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ABSTRACT

of the dissertation for the scientific degree of Doctor of Philosopy

SOME STATİC AND DYNAMİC PROBLEMS OF ELASTİCİTY THEORY FOR A TRANSVERSALLY-İSOTROPİC SPHERİCAL SHELL

Specialty: 2002.01- Mechanics of deformable solid

Field of science: Mathematics

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GENERAL CHARACTERİSTİCS OF THE WORK

Rationale and development degree of the topic. Covers are widely used constructions in nuclear reactors, space vehicles, aviation, rocket making, shipbuilding, making reservoirs, and construction in modern technology. Applied theories based on various hypotheses are used during the calculation of covers. Determining the areas of application of those theories and the need to create new applied theories requires the study of coatings based on the equations of elasticity theory. The study of the stress-strain state of coatings based on the equations of the theory of elasticity is a difficult mathematical problem. Methods of solving three-dimensional problems for coatings are given in the works of A.Byuffler, K.Friedrichs, R.Dressler, I.I.Vorovich, Y.A.Ustinov, A.L.Goldenweiser, V.A.Lomakin, V.P.Plevako, M.F.Mehdiyev, S.D.Akbarov and other scientists.

Object and subject of the study. Applicantion of asymptotic methods for studying stress-strain state of a small-thickness, radial, inhomogeneous transversally isotropic spherical shell.

Goals and objectives of the study. To define the character of stress-strain state of a radial inhomogeneous transversally isotropic spherical shell with no $0,\ \pi$ poles whose elasticity radius change according to the linear law with respect to the radius. On the basis of elasticity theory equations to obtain asymptotic expressions; to calculate the displacement vector and stress tensor components; to study dynamic problem of elasticity theory with respect to the axis for a transverse-isotropic spherical shell with a lateral surface free from load; to construct exact and asymptotic solutions to the problems of torsional vibration of a transverse-isotropic spherical shell.

Research methods. The research methods are based on the metods of asymptotic integration of elasticity theory equations, homogeneous solutions.

The main thesis to be definded.

- Studying torsional problems of elasticity theory for a radial inhomogeneous transversally isotropic spherical shell symmetric with respect to the axis.

- Constructing inhomogeneous solutions and their classification.

- Estimating the influence of inhomogeneity of the material of a transversally-isotropic spherical shell on the stress-strain state.

- Studying torosional vibrations of a transverssaly-isotropic spherical shell.

- Analysing dynamical problems of elasticity theory for a transversally-isotropic spherical shell with respect to the axis.

- defining asymptotic expressions enabling to calculate stress-strain state for finite and high frequency vibrations.

Scientific novelty of the study. The main results obtained in the dissertation work are the followings:

- A symmetric problem of elasticity theory for a radial inhomogeneous transversally-isotropic spherical shell with no 0, π poles and with a lateral surface free of load, with studied. Exact and asymptotic solutions of the problem were structured.

Asymptotic expressions for the displacement vector and stress tensor components were obtained. It was shown that the homogeneous solution consists of the sum of expanded solutions that is equivalent to the principal vector of stresses acting in the section $\theta = const$ of the spherical shell, simple boundary effect and boundary layer character solutions. The character of the stress-strain state defined by these solutions is studied. The possibility of the change in stress-strain state of the boundary layer character solution located on the seats of the spherical shell (in conic sections) expanding interior to the domain for from the seats, was defined.

- A problem of elasticity theory symmetric with respect to the axis for a radial inhomogeneous transversally isotropic spherical shell with no 0, π poles and with fixed lateral surface was studied. It was shown that the homogeneous solution consists of a boundary layer character solutions equivalent to the Sain-Venant boundary effect for inhomogeneous transversal-isotropic plates localized only in conic sections.

- Torsional problems were studied for a radial inhomogeneous shperical shell in the case when the lateral surface of the spherical shell is free from stress and the lateral surface of the spherical shell is closed.In the case when the lateral surface of the shell is free from stress, it was shown that the ingomogeneous solution consists of the expanded solution propertional to the torgue of stresses acting in the section $\theta = const$ and boundary layer solutions damping by the exponential law by moving a way from conical section. When the lateral surface is closed, in was defined that the problem has only a boundary layer character.

- The dynamic problem of elasticity theory symmetric with respect to the axis was studied for a transversally-isotropic spherical shell with lateral surface free from load by the asymptotic integration method, finite and high frequency vibrations of the spherical shell was studied. Asymptotic expressions enabling to calculate the stress-strain state at various values of frequency were obtained.

- Given various boundary conditions in the lateral surface of a transversally-isotropic shperical shell, the torsional vibrations of the spherical shell were studied.

General methodology of the study. The methodology of thye research is based on asymptotic integration of elasticity theory equations.

Theoretical and practical value of the study. This work is of theoretical character. A new class of solutions that can not be described by applied theories of shells are determined. The obtained asymptotic formulas enable to calculate stress-strain state of a radial inhomogeneous cylindrical shell, to estimate the application domain of various applied theories existing for a cylindrical shell and to build more exact applied theories.

Approbation and application of the work. The results of the dissertation work was reported at the conference ”Classic and modern problems of mechanics” devoted to 100-th jubilee of prof. A.Amenzadeh (Baku, 2014), The XLIV International conference “Russian science in modern world” (Moscow, 2022, 28 February), in the XIV scientific practical conference “Young scientists of Russian Federation” (2022, 23 August), in the 7th International conference on “Control and optimization with industrial applications” (COİA-2020, Baku, 26-28 august 2020), at the scientific seminar of the chair of “Mathematics and Statistics” of UNEC.

Author’s personal contribution. All the results of the work except the problem statement belong to the author.

Author's publications. 5 papers in the publications recommended by the HAC at the President of the Republic of Azerbaijan, 4 in proceeding of conferences.

The name of the organization where the work was executed. Dissertation work was carried out at the "Deformable solid mechanics" department of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

Total volume of the dissertation work indicating separately the volume of structural units in signs.

The 138 page dissertation work consists of introduction, 2 chapter and the used references. The total volume of the work is 233823 signs (title page -409, contents -2447, introduction-18967, chapter I-124000, chapter II -88000). The work contains 1 figure, 10 graphs and a list of refernces with 98 names.

THE MAIN CONTENT OF THE DISSERTATION WORK

Rationale of the topic, review of the works related to the topic, goal of the study and brief content of the work are given in the **introduction.**

Chapter I is called "Static problems of elasticity theory for radially inhomogeneous transversally-isotropic spherical coating".

In chapter asymptotic theory of a radial inhomogeneous small-thickness transversally isotropic spherical shell whose elasicity modulus change with respect to the radius changing with a linear law, is interpreted.

In **1.1** we state a boundary value problem for a radial inhomogeneous transversally-isotropic spherical shell with no $0\,,\pi$ poles and with a lateral surface free from load.

In the spherical coordinate system we consider a problem of elasticity theory for a radial inhomogeneous small-thiskness transversally –isotropic shperical-shell with volume $\Gamma=\{r\in[r_1;\, r_2],\ \theta\in[\theta_1;\, \theta_2],\ \varphi\in[0;\, 2\pi]\}$ and with no 0, π poles, symmetric with respect to the axis.

İt is assumed that the elasticity modulus change with respect to the radius by the linear law

$$A_{11}=a_{11}^{(0)}r,\ \ A_{12}=a_{12}^{(0)}r,\ \ A_{22}=a_{22}^{(0)}r,\ \ A_{23}=a_{23}^{(0)}r,\ \ A_{44}=a_{44}^{(0)}r,$$

($a_{11}^{(0)}, a_{12}^{(0)}, a_{22}^{(0)}, a_{23}^{(0)}, a_{44}^{(0)}$ are constant values).

The expression for equilibrium equations in the spherical coordinate system is as follows:

$$b_{11}^{(0)}\left(\frac{\partial^2 u_\rho}{\partial\rho^2}+2\varepsilon\frac{\partial u_\rho}{\partial\rho}\right)+2\left(2b_{12}^{(0)}-b_{22}^{(0)}-b_{23}^{(0)}\right)\varepsilon^2 u_\rho+$$

$$+\left(2b_{12}^{(0)}-b_{23}^{(0)}-b_{22}^{(0)}-b_{44}^{(0)}\right)\times$$

$$\times\varepsilon^2\left(\frac{\partial u_\theta}{\partial\theta}+u_\theta ctg\theta\right)+\varepsilon\left(b_{44}^{(0)}+b_{12}^{(0)}\right)\left(\frac{\partial u_\theta}{\partial\rho}ctg\theta+\frac{\partial^2 u_\theta}{\partial\theta\partial\rho}\right)+$$

$$+\varepsilon^2 b_{44}^{(0)}\left(\frac{\partial^2 u_\rho}{\partial\theta^2}+\frac{\partial u_\rho}{\partial\theta}ctg\theta\right)=0, \tag{1}$$

$$b_{44}^{(0)}\left(\frac{\partial^2 u_\theta}{\partial\rho^2}+2\varepsilon\frac{\partial u_\theta}{\partial\rho}-3\varepsilon^2 u_\theta+\varepsilon\frac{\partial^2 u_\rho}{\partial\rho\partial\theta}\right)+\left(3b_{44}^{(0)}+b_{22}^{(0)}+b_{23}^{(0)}\right)\times$$

$$\times\varepsilon^2\frac{\partial u_\rho}{\partial\theta}+\varepsilon^2 b_{22}^{(0)}\left(\frac{\partial u_\theta}{\partial\theta}ctg\theta+\frac{\partial^2 u_\theta}{\partial\theta^2}\right)+\varepsilon b_{12}^{(0)}\frac{\partial^2 u_\rho}{\partial\rho\partial\theta}-$$

$$-\varepsilon^2\left(b_{22}^{(0)}ctg^2\theta+b_{23}^{(0)}\right)u_\theta=0, \tag{2}$$

$$b_{44}^{(0)}\left(\frac{\partial^2 u_\varphi}{\partial\rho^2}+2\varepsilon\frac{\partial u_\varphi}{\partial\rho}-3\varepsilon^2 u_\varphi\right)+$$

$$+\frac{\left(b_{22}^{(0)}-b_{23}^{(0)}\right)\varepsilon^2}{2}\left(\frac{\partial^2 u_\varphi}{\partial\theta^2}+\frac{\partial u_\varphi}{\partial\theta}ctg\theta-\frac{\cos 2\theta}{\sin^2\theta}u_\varphi\right)=0. \tag{3}$$

Here $\rho=\frac{1}{\varepsilon}\ln\left(\frac{r}{r_0}\right)$ is a new pure variable; $\varepsilon=\frac{1}{2}\ln\left(\frac{r_2}{r_1}\right)$ is a small parameter characterizing the thickness of the spherical shell;

$r_0 = \sqrt{r_1 r_2}$; $\rho \in [-1; 1]$; $b_{ij}^{(0)} = \dfrac{a_{ij}^{(0)} r_0}{G_0}$ are pure variables; G_0 is a quantity with the dimension of the elastic module.

It is assumed that the lateral surface of the spherical shell is free from stresses:

$$\sigma_{\rho\rho} = \frac{1}{\varepsilon}\left[b_{11}^{(0)} \frac{\partial u_\rho}{\partial \rho} + \varepsilon b_{12}^{(0)} \left(u_\theta ctg\theta + 2u_\rho + \frac{\partial u_\theta}{\partial \theta} \right) \right]\Bigg|_{\rho=\pm 1} = 0, \tag{4}$$

$$\sigma_{\rho\theta} = \frac{1}{\varepsilon} b_{44}^{(0)} \left[\frac{\partial u_\theta}{\partial \rho} + \varepsilon \left(\frac{\partial u_\rho}{\partial \theta} - u_\theta \right) \right]\Bigg|_{\rho=\pm 1} = 0, \tag{5}$$

$$\sigma_{\rho\varphi} = \frac{b_{44}^{(0)}}{\varepsilon} \left(\frac{\partial u_\varphi}{\partial \rho} - \varepsilon u_\varphi \right)\Bigg|_{\rho=\pm 1} = 0 \tag{6}$$

and on the seats of the spherical shell (on conic sections) the boundary conditions

$$\sigma_{\theta\theta}\big|_{\theta=\theta_s} = f_{1s}(\rho), \sigma_{\rho\theta}\big|_{\theta=\theta_s} = f_{2s}(\rho), \tag{7}$$

$$\sigma_{\varphi\theta}\big|_{\theta=\theta_s} = f_{3s}(\rho), \tag{8}$$

retaining it in equilibrium, are given.

Here $f_{1s}(\rho)$, $f_{2s}(\rho)$, $f_{3s}(\rho)$ $(s = 1;2)$ are smooth functions satisfying the equilibrium condition and of order $O(1)$ with respect to the parameter ε.

Problem (3), (6), (8) characterzes the torsion of a radial inhomogeneous spherical shell. The solution of the problem (1), (2), (4), (5) is sought in the form of

$$u_\rho(\rho,\theta) = a(\rho) m(\theta); \quad u_\theta(\rho,\theta) = d(\rho) m'(\theta), \tag{9}$$

In (9) the function $m(\theta)$

$$m''(\theta) + ctg\theta \cdot m'(\theta) + \left(z^2 - \frac{1}{4} \right) m(\theta) = 0, \tag{10}$$

is the solution of the Legendre equation.

Substituting (9) in (1),(2),(4),(5) and take into account (10):

$$b_{11}^{(0)}a''(\rho)+2\varepsilon b_{11}^{(0)}a'(\rho)+\varepsilon^2\times$$
$$\times\left[\left(4b_{12}^{(0)}-2b_{22}^{(0)}-2b_{23}^{(0)}\right)-\left(z^2-\frac{1}{4}\right)b_{44}^{(0)}\right]a(\rho)-\left(z^2-\frac{1}{4}\right)\times$$
$$\times\varepsilon\left[\left(b_{44}^{(0)}+b_{12}^{(0)}\right)d'(\rho)-\varepsilon\left(b_{44}^{(0)}+b_{22}^{(0)}+b_{23}^{(0)}-2b_{12}^{(0)}\right)d(\rho)\right]=0, \quad (11)$$
$$b_{44}^{(0)}\left(d''(\rho)+2\varepsilon d'(\rho)\right)-$$
$$-\varepsilon^2\left[\left(z^2-\frac{1}{4}\right)b_{22}^{(0)}+\left(b_{23}^{(0)}-b_{22}^{(0)}+3b_{44}^{(0)}\right)\right]d(\rho)+$$
$$+\varepsilon^2\left(3b_{44}^{(0)}+b_{22}^{(0)}+b_{23}^{(0)}\right)a(\rho)+\varepsilon\left(b_{44}^{(0)}+b_{12}^{(0)}\right)a'(\rho)=0, \quad (12)$$
$$b_{11}^{(0)}a'(\rho)+\varepsilon b_{12}^{(0)}\left(2a(\rho)-\left(z^2-\frac{1}{4}\right)d(\rho)\right)=0, \ \ \rho=\pm1 \ \text{ for } \quad (13)$$
$$b_{44}^{(0)}\left[d'(\rho)+\varepsilon\left(a(\rho)-d(\rho)\right)\right]=0, \ \ \rho=\pm1 \ \text{ for } \quad (14)$$
The solution of the system (11), (12) is as follows:
$$a(\rho)=e^{-\varepsilon\rho}\left[p_1e^{\varepsilon s_1\rho}C_1+p_1e^{-\varepsilon s_1\rho}C_2+p_2e^{\varepsilon s_2\rho}C_3+p_2e^{-\varepsilon s_2\rho}C_4\right], \quad (15)$$
$$d(\rho)=e^{-\varepsilon\rho}\left[h_1e^{\varepsilon s_1\rho}C_1+q_1e^{-\varepsilon s_1\rho}C_2+h_2e^{\varepsilon s_2\rho}C_3+q_2e^{-\varepsilon s_2\rho}C_4\right], \quad (16)$$
here C_n $(n=\overline{1,4})$ are arbitrary constants,
$$p_k=b_{44}^{(0)}s_k^2-\left(z^2-\frac{1}{4}\right)b_{22}^{(0)}+\left(b_{22}^{(0)}-b_{23}^{(0)}-4b_{44}^{(0)}\right),$$
$$h_k=-\left(b_{44}^{(0)}+b_{12}^{(0)}\right)s_k-\left(2b_{44}^{(0)}+b_{22}^{(0)}+b_{23}^{(0)}-b_{12}^{(0)}\right),$$
$$q_k=\left(b_{44}^{(0)}+b_{12}^{(0)}\right)s_k-\left(2b_{44}^{(0)}+b_{22}^{(0)}+b_{23}^{(0)}-b_{12}^{(0)}\right),$$
s_k – are the roots of the biquadratic equation
$$b_{11}^{(0)}b_{44}^{(0)}s^4+\left[\left(z^2-\frac{1}{4}\right)\left((b_{12}^{(0)})^2+2b_{44}^{(0)}b_{12}^{(0)}-b_{11}^{(0)}b_{22}^{(0)}\right)+b_{11}^{(0)}b_{22}^{(0)}-\right.$$
$$-2b_{22}^{(0)}b_{44}^{(0)}-b_{11}^{(0)}b_{23}^{(0)}-5b_{11}^{(0)}b_{44}^{(0)}+4b_{12}^{(0)}b_{44}^{(0)}-$$
$$\left.-2b_{23}^{(0)}b_{44}^{(0)}\right]s^2+\left[\left(z^2-\frac{1}{4}\right)b_{44}^{(0)}-4b_{12}^{(0)}+2b_{22}^{(0)}+2b_{23}^{(0)}+b_{11}^{(0)}\right]\times$$

$$\times\left[\left(z^2-\frac{1}{4}\right)b_{22}^{(0)}+b_{23}^{(0)}-b_{22}^{(0)}+4b_{44}^{(0)}\right]-$$

$$-\left(z^2-\frac{1}{4}\right)\left(2b_{44}^{(0)}+b_{22}^{(0)}+b_{23}^{(0)}-b_{12}^{(0)}\right)^2=0$$

Having substituted (15), (16) in the boundary conditions (13), (14), from the existence of nontrivial solution of the system of linear equations obtained with respect to the constants C_1, C_2, C_3, C_4 we define the following characteristic equation

$$\Delta_1(z,\varepsilon)=(Q_{21}D_{22}-Q_{22}D_{21})(Q_{12}D_{11}-Q_{11}D_{12})sh^2(\varepsilon(s_1+s_2))+$$
$$+(Q_{21}D_{12}-Q_{12}D_{21})(Q_{11}D_{22}-Q_{22}D_{11})sh^2(\varepsilon(s_2-s_1))=0. \qquad (17)$$

Here

$$Q_{ik}=(-1)^{i-1}b_{11}^{(0)}b_{44}^{(0)}s_k^3+b_{44}^{(0)}\left(2b_{12}^{(0)}-b_{11}^{(0)}\right)s_k^2+$$
$$+(-1)^{i-1}\left[b_{11}^{(0)}\left(b_{22}^{(0)}-b_{23}^{(0)}-4b_{44}^{(0)}\right)+\left(z^2-\frac{1}{4}\right)\left(b_{12}^{(0)}b_{44}^{(0)}+(b_{12}^{(0)})^2-\right.\right.$$
$$\left.\left.-b_{11}^{(0)}b_{22}^{(0)}\right)\right]s_k+\left(z^2-\frac{1}{4}\right)\left(b_{11}^{(0)}b_{22}^{(0)}-b_{12}^{(0)}b_{22}^{(0)}+2b_{44}^{(0)}b_{12}^{(0)}+b_{23}^{(0)}b_{12}^{(0)}-\right.$$
$$\left.-(b_{12}^{(0)})^2\right)+2b_{12}^{(0)}\left(b_{22}^{(0)}-b_{23}^{(0)}-4b_{44}^{(0)}\right)-b_{11}^{(0)}\left(b_{22}^{(0)}-b_{23}^{(0)}-4b_{44}^{(0)}\right);$$

$$D_{ik}=-b_{12}^{(0)}s_k^2+(-1)^{i-1}\left(3b_{12}^{(0)}-b_{22}^{(0)}-b_{23}^{(0)}\right)s_k-$$
$$-\left(z^2-\frac{1}{4}\right)b_{22}^{(0)}+3b_{22}^{(0)}+b_{23}^{(0)}-2b_{12}^{(0)};$$

$i=1;2$ and $k=1;2$.

The transcendental equation (17) has a denumerable number roots z_k.

In **1.2** as $\varepsilon\to 0$ the roots of the equation (17) is classified.

Theorem: as $\varepsilon\to 0$ the of roots $\Lambda(z)$ of the equation (17) is a denumrable set and the expansion

$$\Lambda(z)=\Lambda_1(z)\cup\Lambda_2(z,\varepsilon)\cup\Lambda_3(z,\varepsilon).$$

is valid:

1. The set $\Lambda_1(z)$ consists of the roots $z=\pm 1.5$.

2. The set $\Lambda_2(z,\varepsilon)$ consists of $O(\varepsilon^{-1/2})$ -th order four roots

$$z_k = \varepsilon^{-\frac{1}{2}}\left(z_{0k} + \varepsilon z_{1k} + ---\right) \tag{18}$$

Here

$$z_{0k}^4 = -\frac{3\ell_1}{\ell_2}; \quad z_{1k} = \frac{5\ell_0\ell_2^2 - \ell_1\left(5\ell_2\ell_3 - 2\ell_1\ell_6\right)}{20\ell_1\ell_2^2 z_{ok}};$$

$$\ell_i = \ell_i\left(b_{11}^{(0)}, b_{12}^{(0)}, b_{22}^{(0)}, b_{23}^{(0)}, b_{44}^{(0)}\right).$$

3. The set $\Lambda_3(z,\varepsilon)$ consists of $O(\varepsilon^{-1})$ th order denumerable number roots

$$z_k = \frac{\delta_{0k}}{\varepsilon} + O(\varepsilon). \tag{19}$$

δ_{0k} contained in (19) are determined as follows:

1^0. for $b_1 > 0,\ b_1^2 - b_2 > 0$

$$(g_1 + g_2)\sin\left((g_1 - g_2)\delta_{0k}\right) \pm (g_1 - g_2)\sin\left((g_1 + g_2)\delta_{0k}\right) = 0, \tag{20}$$

here

$$g_1 = \sqrt{b_1 + \sqrt{b_1^2 - b_2}}; \quad g_2 = \sqrt{b_1 - \sqrt{b_1^2 - b_2}};$$

$$b_1 = \left(2b_{44}^{(0)}b_{11}^{(0)}\right)^{-1}\left(2b_{44}^{(0)}b_{12}^{(0)} + (b_{12}^{(0)})^2 - b_{11}^{(0)}b_{22}^{(0)}\right); \ b_2 = b_{22}^{(0)}(b_{11}^{(0)})^{-1}.$$

2^0. for $b_1 > 0,\ b_1^2 - b_2 < 0$

$$\alpha\sin(2\beta\delta_{0k}) \pm \beta sh(2\alpha\delta_{0k}) = 0, \tag{21}$$

here

$$g_1 = \sqrt{b_1 + \sqrt{b_1^2 - b_2}} = \pm(\alpha + i\beta);$$

$$g_2 = \sqrt{b_1 - \sqrt{b_1^2 - b_2}} = \pm(\alpha - i\beta).$$

3^0. for $b_1 > 0,\ b_1^2 = b_2$

$$\sin(2g\delta_{0k}) \pm 2g\delta_{0k} = 0, \tag{22}$$

here $g = \sqrt{b_1}$.

4^0. for $b_1 < 0,\ b_1^2 - b_2 > 0$

$$(g_1+g_2)sh\left((g_1-g_2)\delta_{0k}\right)\pm(g_1-g_2)sh\left((g_1+g_2)\delta_{0k}\right)=0, \qquad (23)$$

here $\qquad g_1=\sqrt{|b_1|-\sqrt{b_1^2-b_2}}; \quad g_2=\sqrt{|b_1|+\sqrt{b_1^2-b_2}}.$

5^0. for $b_1<0,\ b_1^2-b_2<0$

$$\alpha\, sh\left(2\beta\delta_{0k}\right)\pm\beta\sin\left(2\alpha\delta_{0k}\right)=0, \qquad (24)$$

here

$$g_1=\sqrt{|b_1|-\sqrt{b_1^2-b_2}}=\pm(\alpha-i\beta);$$

$$g_2=\sqrt{|b_1|+\sqrt{b_1^2-b_2}}=\pm(\alpha+i\beta).$$

6^0. for $b_1<0,\ \ b_1^2=b_2$

$$sh(2g\delta_{0k})\pm 2g\delta_{0k}=0, \qquad (25)$$

here $g=\sqrt{|b_1|}$.

In 1.3, the solutions corresponding to the above defined roots of the characteristic equation and the general solution

$$u_\rho(\rho,\theta)=u_\rho^{(1)}+u_\rho^{(2)}+u_\rho^{(3)}, \qquad (26)$$

$$u_\theta(\rho,\theta)=u_\theta^{(1)}+u_\theta^{(2)}+u_\theta^{(3)}, \qquad (27)$$

of the problem (1), (2), (4), (5) are constructed.

In 1.4, the solutions are classified and the nature of the stress-strain state corresponding to them is determined. It is shown that the solution of (17) corresponding to the root $z=1.5$ characterizes the motion of the sphere as an absolute solid body. According to the root $z=-1.5$ of equation (17)

$$u_\rho^{(1)}(\rho,\theta)=B\left(\cos\theta\cdot\ln\left(ctg\frac{\theta}{2}\right)-1\right), \qquad (28)$$

$$u_\theta^{(1)}(\rho,\theta)=-B\left(\sin\theta\cdot\ln\left(ctg\frac{\theta}{2}\right)+ctg\theta\right), \qquad (29)$$

the solution is the spreading solution and is equivalent to the principal vector of the stresses acting on the section $\theta=const$:

$$P = 2\pi\left(b_{23}^{(0)} - b_{22}^{(0)}\right)B\, sh(2\varepsilon). \tag{30}$$

The solution

$$.\, u_{\rho}^{(2)}(\rho;\theta) = \sum_{k=1}^{4} M_k \left[1 + \left(\frac{2(e_2 - e_1)}{e_1}\rho - z_{0k}^2\rho^2\right)\frac{b_{12}^{(0)}}{2b_{11}^{(0)}}\varepsilon + O(\varepsilon^2)\right] m_k(\theta), \tag{31}$$

$$u_{\theta}^{(2)}(\rho;\theta) = \sum_{k=1}^{4} \varepsilon M_k \left[-\rho + \frac{e_1 + e_2}{z_{0k}^2 e_1} + O(\varepsilon)\right] m_k'(\theta), \tag{32}$$

corresponding to the roots of (18) included in the set $\Lambda_2(z;\varepsilon)$ is a boundary effect solution is a boundary effect solution.

The stresses determined by the solutions corresponding to the roots included in sets $\Lambda_2(z;\varepsilon)$ and $\Lambda_3(z;\varepsilon)$ are self-equilibrating in arbitrary section $\theta = const$.

Solutions (28), (29), (31), (32) determine the internal stress-strain state of a non-homogeneous transverse-isotropic spherical coating of small thickness. The first limits of the deviations of the sum of those solutions to the parameter ε are the same as the solutions obtained from the theories of application of covers. The solutions corresponding to the roots of (19) included in the set $\Lambda_3(z;\varepsilon)$ have a boundary layer character and those solutions are not determined by any applied theory. Boundary layer solutions are localized around the seats (conical sections) of the spherical cover, and the first limit of those solutions is equivalent to the Saint-Venant boundary effect in the theory of inhomogeneous transverse-isotropic plates. For real δ_{0k}, the boundary-layer characteristic solutions are weakly damped, and those solutions propagate into the domain, changing the stress-strain state far from the conic sections. In this case, the stress-strain states of non-homogeneous transverse-isotropic and non-homogeneous isotropic spherical coatings differ qualitatively. When δ_{0k} are purely imaginary and take complex values, the stress-strain state of the inhomogeneous transverse-isotropic spherical coating is qualitatively the same as the stress-strain state of the inhomogeneous isotropic spherical coating, and they differ only in the extinction rates of the boundary layer solutions.

$\theta = \theta_j \, (j = 1,2)$ when moving away from the conic sections, the boundary effect and boundary layer characteristic solutions disappear by exponential law.

In 1.5, the issue of satisfying the given boundary conditions on the seats (conic sections) of the spherical cover is considered. According to (30), the constant B is determined by means of the head vector P. A system of finite and infinite linear algebraic equations known from the theory of transversal-isotropic inhomogeneous plates is obtained in accordance with the determination of the constants included in the simple boundary effect and boundary layer characteristic solutions.

In 1.6, the problem of torsion of a radially inhomogeneous transversal-isotropic small-thickness spherical cover (3), (6), (8) is studied, where the lateral surface is free of load, and the boundary conditions that keep it in equilibrium at its seats are given. It is shown that the general solution of the problem consists of the sum of the spreading solution, which is proportional to the torsional moment of the stress acting on the $\theta = const$ cross-sections, and the localized boundary layer solutions in the conical sections. In contrast to the isotropic inhomogeneous spherical coating, it is determined that some boundary layer characteristic solutions are weakly damped in a small-thickness transversal-isotropic inhomogeneous spherical coating.

In 1.7 solving numerically the boundary value problems for a radial inhomogeneous and homogeneous transversally isotropic spherica shell, distribution of displacement vector and stress tensor components along the radius, is studied.

In 1.8, the axisymmetric problem of elasticity theory is studied for a transverse-isotropic radial inhomogeneous spherical cover with a small thickness, whose lateral surface is closed, and the boundary conditions that keep it in equilibrium at its seats (conic sections) are given. It is shown that the homogeneous solution consists only of the boundary layer solution.

Chgapter II is called ”Asymptotic analysis of dynamic problems of elasticity theory for a transversally-isotropic spherical shell”. Here we give asymptotic analysis of dynamical problem for a small-thickness transversally-isotropic spherical shell.

In 2.1 we state a dynamical problem of elasticity theory symmetric with respect to the axis for a small-thickness transversally isotropic spherical shell with volume $\Gamma=\{r\in[r_1;\,r_2],\quad \theta\in[\theta_1;\,\theta_2],\ \varphi\in[0;\,2\pi]\}$ and with no 0, π poles.

The expression of the equation of motion in coordinate system with displacements is as follows :

$$b_{11}\frac{\partial^2 u_\rho}{\partial\rho^2}+\frac{2\varepsilon}{1+\varepsilon\rho}b_{11}\frac{\partial u_\rho}{\partial\rho}+\frac{\varepsilon^2}{(1+\varepsilon\rho)^2}\times$$
$$\times\left[\frac{\partial^2 u_\rho}{\partial\theta^2}+\frac{\partial u_\rho}{\partial\theta}ctg\theta+2(b_{12}-b_{22}-b_{23})u_\rho\right]+(1+b_{12})\frac{\varepsilon}{1+\varepsilon\rho}\times$$
$$\times\left(\frac{\partial u_\theta}{\partial\rho}ctg\theta+\frac{\partial^2 u_\theta}{\partial\theta\partial\rho}\right)+(b_{12}-b_{22}-b_{23}-1)\frac{\varepsilon^2}{(1+\varepsilon\rho)^2}\times$$
$$\times\left(\frac{\partial u_\theta}{\partial\theta}+u_\theta ctg\theta\right)=\varepsilon^2\frac{\partial^2 u_\rho}{\partial\tau^2}, \tag{33}$$

$$\frac{\partial^2 u_\theta}{\partial\rho^2}+\frac{2\varepsilon}{1+\varepsilon\rho}\frac{\partial u_\theta}{\partial\rho}+\frac{\varepsilon^2}{(1+\varepsilon\rho)^2}\times$$
$$\times\left[b_{22}\left(\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{\partial u_\theta}{\partial\theta}ctg\theta\right)-(b_{23}+b_{22}ctg^2\theta+2)u_\theta\right]+$$
$$+(1+b_{12})\frac{\varepsilon}{1+\varepsilon\rho}\frac{\partial^2 u_\rho}{\partial\theta\partial\rho}+(b_{22}+b_{23}+2)\frac{\varepsilon^2}{(1+\varepsilon\rho)^2}\frac{\partial u_\rho}{\partial\theta}=\varepsilon^2\frac{\partial^2 u_\theta}{\partial\tau^2}. \tag{34}$$

Here $\rho=\dfrac{r-r_0}{\varepsilon r_0}$ is a new pure variable, $\tau=\dfrac{t}{r_0}\sqrt{\dfrac{A_{44}}{g}}$ is a dimensionless time; $\varepsilon=\dfrac{r_2-r_1}{2r_0}$ is a small parameter characterizing the thickness of the spherical shell; $r_0=\dfrac{r_1+r_2}{2}$ is a radius of the mediam surface of the spherical shell; g is the density of the spherical shell material;

$\rho \in [-1;1]$; $u_\rho = u_\rho(\rho,\theta,\tau)$, $u_\theta = u_\theta(\rho,\theta,\tau)$ are displacement vector components.

It is assumed that the lateral surface of the cylindrical shell is free from stresses:

$$\begin{cases} \sigma_{\rho\rho} = \left.\left(\dfrac{1}{\varepsilon} b_{11} \dfrac{\partial u_\rho}{\partial \rho} + \dfrac{b_{12}}{1+\varepsilon\rho}\left(u_\theta ctg\theta + 2u_\rho + \dfrac{\partial u_\theta}{\partial \theta}\right)\right)\right|_{\rho=\pm 1} = 0, \\ \sigma_{\rho\theta} = \left.\left(\dfrac{1}{\varepsilon} \dfrac{\partial u_\theta}{\partial \rho} + \dfrac{1}{1+\varepsilon\rho}\left(\dfrac{\partial u_\rho}{\partial \theta} - u_\theta \right)\right)\right|_{\rho=\pm 1} = 0, \end{cases} \tag{35}$$

and the boundary conditions

$$\sigma_{\theta\theta}\big|_{\theta=\theta_s} = f_{1s}(\rho)e^{i\lambda\tau}, \ \sigma_{\rho\theta}\big|_{\theta=\theta_s} = f_{2s}(\rho)e^{i\lambda\tau}, \ (s=1;2) \tag{36}$$

are given on the seats (conic sections) of the spherical shell (here λ is vibration frequency).

The solution of the boundary value problem (33)-(35) is sought in the form of

$$u_\rho = a(\rho)m(\theta)e^{i\lambda\tau}, \quad u_\theta = c(\rho)m'(\theta)e^{i\lambda\tau}, \tag{37}$$

Substituting (37) in (33)-(35) as a result we obtain the following spectral problem

$$b_{11}a''(\rho) + \frac{2\varepsilon}{1+\varepsilon\rho} b_{11}a'(\rho) + \left[\left(2(b_{12} - b_{22} - b_{23}) - \left(z^2 - \frac{1}{4}\right)\right) \times \right.$$

$$\left. \times \frac{1}{(1+\varepsilon\rho)^2} + \lambda^2 \right]\varepsilon^2 a(\rho) + \left(z^2 - \frac{1}{4}\right)\left[(b_{22} - b_{12} + b_{23} + 1)\frac{\varepsilon^2}{(1+\varepsilon\rho)^2} \times \right.$$

$$\left. \times c(\rho) - (1+b_{12})\frac{\varepsilon}{1+\varepsilon\rho} c'(\rho) \right] = 0, \tag{38}$$

$$c''(\rho) + \frac{2\varepsilon}{1+\varepsilon\rho} c'(\rho) + \left[\lambda^2 - \left(\left(z^2 - \frac{5}{4}\right) b_{22} + b_{23} + 2 \right)\frac{1}{(1+\varepsilon\rho)^2} \right]\varepsilon^2 c(\rho) +$$

$$+\frac{\varepsilon}{1+\varepsilon\rho}(1+b_{12})a'(\rho)+\frac{\varepsilon^2}{(1+\varepsilon\rho)^2}(2+b_{22}+b_{23})a(\rho)=0, \quad (39)$$

$$\left[b_{11}a'(\rho)+\frac{\varepsilon}{1+\varepsilon\rho}b_{12}\left(2a(\rho)-\left(z^2-\frac{1}{4}\right)c(\rho)\right)\right]\Bigg|_{\rho=\pm1}=0, \quad (40)$$

$$\left[c'(\rho)+\frac{\varepsilon}{1+\varepsilon\rho}(a(\rho)-c(\rho))\right]\Bigg|_{\rho=\pm1}=0 \quad (41)$$

In **2.2** as $\varepsilon\to0$ the spectral problem (38)-(41) is studied for finite λ by the asymptotic integration method. The following asymptotic expressions are obtained for the amplitude values of displacement vector components:

1) $$z=z_0+\varepsilon z_1+\varepsilon^2 z_2+\cdots$$

$$u_\rho^{(a)}=\sum_{k=1}^{2}T_k\left[\lambda^2+t_1+\left(z_{0k}^2-\frac{1}{4}\right)t_2+O(\varepsilon)\right]m_k(\theta), \quad (42)$$

$$u_\theta^{(a)}=\sum_{k=1}^{2}T_k\left[-t_3+O(\varepsilon)\right]m_k'(\theta), \quad (43)$$

here

$$z_{0k}^2=\left(t_2\lambda^2-t_3t_4\right)^{-1}\left(-\lambda^4+\frac{3}{4}\lambda^2t_5-\frac{1}{4}t_3t_6\right), \quad (44)$$

According to (44) mainly two numbers are determined as real or two numbers are determined as purely imaginary z_{0k}. The purely imaginary z_{0k} determine the extended solutions.

2) $$z=\varepsilon^{-\frac{1}{2}}\left(\alpha_0+\varepsilon\alpha_1+\varepsilon^2\alpha_2+\cdots\right)$$

$$u_\rho^{(a)}=\sum_{k=1}^{4}B_k\left[t_2\alpha_{0k}^4+(t_3+2t_2)\alpha_{0k}^2+O(\varepsilon)\right]m_k(\theta), \quad (45)$$

$$u_\theta^{(a)}=-\sum_{k=1}^{4}\varepsilon B_k\left\{\left[t_2\alpha_{0k}^4+(t_3+2t_2)\alpha_{0k}^2\right]\rho+t_9+\lambda^2-t_7+\right.$$

$$\left.+\frac{1}{3}t_2\alpha_{0k}^4+t_3\alpha_{0k}^2+O(\varepsilon)\right\}m_k'(\theta), \quad (46)$$

here

$$\alpha_{0k}^4 = 3b_{11}^{-1}t_2^{-2}\left(t_1\left(2b_{11}b_{22} - b_{11}t_3 - 2b_{12}^2\right) - b_{11}t_2\lambda^2\right). \qquad (47)$$

From (47) we determine four complex roots or two real, two purely imaginary roots. The complex roots correspond to the damping solution that is the analog of a simple boundary effect in the shell statics, the real roots to the expanded solution.

3)

$$z_k = \varepsilon^{-1}\left(\delta_{0k} + \varepsilon\delta_{1k} + \cdots\right) \qquad (48)$$

1. $e_1 > 0, \quad e_1^2 - e_2 > 0$,

$$S_k = \sqrt{e_1 + (-1)^{k+1}\sqrt{e_1^2 - e_2}}; \quad (k = 1,2)$$

We obtain the following two groups for the displacements :

a)

$$u_\rho^{(a)} = \sum_{k=1}^{\infty} D_k^{(1)}(1 + b_{12})S_1 S_2 \delta_{0k}^3 \left[\left(b_{12} - b_{11}S_2^2\right)\cos(\delta_{0k}S_2)\cos(\delta_{0k}S_1\rho) - \right.$$

$$\left. - \left(b_{12} - b_{11}S_1^2\right)\cos(\delta_{0k}S_1)\cos(\delta_{0k}S_2\rho) + O(\varepsilon)\right] m_k(\theta), \qquad (49)$$

$$u_\theta^{(a)} = \sum_{k=1}^{\infty} D_k^{(1)}\delta_{0k}^2 \left[-S_2\left(1 + b_{11}S_1^2\right)\left(b_{12} - b_{11}S_2^2\right)\sin(\delta_{0k}S_1\rho)\cos(\delta_{0k}S_2) + \right.$$

$$\left. + S_1\left(b_{12} - b_{11}S_1^2\right)\left(1 + b_{11}S_2^2\right)\cos(\delta_{0k}S_1)\sin(\delta_{0k}S_2\rho) + O(\varepsilon)\right] m_k'(\theta), \qquad (50)$$

here δ_{0k} are the solutions of the equation

$$(S_1 + S_2)\sin((S_1 - S_2)\delta_{0k}) - (S_1 - S_2)\sin((S_1 + S_2)\delta_{0k}) = 0$$

b)

$$u_\rho^{(a)} = \sum_{k=1}^{\infty} D_k^{(2)}(1 + b_{12})S_1 S_2 \delta_{0k}^3 \left[\left(b_{12} - b_{11}S_2^2\right)\sin(\delta_{0k}S_1\rho)\sin(\delta_{0k}S_2) - \right.$$

$$\left. - \left(b_{12} - b_{11}S_1^2\right)\sin(\delta_{0k}S_2\rho)\sin(\delta_{0k}S_1) + O(\varepsilon)\right] m_k(\theta), \qquad (51)$$

$$u_\theta^{(a)} = \sum_{k=1}^{\infty} D_k^{(2)}\delta_{0k}^2 \left[S_2\left(1 + b_{11}S_1^2\right)\left(b_{12} - b_{11}S_2^2\right)\cos(\delta_{0k}S_1\rho)\sin(\delta_{0k}S_2) - \right.$$

$$\left. - S_1\left(1 + b_{11}S_2^2\right)\left(b_{12} - b_{11}S_1^2\right)\cos(\delta_{0k}S_2\rho)\sin(\delta_{0k}S_1) + O(\varepsilon)\right] m_k'(\theta), \qquad (52)$$

here δ_{0k} are the solutions of the equation

$$(S_1 - S_2)\sin((S_1 + S_2)\delta_{0k}) + (S_1 + S_2)\sin((S_1 - S_2)\delta_{0k}) = 0,$$

2. For $e_1 > 0, \quad e_1^2 = e_2$,

$$S = \sqrt{e_1}.$$

for the displacements we obtain :

a)

$$u_\rho^{(a)} = \sum_{k=1}^{\infty} Q_k^{(1)} \left\{ 2b_{11} S^2 \cos(\delta_{0k} S) \cos(\delta_{0k} S\rho) + \left(b_{12} - b_{11} S^2 \right) \delta_{0k} S \times \right.$$

$$\left. \times \left[\sin(\delta_{0k} S) \cos(\delta_{0k} S\rho) - \rho \cos(\delta_{0k} S) \sin(\delta_{0k} S\rho) \right] + O(\varepsilon) \right\} m_k(\theta), \quad (53)$$

$$u_\theta^{(a)} = \sum_{k=1}^{\infty} Q_k^{(1)} \left\{ \frac{\left(1 + b_{11} S^2\right)\left(b_{11} S^2 - b_{12}\right)}{1 + b_{12}} \times \right.$$

$$\left[\sin(\delta_{0k} S) \sin(\delta_{0k} S\rho) + \rho \cos(\delta_{0k} S) \cos(\delta_{0k} S\rho) \right] +$$

$$\left. + \frac{b_{11}^2 S^4 + \left(3b_{11} + b_{12} b_{11}\right) S^2 - b_{12}}{\delta_{0k} S \left(1 + b_{12}\right)} \cos(\delta_{0k} S) \sin(\delta_{0k} S\rho) + O(\varepsilon) \right\} m_k'(\theta),$$

(54)

here δ_{0k} are the solutions of the equation

$$\sin(2\delta_{0k} S) - 2\delta_{0k} S = 0$$

b)

$$u_\rho^{(a)} = \sum_{k=1}^{\infty} Q_k^{(2)} \left\{ -2b_{11} S^2 \sin(\delta_{0k} S) \sin(\delta_{0k} S\rho) + \left(b_{12} - b_{11} S^2 \right) \delta_{0k} S \times \right.$$

$$\left. \times \left[\cos(\delta_{0k} S) \sin(\delta_{0k} S\rho) - \rho \sin(\delta_{0k} S) \cos(\delta_{0k} S\rho) \right] + O(\varepsilon) \right\} m_k(\theta), \quad (55)$$

$$u_\theta^{(a)} = \sum_{k=1}^{\infty} Q_k^{(2)} \left\{ \frac{\left(1 + b_{11} S^2\right)\left(b_{12} - b_{11} S^2\right)}{1 + b_{12}} \left[\cos(\delta_{0k} S\rho) \cos(\delta_{0k} S) + \right. \right.$$

$$\left. + \rho \sin(\delta_{0k} S) \sin(\delta_{0k} S\rho) \right] - \frac{b_{11}^2 S^4 + \left(b_{12} b_{11} + 3b_{11}\right) S^2 - b_{12}}{\left(1 + b_{12}\right) \delta_{0k} S}$$

$$\left. \times \sin(\delta_{0k} S) \cos(\delta_{0k} S\rho) + O(\varepsilon) \right\} m_k(\theta), \quad (56)$$

here δ_{0k} are the solutions of the equation

$$\sin(2\delta_{0k} S) + 2\delta_{0k} S = 0$$

3. For $e_1 < 0, \quad e_1^2 - e_2 > 0$, $\quad S_k = \sqrt{|e_1| + (-1)^{k+1}\sqrt{e_1^2 - e_2}}$ for the displacements we obtain:

a) $$u_\rho^{(a)} = \sum_{k=1}^{\infty} D_k^{(3)}(1+b_{12})S_1 S_2 \delta_{0k}^3 \left[\left(b_{12} + b_{11}S_1^2\right)ch(\delta_{0k}S_1)ch(\delta_{0k}S_2\rho) - \right.$$
$$\left. -\left(b_{12} + b_{11}S_2^2\right)ch(\delta_{0k}S_2)ch(\delta_{0k}S_1\rho) + O(\varepsilon)\right]m_k(\theta), \tag{57}$$
$$u_\theta^{(a)} = \sum_{k=1}^{\infty} D_k^{(3)}\delta_{0k}^2 \left[S_2\left(1 - b_{11}S_1^2\right)\left(b_{12} + b_{11}S_2^2\right)sh(\delta_{0k}S_1\rho)ch(\delta_{0k}S_2) - \right.$$
$$\left. - S_1\left(b_{12} + b_{11}S_1^2\right)\left(1 - b_{11}S_2^2\right)ch(\delta_{0k}S_1)sh(\delta_{0k}S_2\rho) + O(\varepsilon)\right]m_k'(\theta), \tag{58}$$
here δ_{0k} are the solutions of the equation
$$(S_1+S_2)sh((S_1-S_2)\delta_{0k}) - (S_1-S_2)sh((S_1+S_2)\delta_{0k}) = 0,$$

b) $$u_\rho^{(a)} = \sum_{k=1}^{\infty} D_k^{(4)}(1+b_{12})S_1 S_2 \delta_{0k}^3 \left[\left(b_{12} + b_{11}S_2^2\right)sh(\delta_{0k}S_2)sh(\delta_{0k}S_1\rho) - \right.$$
$$\left. -\left(b_{12} + b_{11}S_1^2\right)sh(\delta_{0k}S_1)sh(\delta_{0k}S_2\rho) + O(\varepsilon)\right]m_k(\theta), \tag{59}$$
$$u_\theta^{(a)} = \sum_{k=1}^{\infty} D_k^{(4)}\delta_{0k}^2 \left[\left(1 - b_{11}S_2^2\right)\left(b_{12} + b_{11}S_1^2\right)S_1 sh(\delta_{0k}S_1)ch(\delta_{0k}S_2\rho) - \right.$$
$$\left. -\left(1 - b_{11}S_1^2\right)\left(b_{12} + b_{11}S_2^2\right)S_2 sh(\delta_{0k}S_2)ch(\delta_{0k}S_1\rho) + O(\varepsilon)\right]m_k'(\theta), \tag{60}$$
here δ_{0k} are the solutions of the equation
$$(S_1-S_2)sh((S_1+S_2)\delta_{0k}) + (S_1+S_2)sh((S_1-S_2)\delta_{0k}) = 0,$$

4. For $e_1 < 0, \quad e_1^2 = e_2$, $\quad S = \sqrt{|e_1|}$. for the displacements we obtain:

a) $$u_\rho^{(a)} = \sum_{k=1}^{\infty} Q_k^{(3)}\left\{\left(b_{12} + b_{11}S^2\right)\delta_{0k}S\left[\rho\, ch(\delta_{0k}S)sh(\delta_{0k}S\rho) - \right.\right.$$
$$\left.\left. - sh(\delta_{0k}S)ch(\delta_{0k}S\rho)\right] + O(\varepsilon)\right\}m_k(\theta), \tag{61}$$
$$u_\theta^{(a)} = \sum_{k=1}^{\infty} Q_k^{(3)}\left\{\frac{\left(1 - b_{11}S^2\right)\left(b_{12} + b_{11}S^2\right)}{1 + b_{12}} \times \right.$$
$$\times\left[sh(\delta_{0k}S)sh(\delta_{0k}S\rho) - \rho ch(\delta_{0k}S)ch(\delta_{0k}S\rho)\right] +$$

$$+\frac{b_{11}^2 S^4-(3b_{11}+b_{12}b_{11})S^2-b_{12}}{\delta_{0k}S(1+b_{12})}ch(\delta_{0k}S)sh(\delta_{0k}S\rho)+O(\varepsilon)\Big\}m_k'(\theta), \tag{62}$$

here δ_{0k} are the solutions of the equation

$$sh(2\delta_{0k}S)-2\delta_{0k}S=0\ ,$$

b) $$u_\rho^{(a)}=\sum_{k=1}^{\infty}Q_k^{(4)}\Big\{(b_{12}+b_{11}S^2)\delta_{0k}S[\rho\, sh(\delta_{0k}S)ch(\delta_{0k}S\rho)-$$

$$-ch(\delta_{0k}S)sh(\delta_{0k}S\rho)]-2b_{11}S^2 sh(\delta_{0k}S)sh(\delta_{0k}S\rho)+O(\varepsilon)\Big\}m_k(\theta), \tag{63}$$

$$u_\theta^{(a)}=\sum_{k=1}^{\infty}Q_k^{(4)}\left\{\frac{(1-b_{11}S^2)(b_{12}+b_{11}S^2)}{1+b_{12}}\times\right.$$

$$\times[ch(\delta_{0k}S)ch(\delta_{0k}S\rho)-\rho sh(\delta_{0k}S)sh(\delta_{0k}S\rho)]-$$

$$\left.-\frac{b_{11}^2S^4-b_{11}(b_{12}+3)S^2-b_{12}}{(1+b_{12})\delta_{0k}S}sh(\delta_{0k}S)ch(\delta_{0k}S\rho)+O(\varepsilon)\right\}m_k(\theta), \tag{64}$$

here δ_{0k} are the solutions of the equation

$$sh(2\delta_{0k}S)+2\delta_{0k}S=0,$$

$$2e_1=(2b_{12}+b_{12}^2-b_{11}b_{22})b_{11}^{-1},\ e_2=b_{11}^{-1}b_{22}\text{-dir.}$$

The solutions of (49)-(64) are of boundary layer character and these solutions are localized around the conic sections $\theta=\theta_j\quad(j=1;2)$.

Boundary layer character solutions for δ_{0k} damp weakly, and these solutions are considered to be expanded solutions. When δ_{0k} take purely imaginary and complex values, the stress strain state of a transversally isotropic cylindrical shell by its quality is identical to the stress-strain state of an isotropic spherical shell.

We consider a high frequency vibration of a transversally-isotropic spherical shell. As $\varepsilon\to 0$ we study the spectral problem (38)-(41) for λ satisfying the condition $\lambda\to\infty$. As $\varepsilon\to 0$ this possible cases a) $\lambda\to\infty$, $\lambda\varepsilon\to 0$, b) $\lambda\to\infty$, $\lambda\varepsilon\to const$, c) $\lambda\to\infty$, $\lambda\varepsilon\to\infty$, are studied.

Asymptotic expressions are derived for the amplitude values of the displacement vector and the components of the stress tensor.

In **2.4** we study satisfaction of boundary conditions (36) in the seats of the spherical shell (in conic sections). For $\lambda = O(1)$, $\lambda = O\left(\varepsilon^{-\gamma}\right)$, $\lambda = O\left(\varepsilon^{1-2\gamma}\right)$, $(0 < \gamma < 1)$ the system of equations obtained for defining constants coincide with the finite system of linear algebraic equations obtained in the static problem of elasticity theory for a transversally isotropic cylindrical shell. For $\lambda = O\left(\varepsilon^{-\gamma}\right)$, $(\gamma \geq 1)$ the obtained system of infinite linear algebraic equations is identical to the system of linear algebraic equations obtained in dynamic problem of elasticity theory for an elastic strip.

In 2.5, the issue of torsional oscillation of a transverse-isotropic spherical coating free from side surface load is studied. Asymptotics of the dispersion equation obtained as a result of satisfying the given homogeneous boundary conditions on the side surface are determined for frequencies that satisfy conditions $\lambda = O(1)$ and $\lambda \to \infty$ when the roots are $\varepsilon \to 0$. Asymptotic expressions corresponding to those roots are determined for displacements and stresses.

In 2.6, the problem of torsional vibration of a transverse-isotropic spherical cover with a closed side surface is studied. The exact and asymptotic solution of the problem is established.

Conclusion

1. Asymptotic theory of a radial inhomogeneous small-thickness transversally-isotropic spherical shell whose elasticity modulus change with respect to the axis, is commented. A problem of elasticity theory symmetric with respect to the axis is studied in the case when the lateral surface of a radial inhomogeneous transversally-isotropic cylindrical shell is free from load. Homogeneous solutions are constructed and classified. It is shown that the homogeneous solution consists of the sum of the expanded solution equivalent to the principial vector of stresses acting in the section $\theta = const$ of the spherical shell, a boundary effect character solution determining the boundary effect in the applied theory of shells

and boundary effect character boundary layer localized around the seats of the spherical shell (of conic sections).

2. A new class of solutions called boiundary layer character solutions that can not be described by the existing applied theories for a spherical shell, is determined. İt is shown that the stress-strain state can change far from the conic sections, where the boundary layer character solutions weakly damp spreading throughout the domain.
3. A problem of elasticity theory symmetric with respect to the axis was studied for a transversally-isotropic radial inhomogeneous spherical shell with fixed lateral surface.
4. The boundary value problems for a radial, inhomogeneous and homogeneous transversally-isotropic spherical shell were numerically solved; the distribution of the displacement vector and stress tensor components in radial direction, was studied. The influence of in homogeneity on the stress-strain state was estimated.
5. A problem of torsion of an inhomogeneous transversally-isotropic spherical shell was studied for the case when the lateral surface of the spherical shell is free from load. It was obtained that the homogeneous solution consists of the expanded solution proportional to the torgue of the stresses acting in the section $\theta = const$ and a boundary layer character solutions damping exponentially moving away from the conic sections. The possibility of weakly damping boundary layer character solutions that make serious additions was specified.
6. A problem of torsion of an inhomogeneous transversally-isotropic spherical shell was studied in the case when the lateral surface of the spherical shell is fixed. The character of the stress-strain state is determined.
7. A dynamic problem of elasticity theory is studied by the asymptotic integration method for a small-thickness transversally-isotropic spherical shell with a lateral surface free from the load. Asymptotic expressions that enable to calculate the stress-strain state of a transversally-isotropic spherical shell for finite and high frequencies, are determined.
8. The torsional vibrations problems of a shell are studied in the case when the lateral surface of the transversally-isotropic spherical shell is free from load. The asymptotic of the roots of dispersion

equations obtained as a result of satisfaction of homogeneous boundary conditions on the lateral surface at small values of the parameter characterizing the thickness of the shell was determined for finite and high frequencies. Asymptotic expressions for stress tensor components and displacement corresponding to these roots, were constructed.

The results of the dissertation have been published in the following scientific works:

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