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ABSTRACT

of the dissertation for the degree Doctor of Philosophy

VIBRATIONS OF RECTANGULAR PANEL AND REINFORCED CYLINDRICAL PANELS IN CONTACT WITH THE ENVIRONMENT

Specialty: 2002.01 - Mechanics of deformable solids

Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THE WORK

Relevance of the topic and degree of elaboration.

Panels and covers with various configurations are widely used in many fields of technology, including construction of construction complexes, machine building, construction of highway pipelines, shipbuilding, etc. One of the most important issues in the reporting of such constructions is to assess the mechanical properties of the structural element as correctly as possible, to take into account the mode of operation during their operation and the influence of the environment they are in contact with. Depending on the mechanical and thermal processing, the type of technology, the composition of the material, the characteristics of homogeneity and anisotropy arise in the materials of the constructions. On the other hand, such constructions are in contact with environments of different nature. Conducted theoretical and experimental studies show that it is important to take into account the resistance of the environment in solving dynamic problems. In many cases, such thin-walled structures require reinforcement to increase their serviceability. Reinforcement is carried out with the help of spindles, rings or spindles and rings. Shafts and rings are reinforced in different ways In the first method, it is assumed that the spindles and the rings are fairly close together. In this method, the study of the stress-deformation state of the structure under the influence of static and dynamic forces is brought to the construction of an equivalent smooth cylindrical cover. In the second method, the system of equations characterizing the deformation process of the structure is based on taking into account the discrete location of the ribs. Thin-walled rectangular and cylindrical panels weakened by holes are widely used in many production areas, including machine building and shipbuilding. It is of great importance to study the effect of the dimensions of the holes on the performance of such structures subjected to dynamic effects.

It should be noted that in order to prevent the collapse of forest massifs and landslides, dams consisting of rectangular and cylindrical panels are built with the help of reinforcement and cement mortar. The property of anisotropy manifests itself prominently in such panels. Therefore, it is necessary to consider the anisotropy property in the reporting of such dams subjected to dynamic influence. The dissertation work is relevant precisely because such issues are devoted to the study of the stress-deformation state of anisotropic, non-homogeneous, reinforced or smooth rectangular, cylindrical smooth panels or panels weakened by holes in contact with a viscoelastic medium.

One of the outstanding scientists in the study of the dynamic stiffness characteristics of panels reinforced with ribs or smooth rectangular, cylindrical panels N.P. Abovsky, I.Y. Amirov, M.B. Akhundov, A.N. Alizade, Y. Sevdimalyev, M.F. Mehdiyev, I.T. Pirmammadov, F.S. Latifov, A.I. Lurye, R.A. Isgandarov, S.R. Timoshenko, V.Z. Vlasov, A.S. Volmir, V.A. Zaruskin, V.C. Hajiyev, V.A. Bajenov, J.J. Goldenblat, V.V. Moskvint, J.V. Gorshkov and others services can be mentioned.

The subject and object of the study.

Dissertation work consists of the study of oscillations of anisotropic, non-homogeneous, rectangular plate in contact with viscoelastic medium, integral or hole-weakened cylindrical plates with reinforced rectangular profile, which are used in many fields of technology.

Research goals and objectives. To study the forced oscillations of anisotropic, non-homogeneous, reinforced or smooth rectangular, smooth or hole-weakened cylindrical plates in contact with a visco-elastic medium under the influence of a force regularly distributed on the free and surface, the oscillation frequencies in case of free oscillation, and cylindrical in case of forced oscillation finding the displacements of the points of the plate consists of studying the influence of the parameters characterizing the structure on these frequencies and displacements.

Research methods. The issues were solved on the basis of the theory of elasticity and coatings, theoretical fundamental laws and methods of mathematical-physical equations. The method based on finding the sign change according to the values calculated from point to point was used.

Main provisions of dissertation.

1. A physical and mathematical model of the problem a mathematical model has been established to find the free oscillation frequencies of cylindrical panels with a rectangular plane and reinforced rectangular profile;
2. A frequency equation was established to calculate the free oscillation frequencies of a rectangular plane panel, its roots were found, and the effect of physical, mechanical, geometrical and inhomogeneity, viscosity parameters characterizing the panel and the external environment on these frequencies was studied;
3. A frequency equation was established to calculate the free oscillation frequencies of reinforced undamaged cylindrical panels in contact with the viscoelastic medium and the effect of the parameters was studied;
4. A frequency equation was established to calculate the free oscillation frequencies of reinforced cylindrical panels with a rectangular profile in contact with visco-elastic medium weakened by holes, the roots were found and the effect of physical, mechanical, geometrical and inhomogeneity, viscosity parameters characterizing the cylindrical panel, external environment, shaft and rings, geometric dimensions of holes was studied.

Scientific novelty of the research.

1. In the dissertation, for the first time, a mathematical and physical model of the problem was established to find the free oscillation frequencies of anisotropic, inhomogeneous rectangular panels in contact with a viscoelastic medium;
2. Mathematical and physical models of the problem were established to find the free oscillation frequencies of anisotropic, inhomogeneous, discretely distributed shafts and rings, undamaged and damaged cylindrical panels with a rectangular profile in contact with a viscoelastic medium. Three cases of cylindrical panel strengthening were considered in the dissertation: with spindles

placed along the stem; with spindles placed along the stem; with shafts and rings forming an orthogonal network;

3. In order to find the free oscillation frequencies of the system in all three reinforcement cases, the frequency equation was established, the roots were found, and the effect of the geometrical, mechanical and inhomogeneity, viscosity parameters characterizing the system on the found frequencies was studied;

4. The formula for finding the deflection of a non-homogeneous anisotropic rectangular panel located on a visco-elastic base has been obtained. It was determined that as the value of the inhomogeneity parameter increases along the thickness of the orthotropic panel, the deflection of the panel increases;

5. The expressions for calculating the oscillation frequencies of the cylindrical cover were obtained, which is in contact with the visco-elastic medium, have an anisotropic inhomogeneity property, which varies linearly and exponentially.

Theoretical and practical significance of research.

The results obtained in the dissertation can be used in the study of vibrations of reinforced belts, hydrotechnical equipment elements, constructions and construction elements used in industrial and civil construction, panels built in fastening, affected by dynamic force.

Approbation and implementation.

The main scientific results of the dissertation work are always in the "Applied-mathematics" department of the Institute of Mathematics and Mechanics (2018-2022), "Higher Mathematics" of AzMIU, in the seminars of the Departments of "Theoretical and Whole Environment Mechanics" of BSU (2020-2022), in Dedicated to the 95th anniversary of the great leader Heydar Aliyev "The basis of our achievements is connected with Heydar Aliyev's reforms" (2018), International conference devoted to the 60th anniversary of the Institute of Mathematics and Mechanics of the Azerbaijan National Academy of Sciences (2019), IV International Ukrainian-Azerbaijani scientific-practical "Building innovations-2021" (2021), at the 17th "Technical and Physical Problems of Engineering" International Conference (2021), in the XI International Science

Conference “Implementation of modern science practice” (2021) discussed and liked.

The name of the organization where the dissertation was conducted.

The dissertation work was completed at the "Applied mathematics" department of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

The volume of the dissertation's structural sections separately and the general volume.

The dissertation consists of a table of contents, an introduction, three chapters, a conclusion, and a list of used literature, with a volume of 134 pages. The total volume of the dissertation is 230862 marks (title page - 343 marks, table of contents - 4116 marks, introduction - 21704 marks, first chapter - 54289 marks, second chapter - 84912 marks, third chapter - 65498 marks). There are 8 pictures, 4 tables, 27 graphs, 74 titles of literature in the dissertation.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction the relevance of the topic, the purpose of the research, research methods, scientific innovations, the obtained results and their practical significance are explained, and a brief summary of the works close to the topic of the dissertation is reflected.

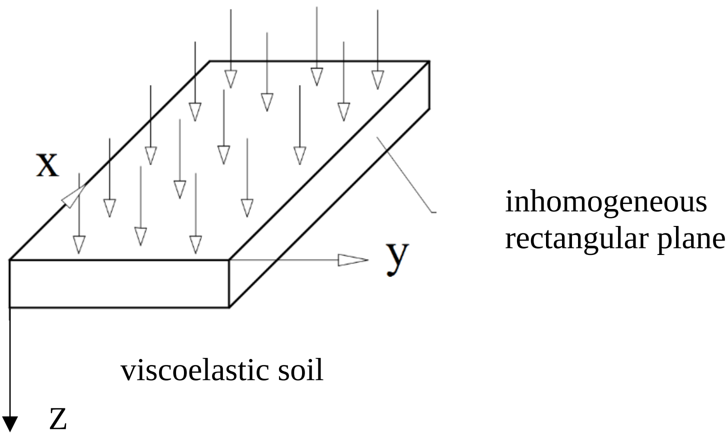
The first chapter consists of five sub-chapters. In the first sub-chapter, the problem of finding the deflection of non-homogeneous rectangular-shaped panels in contact with a visco-elastic medium, subjected to the influence of an external force, was solved. A deflection was found in the center of a non-homogeneous rectangular panel with joints on all sides, and a graph of the dependence of the deflection on the inhomogeneity parameter was constructed. In modern times, rectangular panels made of non-uniform natural and artificial materials throughout their thickness are widely used when building engineering facilities, bridges, overpasses and other fields. In most cases, the inhomogeneity of the rectangular panel is caused by technological preparation, mechanical and thermal processing processes, the presence of various mixtures in its composition, and other reasons.

It should be noted that taking into account the above-mentioned properties and the influence of the external environment makes the mathematical solution of the problem rather difficult, and not taking it into account leads to significant errors.

It was assumed that a non-homogeneous rectangular panel along the thickness rests on a Pasternak-type base. According to this model, the reaction force R acting on the rectangular panel by the environment is expressed by the deflection $W(x, y)$ of the rectangular panel in the following relation:

$$R = K_v W - K_p \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) \qquad (1)$$

Here $K_v\left(\frac{N}{m^3}\right)$ is Winkler's coefficient, $K_p\left(\frac{N}{m}\right)$ is Pasternak's coefficient.



It is assumed that a non-homogeneous rectangular panel throughout its thickness is under the influence of the following type of transverse force:

$$P(x,y) = P_0 \sin\frac{\pi}{a} x \cdot \sin\frac{\pi}{b} y \quad (2)$$

Here, a and b are the dimensions of the inhomogeneous rectangular panel along its thickness.

The differential equation for the curvature of the middle surface of a non-homogeneous anisotropic rectangular panel along its thickness is as follows:

$$\mu L(W) + K_v W - K_p\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) = P_0 \sin\frac{\pi}{a} x \cdot \sin\frac{\pi}{b} y \quad (3)$$

Here

$$L(W) = a_{11}^0 \frac{\partial^4 W}{\partial x^4} + \left(a_{12}^0 + 2a_{12}^0 + a_{32}^0\right)\frac{\partial^4 W}{\partial x^2 \partial y^2} + \\ + a_{22}^0 \frac{\partial^4 W}{\partial y^4} + \left(a_{13}^0 + 2a_{31}^0\right)\frac{\partial^4 W}{\partial x^3 \partial y} + \left(2a_{32}^0 + a_{13}^0\right)\frac{\partial^4 W}{\partial x \partial y^3} \quad (4)$$

$a_{ij}^0 (i, j = 1,2,3) -$ are the constants for a homogeneous anisotropic material,

$$A_3 = \int_{-h/2}^{+h/2} f(z) z^2 dz; \qquad \mu = A_2^2 \cdot A_1^{-1} - A_3,$$

$$A_1 = \int_{-h/2}^{+h/2} f(z)\, dz; A_2 = \int_{-h/2}^{+h/2} f(z)\, z\, dz$$

The deflection of a non-homogeneous along the thickness anisotropic rectangular panel, whose edges are fastened with joints, must satisfy the following boundary conditions:

$$W = 0; \frac{\partial^2 W}{\partial x^2} = 0 \qquad x = 0;\ x = a \ olduqda \quad (5)$$

$$W = 0; \frac{\partial^2 W}{\partial y^2} = 0 \qquad y = 0;\; y = b \;\; olduqda$$

Let us take the solution of equation (3) satisfying the boundary conditions (5) as follows:

$$W(x, y) = W_0 \sin\frac{\pi}{a} x \cdot \sin\frac{\pi}{b} y \tag{7}$$

Considering the solution of (6) in (3), we can find the relation between W_0 and P_0:

$$W_0 = \frac{P_0}{\mu(-a_{11}^0\lambda^4 - a_{22}^0\beta^4 - \lambda\beta\left(D_1\lambda^2 + D_2\beta^2\right) + D_3\lambda^2\beta^2) + K_\nu + K_p\left(\lambda^2 + \beta^2\right)} \tag{8}$$

μ_0 quantity was numerically calculated with the help of expression (8).

In the calculation, the following values were taken for the parameters characterizing the orthotropic, inhomogeneous panel and the medium:

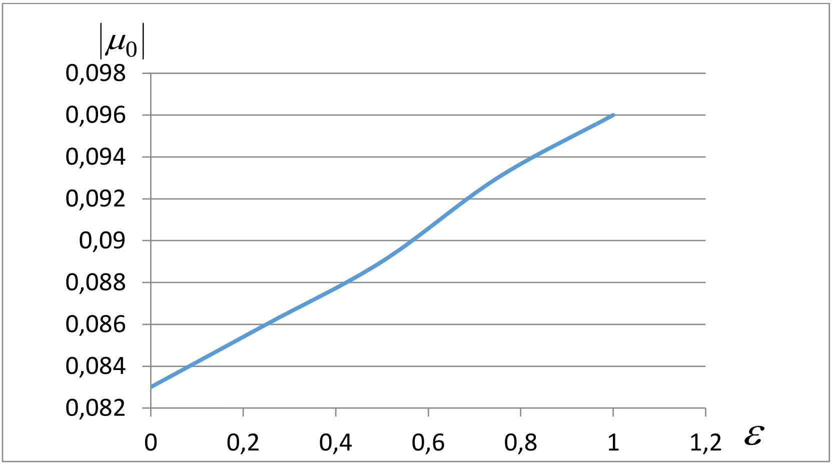
$$f(z) = 1 + \varepsilon\left(\frac{z}{h}\right)^2; \quad \varepsilon \in [0,1]; \tag{9}$$

$$E_1/E_2 = 10; \quad \nu_1 = 0,25; \quad \left(\frac{a}{b}\right) = 1; K_\vartheta = 10^6\, N/m^3, \quad K_p = 10^4\, N/m$$

The results of the calculations are given in table 1 and graph 1. As can be seen from graph 1, as the value of the inhomogeneity parameter increases throughout the thickness of the orthotropic panel, the deflection of the panel increases.

Tabel 1.

ε	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$, $\lvert\overline{\mu_0}\rvert$
0	1	0	0,083
0,25	1	0	0,086
0,5	1	0	0,089
0,75	1	0	0,093
1	1	0	0,096



Graph 1. Dependence of the plate deflection on the inhomogeneity parameter along the thickness

In the second paragraph, the free oscillations of rectangular plates whose inhomogeneity property changes with an exponential law in contact with a viscoelastic medium were studied. It was calculated that the modulus of elasticity, density of the material of the rectangular panel is a continuous function of the coordinates in the directions of thickness and length. Separation of variables and Bubnov-Galyorkin methods were used to solve the problem. In the first approximation, an analytical expression was obtained to calculate the oscillation frequency.

Rectangular panels are widely used in various fields of modern technology, including machine building, shipbuilding, civil and industrial fields, and in the construction of other huge engineering facilities. Such panels are usually made of various composite, polymer, glass plastic materials. Due to the presence of mixtures in the composition, depending on the manufacturing technology, non-homogeneity occurs in the boards.

For these reasons, the modulus of elasticity and density of panels in sliding are a function of one, two, and three spatial coordinates. It is assumed here that the elastic properties and density of the panel are function of two (x, y) coordinates. The coordinate axes are chosen so that x and y in the middle plane of the rectangular panel, the axis z is located perpendicular to this plane.

In this case, the elastic constants and density are taken as follows.

$$a_{ij} = a_{ij}^{0} f(z)\left(1+\varepsilon e^{\bar{x}}\right); \quad \rho = \rho_0 \eta_1(z) \eta_2(x) \tag{10}$$

Here a_{ij}^{0} and ρ_0 are the constants corresponding to the homogeneous panel and $\bar{x} = xa^{-1}$. It is considered that the reaction force R acting on the rectangular panel by the viscoelastic medium is expressed by the deflection $W(x, y)$ of the rectangular panel in the following relation:

$$R = k_1 w + k_2 \frac{\partial^2 w}{\partial t^2} \tag{11}$$

Here k_1 and k_2 are the characteristics of the viscoelastic medium $t-$ is time. In this case, the differential equation for the curvature of the middle surface of the anisotropic rectangular panel inhomogeneous along its thickness is as follows:

$$L_1(w) + L_2(w) + L_3(w) + k_1 w + \left(\bar{\rho}\psi(x) + k_2\right)\frac{\partial^2 w}{\partial t^2} = 0 \tag{12}$$

Here

$$L_1(w) = \left(1+\varepsilon e^{\bar{x}}\right)\left[a_{11}^0 \frac{\partial^4 w}{\partial x^4} + a_{22}^0 \frac{\partial^4 w}{\partial y^4} + \left(a_{12}^0 + a_{21}^0 + a_{31}^0\right)\frac{\partial^2 w}{\partial x^2 \partial y} + \right.$$

$$\left. + a_{13}^0 \frac{\partial^4 w}{\partial x^3 \partial y} + a_{22}^0 \frac{\partial^4 w}{\partial y^3 \partial x}\right]$$

$$L_2(w) = 2\varepsilon a^{-1} e^{\bar{x}}\left(a_{11}^0 \frac{\partial^3 w}{\partial x^3} + a_{12}^0 \frac{\partial^3 w}{\partial y^2 \partial x} + a_{13}^0 \frac{\partial^3 w}{\partial x^2 \partial y}\right)$$

$$L_3(w) = \varepsilon a^{-2} e^{\bar{x}}\left(a_{11}^0 \frac{\partial^2 w}{\partial x^2} + a_{12}^0 \frac{\partial^2 w}{\partial y^2} + a_{13}^0 \frac{\partial^2 w}{\partial x \partial y}\right)$$

Expressions of operators $L_i\,(i=1,2,3)$ included in equation (12) are given in the dissertation. Looking for the solution of equation (12) in the form $\mathrm{w}(x,y,t) = V_0 \varphi(y)\psi(x)e^{i\omega t}$, and using the Bubnov-Galyorkin orthogonalization method, we get the following expression to find ω^2:

$$\omega^2 = \frac{\int\limits_0^a\int\limits_0^b \left(\bar{L}_1(\varphi,\psi) + \bar{L}_2(\varphi,\psi) + \bar{L}_3(\varphi,\psi) + k_1\varphi_1 \cdot \psi_1\right)\varphi_1\psi_1 dxdy}{\int\limits_0^a\int\limits_0^b \left(k_2 + \bar{\rho}\varphi(x)\right)\varphi_1^2(x)\psi_1^2(x)dxdy}$$

In the third paragraph, the free oscillations of the inhomogeneous orthotropic cylindrical coating along its thickness in contact with the viscoelastic medium were studied. It was calculated that the density of the medium and the coating is a function of the coordinate along the thickness. Separation of variables and Bubnov-Galyorkin methods were used to solve the problem. For the square of the desired oscillation frequency, an analytical expression depending on the modulus of elasticity of the coating, density, parameters characterizing the environment was obtained and numerical calculations were carried out.

Cylindrical covers with a circular cross-section are the main leading parts of the most common traditional and composite material constructions, such as pipes for various purposes, pressurized gas cylinders, aircraft and ship parts. The wide application of cylindrical coatings in engineering and industrial fields is the simplicity of the equations expressing its stress-strain state. In many cases, depending on the thermal and mechanical processing, and the preparation of composite materials, the material of the cylindrical coating has the property of inhomogeneity. The consideration of such real properties leads to significant errors in solving the problems of strength, stability and vibration of the cylindrical cover. In order to solve the problem, a homogeneous system of equations was obtained taking into account the components of the displacement vector:

$$\begin{aligned} L_1U + L_2V + L_3W &= 0 \\ L_4U + L_5V + L_6W &= 0 \\ L_7U + L_8V + L_9W &= 0 \end{aligned} \tag{13}$$

Expressions of operators $L_i = (1,2,.....9)$ included in system (13) are given in the dissertation.

The system (13) is simplified in the case of axisymmetric oscillation. In this case, the equation of motion falls into the following form:

$$c_1 \frac{\partial^4 W}{\partial x_1^4} + c_2 \frac{\partial^2 W}{\partial x_1^2} + (c_3 + K_1)W + (K_2 + \rho)\frac{\partial^2 W}{\partial t^2} = 0 \tag{14}$$

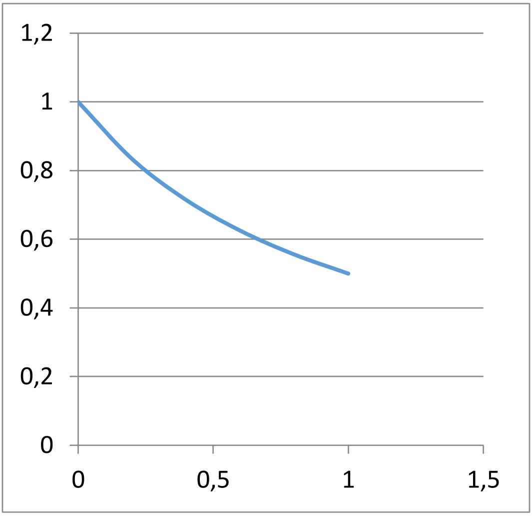
The following notations are adopted here:

$$c_1 = \frac{E_1^0}{1-\nu_1\nu_2}\left(A_2^2 - A_1\right);\ c_2 = -\frac{E_2^0}{1-\nu_1\nu_2}\frac{A_2\nu_1}{R}\left(A_1 - 1\right);$$

$$c_3 = -\frac{E_1^0}{1-\nu_1\nu_2}\frac{A_1^2\nu_1 - A_1\nu_2}{R^2} + K_1$$

Table 2.

μ	$\overline{\omega^{*2}}$
0	1
0,2	0,833333
0,4	0,714286
0,6	0,625
0,8	0,555556
1	0,5



Graph 2. Dependence of the dance frequency on the inhomogeneity parameter

The solution of equation (14), if we look for $T_1 = 0;\ W = 0;\ M_1 = 0$; $x = 0,\ x = l$ boundary conditions in the form $W = W_0 \sin\frac{m\pi}{l} x e^{i\omega t}$, after certain transformations we get:

$$\left(\frac{\omega}{\omega_v^2}\right)^2 = \frac{1}{\mu+1}; \quad \mu = K_2 \bar{\rho}^{-1}.$$

The results of the calculations are given in table 2 and graph 2. As can be seen from Table 2, as the value of the parameter increases, the oscillation frequency decreases. Graph 2 shows that as the value of the medium's inhomogeneity parameter increases, the oscillation frequency also decreases here.

In the fourth paragraph, the free oscillations of the cylindrical cover, whose inhomogeneity property changes with an exponential law, were studied. The solution of the problem is brought to the system consisting of two linear differential equations, taking into account the stress function and the deflection. Then, the stress function was eliminated, and the differential equation of motion was reduced to a differential equation considering the deflection. The resulting equation was solved by separation of variables and Bubnov-Galyorkin methods.

It is known that cylindrical covers with a circular cross-section are widely used in modern technology, in the field of energy, in machine building, and in various construction areas. In many cases, due to manufacturing technology and various reasons, continuous non-homogeneity of the coating occurs along the surface. These two mentioned factors play an important role in the analysis of the amplitude-frequency characteristics of the coating. It should be noted that the consideration of the material density and modulus of elasticity of the cylindrical coating as a function of the coordinate has a significant effect on the specific oscillation frequencies of the system. Taking into account the influence of the viscoelastic medium makes the solution of the problem a little more difficult. It is assumed that the modulus of elasticity E of the cylindrical coating, the density ρ and the characteristics of the viscoelastic medium are a function of the x coordinate along the generator of the cylindrical coating:

$$E = E_0(1+\varepsilon e^{\bar{x}}), \quad \rho = \rho_0 \Psi(x), \; q = K_1(x)W + K_2(x)\frac{\partial^2 W}{\partial t^2}.$$

The equation of motion of a non-homogeneous cylindrical coating along a conductor in contact with a visco-elastic medium is obtained in the following form:

$$D_0\left[\frac{\partial^6 W}{\partial x^6}+\varepsilon e^{\bar{x}}\left(\frac{\partial^6 W}{\partial x^6}+4e^{-1}\frac{\partial^5 W}{\partial x^5}+6e^{-2}\frac{\partial^4 W}{\partial x^4}+4e^{-3}\frac{\partial^3 W}{\partial x^3}+\varepsilon e^{-4}\frac{\partial^2 W}{\partial x^2}\right)-\right.$$

$$\left.-\left(e^{-4}-\frac{hE_0}{R^2 D_0}\right)\right]-\frac{\partial^2 W}{\partial x^2}\quad-\frac{hE_0}{R^2 D_0}\frac{\partial^2 W}{\partial x^2}+2K_1'(x)\frac{\partial W}{\partial x}+K_2''(x)W+$$

$$+\left(K_2''(x)+h\rho_0\frac{\partial^2 \Psi}{\partial x^2}\right)\frac{\partial^2 W}{\partial t^2}+2\left(K_2'(x)+h\rho_0\frac{\partial \Psi}{\partial x}\right)\frac{\partial^3 W}{\partial t^2 \partial x}+$$

$$+\left(K_2(x)+h\rho_0\Psi(x)\right)\frac{\partial^4 W}{\partial t^2 \partial x^2}=0 \tag{15}$$

Obtained for the frequency by looking for the $W(x,t)$ function in the figure $W(x,t)=e^{i\omega t}\sum_{i=1}^{n}C_i\theta_i(x)$:

$$\omega^2=\frac{\int_0^l L_1(\theta_i)\theta_i dx}{\int_0^l L_2(\theta_i)\theta_i dx} \tag{16}$$

To find ω^2 the approximation of the $\theta_i(x)$ function and , the functions $K_1(x),\ K_2(x)$, $\Psi(x)$ must be known. The following cases were considered:

$$\theta_i(x)=\sin\frac{\pi}{l}x;\qquad K_1(x)=K_1^0(1+\alpha\bar{x});\qquad K_2(x)=K_2^0(1+\alpha\bar{x})$$

$$\Psi(x)=1+\mu\bar{x};\qquad \bar{x}=x\cdot l^{-1};\qquad \alpha\in[0,1];\qquad \mu\in[0,1]$$

In the fifth paragraph, axisymmetric transverse free oscillations of an orthotropic, inhomogeneous circular cross-section cylindrical coating in contact with a viscoelastic medium were studied. It was considered that the elastic modulus and density of the material of the orthotropic cylindrical coating is a continuous function of the coordinate along the thickness. Numerical calculation was carried out for the specific case of homogeneity, while the edges of the orthotropic cylindrical cover were free, and a tabular and graphical dependence was established between the dimensionless frequency parameter and the non-homogeneity parameter.

$$C_1 = \frac{E_1^0}{1-\nu_1\nu_2}\left(A_2^2 - A_1\right), \quad C_2 = \frac{E_1^0}{1-\nu_1\nu_2}\frac{A_2}{R}\left((A_1-1)\nu_1\right),$$

$$C_3 = \frac{E_1^0}{1-\nu_1\nu_2}\frac{A_1^2\nu_1 - A_1\nu_1}{R^2} + K_1 \tag{17}$$

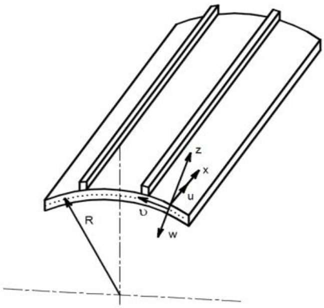
accepting notations, the equation of motion is obtained in the following form:

$$C_1\frac{\partial^4 W}{\partial x^4} + C_2\frac{\partial^2 W}{\partial x^2} + \left(C_3 + K_1\right)\cdot W + \left(K_2 + \bar{\rho}\right)\frac{\partial^2 W}{\partial t^2} = 0 \tag{18}$$

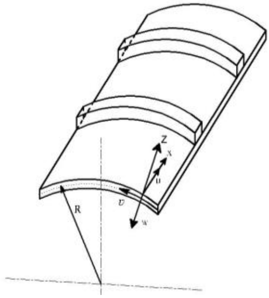
Integrating equation (18) within the boundary conditions $T_1 = 0; \quad W = 0; \quad M_1 = 0; x = 0, \ x = l$, the following expression was obtained to calculate the frequencies:

$$\omega^2 = \frac{c_3 - 0{,}25\ c_2^2 \cdot c_1^{-1}}{\overline{K_2} + \bar{\rho}}$$

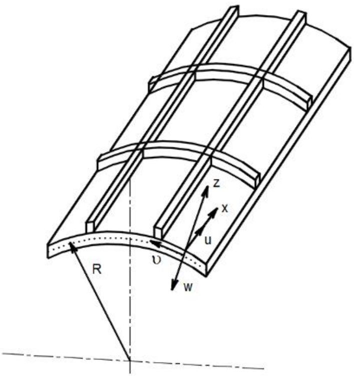
The second chapter consists of four paragraphs. In the first paragraph, the problem of free oscillations of a cylindrical panel located on a visco-elastic base, reinforced (picture 1 - with spindles, picture 2 - with arched rings, picture 3 - with spindles and arched rings), non-homogeneous throughout the thickness, and fastened with joints on all sides is presented. A rigid connection of the spindle and arc rings to the coating on the surface of the cylindrical panel is assumed. The contact conditions between the viscoelastic medium, shaft and arc rings, and the cylindrical panel, which is inhomogeneous along its anisotropic thickness, and is fixed on all sides with joints, are given. The method of applying the energetic method is given.



Picture 1. Cylindrical panel reinforced with spindles



Picture2. Cylindrical panel reinforced with arcuate rings



Picture 3. Cylindrical panel reinforced with spindles and arcuate rings

Hamilton-Ostrogradski variation principle was used to solve the problem:

$$\delta W = 0 \tag{19}$$

Here $W = \int_{t'}^{t''} J dt$ – the Hamiltonian effect, t' and t'' are given arbitrary moments of time.

The total energy of the structure consisting of cylindrical panel, shaft and rings, viscoelastic base is as follows:

$$J = V + V_1 + V_2 + A_0 \tag{20}$$

Here V is the total energy of the cylindrical panel:

$$V = \frac{R}{2}\iint_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \rho(z)\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)\right) dx d\varphi dz$$

V_1, V_2 - are the total energy of shafts and rings, respectively.

The potential energy of the forces acting on the orthotropic cylindrical panel by the viscoelastic base is equal to the work done by these forces on the displacements of the points of the panel with the opposite sign:

$$A_0 = -R\int_0^l \int_0^{\phi_0} q_z w dx d\varphi$$

The force q_z acting on the cylindrical cover by the environment is expressed by the deflection $w(x, y, z)$ of the cylindrical cover as follows:

$$q_z = k_\upsilon w - k_p\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \int_0^t \Gamma(t-\tau)w(\tau)d\tau$$

Here, k_ϑ is Winkler's coefficient k_p is Pasternak's coefficient, found by experiment, t is time, $\Gamma(t-\tau)$-viscosity kernel.

Boundary conditions are also added to the (20) energy expression. In the case of a hinged joint

$$u = v = w = M_x = 0 \quad \text{for} \quad x = 0; L$$

$$u = v = w = M_x = 0 \quad \text{for} \quad \varphi = 0; \varphi_0 \tag{21}$$

Thus, the study of free oscillations of a reinforced cylindrical panel located on a visco-elastic base, non-homogeneous throughout its thickness, reinforced by joints is brought to the integration of the (20) total energy within the (21) boundary conditions and the application of the Hamilton-Ostrogradsky variation principle.

In the second paragraph, the free oscillations of the anisotrop cylindrical panel, which is in dynamic contact with a visco-elastic medium, is reinforced with shafts, is non-homogeneous throughout its thickness, and is fastened with joints on all sides. Using the Hamilton-Ostrogradsky variational principle, a frequency equation was established to calculate the free oscillation frequencies of a cylindrical panel, which is non-uniform throughout its thickness, located on a visco-elastic base, and all sides are jointed, and its roots were found, and the physical and mechanical parameters defining the environment, shafts and cover for these roots the effect of the characteristics has been studied. The displacements of the cylindrical panel were searched as follows:

$$u = u_0 \sin\frac{\pi m x}{l} \sin k\frac{\pi\varphi}{\varphi_0} \sin\omega t;\;\; \vartheta = \vartheta_0 \sin\frac{\pi m x}{l} \sin k\frac{\pi\varphi}{\varphi_0} \sin\omega t;$$

$$w = w_0 \sin\frac{\pi m x}{l} \sin k\frac{\pi\varphi}{\varphi_0} \sin\omega t. \qquad (22)$$

Here, u_0, ϑ_0, w_0- are the unknown constants, m, k-respectively, are the wave numbers in the length and width directions of the cylindrical panel.

Considering expressions (22) in (20) ($V_2 = 0$), using the Hamilton-Ostrogradsky variation principle (19), according to variables u_0, ϑ_0, w_0 a homogeneous system of equations was obtained. Since the obtained system is a system of homogeneous linear equations, a necessary and sufficient condition for the existence of its non-trivial solution is that the principal determinant is equal to zero. As a result, the following frequency equation was obtained:

$$\det\left\| a_{ij} \right\| = 0, i, j = 1,3 \qquad (23)$$

Equation (23) is a transcendental equation with respect to frequency ω. Its roots are complex numbers, the real part $\omega_1 = \mathrm{Re}\,\omega$ characterizes the frequencies of the oscillation, and the imaginary part characterizes the damping of the oscillation. The roots of equation (23) were found by numerical method. In the calculations, the following values were taken for the physical, mechanical and geometrical parameters characterizing the panel, the environment, the non-homogeneity of the panel:

$$I_{kpi} = 0,23mm^4; I_{xi} = 5,1mm^4; k_v = 10^6 N/m^3, k_p = 10^4 N/m, \frac{l}{R} = 3, R = 1,6sm,$$

$$E_i = 6,67 \cdot 10^9 N/m^2; I_{zi} = 1,3mm^4, \ \nu = 0,35; m = 1 \ n = 8; h_i = 1,39sm;$$

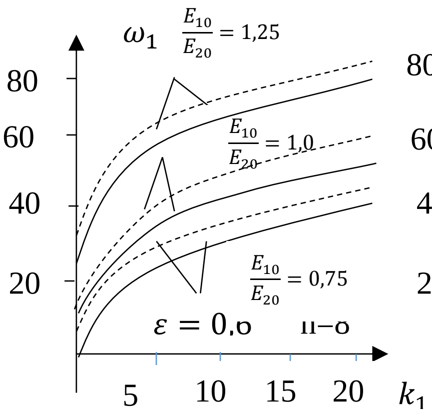
$$\rho_0 = \rho_i = 1850kg/m^3, \quad F_i = 5.2mm^2; A = 0,034; \psi = 0,05;$$

It is assumed that the modulus of elasticity of a cylindrical panel varies linearly with respect to the variable z :

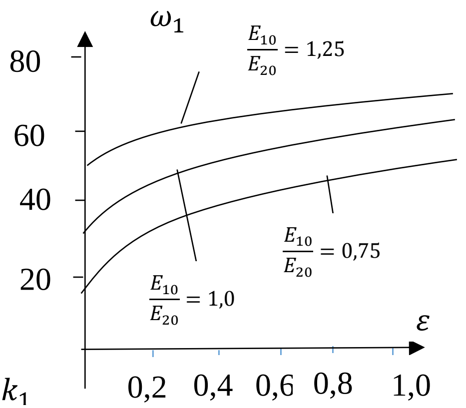
$$E_1(z) = E_{10}(1+\beta z); \ E_2(z) = E_{20}(1+\beta z); G(z) = G_0(1+\beta z)$$

Here β - is the inhomogeneity parameter and $\beta \in [-1;1]$.

The results of the calculations are shown in graph 3 as a function of the ω_1 frequency on the number of miles, and in graph 4 as a function of the inhomogeneity parameter ε. Graph 3 shows that the specific oscillation frequencies of the system increase as the number of spindles and the modulus of elasticity in the direction of the length of the cylindrical panel increases. Graph 4 shows that a suitable result is obtained when the inhomogeneity parameter ε increases.



Grahp 3:
ω_1 frequency dependence
on k_1 the number of miles

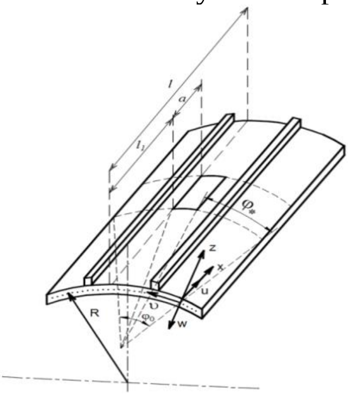


Grahp 4:
ω_1 frequency dependence
on inhomogeneity parameter ε

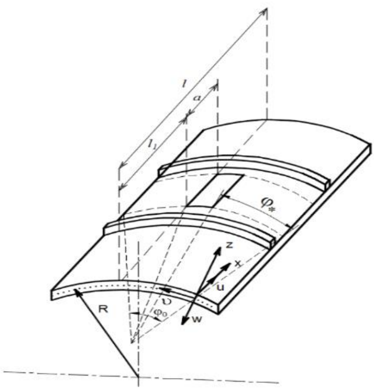
A similar problem is performed in the third paragraph for anisotropic, arcuate rings reinforced cylindrical panel, in the fourth paragraph, on a visco-elastic base reinforced with an orthogonal mesh shaft and arcuate rings, non-homogeneous throughout its thickness, and reinforced with joints on all sides.

In the third chapter, the problem of free oscillations of an orthotropic cylindrical panel located on a visco-elastic base, reinforced in different ways, non-homogeneous throughout the thickness, reinforced with joints, and damaged by rectangular holes was studied. Using the Hamilton-Ostrogradsky variational principle, a frequency equation was established to calculate the free oscillation frequencies of a cylindrical panel, which is non-uniform throughout its thickness, located on a visco-elastic base, and all sides are jointed, and its roots were found, and the physical and mechanical parameters defining the medium, ring, and cover for these roots the effect of the characteristics has been studied.

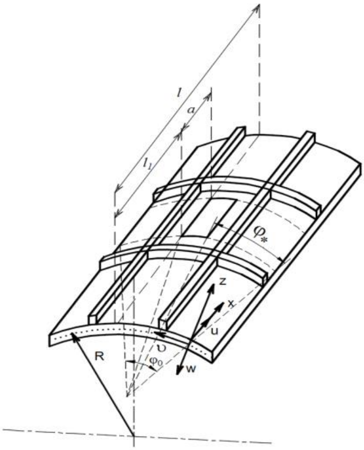
In the first paragraph, the problem of free oscillations of the orthotropic cylindrical panel with a rectangular profile, which is located on a visco-elastic base, is reinforced, is non- homogeneous throughout its thickness, is fixed with joints on all sides, and is damaged by holes. Three cases of cylindrical plate reinforcement are considered.



Picture 4. Damaged cylindrical panel reinforced with spindles



Picture 5. Damaged cylindrical panel reinforced with arched rods



Picture 6. Reinforced with
shafts and arched rods

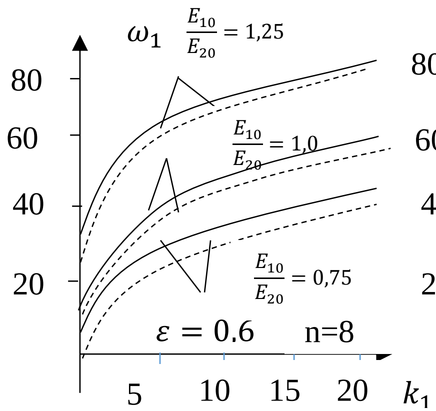
Here, it is assumed by a similar rule that the coordinate axes coincide with the main curvature lines of the given panel, and the rods are in rigid contact with the coating along these lines. In this regard, the conditions mentioned in chapter II between the cylindrical panel, the shaft and the arcuate rods are satisfied.

$$V = \frac{R}{2}\iint\limits_{s-s_0}\int_{-\frac{h}{2}}^{\frac{h}{2}}\Bigg(\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \\ + \rho(z)\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)\Bigg) dx d\varphi dz$$

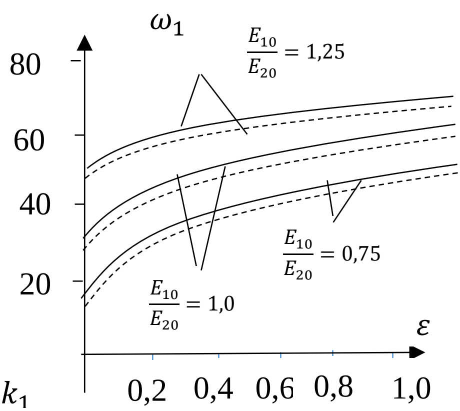
In this case, the total energy of the cylindrical plate is as follows: The expression of the total energy of a structure consisting of a cylindrical panel weakened by a hole in a rectangular shape, shafts and rings, and a viscoelastic base is formally the same as the expression (20). The difference is that in expression (20) of the integration of expressions V and A_0 is not done on the S surface, but on the $S-S_*$ surface. Here, since S_* is a rectangular region $\left(l_1 \le x \le l_1 + a,\ \varphi_* R \le b \le \dfrac{\varphi_0 R}{k_1 + 1} - \varphi_* R\right)$. It should be noted that a cylindrical panel on a visco-elastic foundation, which is non-uniform throughout its thickness, and weakened by a hole on all edges of which is jointed, is removed $V_2 = 0$ only if it is reinforced with shafts, $V_1 = 0$ only if it is reinforced with arched rods. Having performed the integration, using Hamilton-Ostrogradsky's variation principle (19), according to variables u_0, ϑ_0, w_0 a homogeneous system of equations and setting the main determinant of this system equal to zero, the frequency equation was obtained:

$$\det\left\|a_{ij}\right\| = 0, i, j = 1{,}3 \tag{24}$$

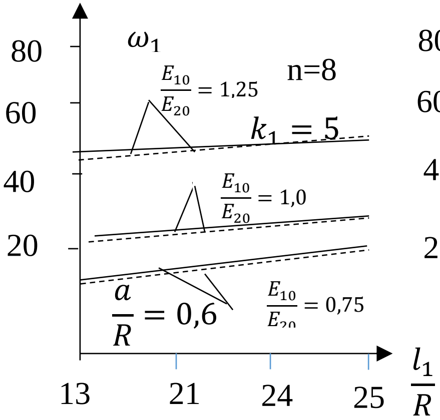
The roots of equation (24) were calculated by numerical method.



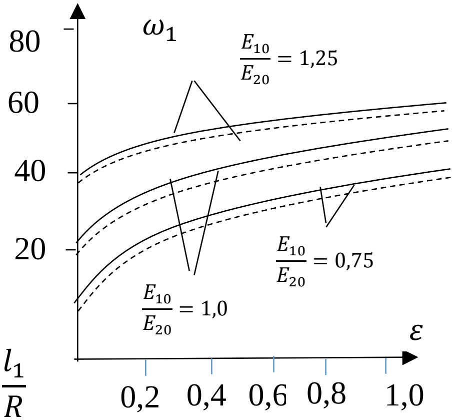
Graph 5. ω_1 frequency dependence on k_1 the number of miles



Grahp 6: ω_1 frequency dependence on inhomogeneity parameter ε



Graph 7. frequency ω_1 dependence on on the ratio $\frac{l_1}{R}$



Graph 8. frequency ω_1 dependence on on the ratio $\frac{a}{R}$

The results of the calculations are shown in graph 5 ω_1 frequency dependence on k_1 the number of miles, graph 6 shows the dependence of the inhomogeneity parameter ω_1, graph 7 the $\frac{l_1}{R}$ ratio, graph 8 the dependence of the $\frac{a}{R}$ ratio.

The broken lines in the graphs correspond to the specific oscillation frequencies of the system consisting of the perforated cylindrical cover, shaft and medium. Graph 5 shows that as the number of shafts and the modulus of elasticity in the direction of the length of the cylindrical panel increases, the specific oscillation frequencies of the system increase. Graph 6 shows that a suitable result is obtained when the inhomogeneity parameter ω_1 increases. Graph 7 and graph 8 show that the influence of the location and length of the section on the specific oscillation frequencies of the system for the given values is quite weak. This allows to use it in practice, for example, in the removal of rainwater from the soil in landslide zones.

A similar issue performed in the third paragraph for anisotropic, reinforced by arcuate rings and in the fourth paragraph for reinforced with shafts and arcuate rings forming an orthogonal network for a damaged cylindrical panel located on a visco-elastic base, non-homogeneous throughout the thickness, jointed on all sides.

RESULTS

The dissertation is devoted to the study of the forced oscillations of anisotropic, non-homogeneous, reinforced or smooth rectangular, smooth or perforated cylindrical panel in contact with a viscoelastic medium under the influence of a force distributed regularly on the free and surface, the oscillation frequencies in the case of free oscillation, the frequency of oscillations in the case of forced oscillation is dedicated to finding the displacements of the points of the cylindrical panel, studying the influence of the parameters characterizing the structure on these frequencies and displacements.

The following results were obtained:

1. A physical and mathematical model of the problem was established to find the free oscillation frequencies of damaged or undamaged cylindrical panels and covers with an anisotropic, inhomogeneous rectangular plane and reinforced rectangular profile in contact with a viscoelastic medium;
2. The formula for calculating the deflection of a inhomogeneous anisotropic in contact with viscoelastic medium rectangular panel is obtained. It is shown that as the value of the inhomogeneity parameter increases along the thickness of the orthotropic panel, the deflection of the panel increases;
3. Expressions were obtained for calculating the oscillation frequencies of the cylindrical cover, whose anisotropic inhomogeneity property is in contact with a viscoelastic medium, and which varies with the linear and exponential law. It is shown that as

the value of the inequality parameter increases, the oscillation frequency decreases;

4. The specific oscillation frequencies of the system increase as the number of spindles and ring-shaped rods and the modulus of elasticity in the direction of the length of the cylindrical panel increases.

5. It was determined that the free oscillation frequencies of cylindrical panels with a rectangular profile damaged by a hole practically do not change compared to the free oscillation frequencies of cylindrical panels with an undamaged rectangular profile.

The main clauses of the dissertation work were published in the following scientific works:

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