \in (t_0, T]$; $s \in I_k^+$, $k \in I$; $P^{ks}(x,t)$, $Q^{ks}(x,t)$ – is pressure and flow rate in (k,s) -th pipeline of the network, at a point $x \in (0, l^{ks})$ in a moment of time t; c – speed of sound in the medium; S^{ks} – internal cross-sectional area of the (k,s)-th section of pipeline; a^{ks} – is the resistance coefficient on the (k,s) –th section (in the case of a laminar regime, we can assume that the kinematic viscosity coefficient ν is independent of pressure, and then $2a^{ks} = 32\nu / (d^{ks})^2 = const$).

$2M$ boundary conditions must be specified for the system (22). The conditions of material balance

$$\sum_{s \in I_k^+} Q^{ks}(l^{ks}, t) - \sum_{s \in I_k^-} Q^{ks}(0,t) = \tilde{q}^k(t), \;\; k \in I^{\text{int}}, \quad t \in [t_0, T], \tag{23}$$

and flow continuity:

$$P^{\hat{k}k}(l^{\hat{k}k}, t) = P^{k\breve{k}}(0,t), \quad \hat{k} \in I_k^+, \;\; \breve{k} \in I_k^-, \;\; k \in I^{\text{int}}, \;\; t \in [t_0, T], \tag{24}$$

are used for I^{int} - the set of internal nodes of the network, where $\tilde{q}^k(t)$ – is the given external inflow $(\tilde{q}^k(t) > 0)$ or outflow $(\tilde{q}^k(t) < 0)$ for the k – the internal node.

At each node from $I^f = I \setminus I^{\text{int}}$, which is not the internal node of the network, we define the pressure value (the set of such nodes is denoted by $I_p^f \subset I^f$) or the flow rate (the set $I_q^f \subset I^f$). Then conditions (23) and (24) are supplemented with the following conditions:

$$\begin{cases} P^{ns}(0,t) = \tilde{P}^n(t), & s \in I_n^+, \; \text{если}\; I_n^- = \varnothing, \\ P^{sn}(l^{sn}, t) = \tilde{P}^n(t), & s \in I_n^-, \; \text{если}\; I_n^+ = \varnothing, \end{cases} \quad n \in I_p^f, \; t \in [t_0, T], \tag{25}$$

$$\begin{cases} Q^{ms}(0,t) = \tilde{Q}^m(t), & s \in I_m^+, \; \text{если}\; I_m^- = \varnothing, \\ Q^{sm}(l^{sm}, t) = \tilde{Q}^m(t), & s \in I_m^-, \; \text{если}\; I_m^+ = \varnothing, \end{cases} \quad m \in I_q^f, \; t \in [t_0, T], \tag{26}$$

and it is important that the conditions are met: $I^f = I_q^f \bigcup I_p^f$, $I_q^f \bigcap I_p^f = \varnothing$, $I_p^f \neq \varnothing$.

In 3.3, numerical schemes are investigated for calculating unsteady modes of fluid flow in pipeline networks.

In 3.3.1, a scheme is proposed for numerically calculating the modes of fluid flow through a pipeline, based on the application of the lines method and reducing the problem to a boundary value problem with respect to ODE systems with non-separated boundary conditions, for the solution of which a scheme of a sweep method is proposed in the first chapter.

In 3.3.2 a numerical scheme based on the use of the grid method is proposed for the numerical computation of the modes of fluid flow through the pipeline. The application of the grid method in two versions of the equation for the fluid flow is investigated. In the first version, a system of two differential equations in each section is reduced to a second-order equation. Approximating the boundary-value problem together with the initial and boundary conditions, a system of block-tridiagonal algebraic equations with nonseparated boundary conditions is obtained, for the solution of which the scheme of sweep method for boundary conditions is used, which proposed in the first chapter. In the second version a finite-difference approximation of the system (22) itself is used. A scheme based on the use of implicit grid methods is proposed for numerically calculating the modes of fluid flow through the pipeline, algorithms are developed based on the obtained formulas that determine the functional dependence of the values of the desired functions at current points on the boundary values at the left or right ends of the corresponding sections.

We introduce uniform grid areas:

$$\omega^{ks} = \{(x_i, t_j):\ x_i = ih,\ \ t_j = j\tau,\ \ i = \overline{0, n_{ks}},\ \ j = \overline{0, n_t}\},$$
$$n_{ks} = [l^{ks}/h],\ \ \ n_t = [T/\tau], \tag{27}$$

on the set $[0, l^{ks}] \times [0, T],\ \ (k,s) \in J$, where h, τ –are the given positive numbers, defining grid steps, [a]–is the integer part of the number a. Using the following notation $\lambda^{ks} = (\frac{h}{\tau} + 2a^{ks}h)$, $\mu = \frac{h}{\tau}$, $\eta = \frac{h}{\tau c^2}$,

$P_{ij}^{ks} = P^{ks}(x_i, t_j),\ \ Q_{ij}^{ks} = \dfrac{\rho}{S^{ks}} Q^{ks}(x_i, t_j),\ \tilde{q}_j^k = \tilde{q}^k(t_j),$ and applying a stable implicit grid method for approximating system (22) on a template $(i, j-1), (i, j), (i-1, j)$ or on $(i, j-1), (i, j), (i+1, j)$, after simple transformations, are obtained:

$$\begin{cases} P_{i-1j}^{ks} = P_{ij}^{ks} + \lambda^{ks} Q_{ij}^{ks} - \mu Q_{ij-1}^{ks}, \\ Q_{i-1j}^{ks} = Q_{ij}^{ks} + \eta P_{ij}^{ks} - \eta P_{ij-1}^{ks}, \quad i = \overline{n_{ks}, 1}, \quad s \in I_k^+,\ k \in I, \end{cases} \tag{28}$$

or

$$\begin{cases} P_{i+1j}^{ks} = P_{ij}^{ks} - \lambda^{ks} Q_{ij}^{ks} + \mu Q_{ij-1}^{ks}, \\ Q_{i+1j}^{ks} = Q_{ij}^{ks} - \eta P_{ij}^{ks} + \eta P_{ij-1}^{ks}, \quad i = \overline{0, n_{ks} - 1}, \quad s \in I_k^+,\ k \in I. \end{cases} \tag{29}$$

To sweep the conditions (23) - (27) from the left end to the right on each $j-$th time layer for $(k,s)-$th section are obtained functional dependences:

$$P_{0j}^{ks} = R(P_{n_{ks}j}^{ks}, Q_{n_{ks}j}^{ks}), \quad Q_{0j}^{ks} = G(P_{n_{ks}j}^{ks}, Q_{n_{ks}j}^{ks}), \quad (k,s) \in J, \tag{30}$$

which are called formulas of the right sweep, and for sweeping conditions from the right end to the left, are obtained dependencies:

$$P_{n_{ks}j}^{ks} = R(P_{0j}^{ks}, Q_{0j}^{ks}), \quad Q_{n_{ks}j}^{ks} = G(P_{0j}^{ks}, Q_{0j}^{ks}), \quad (k,s) \in J, \tag{31}$$

which are called formulas of the left sweep.

For this purpose, for the right sweep, the dependencies in the following form is built:

$$\begin{aligned} P_{0j} &= \alpha_i^p P_{ij} + \beta_i^p Q_{ij} + \theta_i^p, \\ Q_{0j} &= \alpha_i^q Q_{ij} + \beta_i^q P_{ij} + \theta_i^q, \end{aligned} \qquad i = \overline{1, n}, \tag{32}$$

where $\alpha_i^p,\ \beta_i^p,\ \theta_i^p,\ \ \alpha_i^q,\ \beta_i^q,\ \theta_i^q -$are sweep coefficients, satisfying the following recurrence relations:

$$\begin{cases} \alpha_p^{ks(r+1)} = \alpha_p^{ks(r)} + \eta \beta_p^{ks(r)}, & \alpha_p^{ks(1)} = 1, \\ \beta_p^{ks(r+1)} = \alpha_p^{ks(r)} \lambda^{ks} + \beta_p^{ks(r)}, & \beta_p^{ks(1)} = \lambda^{ks}, \\ \theta_p^{ks(r+1)} = \theta_p^{ks(r)} - \alpha_p^{ks(r)} \mu Q_{r+1j-1}^{ks} - \beta_p^{ks(r)} \eta P_{r+1j-1}^{ks}, & \theta_p^{ks(1)} = -\mu Q_{1j-1}^{ks}, \end{cases}$$

$$\begin{cases} \alpha_q^{ks(r+1)} = \alpha_q^{ks(r)} + \lambda^{ks}\beta_q^{ks(r)}, & \alpha_q^{ks(1)} = 1, \\ \beta_q^{ks(r+1)} = \beta_q^{ks(r)} + \eta\alpha_q^{ks(r)}, & \beta_q^{ks(1)} = \eta, \\ \theta_q^{ks(r+1)} = \theta_q^{ks(r)} - \alpha_q^{ks(r)}\eta P_{r+1\,j-1}^{ks} - \beta_q^{ks(r)}\mu Q_{r+1\,j-1}^{ks}, & \theta_q^{ks(1)} = -\eta P_{1j-1}^{ks}, \end{cases} \tag{33}$$

$r = \overline{1, n_{ks} - 1},\ s \in I_k^+, k \in I.$

As a result, according to formulas (32), the following expressions are obtained for $r = n_{ks}$ in the case of a right sweep:

$$\begin{aligned} P_{0j}^{ks} &= \alpha_p^{ks(n_{ks})} P_{n_{ks}j}^{ks} + \beta_p^{ks(n_{ks})} Q_{n_{ks}j}^{ks} + \theta_p^{ks(n_{ks})}, \\ Q_{0j}^{ks} &= \alpha_q^{ks(n_{ks})} Q_{n_{ks}j}^{ks} + \beta_q^{ks(n_{ks})} P_{n_{ks}j}^{ks} + \theta_q^{ks(n_{ks})}. \end{aligned} \tag{34}$$

Formulas are constructed for the variant of the left sweep similar to formulas (32), (33), (34). In formulas (33), the values of the desired functions at the left end are expressed using their values at the right end of the $(k,s)-$th section. Carrying out the operation of sweeping conditions for all sections $(k,s) \in J$ to one end: right (or left), substituting the obtained expressions (34) in (23) - (26) for all sections are obtained all $2M$ conditions only at one left (right) end of the intervals $(0, l_{ks})$, $(k,s) \in J$:

$$\alpha_q^{ks(0)} Q_{0j}^{ks} + \beta_q^{ks(0)} P_{0j}^{ks} + \theta_q^{ks(0)} = \tilde{Q}_s(t_j), \quad s \in I_k^+, \quad k \in I_q^f,$$

$$\alpha_p^{ks(0)} P_{0j}^{ks} + \beta_p^{ks(0)} Q_{0j}^{ks} + \theta_p^{ks(0)} = \tilde{P}_s(t_j), \quad s \in I_k^+, \quad k \in I_p^f,$$

$$\alpha_p^{\hat{k}k(0)} P_{0j}^{\hat{k}k} + \beta_p^{\hat{k}k(0)} Q_{0j}^{\hat{k}k} + \theta_p^{\hat{k}k(0)} = P_{0j}^{k\breve{k}}, \quad \forall \hat{k} \in I_k^+, \ \breve{k} \in I_k^-, \ k \in I,$$

$$\sum_{s \in I_k^+} \alpha_q^{ks(0)} Q_{0j}^{sk} + \beta_q^{ks(0)} P_{0j}^{ks} + \theta_q^{ks(0)} - \sum_{s \in I_k^-} Q_{0j}^{ks} = \tilde{q}_k(t_j), \qquad k \in I.$$

These conditions can be written in compact form in the form of the following system of linear algebraic equations of order $2M$:

$$AX = B, \tag{35}$$

where $X = (x_1, ..., x_{2M})^*$ ($x_s = P_{n_{r_s}}^{r_s}$, $s = 1,..,M$, $x_s = Q_{n_{r_s}}^{r_s}$, $s = M+1,..,2M$ in the case of left or $x_s = P_{0r_s}^{r_s}$, $s = 1,..,M$, $x_s = Q_{0r_s}^{r_s}$ $s = M+1,..,2M$ in the case of right sweep), A-is the $2M \times 2M$ dimensional matrix, B- $2M$ dimensional vector. Solving the system (35) by any numerical method, the pressure and flow rate values at the right (or left) ends of the corresponding intervals (sections) $(0, l^{ks}), \ (ks) \in J$ are found. So, the problem (22) - (26) with unseparated boundary conditions is reduced to a problem with all conditions at one end, for the solution of which using formulas (32) it is possible to find the values of pressure and flow rate at all points of the intervals $(0, l^{ks}), \ (ks) \in J$.

In 3.4, a description of a numerical scheme for calculating fluid flow regimes for a particular pipeline network is given**.**

In 3.5, numerical experiments were conducted on the example of solving model problems using the proposed approaches, an analysis of the results is given.

The fourth chapter, consisting of three sections is devoted to the numerical solution of the problem of optimal control on transient processes in complex hydraulic networks.

In 4.1, the formulation of the problem of optimal transient control in a pipeline network of complex structure is investigated. It lies in controlling the boundary conditions (by pumping stations installed at the beginning and end of the section), to transfer the flow mode of the network sections from the initial stationary state to the required final stationary state, optimizing the specified criterion.

Various indicators can act as criteria for controlling transient modes on a pipeline network. It has adopted the total value of two indicators: the duration of the transition process (performance) and the integral value of the deviation of the values of the state function from predefined:

$$J(\upsilon, T) = T + r_1 \sum_{k \in I} \sum_{s \in I_k^+} \int_0^{l^{ks}} (Q^{ks}(x, T; \upsilon) - Q_T^{*ks})^2 \, dx + r_2 \sum_{k \in I} \sum_{s \in I_k^+} \int_0^{l^{ks}} (P^{ks}(x, T; \upsilon) - P_T^{*ks}(x))^2 \, dx.$$

here r_1, r_2 – are given weight (penalty) factors, the values of which are determined by the desired conditions for the establishment.

In 4.2, the problem of optimal performance of transient control due to boundary conditions is considered. Two methods were used for this. In the first approach, two-stage optimization was used: first, the optimal time T^{opt} is determined for which one of the known methods of one-dimensional minimization is applied, then, at this value T, it is determined

$$J_T^{\ opt} = J(\upsilon_T^{opt}, T) = \min_{\upsilon} J(\upsilon, T),$$

related to the problem of controlling a system of differential equations with a given end time of the process. In another approach, the end time T is considered as a control parameter, to determine the optimal value T and $\upsilon(t)$ at the same time a gradient optimization method is used.

For the numerical solution of these problems, calculation formulas, algorithms, and recommendations for their application are given.

In 4.3, the results of numerical experiments are presented on the example of solving model problems using the formulas obtained in 4.2 and an analysis of the obtained numerical results is given.

The fifth chapter, consisting of seven paragraphs, is devoted to the formulation and numerical solution of the inverse problem associated with the localization of places of fluid leakages in the pipeline network.

In 5.1, analysis is carried out for studies and applied tools, methods for detecting leakages in pipelines

In 5.2 the formulations of the problem of determining the places and volume of leakage of raw materials on a linear section of the pipeline under non-stationary mode of fluid flow are investigated. The identification problem is considered within the class of optimal control problems.

In 5.3 analytical formulas are obtained for the gradient of the functional corresponding to the problem studied in 5.2. These formulas allow us to use effective optimization methods to solve the problem.

In 5.4 the results of computer experiments obtained in solving model problems using the formulas from 5.3 are presented; the results are analyzed.

In 5.5 the formulation of one inverse problem on the hydraulic network of a complex loopback structure is investigated. The problem is to determine the places and volumes of the leakage of raw materials in the presence at any point in the pipeline of the results of additional observations of the modes of unsteady fluid flow. The features of the problem under consideration are the absence of classical initial conditions and the setting of boundary conditions in the form of non-separated relations between the states of the process at the ends of adjacent sections of the pipeline network.

Suppose that at the point $\xi^{k_{i_s},k_{j_s}} \in (0;l^{k_{i_s},k_{j_s}})$ of the (k_{i_s},k_{j_s})-th network segment there is a leakage of raw materials, the volume of which is determined by the function $q^{k_{i_s},k_{j_s}}(t)$. The set of such sections in the network in quantity of Z is denoted by $J^{loss} = \{(k_{i_1},k_{j_1}),...,(k_{i_Z},k_{j_Z})\} \subset J.$

The process of unsteady isothermal laminar flow of a dropping fluid of constant density ρ on the (k,s)–th linear section of the oil pipeline network can be described by the following two linear differential equations of hyperbolic type:

$$-\frac{\partial P^{ks}(x,t)}{\partial x} = \frac{\rho}{S^{ks}}\frac{\partial Q^{ks}(x,t)}{\partial t} + 2a^{ks}\frac{\rho}{S^{ks}}Q^{ks}(x,t),$$

$$-\frac{\partial P^{ks}(x,t)}{\partial t} = \begin{cases} c^2\frac{\rho}{S^{ks}}\left(\frac{\partial Q^{ks}(x,t)}{\partial x} + q^{ks}(t)\delta(x-\xi^{ks})\right), & x\in(0,l^{ks}),\ (k,s)\in J^{loss}, \\ c^2\frac{\rho}{S^{ks}}\frac{\partial Q^{ks}(x,t)}{\partial x}, & x\in(0,l^{ks}),\ (k,s)\notin J^{loss}. \end{cases} \quad (36)$$

As indicated in Chapter 2, if the process (36) is as long enough, then due to the presence of friction determined by the second terms in the first equations of the subsystems (36), the influence of the values of the initial conditions on the modes of fluid flow in the pipeline weakens over time.

Therefore, during long-term monitoring of the process, there is such $\tau,\ \tau > t_0$ that, at $t > \tau$ only the values of the boundary conditions significantly affect the mode of flow in the time interval $[t_0,T]$, where the value of τ is determined by the parameters of the fluid and the geometric dimensions of the pipeline network.

Therefore, we will assume that at the initial moment of time t_0 the initial conditions for the process (36) are not exactly known, and some sets of possible values of the initial modes are specified, which are defined in this case by a parametric set $D \subset R^{M+n}$ of possible values of flow rates in the sections under steady-state flow modes:

$$\begin{aligned} &\hat{Q}^{ks}_{\gamma}(x) = Q^{ks}(x,t_0;\gamma) = \gamma^{ks}_q = const, \\ &\hat{P}^{ks}_{\gamma}(x) = P^{ks}(x,t_0;\gamma) = \gamma^{ks}_p - 2ax\gamma^{ks}_q, \end{aligned} \quad x \in (0,l^{ks}),\ (k,s) \in J\ , \quad (37)$$

$$\gamma = (\gamma_p, \gamma_q) = (\gamma^{ks}_p, \gamma^{ks}_q)_{\substack{ks \in J \\ k \in I}} \in D \subset R^{M+n}\ .$$

Here $\gamma^{ks}_q -$ possible value of flow rates in the $(k,s)-$th section $ks \in J$, $\gamma^{ks}_p -$ possible value of pressures at the nodes $k \in I$ under steady-state flow modes, the corresponding density functions are given, which are written in vector form as $\mu_{\mathrm{D}}(\gamma)$.

Possible set of initial states can also be determined as by finite set of their values, as well as by the set of parametrically given functions:

$$\begin{aligned} &\{\hat{Q}^{ks}_{\gamma_1}(x), \hat{Q}^{ks}_{\gamma_2}(x), ..., \hat{Q}^{ks}_{\gamma_N}(x)\}, \\ &\{\hat{P}^{ks}_{\gamma_1}(x), \hat{P}^{ks}_{\gamma_2}(x), ..., \hat{P}^{ks}_{\gamma_N}(x)\}, \end{aligned} \quad x \in (0,l^{ks}),\quad (k,s) \in J\ .$$

Based on the meaning of the problem, there are restrictions on identifiable functions and parameters:

$$0 \le \xi^{ks} \le l^{ks},\ \underline{q} \le q^{ks}(t) \le \overline{q}, \quad t \in [t_0,T],\ (k,s) \in J^{loss}. \quad (38)$$

To determine the unknown locations and volumes of leakages $(\xi^{ks}, q^{ks}(t)),\ (k,s) \in J^{loss}$, we will assume that at certain points in various sections of the network, the number of which must exceed $2Z$ – the number of identified parameters, there are results of monitoring of pressure and / or flow values.

Suppose that the points of additional measurements are again the nodes of the inlet and outlet of the pipeline network, i.e. from I^f. Moreover, if in the boundary conditions (23) and (24) the pressure measurements were used for the nodes from the set I^f_p, and flow rate measurements for the nodes from I^f_q, then the additional information will be the results of pressure measurements at some nodes of the subset I_{qp}^{f} of the set I_{q}^{f}, i.e. $I_{qp}^{f} \subset I_{q}^{f}$, and flow measurements at the vertices of a subset I_{pq}^{f} of the set I_{p}^{f}, т.е. $I_{pq}^{f} \subset I_{p}^{f}$:

$$\begin{cases} P^{ns}(0,t) = P_{mes}^{n}(t), \quad s \in I_n^+, \ если \, I_n^- = \varnothing, \\ P^{sn}(l^{sn},t) = P_{mes}^{n}(t), \ s \in I_n^-, \ если \, I_n^+ = \varnothing, \end{cases} \qquad n \in I_{qp}^{f} \subset I_{q}^{f}, \tag{39}$$

$$\begin{cases} Q^{ms}(0,t) = Q_{mes}^{m}(t), \quad s \in I_m^+, \ если \, I_m^- = \varnothing, \\ Q^{sm}(l^{sm},t) = Q_{mes}^{m}(t), \ s \in I_m^-, \ если \, I_m^+ = \varnothing, \end{cases} \qquad m \in I_{pq}^{f} \subset I_{p}^{f}. \tag{40}$$

In 5.6, the inverse problem posed in 5.5 on the hydraulic network of a complex loopback structure is reduced to the parametric optimal control problem with inaccurate given initial conditions and with nonseparated boundary conditions.

To form the objective functional of the problem, without detracting from the generality, it is assumed that, in conditions (39) the sets I_{qp}^{f} and I_{q}^{f} coincide:

$$\Im(\xi,q) = \int_D \left[\Phi(\xi,q;\gamma) + \Re(\xi,q)\right]\mu_D(\gamma)d\gamma \to \min, \tag{41}$$

$$\Phi(\xi,q;\gamma) = \sum_{m\in\tilde{I}_q^f} \int_\tau^T [Q^m(t;\xi,q(t),\gamma) - Q_{mes}^m(t)]^2 dt,$$

$$\Re(\xi,q) = \varepsilon_1 \left\| q(t) \text{-} \hat{q} \right\|^2_{L_2^Z[t_0,T]} + \varepsilon_2 \left\| \xi \text{ - } \hat{\xi} \right\|^2_{R^Z},$$

where $Q^m(t;\xi,q(t),\gamma)$, $m \in \tilde{I}_q^f$ are the calculated values of the flow rate at the observed points as a result of solving the boundary value problem for any possible initial conditions $\hat{Q}_\gamma = \{\hat{Q}_\gamma^{ks}(x)\}_{(k,s)\in J}$, $\hat{P}_\gamma = \{\hat{P}_\gamma^{ks}(x) \in \mathrm{P}^{ks}\}_{(k,s)\in J}$ and given admissible places and volumes of leakages $(\xi,q(t))$; $[\tau,T]$– the time interval for monitoring the process, the modes of which no longer depend on the initial conditions; $\hat{\xi},\hat{q} \in R^m, \varepsilon_1, \varepsilon_2$ are the regularization parameters.

Since the initial conditions determined at the time instant t_0 do not affect the process in the interval $[\tau,T]$, the exact knowledge of the initial value t_0 and the initial functions $Q^{ks}(x,t_0;\gamma)$,

$P^{ks}(x,t_0;\gamma), (k,s)\in J,\ \gamma\in D$ does not affect the value of functional (42) is not fundamental importance.

The following theorem is proved.

Theorem 5.1 In problem (36) - (41), the components of the gradient of functional (41) with respect to admissible places and volumes of leakages $(\xi^{\bar{k}\bar{s}}, q^{\bar{k}\bar{s}}(t)),\ (\bar{k},\bar{s})\in J^{loss}$ are determined by the formulas:

$$grad_{\xi^{\bar{k}\bar{s}}}\Im(\xi,q)=\frac{1}{N}\sum_{i=1}^{N}c^2\frac{\rho}{S^{\bar{k}\bar{s}}}\int_{t_0}^{T}q^{\bar{k}\bar{s}}(t)(\psi^{\bar{k}\bar{s}}(x,t;\gamma))'_x\,|_{x=\xi^{\bar{k}\bar{s}}}\ dt+2\varepsilon_2(\xi^{\bar{k}\bar{s}}-\hat{\xi}^{\bar{k}\bar{s}}),$$

$$grad_{q^{\bar{k}\bar{s}}}\Im(\xi,q)=\int_D\left\{c^2\frac{\rho}{S^{\bar{k}\bar{s}}}\psi^{\bar{k}\bar{s}}(\xi^{\bar{k}\bar{s}},t;\gamma)+2\varepsilon(q^{\bar{k}\bar{s}}(t)-\tilde{q}^{\bar{k}\bar{s}}(t))\right\}\mu_D(\gamma)d\gamma\ .$$

Here the functions $\psi^{ks}(x,t)=\psi^{ks}(x,t;\gamma),\ \ (k,s)\in J$ are the solutions of the conjugate initial-boundary value problem with nonlocal boundary conditions, corresponding to the direct problem (36), (37).

In 5.7 the results of numerical experiments are presented on the example of solving model problems using the formulas obtained in 5.6, an analysis of the results is given.

The Appendix provides a description, structure and listings of main modules of software developed by the author for the numerical solution of optimal control problems, boundary value problems with respect to systems of differential equations with independent subsystems connected only by boundary conditions.

The author is grateful to the scientific consultant, corresponding member of NAS of Azerbaijan, doctor of physics and mathematics, prof. Aida-zade K.R. for permanent attention to work.

MAIN RESULTS OF WORK

In the dissertation, the following results were obtained.

1. The numerical methods have been developed for the solution to the systems of linear differential equations of large dimension of a block structure, the subsystems of which are connected only by unseparated boundary conditions.
2. A numerical method has been developed for solving the systems of linear discrete equations of large dimension of a block structure, the blocks of which are connected only by initial and final variables in an arbitrary order.
3. A numerical algorithm for the solution to the problem of optimal control by a large ODE system of a block structure is proposed.
4. For the processes described as by parabolic, as well as hyperbolic partial differential equations, the duration time of the influence of the initial conditions on their state is studied depending on the values of the parameters involved in the formulation of the problem.
5. The formulation and formulas for the numerical solution of problems of optimal control of evolutionary processes under inaccurate given initial conditions are proposed.
6. The numerical methods based on lines and grid methods and proposed transfer schemes of boundary conditions have been developed to solve a system of differential equations of a hyperbolic type of large dimension with unseparated boundary conditions.
7. The formulation is investigated and formulas are proposed for the numerical solution to the problem of optimal control by the process described by a large system of differential equations of hyperbolic type with unseparated boundary conditions and inaccurate given initial conditions.
8. The formulation, formulas, and algorithm for the numerical solution to the inverse problem on determination the locations and powers of sources for processes described by a large system of

differential equations with partial derivatives of a hyperbolic type with unseparated boundary and inaccurate given initial conditions are proposed.

9. The statements of the problems of optimal control of transients in complex hydraulic networks are investigated, the formulas and algorithms for their numerical solution are proposed.
10. Formulations and algorithms are proposed for the numerical solution to the inverse problems on determination sections, places on them, and volumes of leakage of raw materials in networks of a complex looped structure under nonsteady fluid flow mode.
11. To solve the problems considered in the work, software was developed using modern software technologies.

Basic dissertation results published in the following works:

1. Aida-zade K.R., Ashrafova Y.R. Control of Systems with Concentrated Parameters in a Class of Special Control Functions // Automatic Control and Computer Sciences, Springer, 2009, v. 43, no 3, pp. 148-155.
2. Ашрафова Е.Р. О задаче оптимизации границы области и длительности процесса нагрева //Известия НАНА, сер. «Информатика и проблемы управления», Баку, 2010, т. XXX, № 3. с. 13-18.
3. Ashrafova Y.R., Rahimov A.B. Optimal Control For Systems On Some Classes Of Control Functions //Selected Papers of Proceedings of 24th Mini Euro Conference "Continuous Optimization and Information- Based Technologies in the Financial Sector, (MEC EurOPT), Izmir, Turkey, 2010, pp. 141-147.
4. Ашрафова Е.Р., Рагимов А.Б. Оптимизация мест размещения сосредоточенных источников и управление ими в распределенных системах / Матер. Межд. Российско-Болгарского симпозиума «Уравнения смешанного типа и родственные проблемы анализа и информатики», Нальчик-Хабез, 2010, с.42-44.
5. Ashrafova Y.R. Optimal impulsive control in distributed systems / Abstracts of The Third International Conference "Problems of Cybernetics and Informatics", v. III, Baku, Azerbaijan, 2010,p.112-115.
6. Aida-zade K.R., Ashrafova Y.R. Optimal control problems of sources in Distributed systems on the classes of Impulsive, Piecewise constant and Heaviside Functions // J. of Automation and Information Sciences, New York: Begel House, 2011,v.43, No 5, pp. 64-82.
7. Айда-заде К.Р., Ашрафова Е.Р. Определение мест и объема утечек углеводородного сырья в трубопроводе при нестационарном режиме / Материалы Второй межд. Российско-Казахский симпозиум «Уравнения смешанного

типа, родственные проблемы анализа и информатики», Нальчик, 2011, с.16-18.
8. Ashrafova Y.R. Optimal control by boundary of a domain varying in time and by completion time of control processes / Abstracts of the IV Congress of Turkic World Mathematical Society (TWMS), Baku, Azerbaijan, 2011, p.366.
9. Айда-заде К.Р., Ашрафова Е.Р. Определение места разрыва и объема утечки в магистральном нефтепроводе при нестационарном режиме //Труды РГУ нефти и газа имени И.М.Губкина, 2012, № 2/ 267, с. 78-84.
10. Ашрафова Е.Р. Задача локализации места утечки в магистральном трубопроводе //Известия НАНА, сер. «Информатика и проблемы управления», 2012, т. 32, № 3, с.118-125.
11. Aida-zade K.R., Ashrafova Y.R. Localization of points of leakage in main oil pipelines under nonsteady state // Journal of Engineering Physics and Thermo-physics, New York: Springer, 2012, v.85, no 2, pp. 1148-1156.
12. Ashrafova Y.R. Optimal control of lumped sources in distributed-parameter systems on classes of impulse and Heaviside functions //Cybernetics and Systems analysis, New York: Springer, 2012, v.48 no 5, pp. 798-806.
13. Aida-zade K.R., Ashrafova Y.R. Problem of determination of leakages in oil pipeline networks //Selected Papers of IV International Conference "Problems of Cybernetics and Informatics", v.4, IEEE Xplore, Baku, Azerbaijan, 2012, p.32-34.
14. Ashrafova Y.R. A problem of optimal control of the processes with unknown initial conditions //Selected papers of IV International Conference "Problems of Cybernetics and Informatics", v.4, IEEE Xplore, Baku, Azerbaijan, 2012, p.35-37.
15. Aida-zade K.R., Ashrafova Y.R. Optimal control problems without initial conditions / Proceedengs of III Internatioanl

Conference «Optimization and applications», Koshta-da-Kaparika, Protuqie, 2012, p.21-24.

16. Ашрафова Е.Р., Мамедов В.М. Численное исследование состояния эволюционных процессов при незаданных начальных условиях //Известия НАНА, сер. «Информатика и проблемы управления», 2013, т. XXXIII, № 6, с. 30-38.
17. Ашрафова Е.Р. Метод расчета неустановившихся режимов транспортировки жидкости в трубопроводных сетях сложной структуры / Материалы IV межд. конф. «Нелокальные краевые задачи и родственные проблемы математической биологии, информатики и физики», Нальчик-Терскол, 2013, С. 59-62.
18. Ашрафова Е.Р. Исследование схем метода прямых высокого порядка точности / Тезисы Межд. конф., посв. 90-летию со дня рождения Гейдара Алиева, Азербайджан, 2013, с.255-256.
19. Ashrafova Y.R. Calculation and optimization of regimes of fluid flow in oil pipeline networks of complicated structure/ Abstracts of IV International Conference on Optimization Methods and Applications “Optimization and applications”, Montenegro, 2013, pp. 29-30.
20. Ashrafova Y.R. Numerical investigation of problems of control by evolutional processes without initial conditions / Extended Abstracts of 4th Conference on Control and Optimization with Industrial Application, Borovets, Bulgaria, COIA-2013, p.96-97.
21. Ashrafova Y.R. Optimal control by the systems of evolutional equations with unseparated boundary conditions without initial conditions / Extended Abstracts of 4th Conference on Control and Optimization with Industrial Application. Borovets, Bulgaria, COIA 2013, p.95.
22. Aida-zade K.R., Ashrafova Y.R. Optimal control of sources on some classes of functions // Optimization: A Journal of Mathematical Programming and Operations Research, England: Taylor & Francis, 2014, v.63, no 7, pp. 1135-1152.

23. Айда-заде К.Р., Ашрафова Е.Р. Численное исследование свойств решения краевых задач без начальных условий //Прикладная математика и фундаментальная информатика, Омск, 2014, №1, с. 20-23.
24. Ашрафова Е.Р. Численная схема расчета режимов течения жидкости в трубопроводных сетях сложной структуры //Известия НАНА, сер. «Информатика и проблемы управления», Баку, 2014, т. 34, №3, с. 153-163.
25. Ашрафова Е.Р. Численное решение системы независимых уравнений второго порядка при неразделенных граничных условиях //Известия НАНА, сер. «Информатика и проблемы управления», 2014, т.34, №6, с. 11-19.
26. Ашрафова Е.Р. Оптимальное управление переходными процессами в нефтепроводах сложной структуры с незаданными начальными условиями / Матер. Пятой Межд. конф. «Математика, ее приложения и математическое образование» (МПМО)2014.с.34-36.
27. Ашрафова Е.Р. Локализация мест утечек в трубопроводах сложной структуры / Третий Межд. Российско-Казахский симпозиум «Уравнения смешанного типа, родственные проблемы анализа и информатики», Россия, Нальчик, 2014, с.36-38.
28. Айда-заде К.Р., Ашрафова Е.Р. Расчет переходных режимов движения жидкости в трубопроводных сетях //Сибирский журнал индустриальной математики, 2015, т. XVIII, № 2, с. 12-23.
29. Ашрафова Е.Р. Анализ длительности зависимости режима движения жидкости в трубопроводе от начальных режимов //Доклады Адыгской (Черкесской) Международной Академии Наук, 2015, № 2, с. 9-16.
30. Aida-zade K.R., Ashrafova Y.R. Solving Systems of Differential Equations of Block Structure with Nonseparated Boundary Conditions //Journal of Applied and Industrial Mathematics, Springer, 2015, v. 9. No 1, pp. 1-10.

31. Ashrafova E.R. Numerical investigation of the duration of the effect exerted by initial regimes on the process of liquid motion in a pipeline //Journal of Engineering Physics and Thermophysics, Springer, 2015, v.88, no 5, pp. 1-9.
32. Ashrafova Y.R. Numerical calculation of processes, described by block-diagonal system with nonseparated initial-boundary conditions between blocks / Book of Abstracts of the 5th International Conference on Control and Optimization with Industrial Applications, Baku, Azerbaijan, COIA-2015, pp. 252-253
33. Aida-zade K.R., Ashrafova Y.R. Localization of Leakage Points in Pipelines of Complex Structure / Abstracts of VI International Conference on Optimization Methods and Applications Optimization and applications (OPTIMA-2015), Petrovac, Montenegro, 2015, pp.24-25.
34. Aida-zade K.R., Ashrafova Y.R. Calculation of the state of a system of discrete linear processes connected by nonseparated boundary conditions // Journal of Applied and Industrial Mathematics, 2016, v. 10 no 4, pp. 1-13.
35. Ашрафова Е.Р. Нахождение мест утечек сырья в гидравлических сетьях //“Прикладная математика и фундаментальная информатика”, Омск, 2016, № 1, с. 77-81.
36. Ашрафова Е.Р. Оптимальное управление эволюционными процессами при отсутствии точной информации об их начальных условиях //Известия НАНА, сер. «Информатика и проблемы управления», Баку, 2016, Т.36, №3, с. 24-34.
37. Aida-zade K.R., Ashrafova Y.R. Optimal control of wave process without accurate information about initial conditions / Материалы V Всероссийской науч. практ. конф. Математическое моделирование процессов и систем, г. Стерлитамак, Башкортостан, 2016, с.5-10.
38. Aida-zade K.R., Ashrafova E.R. Numerical Leak Detection in a Pipeline Network of Complex Structure with Unsteady Flow //Computational Mathematics and Mathematical Physics, 2017, Vol. 57, No. 12, pp. 1919–1934.

39. Айда-заде К.Р., Ашрафова Е.Р. Исследование задач оптимального управления пучками эволюционных процессов //Информационный бюллетень Омского научно-образовательного центра ОмГТУ и ИМ СО РАН в области математики и информатики, Изд-во ОмГТУ, 2017. – Т. 1, № 1. с. 12-16.
40. Ашрафова Е.Р. Определение участков трубопровода сложной структуры, содержащих утечки сырья //Информационный бюллетень Омского научно-образовательного центра ОмГТУ и ИМ СО РАН в области математики и информатики, Изд-во ОмГТУ, 2017. – Т. 1, № 1. с. 98-101
41. Aida-zade K.R., Ashrafova Y.R. Investigation of optimal control problem without initial conditions // CEUR Workshop Proceedings (VIII Intern. Conf. on Optimization Methods and Applications (OPTIMA-2017), Petrovac, Montenegro, 2017, Vol. 1987, p. c.24-30.
42. Ашрафова Е.Р. Численный метод определения мест утечек сырья в сложных трубопроводных системах транспорта / Материалы XI Межд. Четаевской науч. конф. «Аналитическая механика, устойчивость и управление», Казань 14-18 июня 2017 г., с.25-28.
43. Ашрафова Е.Р. Поблочный перенос условий для системы дискретных процессов, связанных лишь краевыми условиями //Прикладная математика и фундаментальная информатика, Изд-во ОмГТУ, 2018 – Т.5, № 2. с.34-42.
44. Aida-zade K.R., Ashrafova Y.R. Numerical solution of inverse problem on determination of leakage for unsteady flow in a pipeline network //IFAC PapersOnLine 51-30 (2018) pp. 21–26.
45. Aida-zade K.R. Ashrafova Y.R. Numerical solution to optimal control problem with the set of initial conditions / Abstracts of the 6th Intern. Conf. on Control and Optimization with Industrial Applications, Baku, (COIA-2018), July 11-13, 2018, pp.110-112.

46. Ашрафова Е.Р. Задача расчета состояния сложных дискретных процессов со связанными краевыми условиями / Материалы V межд. Науч.конф. «Нелокадьные краевые задачи и родственные проблемы математической биологии, информатики и физики» к 80-летию А.М. Нахушева, 4-7 декабря 2018 г. Нальчик, Кабардино-Балкарская Республика, с.41.
47. Ашрафова Е.Р. Решение задач оптимального управления для систем ОДУ блочной структуры с неразделенными граничными условиями между блоками /Материалы IV-ой Межд. конф. «Актуальные проблемы прикладной математики» Россия, г. Нальчик-Эльбрус, 22 – 26 мая 2018 г., с. 50.
48. Aida-zade K.R. Ashrafova Y.R. Optimal control by the system of differential equations with blocks, related by boundary conditions /Abstracts of IX Intern. Conf. on Optimization Methods and Applications (OPTIMA-2018), Petrovac, Montenegro, 2018, pp.26.
49. Aida-zade K.R., Ashrafova Y.R. Numerical method for solving the problem of optimal by system ODE of a block structure /Abstracts of X Intern. Conf. on Optimization Methods and Applications (OPTIMA-2019), Petrovac, Montenegro, 2019, pp.20.
50. Aida-zade K.R., Ashrafova Y.R. Numerical solution to an inverse problem on a determination of places and capacities of Sources in the hyperbolic systems //Journal of industrial and management optimization, 2020, V.16, No.6 pp.3011-3033.
51. Aida-zade K.R., Ashrafova Y.R. Investigation of the problem of optimal control by a system ODE of block structure with blocks connected only by boundary conditions //Optimization and Applications, Part of the Communications in Computer and Information Science book series (CCIS, volume 1145), 2020, Springer, Vol. 1145, p.367-378.

Personal contribution of the author to the joint work:

- In [16,23,27,28,30,34] the author participated in the discussion of the problem statement formulation, proposed a numerical method for solving the problem, developed software, and conducted numerical experiments.

- In [1,3,4,6,7,9,11,12,15,22,33,37-39,41,44,45,48-51] the author participated in the discussion of the problem statement formulation, obtained the necessary optimality conditions, proposed a numerical method for solving the problem, developed software, and conducted numerical experiments.

The defense will be held on 26 May 2021 at 14^{00} at the meeting of the Dissertation council ED 1.19 of the Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Adress: Az1141, Baku, st. B. Vahabzade, 68.

Dissertation is accessible at the Library of the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Electronic versions of dissertation and its abstract are available on the official website of the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Abstract was sent to the required addresses on 23 April 2021.

Signed for print: 15.04.2021

Paper format: 60x84 1/16

Volume: 78000

Number of hard copies: 20