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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

ALGORITHMS FOR SOLVING THE PROBLEMS OF OPTIMIZATION AND IDENTIFICATION IN THE OIL EXTRACTION PROCESS

Speciality: 3338.01- Systematic analysis, management and information processing

Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THE WORK

The relevance of the topic and the degree of elaboration: Control and optimization issues have developed since the late 1950s, early 60s, and formed a certain scientific direction in the early 1970s. Among them are Pontryagin's "maximum" principle, Bellman's "dynamic programming" method, Kalman's "Method of solving the linear quadratic Gauss problem (Kalman)", Larin's "Method of frequencies", etc. can be noted. The advantages of these works are that they have large applications, and naturally it is reasonable to extend numerical methods for their realization and to simulate them for their applications. The shown above, includes only the optimization problems with initial conditions. Recently, the solution of some robotics problems is brought to the solution of optimal control problems with two-point inseparable boundary condition. Since the problems are strictly nonlinear, one of the most realistic steps is to bring them to a linear quadratic optimal control problem using quasi-linearization methods. Quasi-linearization is noted as one of the most realistic methods if the initial iteration is chosen correctly.

Recently, the development of the gas lift method in oil extraction - the mathematical model, an optimal program trajectory and a controlling, construction of optimal regulator is of great importance. Although the equations of motion here are written as special differential equations of hyperbolic type, they can be reduced to ordinary differential equations using averaging or straight line methods. This is due to the fact that when the motion of the object is written by special derivative differential equations, it is very difficult to construct suitable optimal regulators as well as program trajectories. One of the main difficulties is to derive the controller from the initial conditions.

It should be noted here that the construction of the optimal regulator with the participation of the controller creates many difficulties, because it seems very difficult to bring the solution to standard methods.

Oil production using a rod pump device is described by classical oscillatory equations. However, since the plunger moves inside the Newtonian fluid, it is necessary to write the equation not as a classical second power ordinary differential equation, but to write the auxiliary member as fractional degree. This completely changes both the form and content of the issue. There is no difficulty in determining the coefficients included in the equation, but the determination of the fractional derivative is a very problematic issue, and for this, a certain result was first obtained in the article on its determination based on statistical data.

Contamination of the pipe during oil extraction, that is, accumulation of fuel oil layer on the side walls, makes it necessary to change the optimal mode. Therefore, this topic determines the relevance of the coefficient of hydraulic resistance.

Here, of course, the coefficients of hydraulic resistance in different parts of long-distance pipes will differ. For this, consideration of such issues is of great importance. This is important not only in oil extraction, but also in the transfer of oil products to long-distance facilities.

In problems governed by initial conditions, optimization and optimal stabilization, periodicity are very important, because these types of problems form the basis of the gas lift method. Here, the more difficult issue - issues of partial periodicity play an important role, because the volume of gas injected at the beginning should be such that it can remove the mass of oil from the bottom of the well as a debit by moving it through the riser pipe. If a lot of gas is given here, it is possible that the oil will be pushed to the bottom of the well, but the gas will come out on its own, but if it is given too little, no oil can be produced at all. Here, it is required to provide such minimum volume of gas so that the maximum volume of oil can be removed from the bottom of the well as a debit through the riser pipe.

In particular, it is worth noting that the construction of a suitable mathematical model or the solution of the above problems are very complex computational processes, and the development of suitable asymptotic methods for their specification helps to create more general automated systems when creating computer calculations.

The above-mentioned shortcomings require the development of new scientific directions and bring the development of new computer technologies to the agenda. All this confirms the relevance of the dissertation work.

Object and subject of research: The object of the dissertation work is to create a new mathematical model in oil extraction, where it is possible to involve and work with modern optimization and control methods for solving relevant problems. The subject of the research is the development of new algorithms for the oil extraction process and the creation of suitable new calculation algorithms.

Research methods: The theory of optimal control, differential equations, identification methods, asymptotic methods and the theory of matrices form the theoretical and methodological basis of the dissertation work.

Main clauses defended:

1. A numerical algorithm was developed for the solution of the optimization problem with unseparated boundary condition;

2. A new sweep method is proposed for solving the continuous linear quadratic optimization problem with a unseparated boundary condition;

3. A new method has been developed for solving the linear quadratic optimization problem in the discrete case with unseparated boundary conditions;

4. An identification method for determining the coefficient of hydraulic resistance in different parts of pump compressor pipes is proposed;

5. An asymptotic method for finding the coefficient of hydraulic resistance in different parts of pipes for oil products transported over long distances in oil extraction is given;

6. A method for periodic solution of liquid-damper oscillation systems has been developed;

7. A method for solving a continuous optimal control problem satisfying periodicity in a part for systems controlled by initial data;

8. A method of solving the optimization problem controlled by initial data in the discrete case;

A method for the optimal regulator problem (in oil production) controlled by initial data in the stationary case.

Scientific novelty of the research: In the dissertation work, new results named with the following conventional name were obtained:

1. A new method based on the "sweep" algorithm is given for solving the continuous and discrete linear quadratic optimization problem with a non-separating boundary condition, where, unlike the previous ones, the transition at those points in multi-point optimization problems is set to be single-valued;

2. An identification is established and an algorithm is proposed for determining the coefficient of hydraulic resistance in different parts of the pump-compressor pipes. So, in long-distance oil transfer this method is proposed as an asymptotic algorithm by simplifying and by introducing a small parameter;

3. By applying the liquid-damped oscillating systems to oil extraction, the movement of the plunger in the rod pump unit is expressed by the periodic mode, and the solution is established;

4. In continuous and discrete systems controlled by initial data, the solution of the optimization problem satisfying the periodicity in

one part is established, suitable algorithm is proposed and it is proposed to use the gas lift method in oil;

5. The construction of optimal regulators controlled by initial data in the stationary case is considered and applied to oil production.

Theoretical and practical significance of research: In the dissertation, the proposed chasing method is used in two and multi-point linear quadratic optimal control problem, and the construction of a periodic solution in one part is presented as a new method. Also, the construction of a suitable optimal regulator is considered almost for the first time in optimal control, controlled by initial conditions. Finding the coefficient of hydraulic resistance is a new identification method that allows it to be found in different parts of the interval. Apart from theoretical studies of all issues has great applications in oil extraction and transportation of oil products and all issues are taken from these problems.

Approval and application: The main results of the dissertation work were repeatedly discussed at the institute seminars of BSU TRETI and reported at the following international scientific conferences.

1. COİA-2018- 1 reports
2. COİA-2020- 2 reports
3. COİA-2022-1 report

17 scientific works related to the dissertation were published. 12 of them are scientific articles. 7 of the scientific works were published in "Web of Science" indexed journals.

Name of the organization where the dissertation work was performed: The dissertation work was performed at the İnstitute of Applied Mathematics of Baku State University.

Claimant's personal contribution. All the scientific results reflected in the dissertation work are the result of the applicant's personal activity and the application of the idea direction of the scientific supervisor, the setting of the problem to a specific research object.

The total volume of the dissertation, noting the volume of the structural sections of the dissertation separately: Dissertation consists of 122 pages of introduction (34605 marks), chapter I (43905 works), chapter II (38019 works), chapter III (39541 works), conclusion (2573 works), 151 references list and 1 the table consists of 2 graphs. The total volume is 162604 (excluding tables, graphs and bibliography).

CONTENTS OF THE WORK

In the introduction, the relevance of the dissertation work is justified, the object and subject of the research, the purpose and tasks of the work, the scientific innovation and the theoretical and practical importance of the research are listed, the analysis of other studies conducted in this field are carried out.

In the first chapter, the methods for solving of optimization problems with unseparated boundary condition, arising during oil extraction were proposed.

In the first sub-chapter of the **first chapter**, the problem of linear quadratic optimization with unseparated boundary condition is considered. The fundamental matrix of the appropriate Hamiltonian matrix is constructed by Euler's method, and based on it, the numerical calculations of the appropriate phase coordinates and control are given. The results are illustrated by the example of the construction of optimal program trajectories and controls of bipedal self-propelled apparatus with telescopic legs. It is assumed that the motion of the body is described by the following system of ordinary linear differential equations

$$\dot{x} = F(t)x(t) + G(t)u(t) \quad (1)$$

Here it is required to find such controlling influence $u(t)$ that given quadratic funksional

$$J = \frac{1}{2}\int_{t_0}^{T}(x'(t)Q(t)x(t) + u'(t)C(t)u(t))dt \qquad (2)$$

get a minimum price and $x(t)$ satisfies the unseparated boundary condition

$$\Phi_1 x(t_0) - \Phi_2 x(T) = q \qquad (3)$$

at the beginning and end points.

Here x is $n-$ dimensional phase vector, $u(t)$ is m – dimensional vektor of controlling influences, $F(t)$ is an $n \times n$ -dimensional matrix function with continuous elements, $G(t)$ is $n \times m$ -dimensional matrix and the pair $(F(t), G(t))$ is controllable. Furthermore, it is assumed that $Q(t) = Q'(t) \geq 0$, $C(t) = C'(t) > 0$ accordingly are $n \times n$ and $m \times m$ dimensional matrices, Φ_1, Φ_2 are $k \times n$ – dimensional constant matrices, q is constant k -dimensional vektor, $[\Phi_1 - \Phi_2]$ pair with the vector q satisfy the Kronecker–Capelli condition, and the (`) represents the transpose operation.

If we formulate the extended functional of the optimization problem (1)-(3), on its basis we get the Euler-Lagrange equation (5),

$$\begin{vmatrix}\dot{x}\\ \dot{\lambda}\end{vmatrix} = \begin{bmatrix}F(t) & -G(t)C^{-1}(t)G'(t)\\ -Q(t) & -F'(t)\end{bmatrix}\begin{bmatrix}x\\ \lambda\end{bmatrix} = H\begin{bmatrix}x\\ \lambda\end{bmatrix} \qquad (5)$$

which satisfy the boundary conditions (3) and

$$\left.\begin{array}{r}\lambda(t_0) = -\Phi'_1\nu\\ \lambda(T) = \Phi'_2\nu\end{array}\right\} \qquad (4)$$

Here $\lambda(t)$ is an n – dimensional Lagrangr vector-function, ν is k– dimensional vector of Lagrange multipliers corresponding to the boundary condition (3), H is Hamilton matrix.

Now using Euler's method we can found the fundamental matrix (4), (5) in the following form

$$\Phi(t_{i+1}, t_i) = (H(t_i)\Delta + E)\Phi(t_i, t_{i-1}), \quad \Phi(t_0, t_0) = E. \tag{6}$$

Here Δ is the step of numerical integration, E is the unit matrix. Then at $i = 0,1 \dots, N-1$ and $T = t_N$, using the expression (6) we obtain

$$\Phi(T, t_0) = (H(t_N, t_{N-1})\Delta + E) \times$$
$$\times (H(t_{N-1}, t_{N-2})\Delta + E) \dots (H(t_1, t_0)\Delta + E) \tag{7}$$

and if we divide the fundamental matrix $\Phi(t_N, t_0)$ in the following form

$$\Phi(t_N, t_0) = \begin{bmatrix} \Phi_{11}(t_N, t_0) & \Phi_{12}(t_N, t_0) \\ \Phi_{21}(t_N, t_0) & \Phi_{22}(t_N, t_0) \end{bmatrix}, \tag{8}$$

to find the unknowns $x(t_0)$ and ν we take the following system of linear algebraic equations

$$D \begin{bmatrix} x(t_0) \\ \nu \end{bmatrix} = w \tag{9}$$

Here

$$D = \begin{bmatrix} -\Phi_{22}^{-1}(\tau, t_0)\Phi_{21}(\tau, t_0) & & (-\Phi_{22}^{-1}(\tau, t_0)\Phi_2' + \Phi_1')) \\ \Phi_1 - \Phi_2(\Phi_{11}(\tau, t_0) - \Phi_{12}(\tau, t_0)\Phi_{22}^{-1}(\tau, t_0) & \Phi_{21}(\tau, t_0) & \Phi_2\Phi_{12}(\tau, t_0)\Phi_{22}^{-1}(\tau, t_0)\Phi_2' \end{bmatrix} \tag{10}$$

$$w = \begin{bmatrix} 0 \\ q \end{bmatrix}$$

In this case, the coordinates of the object will be designated as

$$x(t_i) = \Phi_{11}(\tau_i, t_0)x(t_0) - \Phi_{12}(t_i, t_0)\Phi_1'\nu, \tag{11}$$

and the controlling influence as

$$u(t_i) = -C^{-1}(t_i)G'(t_i)\Phi_{21}(t_i, t_0)x(t_0) + C^{-1}(t_i)G'(t_i)\Phi_{22}(t_i, t_0)\Phi_1'\nu \tag{12}$$

In the **second sub-chapter** of the first chapter, the sweep method is proposed for solving the linear quadratic optimization in continuous case. Here, the motion of the object is described by a system of linear differential equations (1) and the unseparated boundary condition (3) of the solution is required to be satisfied. In this case, it is required to find such controlling influences $u(t)$ that it minimizes the quadratic quality criterion (2) together with $x(t)$, which is the solution of the problem (1), (3).

Suppose that $\Phi(t, t_0)$ is the fundamental matrix of system (5), i.e

$$\dot{\Phi}(t,t_0) = H(t)\Phi(t,t_0), \quad \Phi(t_0,t_0) = E \tag{13}$$

Here E is $n \times n$ -dimensional unit matrix. If we write the fundamental matrix $\Phi\ (t, t_0)$ (8) n the form (8) the solution of the system (5) can be described as

$$\left.\begin{aligned} x(t) &= \Phi_{11}(t,t_0)\,x(t_0) + \Phi_{12}(t,t_0)\,\lambda(t_0) \\ \lambda(t) &= \Phi_{21}(t,t_0)\,x(t_0) + \Phi_{22}(t,t_0)\,\lambda(t_0) \end{aligned}\right\}. \tag{14}$$

From this we obtain

$$x(t) = \left(\Phi_{11}(t,t_0) - \Phi_{12}(t,t_0)\Phi_{22}^{-1}(t,t_0)\Phi_{21}(t,t_0)\right) x(t_0) + \Phi_{12}(t,t_0)\Phi_{22}^{-1}(t,t_0)\lambda(t), \tag{15}$$

$$u(t) = -C^{-1}(t)G(t)\lambda(t) \tag{16}$$

for $x(t)$ and $u(t)$. Using these relations, if $\Phi_{22}^{-1}(t,t_0)$ has inverse price, after certain conversions the following system of algebraic equations is defining to determine $x(t_0)$ and ν

$$\left.\begin{aligned} &\Phi_{22}^{-1}(\tau,t_0)\Phi_{21}\ (\tau,t_0)x(t_0) + \left(-\Phi_{22}^{-1}(\tau,t_0)\Phi_2' + \Phi_1'\right)\nu = 0 \\ &\left[-\Phi_1 - \Phi_2(\Phi_{11}\ (\tau,t_0) - \Phi_{12}\ (\tau,t_0)\Phi_{22}^{-1}(\tau,t_0)\Phi_{21}\ (\tau,t_0))\right]x(t_0) + \\ &\qquad + \Phi_2\Phi_{12}\ (\tau,t_0)\Phi_{22}^{-1}(\tau,t_0)\Phi_2'\nu = q \end{aligned}\right\} \tag{17}$$

The following is true for the system (17).

Lemma 1. In the system of linear algebraic matrix equations (17), the submatrices $\Phi_{22}^{-1}(\tau,t_0)\ \Phi_{21}(\tau,t_0)$ are symmetric.

Lemma 2. In the system of linear algebraic matrix equations (17), the submatrices $\Phi_2\ \Phi_{12}(\tau,t_0)\ \Phi_{22}^{-1}(\tau,t_0)\ \Phi_2'$ are symmetric.

Lemma 3. In the system of linear matrix equations (17) the symmetry condition

$$\left[\Phi_1 - \Phi_2\left(\Phi_{11}(\tau,t_0) - \Phi_{12}(\tau,t_0)\Phi_{22}^{-1}(\tau,t_0)\Phi_{21}(\tau,t_0)\right)\right] = \left(-\Phi_{22}^{-1}(\tau,t_0)\Phi_2' + \Phi_1'\right)' \tag{18}$$

is satisfied.

Thus, we get the following theorem.

Theorem 1. The matrix D in the system of algebraic equations (17) or (9) is symmetric.

If we solve the system of algebraic equations (9) and find $x(t_0)$ and ν and eliminate the Lagrange product $\lambda(t)$ from expressions (15)-(16), we get for $x(t)$ and $u(t)$

$$x(t) = \Phi_{11}(\tau, t_0)x(t_0) - \Phi_{12}(\tau, t_0)\Phi_1' \,\nu, \quad (19)$$

$$u(t) = -C^{-1}(t)G(t)\Phi_{21}(\tau, t_0)x(t_0) + C^{-1}(t)G(t)\Phi_{22}(\tau, t_0)\Phi_1' \nu \quad (20)$$

The symmetry of the matrix D ensures that the system of equations (11) is well satisfied.

Thus, we can construct the following algorithm.

Alqorithm.

1. The matrices $F(t)$, $G(t)$, Φ_1, Φ_2, $R(t)$, $C(t)$ in (1)-(2) are constructing.

2. The matrix $H(t)$ in (5) is finding.

3. (13) matrix equation is solved and the fundamental matrix $\Phi(t, t_0)$ in the form (8) is obtaining.

4. According to (10) are construct the matrix $D = D'$ and vector $\tilde{W}$, and solving the algebraic matrix equation (9), $x(t_0)$, ν are found.

5. By expressions (19) and (20) the $x(t_0)$, and by means ν the $x(t)$ and $u(t)$ are calculated.

According to the above method, the solution of problem (1) - (2) is brought to find the fundamental matrix of the system (5), that is, the solution of problem (13). However, the possibility of solving of the problem (13) is accompanied by difficulties in the general case. Therefore we will use Zakhar-Itkin method to find $\Phi(\tau, t_0)$. In this case the relationship

$$\begin{cases} x(t) = \psi x(t_0) - W\lambda(t) \\ \lambda(t_0) = Vx(t_0) + \psi'\lambda(t), \end{cases} \quad (21)$$

is used, here ψ, W, V are matrices that satisfy the following differential equations and initial conditions:

$$\begin{cases} \dot{\psi} = (F + WR)\psi, & \psi(t_0) = E \\ \dot{W} = FW + WF' + WRW - GQ^{-1}G', & W(t_0) = 0 \\ \dot{V} = \psi' R\psi, & V(t_0) = 0. \end{cases} \tag{22}$$

If we make some substitutions in (14), we'll get the attitude

$$\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} \psi + W(\psi')^{-1}V & -W(\psi')^{-1} \\ -(\psi')^{-1}V & (\psi')^{-1} \end{bmatrix} \begin{bmatrix} x(t_0) \\ \lambda(t_0) \end{bmatrix}. \tag{23}$$

From (23), the submatrices $\Phi_{11}(t,t_0)$, $\Phi_{12}(t,t_0)$, $\Phi_{21}(t,t_0)$, $\Phi_{22}(t,t_0)$ of the fundamental matrix defined by (8) are defined as

$$\begin{aligned} &\Phi_{11}(t,t_0) = \psi(t,t_0) + W(t,t_0)\cdot(\psi'(t,t_0))^{-1}V(t,t_0), \\ &\Phi_{12}(t,t_0) = -W(t,t_0)\cdot(\psi'(t,t_0))^{-1}, \\ &\Phi_{21}(t,t_0) = -(\psi'(t,t_0))^{-1}V(t,t_0), \\ &\Phi_{22}(t,t_0) = (\psi'(t,t_0))^{-1} \end{aligned} \tag{24}$$

In the third sub-chapter of the first chapter, the sweep algorithm for solving the linear quadratic problem with unseparated boundary conditions in the discrete case is developed. Here it is assumed that the movement of the object is described by a following discrete linear controlled system

$$x(i+1) = \psi(i)x(i) + \Gamma(i)u(i), \quad i = 1, 2, \dots, l-1, \tag{25}$$

and satisfies the boundary condition

$$\Phi_1 x(0) - \Phi_2 x(l) = q. \tag{26}$$

Here $x(i)$ is n-dimensional phase vector, $u(i)$ is m -dimensional controlling influence, $\psi(i)$, $\Gamma(i)$ are known functions and constants Φ_1, Φ_2 are $n \times n$, $n \times m$, and $k \times n$ dimensional matrices respectively, the dimension of the known constant vector q is $k \times 1$.

It is required to find vectors $x(i)$, $u(i)$ such that let the quadratic functional

$$J = \frac{1}{2}\sum_{i=0}^{l-1}(x'(i)R(i)x(i) + u'(i)C(i)u(i)) \qquad (27)$$

take a minimum value within the constraints (25), (26). $R(i) = R'(i) \geq 0$, $C(i) = C'(i) > 0$ are $n \times n$, $m \times m$ -dimensional simmetric matrices, respectively.

Using the necessary and sufficient optimality conditions in the Euler-Lagrange form, it is shown that the solution of the problem (25) - (27) is determined as follows

$$\Phi_1'\nu + \lambda(0) = 0, \quad -\Phi_2'\nu + \lambda(l) = 0 \qquad (28)$$

and is brought to the solution of a system of linear finite-difference equations of 2n composition with boundary conditions (26)

$$\left.\begin{aligned} x(i+1) = \psi(i)x(i) - M(i)\lambda(i+1) \\ \lambda(i) = R(i)x(i) + \psi'(i)\lambda(i+1) \end{aligned}\right\}. \qquad (29)$$

If we mark $\Phi = [\Phi_1, -\Phi_2]$ it can be shown that the matrix Φ' is described as

$$\Phi' = P^{-1}\begin{bmatrix} E \\ 0 \end{bmatrix} Q^{-1},$$

here P and Q are some quadratic, E -unit matrices with $2n \times 2n$, $n \times n$ and $k \times k$ dimension, respectively. Now suppose that the matrix P is separated into blocks

$$P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$$

and here P_1, P_2 and P_3, P_4 are $k \times n$ and $(2n - k) \times n$ -dimensional matrices, respectively. Then the solution of problem (25), (26) is brought to the following solution of equation (29) with 2n numbers boundary condition

$$\begin{bmatrix} \Phi_1 & 0 \\ 0 & -P_3 \end{bmatrix}\begin{bmatrix} x(0) \\ \lambda(0) \end{bmatrix} + \begin{bmatrix} -\Phi_2 & 0 \\ 0 & P_3 \end{bmatrix}\begin{bmatrix} x(l) \\ \lambda(l) \end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix}, \qquad (30)$$

and $u(i)$ control low is defined as

$$u(i) = -C^{-1}(i)\Gamma'(i)\lambda(i+1). \qquad (31)$$

Note that the missing boundary data can be determined from the following system of linear algebraic equations:

$$\begin{bmatrix} \psi(0,l) & 0 & -E & -M(0,l) \\ R(0,l) & -E & 0 & -\psi'(0,l) \\ \Phi_1 & 0 & -\Phi_2 & 0 \\ 0 & -P_3 & 0 & P_4 \end{bmatrix} \begin{bmatrix} x(0) \\ \lambda(0) \\ x(l) \\ \lambda(l) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ q \\ 0 \end{bmatrix} \tag{32}$$

Here $\psi(0,l)$, $M(0,l)$, $R(0,l)$ satisfy the following recurrent relations within the conditions $i = 0,\ j = l$

$$\psi(i,j) = \psi(i+j-1)Q(i,j-1)\psi(i,j-1), \psi(i,1) = \psi(i)$$
$$M(i,j) = M(i+j-1) + \psi(i+j-1)Q(i,j-1)M(i,j-1)\psi'(i+j-1), M(i,1) = M(i)$$
$$R(i,j) = R(i,j-1) + \psi'(i,j-1)R(i+j-1)Q(i,j-1)\psi(i,j-1), \qquad R(i,1) = R(i)$$
$$Q(i,j) = (E + M(i,j)R(i+j))^{-1} \tag{33}$$

Solving the system of linear algebraic equations (SLAE) (32), we find the initial and final values $x(0),\ \lambda(0),\ x(l),\ \lambda(l)$. Then the current values of $x(i),\ \lambda(i)$ is determined from SLAE

$$\begin{bmatrix} E & M(0,i) \\ 0 & \psi'(0,i) \end{bmatrix} \begin{bmatrix} x(i) \\ \lambda(i) \end{bmatrix} = \begin{bmatrix} \psi(0,i) & 0 \\ -R(0,i) & E \end{bmatrix} \begin{bmatrix} x(0) \\ \lambda(0) \end{bmatrix} \tag{34}$$

and

$$\begin{bmatrix} \psi(i,l-i) & 0 \\ R(i,l-i) & -E \end{bmatrix} \begin{bmatrix} x(i) \\ \lambda(i) \end{bmatrix} = \begin{bmatrix} E & M(i,l-i) \\ 0 & -\psi'(i,l-i) \end{bmatrix} \begin{bmatrix} x(l) \\ \lambda(l) \end{bmatrix} \tag{35}$$

depending on singularities of the matrices $\psi(i)$. Thus, by finding from SLAE (32) $x(0)$, $\lambda(0)$, $x(l)$, $\lambda(l)$, we recover the current values of $x(i)$, $\lambda(i)$, depending on at which i points $\psi(i)$ is degenerated , from (34) and (35). Then, the optimal control $u(i)$ is recovered according to the expression (31).

Note that this method may face difficulties if the size of the problem (25) - (27) is large, that is, to find the starting data $x(0)$ (which is an *n*-dimensional vector), it is necessary to solve the 4n-dimensional SLAE (32). Also, to determine $x(i)$-in the general case (when the inverse of $\psi(i)$does not exist) it is required to solve SLAE

(34), (35). In order not to encounter these difficulties, a sweep method is proposed to solve the problem (25)-(27), which greatly reduces the size of the analogous equations.

If we find $\lambda(0)$ and $\lambda(l)$-i from (26) and write it into the second equation of (32), we'll get the relation

$$R(0,l)x(0) + (\Phi_1' - \psi'(0,l)\Phi_2')\nu = 0. \tag{36}$$

Then defining $x(l)$ from the first equation of (32) as

$$x(l) = \psi(0,l)x(0) - M(0,l)\lambda(l) \tag{37}$$

and considering this in (26), after some substitutions we obtain

$$\Phi_1 x(0) - \Phi_2\psi(0,l)x(0) + \Phi_2 M(0,l)\lambda(l) = q\,. \tag{38}$$

Then consider $\lambda(l)$ in the last relation of (28) in (38) we obtain

$$(\Phi_1 - \Phi_2\psi(0,l))x(0) + \Phi_2 M(0,l)\Phi_2'\nu = q \tag{39}$$

and from here for determining of $x(0)$, ν, combining (36) and (39), we get the following SLAE:

$$\begin{bmatrix} R(0,l) & \Phi_1' - \psi'(0,l)\Phi_2' \\ \Phi_1 - \Phi_2\psi(0,l) & \Phi_2 M(0,l)\Phi_2' \end{bmatrix} \begin{bmatrix} x(0) \\ \nu \end{bmatrix} = \begin{bmatrix} 0 \\ q \end{bmatrix} \tag{40}$$

Note that the main matrix of SLAE (40) is symmetric, which allows us to solve the given system more accurately.

We can calculate $x(i)$ and $u(i)$ without using $\lambda(i)$. For this let us assume the presence of $\psi^{-1}(i)$ in (29) and we can find $\lambda(i+1)$ from the second equation of (29) and show it in the following form with the help of mathematical induction:

$$\lambda(i+1) = -\sum_{k=0}^{i}\left(\prod_{j=k}^{i}{\psi'}^{-1}(i+k-1)\right)R_k x_k - \prod_{k=0}^{i}\left({\psi'}^{-1}(i-1)\right)\Phi_1'\nu \tag{41}$$

If we consider expression (41) in the first equation of (29) and (41), we can obtain $x(i+1)$, using $x(i)$ and ν in the following form

$$x(i+1) = \psi(i)x(i) - M(i)\left\{\sum_{k=0}^{i}\left(\prod_{j=k}^{i}\psi'^{-1}(i+k-j)\right)R_k x_k - \left(\prod_{k=0}^{i}\psi'^{-1}_{i-k}\right)\Phi'_1 \nu\right\},$$

$$(i = 0, 1, \dots, l-1) \qquad (42)$$

and $u(i)$ will be in the form

$$u(i) = C^{-1}(i)\Gamma'^{\left\{\sum_{k=0}^{i}\left(\prod_{j=k}^{i}\psi'^{-1}(i+k-j)\right)R_k x_k + \left(\prod_{k=0}^{i}\psi'(i-k)\right)\Phi'_1 \nu\right\}}$$

$$(i = 0, 1, \dots, l-1)\ . \qquad (43)$$

Thus, we construct the following calculation algorithm.

ALQORİTHM.

1. The matrices $\psi(i),\ \Gamma(i),\ \Phi_1,\ \Phi_2,\ C(i),\ R(i)$ matrislǝri and vector q from problem (25)-(27) are forming.

2. Within the $\psi(0,1) = \psi(0),\ M(0,1) = M(0),\ R(0,1) = R(0)$ matrix conditions the $\psi(0,l),\ R(0,l)$ and $M(0,l)$ from relation (33) are calculated.

3. The main matrix on the right side of the SLAE (40) and the vector on the left are constructed.
4. The SLAE (40) is solved and $x(0),\ \nu$ are determined.
5. The control $u(i)$ is found from (43) and the trajectory $x(i)$ is found from (42).

The **fourth subsection** of the **first chapter** proposes a new method for solving the corresponding problem, which does not require solving matrix Riccati equations, linear matrix equations, etc. Here, too, the solution of the two-point unseparated linear quadratic optimization problem for the continuous case is considered. As in the second subchapter, a system of linear algebraic equations is obtained to find the initial data of the problem and the Lagrange constant, and these parameters are found. Then, based on the values found, the phase coordinates x(t) and the controliing influence u(t), i.e., the program trajectory and the controller are constructing.

The **second chapter** of the dissertation work is dedicated to the methods of solving identification and statistical problems that arise during oil extraction.

In the **first sub-chapter** of the second chapter, an algorithm for solving the identification problem was proposed for determining the coefficient of hydraulic resistance (CHR) in different parts of the pump-compressor pipes during the gas lift process.

It is known that the movement of the gas-liquid mixture in pipes is described by the following system of partial differential equations of hyperbolic type:

$$\begin{cases} \frac{\partial P_i}{\partial t} = -\frac{c_i^2}{F_i}\frac{\partial Q_i}{\partial x}, \\ \frac{\partial Q_i}{\partial t} = -F_i\frac{\partial P_i}{\partial x} - 2a_iQ_i, \end{cases} \quad i = 1,2, \tag{44}$$

here $P_i = P_i(x,t)$, Q_i are pressure and volume of gas and gas-liquid mixture, respectively, the parameters $2a_i = \frac{g}{\omega_i} + \frac{\lambda_i\omega_i}{2D_i}$; $c_i,\ a_i,\ \omega_i,\ g_i,\ \lambda_i,\ D_i,\ F_i,\ \ (i = 1,2)$ have concrete practical significance. Using the method of straight lines and noting $l_p = 1/n\,, p = \overline{1,n}$, from (1) we obtain

$$\begin{cases} \frac{dP_k}{dt} = -\frac{c_i^2}{F_il}(Q_k - Q_{k-1}), \\ \frac{dQ_k}{dt} = -\frac{F_i}{l}(P_k - P_{k-1}) - 2a_iQ_k \end{cases} \quad i = 1,2, \quad k = \overline{0,2n} \tag{45}$$

$$F_i = \begin{cases} F_1, & 0 < k \le n \\ F_2, & n < k \le 2n \end{cases}$$

$$c_i = \begin{cases} c_1, & 0 < k \le n \\ c_2, & n < k \le 2n, \end{cases} \quad a_i = \begin{cases} a_1, & 0 < k \le n \\ a_2, & n < k \le 2n \end{cases}.$$

Note that in the case of $k = n + 1$ the equation (45) is as follows

$$\begin{cases} \dfrac{dP_{n+1}}{dt} = -\dfrac{c_2^2}{F_2 l} Q_{n+1} + \dfrac{c_2^2}{F_2 l} Q_n + \dfrac{c_2^2}{F_2 l} Q_{pl}, \\ \dfrac{dQ_{n+1}}{dt} = -\dfrac{F_2}{l} P_{n+1} + \dfrac{F_2}{l} P_n - 2a_2 Q_{n+1} + \dfrac{F_2}{l} P_{pl}, \end{cases} \quad i = 1,2, \quad k = \overline{0,2n}$$

here Q_{pl}, P_{pl} are the volume and layer pressure at the bottom of the well, respectively.

After some transformations, the system (45) can be put in the following form

$$\begin{cases} \dot{x}_1(t) = A_1\big(a_2(\lambda_1)\big)x_1(t) + B_1 u_0 + V \\ \dot{x}_2(t) = A_2\big(a_2(\lambda_2)\big)x_2(t) + B_2 u_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{x}_n(t) = A_n\big(a_2(\lambda_n)\big)x_n(t) + B_n u_{n-1} \end{cases} \qquad (46)$$

with initial condition

$$x_m(0) = \begin{bmatrix} P_m(0) \\ Q_m(0) \end{bmatrix}, \quad m = \overline{1,n},$$

here

$$\dot{x}_m(t) = \begin{bmatrix} \dot{P}_m(t) \\ \dot{Q}_m(t) \end{bmatrix}, \quad x_m(t) = \begin{bmatrix} P_m(t) \\ Q_m(t) \end{bmatrix}$$

$$A_m = \begin{bmatrix} 0 & -\dfrac{c_2^2}{F_2 l_i} \\ -\dfrac{F_2}{l_i} & -2a_2(\lambda_i) \end{bmatrix}, \qquad B_m = \begin{bmatrix} 0 & \dfrac{c_2^2}{F_2 l_i} \\ \dfrac{F_2}{l_i} & 0 \end{bmatrix},$$

$$u_c = \begin{bmatrix} P_c(t) \\ Q_c(t) \end{bmatrix}, \quad V = \begin{bmatrix} 0 & \frac{c_2^2}{F_2 l_1} \\ -\frac{F_2}{l_i} & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{pl} \\ Q_{pl} \end{bmatrix}, \quad m = \overline{1,n}, \; i = \overline{1,2}, \; c = \overline{0, n-1}.$$

Assume that CHR is different in different parts along the entire length of the pipelines and assume that the coefficient of hydraulic resistance λ_m ($m = \overline{1,n}$) in the pump-compressor pipe (PKP) of gas lift wells varies in the interval $0 \leq \lambda_m \leq 1$. Let assume that, from the statistical data λ_m ($m = \overline{1,n}$) taken from different parts of PKP, CHR is known, $Q_0^{S,St}$ is a gas injected at the wellhead, $Q_n^{S,St}$- is the debit at the outlet of the well, $S = \overline{1,k}$ (S is number of the statistical data). İn this case for each $Q_0^{S,St}$ we can solve (45) and calculate $Q_n^s(\lambda_1, \lambda_2, \ldots, \lambda_n, T)$. Thus, the problem is to find such values of λ_m- lərin ($i = \overline{1,m}$) that, at these values the quadratic functional

$$I(\lambda_1, \lambda_2, \ldots, \lambda_n) = \sum_{s=1}^{k} \left[Q_n^s(\lambda_1, \lambda_2, \ldots, \lambda_n, T) - Q_n^{S,St} \right]^2 \quad (47)$$

have a minimal value.

Solving the system (46) with initial conditions $x_m(0) = \begin{bmatrix} P_m(0) \\ Q_m(0) \end{bmatrix}$, $m = \overline{1,n}$ in the pieces $[0 \; l_1], (l_1 l_2], \ldots, (l_{n-1} l_n], \; (l_n = l)$, we obtain the general solution of (46):

$$x_n(t) = e^{A_n t} x_n(0) + \sum_{m=1}^{n-1} (-1)^m \left(\prod_{j=n}^{m+1} A_j^{-1} B_j \left(E - e^{A_j t}\right) \right) e^{A_i t} x_i(0) +$$

$$+(-1)^n \left(\prod_{m=1}^{n} A_m^{-1} B_m (E - e^{A_m t}) \right) u_0 + \quad (48)$$

$$+(-1)^n \left(\prod_{m=2}^{n} A_m^{-1} B_m (E - e^{A_m t}) \right) \left(A_1^{-1} (E - e^{A_i t}) \right) V$$

Here E is a unit matrix.

From the solution of (48) we obtain

$$Q_n(\lambda_1, \lambda_2, \dots, \lambda_n, T) = J x_n(T), \qquad (49)$$

here $J = [0 \;\; 1]'$. Considering (48) and (49) in (47), we obtain

$$I = \sum_{s=1}^{k} \left[Q_n^s(\lambda_1, \lambda_2, \dots, \lambda_n, T) - Q_n^{S,St} \right]^2 = \sum_{s=1}^{k} \left[J x_n^s(T) - Q_n^{S,St} \right]^2 =$$

$$= \sum_{s=1}^{k} \Bigg[J e^{A_n T} x_n^S(0)$$

$$+ J \sum_{m=1}^{n-1} (-1)^m \left(\prod_{j=n}^{m+1} A_j^{-1} B_j (E - e^{A_j t}) \right) e^{A_i T} x_i^s(0) +$$

$$+ J(-1)^n \left(\prod_{m=1}^{n} A_m^{-1} B_m (E - e^{A_m T}) \right) u_0 + \qquad (50)$$

$$+ J(-1)^n \left(\prod_{m=2}^{n} A_m^{-1} B_m (E - e^{A_m T}) \right) \left(A_1^{-1} (E - e^{A_1 T}) \right) V - Q_n^{S,St} \Bigg]^2 .$$

To solve the optimization problem (46) - (47), we find the gradient of the functional $I(\lambda_1, \lambda_2, \dots, \lambda_n)$ and make it equal to zero. Since it is practically impossible to calculate the gradient vector for $I(\lambda_1, \lambda_2, \dots, \lambda_n)$, when calculating the derivatives $\partial I / \partial \lambda_i, i = \overline{1, n}$ we use the following expressions:

$$
\begin{cases}
\dfrac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_1} \approx \dfrac{I(\lambda_1+h_1,\lambda_2,\dots,\lambda_n)-I(\lambda_1-h_1,\lambda_2,\dots,\lambda_n)}{2h_1}, \\
\dfrac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_2} \approx \dfrac{I(\lambda_1,\lambda_2+h_2,\dots,\lambda_n)-I(\lambda_1,\lambda_2-h_2,\dots,\lambda_n)}{2h_2}, \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
\dfrac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_n} \approx \dfrac{I(\lambda_1,\lambda_2,\dots,\lambda_n+h_n)-I(\lambda_1,\lambda_2,\dots,\lambda_n-h_n)}{2h_n},
\end{cases} \quad (51)
$$

here $h_i,\ i=\overline{1,n}$ are sufficiently small parameters.

Thus, the following algorithm is constructed to find CHR λ_i $(i=\overline{1,n})$:

Alqorithm

1. The initial data and parameters of the system (46) are entered;
2. The statistical data $Q_0^{S,St}$, $Q_n^{S,St}$ are entered;
3. $Q_n(\lambda_1,\lambda_2,\dots,\lambda_n,T)$ are defined from (49);
4. The functional (50) is constructed;
5. Using (51) the solutions of following equations are found
$$\frac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_1}=0,\quad \frac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_2}=0,\dots,\frac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_n}=0;$$
6. For sufficiently small ε the conditions
$$\left|\frac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_1}\right|<\varepsilon,\quad \dots,\left|\frac{\partial I(\lambda_1,\lambda_2,\dots,\lambda_n)}{\partial\lambda_n}\right|<\varepsilon,$$

are checked, if the condition is satisfied, the calculation is stopped, otherwise it goes to step 5, decrementing h_i $(i=\overline{1,n})$.

In the **second sub-chapter** of the second chapter, an asymptotic method for finding the coefficient of hydraulic resistance in different parts of pipelines during oil extraction was proposed.. Here, the time-averaged nonlinear ordinary differential equation of the system of hyperbolic equations describing the movement of the gas-liquid mixture in the lift is considered:

$$\dot{Q} = \frac{2a\rho F Q^2}{c^2\rho^2 F^2\mu - Q^2}, \quad Q(0) = Q_0 , \tag{52}$$

here c-is speed of sound in gas and gas-liquid mixture; $2a = \frac{g}{\omega_c} + \frac{\lambda\omega_c}{2D}$; g , λ - are the hydraulic resistance in gas and gas-liquid mixture (GLM); ω_c is cross-sectionally averaged velocity of gas and mixture in the annular space and lift, respectively, D is internal effective diameters of the annular space and the lifting device, $\rho\omega_c = \frac{Q}{F}$, $Q = \rho\omega_c F$ are mass consumption of the gas injected into the annular space and the gas-liquid mixture in the lift, Q – volume of gas and gas-liquid mixture, F is the cross-sectional area of the pump-compressor pipes, which is constant along the axes.

Suppose that the lift of length l (i.e. piece $[l, 2l]$) of a gas lift well is divided into m parts as $[l_i, l_{i+1}]$ $(i = 1,2,...,m-1)$ and the movement of the GLM at each interval $[l_i, l_{i+1}]$ is described by a system of nonlinear ordinary differential equations (52), where $Q(l+0) = Q_l$, and λ_i is the GLM at each $[l_i, l_{i+1}]$ interval.

Now let's assume that the initial data (that is, the measured parameters of the model when observing the object)

$$Q^j(l+0) = \tilde{Q}_l^j, \; j = \overline{1,k} \tag{53}$$

and there are k number of statistics values of debit $\tilde{Q}_{2l}^j$ at the end of the lift corresponding to them. Since the motion of the GLM is continuous

$$Q(l_{i+1} + 0) = Q(l_{i+1} - 0), \tag{54}$$

can be accepted. Here in each piece $[l_i, l_{i+1}]$ it is required to find such $\lambda_i (i = 1,...,m)$ values of CHR that the difference between the solution $Q(2l) = Q_{2l}$ of equation (52) and the given final value $\tilde{Q}_{2l}^j$

at the end $x = 2l$ of the lift is minimal. For solving such identification problem we can use the method of least squares and the quadratic functional

$$J = \sum_{j=1}^{k} \left(Q^j(2l) - \tilde{Q}_{2l}^j \right)^2 + \alpha (a_1^2 + a_2^2 + ... + a_m^2) \cdot \qquad (55)$$

Since it is difficult to find a solution to the problem (52) - (55)[1], let us try to use a small parameter from (52) in the general case, and find its $O(\varepsilon)$ -order solution asymptotically.

At the interval $[l_i, l_{i+1}]$ the solution of equation (52) around the small parameter $\mu = 0$ at the first order can be found as follows

$$Q(x,\mu) = \left(Q_{li}^j - 2a\rho Fx \right) + \frac{2a_i c^2 \rho^2 F^3}{2a_i \rho F x Q_{li} - Q_{li}^2} \mu \ . \qquad (56)$$

Using this expression, it can be shown by mathematical induction that due to the small parameter μ in the first approximation $Q(l_m) = Q(2l)$ we obtain the expression

$$\begin{aligned} Q(2l) &= \left(Q_l - 2a_1\rho F l_1 - 2a_2 \rho F l_2 - ... - 2a_m \rho F l_m \right) + \\ &+ \left(\frac{2a_1 c^2 \rho^2 F^3}{2a_1 \rho F l_1 Q_l - Q_l^2} + \frac{2a_2 c^2 \rho^2 F^3}{2a_2 \rho F l_2 (Q_l - 2a_1 \rho F l_1) - (Q_l - 2a_1 \rho F l_1)^2} + \right. \\ &+ \frac{2a_3 c^2 \rho^2 F^3}{2a_3 \rho F l_3 (Q_l - 2a_1 \rho F l_1 - 2a_2 \rho F l_2) - (Q_l - 2a_1 \rho F l_1 - 2a_2 \rho F l_2)^2} + ... + \\ &+ \left. \frac{2a_m c^2 \rho^2 F^3}{2a_m \rho F l_m (Q_l - 2a_1 \rho F l_1 - 2a_2 \rho F l_2 - ... - 2a_{m-1} \rho F l_{m-1}) - (Q_0 - 2a_1 \rho F l_1 - ... - 2a_{m-1} \rho F l_{m-1})^2} \right) \mu \end{aligned} \qquad (57)$$

If we take this into account in (55) and discard the termsn in which the μ^2 is included, we obtain the following asymptotic expression

[1]Aliev, F.A. Asymptotic method for finding the coefficient of hydraulic resistance in lifting of fluid on tubing. F.A. Aliev, N.A. Ismailov, A.A. Namazov. Journal of Inverse and İLL-Posed Problems,Vol.5, - 2015. - pp.511 -518.

for the functional J in the first approximation considering the small parameter μ:

$$J=\sum_{j=1}^{k}\left(Q_0^j-Q_{2l}^j-2\sum_{i=1}^{m}a_i\rho Fl_i\right)^2+2\left(Q_0^j-Q_{2l}^j-2\sum_{i=1}^{m}a_i\rho Fl_i\right)\times$$
$$\times\left(\frac{2a_1c^2\rho^2F^3}{2a_1\rho Fl_1Q_0^j-Q_0^{j^2}}+\sum_{i=2}^{m-1}\frac{2a_2c^2\rho^2F^3}{2a_i\rho Fl_i\left(Q_0^j-2\sum\limits_{i=1}^{m-1}a_i\rho Fl_i\right)-\left(Q_0^j-2\sum\limits_{i=1}^{m-1}a_i\rho Fl_i\right)^2}\right)\mu. \quad (58)$$

After that, we can calculate the gradient of the functional (58), make it equal to zero and find the a_i. Since the resulting equations are non-linear, their solutions we are looking for in the form

$$\tilde{a}_i=a_i^0+\mu a_i^1 \ . \quad (59)$$

After certain transformations, we get a system of linear algebraic equations to find both a_i^0, and a_i^1. After finding them and determining the $\tilde{a}_i$ from (59) for the calculation of CHR λ_i we can use the expression

$$\lambda_i=\frac{2a_iD}{\omega}-\frac{2gD}{\omega^2}. \quad (60)$$

Here, the results obtained when the piece $[0,l]$ is divided into two parts, i.e. $m=2$ are illustrated with a concrete example. We get the following table for $\lambda_1(\alpha), \lambda_2(\alpha)$ at different values of α.

Table: Values of hydraulic resistance coefficient

α	10^{-2}	10^{-4}	10^{-6}	10^{-8}
$\lambda_1(\alpha)$	0.31034224164	0.31034250636	0.31037253624	0.3103433332
$\lambda_2(\alpha)$	0.31029078492	0.30646617497	0.12635629573	0.09170523594

It can be seen from the table that for a sufficiently small value of α, the results coincide with the known results with an accuracy of 10^{-2}.

In the third sub-chapter of the second chapter, methods of solving fractional differential equations are investigated. This is due to the fact that mathematical models for various problems of oil extraction and metallurgical industry are more adequately described by fractional operator differential equations. Relevant problems in these works are mainly based on equations of fluid-damped oscillation systems, which describe these models by the fractional operator differential equations.

Now let's assume that the equation describing the motion of fluid-damped oscillating systems is given

$$y''(t) + aD^{\alpha}y(t) + by(t) = f(t), t\epsilon(t_0, T), \quad (61)$$

where the coefficient a, b are real constant numbers, $t_0 > 0$, $f(t) -$ is a given real continuous function.

Let us consider the following boundary conditions, i.e. periodic boundary conditions

$$y(t_0) = y(T), \quad y'(t_0) = y'(T), \quad (62)$$

The purpose here is to find a solution to the boundary problem (61), (62)..

Suppose that in (61) $\alpha = \frac{P}{q} \in (1,2)$, here p and q are natural numbers. In this case, perform the following transformation in (61):

$$\begin{aligned}
&y(t) = z_1(t),\\
&D^{1/q}y(t) = D^{1/q}z_1(t) = z_2(t),\\
&D^{2/q}y(t) = D^{2/q}z_1(t) = D^{1/q}z_2(t) = z_3(t),\\
&\quad\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots\\
&D^{\frac{p-1}{q}}y(t) = D^{\frac{p-1}{q}}z_1(t) = \cdots = D^{\frac{1}{q}}z_{p-1}(t) = z_p(t),\\
&\vdots \qquad \vdots \qquad \vdots \qquad \vdots\\
&D^{\frac{2q-1}{q}}y(t) = D^{\frac{2q-1}{q}}z_1(t) = \cdots = D^{1/q}z_{2q-1}(t) = z_{2q}(t),
\end{aligned} \quad (63)$$

$$y''_{(t)} = D^{\frac{2q}{q}} y(t) = D^{\frac{2q}{q}} z_1(t) = \cdots = D^{1/q} z_{2q}(t)$$
$$= f(t) - a\, z_{p+1}(t) - b\, z_1(t),$$

The system (63) can be described in matrix form as follows:

$$D^{1/q} z(t) = Az(t) + B(t), \tag{64}$$

here

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_{p+1}(t) \\ \vdots \\ z_{2q}(t) \end{bmatrix}, \quad \mathrm{A} = \begin{bmatrix} \overbrace{0\ 1\ 0 \ldots 0}^{P+1} \ \ldots 0\ 0 \\ 0\ 0\ 1 \ldots 0 .. \quad 0\ 0 \\ 0\ 0\ 0 \ldots 0 \ldots \ 0\ 1 \\ -b\ 0\ 0 \ldots -a ..\ 0\ 0 \end{bmatrix}, B(t) = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ f(t) \end{bmatrix} \tag{65}$$

and it has periodic boundary conditions:

$$z(t_0) = z(T). \tag{66}$$

Note that the boundary conditions (66) also hold the boundary conditions (62). It can be proved that the set boundary problem (64), (66) is correct.

We will look for the solution of the system (64) in the form of a slipped Mittag-Leffler function with an unknown coefficient

$$z(t) = \sum_{k=1}^{\infty} z_k \frac{t^{-1+\frac{k}{q}}}{(-1+\frac{k}{q})!}, \tag{67}$$

where the coefficients z_k are unknown constant vectors. Let's calculate the $\frac{1}{q}$ -order derivatives:

$$D^{1/q} z(t) = \sum_{k=1}^{\infty} z_k \frac{t^{-1+\frac{k-1}{q}}}{(-1+\frac{k-1}{q})!}. \tag{68}$$

If we accept the marking $k-1=m$, we get from (68):

$$D^{1/q} z(t) = \sum_{m=0}^{\infty} z_{m+1} \frac{t^{-1+\frac{m}{q}}}{(-1+\frac{m}{q})!} = z_1 \frac{t^{-1}}{(-1)!} + \sum_{m=1}^{\infty} z_{m+1} \frac{t^{-1+\frac{m}{q}}}{(-1+\frac{m}{q})!}$$

As can be seen , ${}^{t^{-1}}/_{-1!} = \delta(t)$. Let's take into account that

$t > t_0 > 0,\ \delta(t) = 0$. Therefore, the first sum in the above expression is equal to zero. Then if we accept the marking $k-1 = m$, take into account (68) and (67) with some transformations in (64), we obtain

$$\sum_{m=1}^{\infty} z_{m+1} \frac{t^{-1+\frac{m}{q}}}{(-1+\frac{m}{q})!} = A \sum_{m=1}^{\infty} z_m \frac{t^{-1+\frac{m}{q}}}{(-1+\frac{m}{q})!} + \sum_{m=1}^{\infty} B_m \frac{t^{-1+\frac{m}{q}}}{\left(-1+\frac{m}{q}\right)!} \quad (69)$$

Since the function $\frac{t^{-1+\frac{m}{q}}}{(-1+\frac{m}{q})!}$ in (69) are linearly independent functions at different values of m we otain an attitude

$$z_{m+1} = A\ z_m + B_m\ ,\ m \geq 1\ . \quad (70)$$

Then the general solution of system (64) will be as follows:

$$z(t) = \sum_{k=1}^{\infty} \left[A^{k-1} z_1 + \sum_{S=1}^{k-1} A^{k-1-S} B_S\right] \frac{t^{-1+\frac{k}{q}}}{(-1+\frac{k}{q})!} \quad (71)$$

where the unknown constant vector z_1 is determined from the appropriate boundary conditions.

If we write the general solution (71) in the boundary condition (66), we obtain

$$z(t_0) = \sum_{k=1}^{\infty} \left[A^{k-1} z_1 + \sum_{S=1}^{k-1} A^{k-1-S} B_S\right] \frac{{t_0}^{-1+\frac{k}{q}}}{\left(-1+\frac{k}{q}\right)!} =$$

$$= \sum_{k=1}^{\infty} \left(A^{k-1} z_1 + \sum_{S=1}^{k-1} A^{k-1-S} B_S\right) \frac{T^{-1+\frac{k}{q}}}{(-1+\frac{k}{q})!} = z(T) \quad (72)$$

If we group the received equation with respect to z_1, we obtain

$$\sum_{k=1}^{\infty} A^{k-1}\left(\frac{t_0^{-1+\frac{k}{q}}-T^{-1+\frac{k}{q}}}{\left(-1+\frac{k}{q}\right)!}\right) z_1 = \sum_{k=1}^{\infty}\sum_{s=1}^{k-1} A^{k-1-s} B_s \frac{T^{-1+\frac{k}{q}}-t_0^{-1+\frac{k}{q}}}{(-1+\frac{k}{q})!}$$

(73)

Let's assume that the coefficient z_1 in the left part of the equation (73) is not degenerated, i.e

$$\det \sum_{k=1}^{\infty} A^{k-1} \frac{t_0^{-1+\frac{k}{q}}-T^{-1+\frac{k}{q}}}{\left(-1+\frac{k}{q}\right)!} \neq 0 \qquad (74)$$

Then from (73) we get the following relation for z_1

$$z_1 =$$

$$\left[\sum_{k=1}^{\infty} A^{k-1} \frac{t_0^{-1+\frac{k}{q}}-T^{-1+\frac{k}{q}}}{\left(-1+\frac{k}{q}\right)!}\right]^{-1} \left[\sum_{k=1}^{\infty}\sum_{s=1}^{k-1} A^{k-1-s} B_s \frac{T^{-1+\frac{k}{q}}-t_0^{-1+\frac{k}{q}}}{(-1+\frac{k}{q})!}\right].$$

(75)

If we consider z_1–i from (75) in the general solution of the system (64) given in (71), we get the periodic solution of the boundary problem (64) - (66) as follows:

$$z(t) = \sum_{k=1}^{\propto}\left\{A^{k-1}\left[\sum_{l=1}^{\infty} A^{l-1} \frac{t_0^{-1+\frac{l}{q}} - T^{-1+\frac{l}{q}}}{\left(-1+\frac{l}{q}\right)!}\right]^{-1} \times\right.$$

$$\left. \text{x}\left[\sum_{l=1}^{\infty}\sum_{s=1}^{l-1} A^{l-1-s} B_s \frac{T^{-1+\frac{l}{q}} - t_0^{-1+\frac{l}{q}}}{\left(-1+\frac{l}{q}\right)!}\right] + \sum_{s=1}^{k-1} A^{k-1-s} B_s\right\} \frac{t^{-1+\frac{k}{q}}}{\left(-1+\frac{k}{q}\right)!}. \qquad (76)$$

Thus, we come to the following conclusion.

Theorem: A, B given in the form (65) and within the condition (74) there is a periodic solution of the boundary problem (64) - (66) described in the form (76).

In the **fourth sub-chapter** of the second chapter, the collection of statistical indicators for wells covering the same layer and the formation of a database were considered.

Usually, the creation of a model describing this or that object is based on input-output parameters (or a created database) collected during observations of this object. The creation of a mathematical model is carried out by two methods (or a combination of these methods).

In the first method, the system is divided into subsystems in such a way that their properties and characteristics are known from previously conducted or collected experiments and observations. The formal combination of these subsystems from a mathematical point of view eventually leads to the formation of a mathematical model of the entire system. Within this approach, there is no need for experiments in building a mathematical model of the object, that is, in this case, everything corresponds to the "laws of nature". In this case, the mathematical modeling procedures depend on the specific applied mathematical problem and are usually determined by the customary specific characteristics of the considered application area.

Another method of building a mathematical model is based on materials extracted or collected during experiments. At this time, a model is formed by processing a database built on the basis of the results of input and output signals. And this method is an identification method. System identification, ie. building a model based on observations has three main components.

- Data;
- Models that describe the process;

- Evaluation or selection of a validated model based on experiences.

In the first stage, the statistical data of the wells were processed and compared with the results of the model, and the following works were carried out. The development of information (observations or statistical indicators) plays an important role both in the practice of applying mathematical calculations and in financial and economic calculations[2]. The processing of signals and observations is carried out with the help of some algorithms in the MATLAB package[3].

All of the above was carried out on the basis of the statistical indicators of the well of the Nariman Narimanov Department of SOCAR for the years 1976-2012. Some groupings were made on these statistics, they are given in the form of a table and reflected in the following form in the MATLAB package.

The third chapter of the dissertation work is devoted to the problem of optimization which is periodic in a part of the given domain. It should be noted that such issues can be used in the creation of mathematical models of the gas lift method used in oil production. In other words, this problem provides the development of the optimal program trajectory and the appropriate algorithms for the construction of the controller in the gas lift method.

In the **first sub-chapter** of the third chapter, the partial periodic optimal control problem, where the control parameter is included in the initial condition, is considered. An optimization problem is considered, where the motion of the object in the piece $[0,2l]$ is described by different differential equations in the intervals $[0,l)$ and

[2]Davis, C.J., Statistics and data analysis in geology, John Wiley and Sons. / C.J. Davis. Inc. - Canada, - 1986.
[3]Draper, N.R. Applied regression analysis, John Wiley and Sons. / N.R. Draper, H. Smith. - 1966.

$(l, 2l]$ respectively, and at point l the solution satisfies the finite-difference equations.

In addition, the middle (*l*) and end (*2l*) points are connected with the periodic condition. Suppose that the motion of an object in the piece $[0, l]$ is described by the following system of differential and finite-difference equations:

$$\dot{y}(x) = f_1(y(x), x), \qquad y(0) = u\,, \quad 0 < x < l \tag{77}$$

$$y(l+0) = F_\delta y(l-0) + V\,, \qquad x = l \tag{78}$$

$$\dot{y}(x) = f_2(y(x), x), \qquad l < x < 2l\,, \tag{79}$$

where y is *n*-dimension phase vesctor, u is an *n*-dimensional constant vector that acts as a controlling influence, F_δ - $n \times n$ -dimension constant matrix, V is n −dimension constant vector, f_1, f_2 are n-dimension vector-functions, are also piecewise continuous functions, x is the argument.

It is required to find such u that the functional

$$J = u^T R u + y^T(l-0) Q y(l-0) + \int_0^{2l} L(y(x), x) dx \tag{80}$$

have a minimal value within the condition of partial periodicity with respect to the phase vector

$$y(l+0) = y(2l), \tag{81}$$

where $R = R^T > 0$, $Q = Q^T \leq 0$ - $n \times n$ dimension constant matrices, $L(y(x), x)$ – is given function, T is a transposition operation. If we construct the expanded functional and make its gradient equal to zero

$$\frac{\partial \bar{J}}{\partial u} = 2Ru - \lambda(0) = 0, \tag{82}$$

$$\frac{\partial \bar{J}}{\partial y} = \dot{\lambda}(x) + \frac{\partial L}{\partial y} - \lambda^T(x) \frac{\partial f_1}{\partial y} = 0, \qquad 0 < x < (l-0) \tag{83}$$

$$\frac{\partial \bar{J}}{\partial y} = \dot{\lambda}(x) + \frac{\partial L}{\partial y} - \lambda^T(x)\frac{\partial f_2}{\partial y} = 0, \qquad (l+0) < x < 2l \tag{84}$$

$$\begin{cases} \lambda(l-0) = Qy(l-0) + F_\delta^T \lambda(l+0) \\ \quad y(l+0) = F_\delta y(l-0) + V \end{cases} \tag{85}$$

$$\lambda(l+0) = \lambda(2l)\,, \tag{86}$$

we get the system of Euler-Lagrange equations. Control from (82) we find in the form

$$u = \frac{1}{2} R^{-1} \lambda(0) \tag{87}$$

and taking into account it in (77), (79) and (83), (84), in the interval $(0, l)$

$$\left.\begin{aligned} &\dot{y}(x) = f_1(y(x), x), \quad y(0) = \frac{1}{2} R^{-1} \lambda(0), \\ &\dot{\lambda}(x) = -\frac{\partial L}{\partial y} + \lambda^T \frac{\partial f_1}{\partial y}\,, \end{aligned}\right\} \tag{88}$$

but in the interval $(l, 2l)$

$$\left.\begin{aligned} \dot{y}(x) &= f_2(y(x), x) \\ \dot{\lambda}(x) &= -\frac{\partial L}{\partial y} + \lambda^T \frac{\partial f_2}{\partial y} \end{aligned}\right\}. \tag{89}$$

We get $2n$ differential systems. Here at the point l the discontinuity occurs in the form of the discrete equation (85) and the boundary condition (86).

Thus, for the solution of the problem of optimal boundary control, a system of differential and finite-difference equations with unseparated boundary conditions is obtained, and by solving it using any numerical methods, we obtain the optimal program trajectory y_{pr} and the program control u_{pr}.

Now let's look at the corresponding linear quadratic problem. In this case, the system (77)-(79) is described by linear equations

$$\dot{y}(x) = F_1(x) y(x) + V_1(x), \qquad y(0) = u, \qquad 0 < x < l \tag{90}$$

$$y(l+0) = F_\delta y(l-0) + V_2, \quad (91)$$
$$\dot{y}(x) = F_2(x)y(x) + V_3(x), \qquad l < x < 2l \quad (92)$$
where the matrices F_1, F_2 and vectors V_1, V_3 is obtained from the linearization of the relations (77), (79) and has appropriate dimensions. If we take $L(y(x),x) = y^T(x)Q_1(x)y(x)$ in (80), the quadratic functional takes the form

$$J = u^T R u + y^T(l-0)Qy(l-0) + \int_0^l y^T(x)Q_1(x)y(x)dx, \quad (93)$$

where $Q_1(x) = Q_1^T(x) \geq 0$, and the partial periodicity condition (81) remains unchanged. In this case, the corresponding Euler-Lagrange equations will be in the following matrix form

$$\begin{bmatrix} \dot{y}(x) \\ \dot{\lambda}(x) \end{bmatrix} = \begin{bmatrix} F_1(x) & 0 \\ -Q(x) & -F_1'(x) \end{bmatrix} \cdot \begin{bmatrix} y(x) \\ \lambda(x) \end{bmatrix} + \begin{bmatrix} V_1(x) \\ 0 \end{bmatrix}, \quad 0 < x < l-1$$
(94)

$$\begin{bmatrix} \dot{y}(x) \\ \dot{\lambda}(x) \end{bmatrix} = \begin{bmatrix} F_2(x) & 0 \\ -Q(x) & -F_2'(x) \end{bmatrix} \cdot \begin{bmatrix} y(x) \\ \lambda(x) \end{bmatrix} + \begin{bmatrix} V_3(x) \\ 0 \end{bmatrix}, \quad l+0 < x < 2l$$
(95).

If the fundamental matrices of the system of differential equations (94) and (95) are W_1 and W_2 their solutions can be described as follows:

$$\begin{bmatrix} y(x) \\ \lambda(x) \end{bmatrix} = W_1(x,x_0) \cdot \begin{bmatrix} y(x_0) \\ \lambda(x_0) \end{bmatrix} + K_1(x,x_0), \quad 0 < x < l-0 \quad (96)$$

$$\begin{bmatrix} y(x) \\ \lambda(x) \end{bmatrix} = W_2(x,x_0) \cdot \begin{bmatrix} y(x_0) \\ \lambda(x_0) \end{bmatrix} + K_2(x,x_0), \quad l+0 < x < 2l \quad (97)$$

here

$$K_1(x,x_0) = \int_{x_0}^{x} W_1(x,t) \begin{bmatrix} V_1(t) \\ 0 \end{bmatrix} dt \quad (98)$$

$$K_2(x,x_0) = \int_{x_0}^{x} W_2(x,t) \begin{bmatrix} V_3(t) \\ 0 \end{bmatrix} dt \, . \tag{99}$$

Thus from (96) and (97) we get

$$\begin{bmatrix} y(l-0) \\ \lambda(l-0) \end{bmatrix} = W_1(l-0,0) \cdot \begin{bmatrix} y(0) \\ \lambda(0) \end{bmatrix} + K_1(l-0,0), \tag{100}$$

$$\begin{bmatrix} y(2l) \\ \lambda(2l) \end{bmatrix} = W_2(2l,l+0) \cdot \begin{bmatrix} y(l+0) \\ \lambda(l+0) \end{bmatrix} + K_2(2l,l+0). \tag{101}$$

If we add equation (81), the first equation of (85), as well as (86) and (91) to equations (100), (101), we get the system of 8n equations consisting of 8n numbers of unknowns $y(0)$, $\lambda(0)$, $y(l-0)$, $\lambda(l-0)$, $y(l+0)$, $\lambda(l+0)$, $y(2l)$, $\lambda(2l)$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & E & 0 & -E & 0 \\ 0 & 0 & -Q & E & 0 & -F_\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E & 0 & -E \\ 0 & 0 & -F_\delta & 0 & E & 0 & 0 & 0 \\ -W_{11}^1 & -W_{12}^1 & E & 0 & 0 & 0 & 0 & 0 \\ -W_{21}^1 & -W_{22}^1 & E & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -W_{11}^2 & -W_{12}^2 & E & 0 \\ 0 & 0 & 0 & 0 & -W_{21}^2 & -W_{22}^2 & 0 & E \end{bmatrix} \begin{bmatrix} y(0) \\ \lambda(0) \\ y(l-0) \\ \lambda(l-0) \\ y(l+0) \\ \lambda(l+0) \\ y(2l) \\ \lambda(2l) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_2 \\ K_1^1 \\ K_2^1 \\ K_1^2 \\ K_2^2 \end{bmatrix} \tag{102}$$

where burada

$$W_1(l-0,0) = \begin{bmatrix} W_{11}^1 W_{12}^1 \\ W_{21}^1 W_{22}^2 \end{bmatrix}, \qquad W_2(2l,l+0) = \begin{bmatrix} W_{11}^2 W_{12}^2 \\ W_{21}^2 W_{22}^2 \end{bmatrix},$$

$$K_1(l-0,0) = \begin{bmatrix} k_1^1 \\ k_2^1 \end{bmatrix}, \qquad K_1(2l,l+0) = \begin{bmatrix} k_1^2 \\ k_2^2 \end{bmatrix}.$$

If the principal determinant of this system is different from zero, then we get unique values for the unknowns. Thus, we find $y(0)$ and $\lambda(0)$. Then from (87) we get the optimal control, which we will consider as program optimal control. Then, if we solve the equation (88) within the initial data $y(0)$ and $\lambda(0)$ we find the values of the functions $y(x)$ and $\lambda(x)$ in the interval $(0,\ l-0)$, then using relation (85) we find $y(l+0)$ and $\lambda(l+0)$. Then, using these values from equation (89) as initial data, we find the values of the functions $y(x)$ and $\lambda(x)$ already in the interval $(l+0,\ 2l)$. So, solving the problem, we define the program trajectories and control $y_{pr}(x)$ and u_{pr} .

In the **second sub-chapter** of the third chapter, the general picture of the partial periodic discrete optimal control problem with initial control influences is considered. To solve this problem, an algorithm for finding optimal trajectories and controls is proposed with the help of the discrete Euler-Lagrange equation. Analogously, this work can be viewed as a continuous case.

In the **third sub-chapter** of the third chapter, the issue of establishing control systems for the operation of oil wells using the gas lift method is considered. To solve this problem through an averaging over time derived from special derivative nonlinear differential equations nonlinear ordinary differential equations with initial conditions are studied. An optimal control problem is set for it, and software trajectories and controls are found for the set problem. The issue of appropriate stabilization around these found program trajectories and controls is addressed in a later step.

In the **fourth sub-chapter** of the third chapter, the issue of optimal stabilization around the program trajectory and control given in an infinite interval during the exploitation of oil wells by the gas lift method is of greater interest, which is partially brought to the solution of the issue of periodic optimization.

Let's assume that the movement of gas in an annular space is described by a system of linear differential equations

$$\dot{y} = F_1 y, \qquad 0 < x < l \tag{103}$$

with initial conditions

$$y(0) = y_0 = u \ , \tag{104}$$

where mixing of gas with oil in the reservoir in line l is determined by

$$y(l+0) = F_\delta y_q(l-0) + F_\chi y_\chi(l-0)\ , \tag{105}$$

and giving the liquid-gas mixture to the outlet is described by the equation

$$\dot{y} = F_2 y, \qquad l < x < 2l \ . \tag{106}$$

Let's put the problem of optimal stabilization as follows: choose the initial volume u of the gas from (103) so that, the quadratic functional

$$J = u'Cu + \sum_{k=1}^{\infty} y'(kl+0)\, Q_1 y(kl+0)$$

$$+ \int_0^{\infty} y'(x) Q y(x) dx \tag{107}$$

let extreme grades within the

$$y(l+0) = y(2l) \tag{108}$$

and the asymptotic stability condition of the closed system (104), (105), (106).

It is proved in the thesis work that in solving the problem (103) - (108)

$$u = -C^{-1} F_\delta e^{F_1 l} F_\chi^{1-1} (S(l-0) Q_1) y(l-0) \tag{109}$$

where $S = S(l-0) = S(2l)$ satisfies the following algebraic Riccati equation

$$S = \bar{\psi}(0,2)S[E + \bar{M}(0,2)S]^{-1}\bar{\psi}(0,2) + \bar{R}(0,2) \qquad (110)$$

where

$$\left.\begin{array}{c} \psi(0) = F_{\chi}, \quad \psi(1) = e^{F_2 l} \\ M(0) = F'_{\delta} e^{F'_1 l} C^{-1} F_{\delta} e^{F_1 l}, \qquad M(1) = 0 \\ R(0) = Q_1, \quad R(1) = e^{F_2 l} H_2 \end{array}\right\} \qquad (111)$$

$$\left\{\begin{array}{c} \bar{\psi}(0,2) = \psi(1)Q(0,1)\bar{\psi}(0,1), \quad \bar{\psi}(0,1) = \psi(0), \\ \bar{M}(0,2) = M(1)Q(0,1)\bar{M}(0,1)\psi'(1), \quad \bar{M}(0,1) = M(0) \\ \bar{R}(0,2) = \bar{R}(0,1)\bar{\psi}'(0,1)R(1)Q(0,1)\bar{\psi}(0,1), \quad \bar{R}(0,1) = R(0) \\ Q(0,1) = [E + \bar{M}(0,1)R(1)]^{-1} = [E + M(0)R(1)]^{-1} \end{array}\right. \qquad (112)$$

It is proved that if the eigenvalues of the following system are inside the unit circle

$$y(2l) = (E + M(0,2)S(2l))^{-1}\bar{\psi}(0,2)y(l-0)\,, \qquad (113)$$

then the matrix algebraic Riccati equation (110) has a positive definite solution and vice versa.

If the eigenvalues of the matrix lie within the unit circle, then the matrix u also has this property. It follows that in the case of $k \to \infty$

$$y(2kl) \to 0 \qquad (114)$$

and this means that in the case $k \to \infty$

$$y(x) \to y_{pr}(x). \qquad (115)$$

Here y_{pr} is the solution of a suitable optimization problem in a finite interval.

CONCLUSION

Summarizing the research conducted in the dissertation work, the following can be noted as the main results:

1. A new method based on the "sweep" algorithm is given for solving the continuous and discrete linear quadratic optimization problem with a non-separating boundary condition, where, unlike the previous ones, the transition at those points in multi-point optimization problems is single-valued;

2. An algorithm for determining the coefficient of hydraulic resistance in different parts of the pump-compressor pipes is proposed. So, this method is simplified and proposed as an asymptotic algorithm for long-distance oil transmission by introducing a small parameter;

3. Applying fluid-damped oscillatory systems to oil extraction, the movement of the plunger in the rod pump device is expressed in a periodic rejime and the solution is established;

4. In continuous and discrete systems controlled by initial data, the solution of the optimization problem satisfying the periodicity in one part is established, a suitable algorithm is proposed, and the use of the gas lift method in oil extraction is proposed;

5. The construction of optimal regulators controlled by initial data in the stationary case is considered and applied to oil production.

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