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ABSTRACT

of the dissertation for the degree of Doctor of Science

ON FRAME PROPERTIES OF SYSTEMS IN LINEAR TOPOLOGICAL SPACES

Specialty: 1202.01– Analysis and functional analysis

Field of science: Mathematics

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Baku – 2023

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GENERAL CHARACTERISTICS OF THIS WORK

Rationale of the work. Modern harmonic analysis originates from classical Fourier series and covers many areas of mathematics such as approximation theory, potential theory, theory of singular operators, theory of partial differential equations, abstract harmonic analysis, etc. When constructing the theory of classical Fourier series, the main role is played the orthogonality of trigonometric systems. Along with this, harmonic analyzes generated by non-orthogonal systems subsequently appeared.

The emergence of such theories is caused by various needs. Thus, as an example, we can cite the theory of almost-orthogonal (a.o.) functions, which is used in many areas of mathematics and mechanics, including when controlling an infinite system of vibrators in control theory (see, e.g. [[1]]). It should be noted that the theory of a.o. functions is closely related to the theory of Riesz bases in Hilbert spaces. As another example, we can cite the theory of almost periodic (a.p.) functions, created in 1924-1926 by H. Bohr. Subsequently, Bohr's theory was significantly developed in the works of famous mathematicians such as S. Bochner, G. Weil, A. Besikovich, J. Faber, J. Neumann, V.V. Stepanov, N.N. Bogolyubov and others. This theory gave a strong impetus to the emergence and development of harmonic analysis of functions on groups (a.p. functions, series, Fourier integrals on groups). Subsequently, the theory of a.p. functions developed in connection with problems of differential equations, stability theory, dynamic systems, etc.

Another very important example is the theory of frames, created by Duffin and Schaeffer in 1952 in connection with the study of some issues of non-harmonic Fourier series with respect to perturbed systems of exponents. Interest in frames increased after applications of wavelets were found in various fields of natural science in the 1980s.

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[1] А.Г.Бутковский, Методы управления системами с распределенными параметрами, Москва, Наука, 1975, 568 с.

Another very important example is the theory of *K*-frames; interest increased after the applications of wavelets in various fields of natural science were found in the 80s of the last century. Wavelets are widely used in problems of processing and encoding signals, images of various natures (speech, satellite images, *x*-rays of internal organs, etc.), pattern recognition, in studying the properties of the surfaces of crystals and nanoobjects, and in many other areas.

One of the reasons for the emergence of non-harmonic analysis is the solution of partial differential equations using the Fourier method. Since, when solving many equations of mixed or elliptic types in special domains on the complex plane by the Fourier method, perturbed trigonometric systems of the form

$$\{\cos(nt+\alpha(t))\}_{n\in Z_+}, \tag{1}$$

(N are natural numbers and $Z_+ = \{0\}\cup N$) arise, where $\alpha : [0,\pi] \to C$ is some, generally speaking, complex-valued function (C is a complex plane). Regarding similar problems, one can consider, for example, the works of S.M. Ponomarev [2], E.I. Moiseev [3] and others. Naturally, the justification of the Fourier method requires the study of the basis properties (completeness, minimality, basicity, Riesz basicity) of the corresponding systems of the form (1) in the appropriate Banach space of functions.

The dissertation work as a whole is devoted to the above-formulated direction and issues related to this direction. Therefore, we believe that the topic of the dissertation is relevant and is of particular scientific interest.

Let us now give a brief history of the relevant results. The study of basis properties of systems of type (1) has a rich history. The well-known mathematicians Runge and Walsh drew attention to the study of issues related to the approximative properties of such systems back

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[2] Пономарев С.М. К теории краевых задач для уравнений смешанного типа в трехмерных областях // ДАН СССР, 1979, т.246, №6, с. 1303-1304

[3] Моисеев Е.И. О некоторых краевых задачах для уравнений смешанного типа // Дифф. уравн., 1992, т.28, №1, с. 123-132

in 1930 (regarding these results, one can consider Walsh's monograph [4]). Note that establishing the basicity of a system of the form (1) in Lebesgue spaces of functions $L_p(0,\pi)$, $1<p<+\infty$ (in the extreme cases $p=1$ and $p=+\infty$, the basicity, generally speaking, does not hold), is generally not trivial. To clarify this statement, consider the following double analogue of system (1):

$$\left\{A(t)e^{\mathrm{int}};B(t)e^{-ikt}\right\}_{n\in Z_+;k\in N}, \qquad (2)$$

where $A(t)$ and $B(t)$ are complex-valued functions over the interval $[-\pi,\pi]$. As established in the works of B.T. Bilalov [5;6] the basis properties of system (1) in $L_p(0,\pi)$ are proved using similar properties of system (2) in $L_p \equiv L_p(-\pi,\pi)$. Let us denote the closures of linear span of parts $\left\{e^{\mathrm{int}}\right\}_{n\in Z_+}$, $\left\{e^{-\mathrm{int}}\right\}_{n\in N}$, in $L_p, 1<p<+\infty$, by L_p^+ and ${}_{-1}L_p^-$, respectively. The direct decomposition

$$L_p = L_p^+ \dot{+}_{-1} L_p^-,\ 1<p<+\infty, \qquad (3)$$

holds. Let $T^{\pm}: L_p \to L_p -$ be the multiplication operators defined by the expressions

$$T^+ f = Af;\ T^- f = Bf,\ \forall f \in L_p .$$

Then, under the condition $A^{\pm 1};B^{\pm 1}\in L_\infty(-\pi,\pi)$, the question of the basicity of system (2) in L_p is reduced to the correct solvability of the following equation

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[4] Уолш Дж. Л. Интерполяция и аппроксимация рациональными функциями в комплексной области. М.:, ИЛ, 1961

[5]Билалов Б. Т. Система экспонент со сдвигом и задача А. Г. Костюченко // Сиб. мат. журн. 2009, Т. 50, №2. С. 279–288

[6] Билалов Б.Т. Необходимое и достаточное условие полноты и минимальности системы вида $\left\{A\varphi^n;B\overline{\varphi}^n\right\}$ // Докл. РАН. 1992. Т. 322, №6. С. 1019-1021

$$\left(T^{+}P^{+}+T^{-}P^{-}\right)f=g\ ,\ g\in L_{p}\ , \tag{4}$$

where $P^{+}:L_{p}\rightarrow L_{p}^{+};\ P^{-}:L_{p}\rightarrow_{-1}L_{p}^{-}$ are projectors generated by decomposition (3). The following statement is true.

Statement 1. *Let $A^{\pm1};B^{\pm1}\in L_{\infty}$. System (2) forms a basis for L_{p} ,$1<p<+\infty$, if and only if the problem (4) is correctly solvable in L_{p}.*

Further study requires finding conditions regarding the coefficients $A(\cdot)$ and $B(\cdot)$, under which problem (4) is correctly solvable in L_{p}. Problem (4), in turn, due to the identification of $H_{p}^{+}=L_{p}^{+}$ and ${}_{-1}H_{p}^{-}={}_{-1}L_{p}^{-}$ (the Hardy classes H_{p}^{+} and ${}_{-1}H_{p}^{-}$ will be defined later) is reduced to solving the Riemann boundary value problem of the theory of analytic functions in the Hardy classes $H_{p}^{+}\times{}_{-1}H_{p}^{-}$. The first such idea arose in one note by A.V. Bitsadze [7] in 1950. This idea opened the door to the creation of a new method for studying the basis properties of perturbed trigonometric systems of functions in Lebesgue spaces. Subsequently, this idea was successfully used by S.M. Ponomarev when solving a series of mixed-type equations using the Fourier method.

A peculiar reduction of the question of the completeness of a perturbed trigonometric system in L_{p} ,$1<p<+\infty$, to the trivial solvability of the homogeneous Hilbert problem of the theory of analytic functions in the Hardy classes H_{p}^{+} is given in the works of A.N. Barmenkov and Yu.A. Kazmin. Similar questions were studied in the work of A.A. Shkalikov [8]. Using the method of boundary value problems, E.I. Moiseev established a criterion for the basicity of the

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[7] Бицадзе А.В. Об одной системе функций // УМН, 1950, т.5, в. 4(38), с. 150-151

[8] Шкаликов А.А. Об одной системе функций . Математические заметки, 1975, т. 18, в.6, с. 855-860

system of sines $\{\sin[(n+\alpha)t+\beta]\}_{n\in N}$ and exponents $\{e^{i(n+\alpha\, sign\, n)t}\}_{n\in Z}$ in the space $L_p(0,\pi)$ and $L_p(-\pi,\pi)$, $1<p<+\infty$, respectively, where $\alpha,\beta\in R$ are real parameters. Subsequently, having developed this method, B.T. Bilalov established a criterion for the basicity of a double system of exponentials (2) in $L_p(-\pi,\pi)$ and a unary system of the form

$$\{a(t)e^{\text{int}}+b(t)e^{-\text{int}}\}_{n\in N},$$

in $L_p(0,\pi)$, $1<p<+\infty$, where $a;b:[0,\pi]\to C$ are complex-valued functions. Using these results, he also found criteria for completeness and minimality of double power systems $\{A(t)W^n(t);B\overline{W}^n(t)\}_{n\in Z_+}$, and unary power systems $\{a(t)\varphi^n(t)+b(t)\overline{\varphi}^n(t)\}_{n\in Z_+}$, in Lebesgue spaces $L_p(a,b)$, $1\le p\le+\infty$ ($L_\infty(a,b)\equiv C[a,b]$).

In the dissertation work, the above method is generalized to an abstract case. The direct decomposition of a Banach space in subspaces and its double basis generated by this decomposition is considered. The method of boundary value problems is generalized to the abstract considered case and using this method an abstract analogue of the classical "1/4-Kadets" theorem on the Riesz basicity of a perturbed system of exponents is obtained. The boundary value problem method is applied to the basicity of a perturbed double system. Some vector-valued Lebesgue classes and Hardy classes are considered. Some properties of vector-valued functions from these classes and vector-valued analogues of the Sokhotski-Plemelj formulas for vector-valued Cauchy type integrals are established. Vector-valued Riemann boundary value problems are considered and their solvability is studied. Then this approach is applied to abstract questions of the basicity of an abstract system of exponents.

One of the issues considered in the dissertation is the space of coefficients. The term space of coefficients originated in the theory of bases. Its essence is that each basis generates a corresponding Banach space from scalar sequences, which is isomorphic to the original space.

Subsequently, in the works of B.T. Bilalov it was shown that every non-degenerate system in a Banach space (i.e., each member of the system is non-zero) generates a corresponding Banach space of coefficients with a canonical basis. The space of coefficients plays an exceptional role in approximation theory. Classical concepts such as Bessel, Hilbert systems and Riesz bases, introduced by N.K. Bari, are defined based on the space of coefficients, where the space of coefficient is l_2. The fact that an arbitrary non-degenerate system in B-space generates a corresponding B-space from scalar sequences was also noted in the monograph Ch. Heil [9]. In the work of B.T. Bilalov and Z.G. Guseynov [10;11] using the concept of the space of coefficients, all the results of N.K. Bari concerning Bessel-Hilbert systems and Riesz bases were transferred to the general Banach case of the space under consideration. The concept of coefficient space plays a special role in frame theory. Since the concepts of atomic decomposition and Banach (or Hilbert) frame are defined due to the space of coefficients from scalar sequences. In the case of a Hilbert frame, l_2 is taken as the space of coefficients.

One of the key problems considered in the dissertation is frames in Banach spaces. Due to numerous applications in various fields of natural science, frame theory is rapidly developing and interest in it is growing every day. Monographs and review articles by various mathematicians are devoted to it. Many results have been obtained in this direction in the context of the classical Paley–Wiener theorem on the perturbation of an orthonormal basis. More details regarding these results can be found in the monographs of O. Christensen and Ch.Heil.

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[9] Heil Ch. A Basis Theory Primer. Springer, 2011, 534 p.

[10] Билалов Б.Т., Гусейнов З.Г. Критерий базисности возмущенной системы экспонент в лебеговых пространствах с переменным показателем суммируемости // Доклады Академии Наук, 2011, т.436, №5, с.586-589.

[11] Bilalov B.T., Guseynov Z.G. Basicity of a system of exponents with a piece-wise linear phase in variable spaces // Mediterr. J. Math., 2012, vol. 9 , no. 3, pp. 487–498

The dissertation considers an atomic decompositions of Lebesgue spaces and Hardy classes in degenerate systems of exponents. The study of the basis properties of degenerate trigonometric systems originates from the fundamental work of K.I. Babenko regarding the system $\left\{|x|^{\alpha} e^{inx}\right\}_{n \in Z}$. The works of the authors B.Muckenhoupt, R.A.Hunt, W.S.Young, E.I.Moiseev, K.S.Kazaryan, P.I.Lizorkin, B.T.Bilalov, S.S.Pukhov, A.M.Sedletsky et al. are devoted to the studies of the basis properties of trigonometric systems in weighted spaces. In all these works, the weight functions satisfy the well-known Muckenhoupt condition. The question naturally arises: if the coefficient of degeneracy does not satisfy the Muckenhoupt condition, then what can be said regarding the basis properties of the considered system. Apparently, such a case was considered for the first time in the work of E.S.Golubeva in $L_2(-\pi, \pi)$ with respect to the system $\left\{|x| e^{inx}\right\}_{n \in Z}$. This direction was developed in the works of B.T.Bilalov, F.A.Gulieva, S.R.Sadigova, Z.V.Mamedova. They considered the general weight of the power form in the space $L_p(-\pi, \pi)$, $1 < p < +\infty$, relative to systems of sines and exponents. Significant results in this direction were obtained in the works of A.Sh.Shukyurov. He considered trigonometric systems with arbitrary degeneracy and generalized known results to this case.

Another object considered in the dissertation is the issue of convergence of the Fourier-Stieltjes transform. To this end, the concept of statistical convergence is first generalized to the continuous case in different directions, and then this point of view is applied to the question of convergence of the Fourier-Stieltjes transform.

Object and subject of research. Lebesgue spaces of Banach-valued functions, Banach-valued Hardy classes, abstract analogues of Riemann boundary value problems, spaces of coefficients of systems in linear topological spaces, atomic decompositions of Hardy classes, bases from Faber polynomials in weighted Lebesgue spaces, Riemann problems in weighted Smirnov classes, continuous analogue of statistical convergence and its applications to the Fourier - Stieltjes transforms.

The goal and objectives of the study. The purpose and objectives of the dissertation work are to study frame properties (basicity, atomic decomposition, frameness) of systems of elements and the acquisition of new methods of summation and convergence in various linear topological spaces.

General technique of studies. To obtain the main results, methods of functional analysis, function theory, methods of boundary value problems of the theory of analytic functions, the theory of series in Faber polynomials, the theory of singular integral operators, the theory of bases and frames, and the theory of sets with measure are used.

Main provisions of dissertation. The following main results were obtained in this work.

- an abstract generalization of double bases, an abstract analogue of the well-known "$\frac{1}{4}$-Kadets" theorem;
- $H_p(X)$ Hardy vector classes and their main properties;
- concept of space of coefficients of systems of elements in linear topological spaces: sequential and general cases and its properties;
- basicity of half of degenerate systems of exponents in Hardy classes when the degeneracies satisfy the Muckenhoupt condition;
- atomic decompositions in degenerate systems of exponents of Hardy classes when the degeneracies do not satisfy the Muckenhoupt condition;
- solvability questions of Riemann boundary value problems in weighted Smirnov classes;
- basicity of systems from Faber polynomials in weighted Smirnov classes when the weights satisfy the Muckenhoupt condition;
- basicity of a double system from Faber polynomials in Lebesgue spaces on Lyapunov or Radon curves;
- continuous analogue of the concept of statistical convergence;

- concept of μ -strong Cesaro summability, its properties and applications to Fourier transforms.

Scientific novelty. The following main results were obtained in this work.

- an abstract generalization of double bases is given and an abstract analogue of the well-known "$\frac{1}{4}$-Kadets" theorem is obtained;

- based on the abstract space $L_p(X)$ the Hardy vector classes $H_p(X)$ are determined and their main properties are studied;

- the concept of space of coefficients of systems of elements is defined in linear topological spaces in two cases - sequential and general cases, its properties are studied;

-the basicity of half of degenerate systems of exponents in Hardy classes are proved when the degeneracies satisfy the Muckenhoupt condition;

- atomic decompositions in degenerate systems of exponents of Hardy classes are studied when the degeneracies do not satisfy the Muckenhoupt condition;

-the solvability of Riemann boundary value problems in weighted Smirnov classes is studied;

- the basicity of systems from Faber polynomials in weighted Smirnov classes are proved when the weights satisfy the Muckenhoupt condition;

-conditions for the basicity of a double system from Faber polynomials in Lebesgue spaces on Lyapunov or Radon curves are found;

- a continuous analogue of the concept of statistical convergence is found and the main provisions of this theory are transferred to this case;

- concept of μ -strong Cesaro summability is introduced, its properties and applications to Fourier transforms are given;

- connections are established between atomic decompositions of double and unary systems connected with each other by certain relations in Lebesgue spaces.

Theoretical and practical value of the study. The dissertation is of a theoretical character. Its results can be used in the approximation theory, to justify the Fourier method in solving partial differential equations, in bases and frame theories, in the theory of summation, in the theory of statistical convergence, in the theory of Riemann boundary value problems, etc.

Approbation and application. The main results of the dissertation work were reported: at the General All-Institute Seminar of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of Azerbaijan Republic (head of the seminar, corresponding member of ANAS, Prof. M.J. Mardanov), at the seminar of the department "Non-harmonic Analysis" of the IMM (head of the seminar corresponding member of ANAS, Prof. B.T. Bilalov), International conference "Mathematical analysis, differential equations and their applications" (2012, Mersin), International conference on "Actual problems of mathematics and informatics" (2013, Baku), IV Annual Conference of the Georgian Mathematical Union (2014, Tbilisi and Batumi), International mathematical conference on "Theory functions "Modern methods of the theory of boundary value problems" (2017, Ufa). "Pontryagin Readings – XXIX" (2018, Moscow), Differential equations and related problems. International Scientific Conference (2018, Sterlitamak), International Scientific Conference "Modern problems of mathematics and mechanics" (2019, Moscow), 3rd International Conference on "Mathematical Advances and Applications" (2020, Istanbul), 4th International Conference on "Mathematical Advances and Application", (2021, Istanbul), etc.

Personal contribution of the author. All results obtained in the dissertation are the personal contribution of the author.

Publications of the author. The main results of the dissertation were published in 30 works.

The name of the institution where the dissertation was completed. The work was carried out in the department "Non-harmonic analysis" of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of Azerbaijan Republic.

Volume and structure of the dissertation (in signs, indicating the volume of each structural unit separately). The total volume of dissertation work is –369139 characters (title page – 372 characters, content – 3432 characters, introduction – 90000 characters, first chapter – 64000 characters, second chapter – 28000 characters, third chapter –52000 characters, fourth chapter –54000 characters, fifth chapter –76000 characters, conclusions –1335 characters). The list of used literature consists of 186 items.

THE CONTENT OF THE DISSERTATION

The dissertation work consists of an introduction, five chapters, conclusions and a list of references.

The introduction substantiates the relevance of the topic of the dissertation work, provides a brief history of the issues involved, as well as a brief content of the dissertation work.

Chapter I considers the direct decomposition of a Banach space in subspaces and its double basis generated by this decomposition. The method of boundary value problems is generalized to the considered abstract case and using this method an abstract analogue of the classical “$\frac{1}{4}$-Kadets” theorem on the Riesz basicity of a perturbed system of exponents is obtained. The method of boundary value problem is applied to the basicity of a perturbed double system. At the end of the chapter, some vector-valued Lebesgue and Hardy classes are considered. Some properties of vector-valued functions from these classes and vector-valued analogues of the Sokhotski-Plemelj formulas for vector-valued Cauchy type integrals are established. Vector-valued Riemann boundary value problems are considered and their solvability is studied. Then this approach is applied to abstract questions of the basicity of an abstract system of exponents.

Section **1.1** provides the necessary concepts and facts". Let us give some standard notation. B-space – Banach space; H-space– Hilbert space; $\|\cdot\|_X$ is a norm in X (if there are no misunder-standings, then sometimes we will omit the index); I is an identity operator.

The basicity of the double system is understood in the following sense.

Definition 1. *The double system* $\{x_n^+; x_n^-\}_{n\in N}$ *is said to be basis for* X, *if* $\forall x \in X$, $\exists! \{\lambda_n^\pm\}_{n\in N} \subset C$:

$$x = \sum_{n=1}^{\infty} \lambda_n^+ x_n^+ + \sum_{n=1}^{\infty} \lambda_n^- x_n^- .$$

We will also use the following concept.

System $\{x_n\}_{n\in N} \subset X$ *is called a uniformly-minimal in* X, *if* $\exists \delta > 0$:

$$\inf_{\forall u \in L[\{x_n\}_{n\neq k}]} \|x_k - u\|_X \geq \delta \|x_k\|_X \text{ , } \forall k \in N . \quad (5)$$

Section **1.2** proposes one abstract method for establishing the Riesz basicity of double systems in Hilbert spaces. A concrete case $\{e^{i(nt+\alpha(t)\,sign\,n)}\}_{n\in Z}$ is considered, where $\alpha \in L_\infty$ – is some measurable function. In particular, for $\alpha(t) \equiv \alpha t$ the previously known final result regarding the Riesz basicity of the system $\{e^{i(n+\alpha\,sign\,n)t}\}_{n\in Z}$, in L_2, is obtained. Let us give some of them.

Let B-space X have a direct decomposition

$$X = X^+ \dot{+} X^- , \quad (6)$$

by subspaces X^+ and X^-. Let $T^\pm \in L(X)$ be some automorphisms. It is known that $\exists m > 0$:

$$\|x^+\| + \|x^-\| \leq m\|x\| , \forall x \in X . \quad (7)$$

$\inf\{m : satisfting\ \ (7)\}$ *denote by* $\theta(X^+;X^-)$ *and call it the direct norm of expansion (6).*

The following theorem is true.

Theorem 1. *Let* B*-space* X *have the decomposition* $X = X^+ \dot{+} X^-$, $\{x_n^\pm\}_{n\in N} \subset X^\pm$ *form a basis for* $X^\pm$ *and* $\theta(X^+;X^-)$ *be the direct norm of expansion (6). If* $\eta < \dfrac{1}{\theta(X^+;X^-)}$, *then the system* $\{T^+x_n^+;T^-x_n^-\}_{n\in N}$ *forms a basis for* X, *where* $\eta = \max\{\|\Delta T^+\|;\|\Delta T^-\|\}$, $\Delta T^\pm = I - T^\pm$.

Particular cases. Let us take as X the space L_p, $1 < p < +\infty$, $X^\pm = H_p^\pm$, where $H_p^\pm$ are the usual Hardy classes of analytic functions inside and outside (vanishing at infinity), respectively. Let $T^\pm f = A^\pm(t)\cdot f(t)$ be the multiplication operator in L_p and $\theta(H_p^+;H_p^-)$ be the direct norm of expansion $L_p = H_p^+ \dot{+} H_p^-$. From Theorem 1 we directly obtain

Corollary 1. *Let the inequality*

$$\max\left(\|1-A^+(t)\|_\infty;\|1-A^-(t)\|_\infty\right) < \frac{1}{\theta(H_p^+;H_p^-)},$$

be satisfied. Then the system

$$\{A^+(t)e^{\mathrm{int}};A^-(t)e^{-ikt}\}_{n\ge 0;k\ge 1}, \tag{8}$$

forms a basis for L_p *isomorphic to* $\{e^{\mathrm{int}}\}_{n\in Z}$, *where* $\|\cdot\|_\infty$ *is the ordinary norm in* $L_\infty(-\pi,\pi)$.

In particular, for $p=2$ it is clear that $\theta(H_2^+;H_2^-) = \sqrt{2}$ and as a result we obtain

Corollary 2. *Let the inequality*

$$\max\left(\|1-A^+(t)\|_\infty;\|1-A^-(t)\|_\infty\right) < \frac{1}{\sqrt{2}},$$

be satisfied. Then the system (8) forms a Riesz basis for $L_2(-\pi,\pi)$.

Let us consider the particular case $A(t)\equiv e^{i\alpha(t)}$, $B(t)\equiv e^{-i\alpha(t)}$. In this case, it is possible to obtain an exact result.

Statement 2. *Let* $\|\alpha\|_\infty < \frac{\pi}{4}$. *Then the system of exponents* $\left\{e^{i(nt+\alpha(t)\operatorname{sign} n)}\right\}_{n\in Z}$ *forms a Riesz basis for* L_2.

In the case $\alpha(t)=\alpha\, t$, where $\alpha\in R$ is a real parameter, we obtain the previously known final result regarding the basicity of the system $\left\{e^{i(n+\alpha \operatorname{sign} n)t}\right\}_{n\in Z}$ in L_2.

Section **1.3** considers the vector-valued Lebesgue $L_p(X)$ and Hardy classes $H_p(X)$, where X is a Banach space. They are generalizations of the analogous Lebesgue and Hardy spaces of the scalar case. For the Hardy class, two definitions are proposed and their equivalence is proved. Riemann boundary value problems in different formulations are considered. Under certain conditions, their correct solvability is proven. The questions of bases from subspaces in $L_p(X)$ are also considered.

Let X be a B-space. System of subspaces $\left\{X_n^+; X_k^-\right\}_{n,k\in N} \subset X$ is said to be a basis for X, if $\forall x\in X$, $\exists!\left\{x_n^\pm\right\}_{n\in N}$, $x_n^\pm \in X_n^\pm$:

$$x=\sum_{n=1}^{\infty} x_n^+ + \sum_{n=1}^{\infty} x_n^- .$$

Let X be a separable B-space. Denote by $L_p(X)$ class of measurable (strongly or weakly, indifferently due to separability of X) functions $\vartheta:[0,2\pi]\to X$ and such that

$$\|\vartheta\|_p^p = \frac{1}{2\pi}\int_0^{2\pi} \|\vartheta(t)\|_X^p\, dt < +\infty,\ 1\le p<+\infty,$$

where $\|\cdot\|_X$ is a norm in X. With this definition of the norm,, $L_p(X)$ turns into a separable B-space. Functions from $L_p(X)$, that coincide almost everywhere (according to the Lebesgue measure) are identified. Denote by $L_p^{(k)}(X)$ the subspace of the space $L_p(X)$ generated by functions of the form $e^{ikt}a,\ a \in X$, where $k \in Z$ is an integer. Take $\forall f \in L_p(X)$. Let

$$f_k = \frac{1}{2\pi}\int_0^{2\pi} f(t)e^{-ikt}dt \in X,\ \forall k \in Z \cdot$$

Consider $P_k : L_p(X) \to L_p^{(k)}(X),\ k \in Z:\ P_k f = e^{ikt} f_k$. It is easy to see that

$$P_i P_j = \delta_{ij} P_j,\ \ \forall i, j \in Z,$$

holds, where δ_{ij} is the Kronecker symbol. The following theorem is true.

Theorem 3. *The family of projectors* $\{P_n\}_{n \in Z}$ *forms a strong basis for* $L_p(X)$*, i.e.*

$$I = \sum_{n=-\infty}^{+\infty} P_n \Leftrightarrow L_p(X) = \sum_{n=-\infty}^{+\infty} \dot{+} L_p^{(n)}(X).$$

Assume

$$L_p^+(X) \equiv \{ f \in L_p(X) : P_n f = 0,\ \forall n < 0 \}.$$

$L_p^+(X)$ is a subspace of the space $L_p(X)$. Denote by $P_r(\cdot)$ the Poisson kernel for the circle

$$P_r(t) = \frac{1 - r^2}{1 - 2r\cos t + r^2}.$$

The following theorem is true.

Theorem 4. *Let* X *be a separable* B*-space with separable dual space* X^* *and* $f \in L_p(X), 1 < p < +\infty$*. Then with respect to the function* $F(z)$*, defined by the Cauchy type integral expression*

$$F(z) = \frac{1}{2\pi i}\int_{\partial\omega} \frac{f(\tau)d\tau}{\tau - z},$$

the Sokhotski-Plemelj formulas

$$F^{\pm}(\tau)=\pm\frac{1}{2}f(\tau)+[Sf](\tau)\ ,\ \text{a.e. } \tau\in\partial\omega,$$

holds. Moreover, if $f\in L_p^+(X)$ *, then*

$$\lim_{r\to 1-0}\left\|[T_r(f)](t)-f(t)\right\|_p=0\ ,\ \left\|T_r(f)\right\|_p\le M\|f\|_p\ ,$$

holds, where $T_r=P_r$ *is either a Poisson operator*

$$[P_r(f)](t)\equiv\frac{1}{2\pi}\int_{-0}^{2\pi}P_r(t-s)f(s)ds\ ,$$

or $T_r=\mathscr{K}_r$ *is a Cauchy operator*

$$[K_r(f)](t)\equiv\frac{1}{2\pi i}\int_{\gamma}\frac{f(\tau)d\tau}{\tau-re^{it}}\ ,$$

$r:0\le r<1$ *and* M *is a constant independent of* f .

We define the class $L_p^-(X)$ in a similar way*:*

$$L_p^-(X)\equiv\left\{f\in L_p(X):P_n f=0,\ \forall n\ge 0\right\}.$$

By $H_p^+(X)$ $\left({}_{-1}H_p^-(X)\right)$ we denote the Hardy space defined by the following relation

$$H_p^+(X)=\left\{F:\exists f\in L_p^+(X)\Rightarrow F(z)=\frac{1}{2\pi i}\int_{\gamma}\frac{f(z)}{\xi-z}d\xi,\ z\in\omega\right\}$$

$$\left({}_{-1}H_p^-(X)=\left\{F:\exists f\in{}_{-1}L_p^+(X)\Rightarrow F(z)=\frac{1}{2\pi i}\int_{\gamma}\frac{f(z)}{\xi-z}d\xi,\ z\in C\setminus\omega\right\}\right).$$

Assume

$$H_p^{+,0}(X)\equiv\left\{F\in H_p^+(X):F(0)=0\right\},$$

and let $L_p^{+,0}(X)\equiv H_p^{+,0}(X)/_{\partial\omega}$. Denote

$$\hat{L}_p^{+,0}(X)\equiv\left\{g:\hat{g}(t)=g(-t)\in L_p^{+,0}(X)\right\}.$$

Let us denote by $\tilde{H}_p^+(X)$ the class of analytic in ω X - valued functions f, for which

$$\|f\|_{\tilde{H}_p}^p \equiv \sup_{0<r<1} \int_{-\pi}^{\pi} \left\| f\left(re^{it}\right) \right\|^p dt < +\infty \cdot$$

The following theorem is true.

Theorem 5. *Let X be a separable B-space with separable dual space X^*. Then the spaces $H_p^+(X)$ and $\tilde{H}_p^+(X)$ defined above are isometrically isomorphic.*

This chapter considers Riemann boundary value problems in different formulations. Let's consider one of them.

Conjugation problem with operator coefficient. Let $\mathscr{L}_p$ denote the Banach space of bounded operators acting from $L_p(X)$ to $L_p(X)$, i.e. $\mathscr{L}_p \equiv L(L_p(X); L_p(X))$. Let $A, B \in \mathscr{L}_p(X) -$ be some operators. Take $g \in L_p(X)$ and consider the equation

$$AF^+(\tau) + BF^-(\tau) = g(\tau), \text{ a.e. } \tau \in \partial\omega, \tag{9}$$

where $F^{\pm} \in L_p^{\pm}(X)$, i.e. a pair $(F^+; F^-) \in L_p^+(X) \times L_p^-(X)$, is sought for which relation (9) holds. Direct expansion

$$L_p(X) = L_p^+(X) \dot{+} L_p^-(X), \; 1 < p < +\infty \; , \tag{10}$$

is valid.

Put $\theta_X^{+,-} = \theta_X(L_p^+; L_p^-)$. The following theorem is true.

Theorem 6. *Let B-space X satisfy all conditions of Theorem 5, operators A and B satisfy the inequality $\eta(A;B) < \left(\theta_X^{+,-}\right)^{-1}$,where $\eta(A;B) = \max\{\|I-A\|; \|I-B\|\}$. Then equation (9) has a unique solution for $\forall g \in L_p(X)$, $1 < p < +\infty$. Moreover, $\exists M > 0$:*

$$\left\|F_g^{\pm}\right\|_p \le M \|g\|_p, \; \forall g \in L_p(X),$$

where $F_g^{\pm} -$ solutions of equation (9) corresponding to g.

The following theorem is also true.

Theorem 7. *Let all the conditions of Theorem 6 hold. Then the system of subspaces* $\left\{AL_p^{(n)}(X); BL_p^{(-k)}(X)\right\}_{n\geq 0,\ k\geq 1}$ *forms a basis for* $L_p(X)$.

Let's consider the case when X is a H-space with scalar product $(\cdot;\cdot)$. In this case, $L_2(X)$ is also a H- space with scalar product

$$(f;g)_{L_2(X)} = \int_{-\pi}^{\pi} (f(t); g(t))dt,\ \forall f, g \in L_2(X). \qquad (11)$$

It is easy to notice that the subspaces $L_2^+(X)$ and $L_2^-(X)$ are orthogonal, as a result $\theta_X^{+,-} = \sqrt{2}$, and it means that the following corollary is true.

Corollary 3. *Let* X *be a separable* H*-space and operators* A, B *satisfy the condition*

$$\eta_\infty(A;B) = \max\left\{\|I-A\|_\infty; \|I-B\|_\infty\right\} < \frac{1}{\sqrt{2}}.$$

Then the system $\left\{AL_p^{(n)}(X); BL_p^{(-k)}(X)\right\}_{n\geq 0,\ k\geq 1}$, *forms a basis for* $L_2(X)$.

The second chapter is devoted in general to the concept of the space of coefficients.

Section **2.1** provides the necessary concepts and facts used in this chapter. Let us present some of them. By a linear topological space $(X;\tau)$ (abbreviated LTS) we mean a linear space X over a field F ($F \equiv R$ or $F \equiv C$) with topology τ, in which linear operations are continuous and each point in it is a closed set.

Let $(X;\tau)$ be some LTS over the field K. We denote the linear span of a set $M \subset X$ by $L[M]$, and its closure according to the topology τ by $\overline{M}$.

We will use the following concept.

Definition 2. *System* $\{x_n\}_{n\in N} \subset X$ *is said to be non-degenerate if* $x_n \neq 0$, $\forall n \in N$.

Section **2.2** is devoted to the study of the topological properties of the space of coefficients generated by a non-degenerate system in Hausdorff linear topological spaces. It is proved that an arbitrary non-degenerate system in sequentially complete linear topological spaces generates a similar complete linear topological space of coefficients with a canonical basis. A criterion for the basicity of systems in similar spaces is given in the term of the coefficient operator.

Let $(X;\tau)$ be a sequentially complete linear topological space and $\{x_n\}_{n\in N} \subset X$ be some non-degenerate system. Put

$$\mathscr{K}_{\bar{x}} \equiv \left\{\{\lambda_n\}_{n\in N} \subset F : the\ series\ \sum_{n=1}^{\infty} \lambda_n x_n\ \ converges\ in\ \ X\right\}.$$

It is obvious that with respect to the usual operations of component-wise addition and multiplication by a scalar, $\mathscr{K}_{\bar{x}}$ turns into a linear space. Each neighborhood of zero O_ε in X generates a corresponding neighborhood of zero $O_\varepsilon^{\mathscr{K}}$ in $\mathscr{K}_{\bar{x}}$:

$$O_\varepsilon^{\mathscr{K}} \equiv \left\{\bar{\lambda} \equiv \{\lambda_n\}_{n\in N} \in \mathscr{K}_{\bar{x}} : \sum_{n=1}^{m} \lambda_n x_n \in O_\varepsilon, \forall m \in N\right\}.$$

The set of neighborhoods $O_\varepsilon^{\mathscr{K}}$ of zero in $\mathscr{K}_{\bar{x}}$ generates the corresponding topology $\tau_{\mathscr{K}}$ in $\mathscr{K}_{\bar{x}}$⋅.

The following theorem is proved.

Theorem 8. *The space* $\mathscr{K}_{\bar{x}}$ *, generated in it by the topology* $\tau_{\mathscr{K}}$ *, has the following properties: 1) it is sequentially complete; 2) every one-point set is closed in it; 3) linear operations are sequentially continuous in it.*

We will assume that X is a F -space (i.e. Fréchet space) and consider the operator $T : \mathscr{K}_{\bar{x}} \to X$ defined by the expression

$$T\bar{\lambda} = \sum_{n=1}^{\infty} \lambda_n x_n, \bar{\lambda} \equiv \{\lambda_n\}_{n\in\mathbb{N}} \in \mathscr{K}_{\bar{x}}.$$

Operator $T:\mathscr{K}_{\bar{x}}\to X$ is called the coefficient operator of the system $\{x_n\}_{n\in N}\subset X$.

In the term of the space of coefficient, the following basis criterion is valid.

Theorem 9. *Let $(X;\tau)$ be a F−space, $\{x_n\}_{n\in N}\subset X$ be a non-degenerate system, $\left(\mathscr{K}_{\bar{x}};\tau_{\mathcal{K}_{\bar{x}}}\right)$ be its corresponding space of coefficients and $T:\mathscr{K}_{\bar{x}}\to X$ be the corresponding coefficient operator. The system $\{x_n\}_{n\in N}$ forms a basis for X if and only if the following conditions are met: 1) this system is complete in X ; 2) this system is ω -linear independent; 3) $ImT=\overline{ImT}$.*

In **2.3**, the concept of the space of coefficients of systems is defined in linear topological spaces in the general case and similar results are obtained in these spaces.

Chapter III is devoted to atomic decompositions of Lebesgue spaces and Hardy classes in degenerate systems of exponents. This chapter considers parts of system of exponents with degenerate coefficients. The questions of atomic decomposition of Lebesgue spaces and Hardy classes in these systems are studied. Defect indices of these systems are calculated.

Section **3.1** provides the necessary concepts and facts that are used to obtain the results of this chapter.

Definition 3. *A sequence $\{f_k\}_{k\in N}\subset X$ in H-space X with scalar product $(\cdot\,;\cdot)$ is called a frame (or forms a H -frame) if* $\exists A;B>0:$

$$A\|f\|^2\le\sum_{k=1}^{\infty}\left|\left(f;f_k\right)\right|^2\le B\|f\|^2,\forall f\in X,$$

where $\|\cdot\|=\sqrt{(\cdot;\cdot)}$.

A direct generalization of the concept of a frame in a Hilbert space to the Banach case is the atomic decomposition. Let us define it.

Definition 4. *Let* X *be a* B*-space and* $\mathscr{K}$ *is a* B*-space of sequences of scalars. Let*

$$\{f_k\}_{k\in N}\subset X\,,\ \{g_k\}_{k\in N}\subset X^*.$$

Then, if the following conditions hold, $\left(\{g_k\}_{k\in N}\,,\,\{f_k\}_{k\in N}\right)$ *is said to be an atomic decomposition of* X *with respect to* $\mathscr{K}$ *:*

(i) $\{g_k(f)\}_{k\in N}\in\mathscr{K}\,,\ \forall f\in X$ *;*

(ii) $\exists A,B>0: A\|f\|_X\le\left\|\{g_k(f)\}_{k\in N}\right\|_{\mathscr{K}}\le B\|f\|_X\,,\quad\forall f\in X$ *;*

(iii) $f=\sum_{k=1}^{\infty}g_k(f)\,f_k\,,\ \forall f\in X$.

Let us pay attention to the fact that in the Hilbert case l_2 is involved as the space $\mathscr{K}$ and properties (i) and (iii) in Definition 4 are consequences of property (ii).

Section **3.2** provides the necessary notation, concepts and some facts from the theory of bases and frames used in this chapter. Parts of the system of exponents with degenerate coefficients corresponding to a positive value of index are considered. It is proved that if the coefficients satisfy the Muckenhoupt condition, then these parts form bases in the corresponding Hardy classes of analytic functions H_p^+. The frame property (frameness, atomic decomposition) of the part corresponding to the positive value of index is studied when the coefficient, generally speaking, does not satisfy the Muckenhoupt condition.

The restrictions of the classes H_p^+ and ${}_mH_p^-$ to the unit circle $\partial\omega$ denote by L_p^+ and ${}_mL_p^-$, respectively.

Basicity. Let us consider the system $E_+^{(k)}(\rho)\equiv\{\rho(t)e^{\mathrm{int}}\}_{n\ge k}$. We will assume that the degenerate coefficient ρ has a power form

$$\rho(t)=\left(e^{it}-1\right)^{\alpha_0}\prod_{k=1}^{r}\left(e^{it}-e^{it_k}\right)^{\alpha_k}\,,$$

where $\{t_k\}_1^r \subset (-\pi,\pi]\backslash\{0\}-$ are different points and $\{\alpha_k\}_0^r \subset R$. We will assume throughout that the weight function $\rho(\cdot)$ is periodically (with period 2π) extended to the entire real axis R.

Let $\mathscr{A}_p$ denote the class of weights $\nu(t)$, satisfying the Muckenhoupt condition

$$\sup_{I\subset[-\pi,\pi]}\left(\frac{1}{|I|}\int_I \nu(t)dt\right)\left(\frac{1}{|I|}\int_I [\nu(t)]^{-\frac{1}{p-1}}dt\right)^{p-1} < +\infty,$$

where $\sup$ is taken over all intervals $I \subset R$ and is the Lebesgue measure of I. Note that the expression $|\rho|^{\frac{1}{p}} \in \mathscr{A}_p$ is true if and only if the following inequalities

$$-\frac{1}{p} < \alpha_k < 1-\frac{1}{p},\ k=\overline{0,r}, \qquad (12)$$

holds.

The following theorem is true.

Theorem 10. *Let inequalities (12) be satisfied. Then the system* $E_+^{(0)}(\rho)$ *forms a basis for* L_p^+, $1<p<+\infty$.

Defect case. We will consider the case when $|\rho|^{\frac{1}{p}} \notin \mathscr{A}_p$. Let the inequalities

$$1-\frac{1}{p} \le \alpha_0 < 2-\frac{1}{p},\ -\frac{1}{p} < \alpha_k < 1-\frac{1}{p},\ k=\overline{1,r},$$

hold.

The following theorem is true.

Theorem 11. *Let the inequalities*

$$m-\frac{1}{p} \le \alpha_0 < m+\frac{1}{q},\ -\frac{1}{p} < \alpha_k < \frac{1}{q},\ k=\overline{1,r}.$$

hold. Then the system $E_+^{(0)}(\rho)$ *has a defect equal to* m *in* L_p^+. *Moreover, the system* $E_+^{(k)}(\rho)$ *is complete and minimal in* L_p^+, *but is not uniformly minimal in it, and therefore does not form a basis.*

The following theorem is also proved.

Theorem 12. *The system* $E_+^{(0)}(\rho)$ *is an atomic decomposition in* L_p^+, $1 < p < +\infty$, *if and only if the degenerate coefficient* ρ *satisfies Muckenhoupt's condition (12).*

In **3.3** it is considered another part of the system of exponents with a degenerate coefficient. Frame properties (such as completeness, minimality, basicity, atomic decomposability) of this system in the Hardy classes ${}_{-1}H_p^-$ are studied in the case where the coefficient may not satisfy the Muckenhoupt condition.

Basicity. Let $E_\pm^{(k)}(\rho) \equiv \{\rho(t)e^{\pm int}\}_{n\ge k}$. Completely similar to the previous section, the following theorem is proved:

Theorem 13. *Let the inequalities*

$$-\frac{1}{p} < \alpha_k < 1 - \frac{1}{p}, k = \overline{0, r},$$

be satisfied. If $\sum_{k=0}^{r} \alpha_k < 1$, *then the system* $E_-^{(1)}(\rho)$ *forms a basis for* ${}_{-1}L_p^-$, $1 < p < +\infty$.

The following theorem is also true:

Theorem 14. *Let the inequalities*

$$\alpha_0 > -\frac{1}{p}, -\frac{1}{p} < \alpha_k < 1 - \frac{1}{p}, k = \overline{1, r},$$

be satisfied. Then the system $E_-^{(1)}(\rho)$ *is complete and minimal in* $\overline{span E_-^{(1)}(\rho)}$. *Moreover, this system is an atomic decomposition of*

$\overline{span\left(E_{-}^{(1)}(\rho)\right)}$ *with respect to the space of coefficients* $\mathcal{K}_{\rho}^{-}$ *if and only if* $-\frac{1}{p} < \alpha_0 < 1 - \frac{1}{p}$.

Chapter IV is devoted to the study of bases for systems from generalized Faber polynomials in weighted Smirnov and Lebesgue spaces. Section **4.1** provides some necessary information regarding generalized Faber polynomials.

Let us give the definition of the generalized p-Faber polynomials $F_{p,n}^{+}$ and $F_{p,n}^{-}$. Let D be a bounded region with the boundary Γ and the simple-connected complement $D^{-} = C \setminus \overline{D}$ ($\overline{D}$ – is the closure of D). Let $w = \varphi(z)$ be a single-valued conformal mapping the domain D^{-} on $C \setminus \overline{O_1(0)} \equiv O_1^{-}(0)$: $\varphi(\infty) = \infty$, $\varphi'(\infty) = \gamma > 0$. $\varphi(z)$ has a Laurent expansion in the neighborhood of the point $z = \infty$

$$\varphi(z) = \gamma z + \gamma_0 + \gamma_1 z^{-1} + \ldots .$$

Let us take a single-valued branch of the function $\sqrt[p]{\varphi'(z)}$ so that $\sqrt[p]{\varphi'(\infty)} > 0$. The main part of the Laurent expansion of the function $[\varphi(z)]^{n} \sqrt[p]{\varphi'(z)}$ in the neighborhood of the point $z = \infty$ by $F_{p,n}^{+}$, i.e.

$$[\varphi(z)]^{n} \sqrt[p]{\varphi'(z)} \equiv F_{p,n}^{+}(z) + E_{p,n}^{+}(z),\ z \in D^{-},$$

where $E_{p,n}^{+}(\infty) = 0$.

The p - Faber polynomial $F_{p,n}^{-}$ is defined in a similar way.

As usual, by $L_p(\Gamma;\omega)$ we denote the weighted Lebesgue space of functions with norm $\|\cdot\|_{p,\omega}$:

$$\|f\|_{L_p(\Gamma;\omega)} = \left(\int_{\Gamma} |f(\xi)|^{p} \omega(\xi) |d\xi| \right)^{\frac{1}{p}} .$$

Let $A(\xi) \equiv |A(\xi)| e^{i\alpha(\xi)}$, $B(\xi) \equiv |B(\xi)| e^{i\beta(\xi)}$ be complex-valued functions defined on the curve Γ. We will assume that the following basic conditions hold.

i) $|A|^{\pm 1}, |B|^{\pm 1} \in L_\infty(\Gamma)$;

ii) $\alpha(\xi)$, $\beta(\xi)$ – *are piecewise continuous on* Γ *and let* $\{\xi_k, k=\overline{1,r}\} \subset \Gamma$ – *be discontinuity points of the function* $\theta(\xi) \equiv \beta(\xi) - \alpha(\xi)$.

Regarding the curve Γ, we require the following condition to be satisfied.

iii) Either $\Gamma -$ is a piecewise Lyapunov or Radon curve (i.e. is a curve with bounded rotation with no cusps. As the direction over Γ we accept the positive direction, i.e. the direction when moving in it the domain D remains on the left. Let $a \in \Gamma$ be the initial (also the end) point of the curve Γ. *We will assume that* $\xi \in \Gamma$ *follows the point* $\tau \in \Gamma$, *i.e.* $\tau \prec \xi$, *if* ξ *comes after* τ when moving in the positive direction on $\Gamma \setminus a$, *where* $a \in \Gamma$ *are two stuck together points* $a^+ = a^-$, $a^+ -$ *is the beginning,* $a^- -$ *is the end of the curve* Γ.

The class of curves satisfying condition iii) is denoted by $\mathscr{LR}$.

So, without loss of generality, we will assume that $a^+ \prec \xi_1 \prec ... \prec \xi_r \prec b = a^-$. The one-sided limits $\lim\limits_{\substack{\xi \to \xi_0 \pm 0 \\ \xi \in \Gamma}} g(\xi)$ of the function $g(\xi)$ generated by this order at the point $\xi_0 \in \Gamma$ will be denoted by $g(\xi_0 \pm 0)$, respectively. We denote jumps of the function $\theta(\xi)$ at points $\xi_k, k=\overline{1,r}$, by $h_k : h_k = \theta(\xi_k + 0) - \theta(\xi_k - 0)$, $k=\overline{1,r}$. Regarding the jump, we will assume that the following condition is satisfied:

iv) $\left\{h_k - \frac{2\pi}{p} : k = \overline{0,r}\right\} \cap Z = \varnothing$, *where* $h_0 = \theta(a+0) - \theta(a-0)$, $p \in (1,+\infty)$ *– is some number.*

Let $D^+ \subset C-$ be a bounded domain with boundary $\Gamma = \partial D^+$, with respect to which condition i) holds. Let $E_p(D^+), 1 < p < \infty$ denote the Banach Smirnov space of analytic functions in D^+ with norm $\|\cdot\|_{E_p(D^+)}$:

$$\|f\|_{E_p(D^+)} =: \|f^+\|_{L_p(\Gamma)}, \forall f \in E_p(D^+), \qquad (13)$$

where $f^+ = f/_\Gamma -$ is nontangential boundary values of the function f on Γ.

Based on norm (13), the weighted Smirnov class is defined. Let $\rho \in L_1(\Gamma)$ be some weight function. Let us define the weighted Smirnov class $E_{p,\rho}(D^+)$:

$$E_{p,\rho}(D^+) \equiv \left\{ f \in E_1(D^+) : \|f^+\|_{L_{p,\rho}(\Gamma)} < +\infty \right\},$$

and put

$$\|f\|_{E_{p,\rho}(D^+)} = \|f^+\|_{L_{p,\rho}(\Gamma)}.$$

The Smirnov classes in an unbounded domain are defined in a similar way.

In **4.2,** the homogeneous Riemann problem of the theory of analytic functions in weighted Smirnov classes is considered and, under certain conditions on the weight function, on the coefficient of the problem and on the boundary of the domain, a general solution to this problem is constructed

Consider the following homogeneous Riemann problem in weighted classes $E_{p,\rho}(D^+) \times_m E_{p,\rho}(D^-)$:

$$A(\xi)F^+(\xi) + B(\xi)F^-(\xi) = 0, \text{ a.e. } \xi \in \Gamma. \qquad (14)$$

Under the solution of the problem (14) is understood as a pair of analytical functions $\left(F^{+};F^{-}\right)\in E_{p,\rho}\left(D^{+}\right)\times_{m} E_{p,\rho}\left(D^{-}\right)$ whose non-tangent boundary values $F^{\pm}\left(\xi\right)$ a.e. on Γ satisfy equality (14). In the weightless case, this problem has been well studied and its theory is covered in the monograph by I.I. Danilyuk.

Let $S-$be the length of the curve Γ and $z=z\left(s\right),0\le s\le S$, be a parametric representation of Γ with respect to the length of the arc s. Put

$$\Omega\left(s\right)\equiv\theta\left(z\left(s\right)\right),0\le s\le S,$$

and let $h_k=z\left(s_k+0\right)-z\left(s_k-0\right),\ \ k=\overline{1,r};h_0=\Omega\left(+0\right)-\Omega\left(S-0\right)$.
Let us consider piecewise holomorphic functions

$$Z_{(1)}(z)=\exp\left\{\frac{1}{2\pi i}\int_{\Gamma}\ln\left|D(s)\right|\frac{dz(s)}{z(s)-z}\right\},$$

$$\tilde{Z}(z)=\exp\left\{\frac{1}{2\pi}\int_{\Gamma}\Omega(s)\frac{dz(s)}{z(s)-z}\right\}.$$

We denote the product of these functions by Z_{Ω}

$$Z_{\Omega}(z)\equiv Z_{(1)}(z)\tilde{Z}(z).$$

$Z_{\Omega}(\cdot)$ *will be called the canonical solution of the homogeneous problem (14), corresponding to the argument* $\Omega(\cdot)$.

The following theorem is true.

Theorem 15. *Let conditions i)-iii) be satisfied with respect to the complex-valued functions* $A\left(\xi\right),B\left(\xi\right)$ *and the curve* Γ. *Let us assume that the conditions*

$$\frac{h_k}{2\pi} < 1,\ k = \overline{0, r}\ ;$$

$\exists p_1;\ p_2 \in (1, +\infty)$:

$$\int_0^S \sigma^{pp_1} \rho^{p_1}(z(s))ds < +\infty, \quad \int_0^S \sigma^{-qp_2}(s) \rho^{-\frac{q}{p}p_2}(z(s))ds < +\infty, \tag{15}$$

are satisfied with respect to the jumps $\{h_k\}$ *and the weight function* $\rho(\xi)$. *Then the general solution of the homogeneous problem (14) in the classes* $E_{p,\rho}(D^+) \times_m E_{p,\rho}(D^-)$ *has the form*

$$F(z) \equiv Z_\Omega(z) P_m(z), \tag{16}$$

where $Z_\Omega(\cdot)$ *is a canonical solution, and* $P_m(z)$ *is an arbitrary polynomial of degree* $k \le m$.

Let us consider some special cases regarding the weight function ρ .

Example 1. Let the weight $\rho(\cdot)$ have the following power form

$$\rho(z(s)) = \prod_{k=1}^{m} |s - t_k|^{\alpha_k} \ , \tag{17}$$

where $\{t_k\}_1^m \subset (0, S) -$ are different points, $\{\alpha_k\}_1^m \subset R$ are some numbers. We denote the union of the sets $\{s_k\}_0^r$ and $\{t_k\}_1^m$ by $\{\tau_k\}_1^l : \{\tau_k\}_1^l \equiv \{s_k\}_0^r \cup \{t_k\}_1^m$. Let $\chi_A(\cdot)$ be the characteristic function of the set A. We denote the single-point sets of $\{\tau_k\}, k = \overline{1, l}$ by $T_k : T_k \equiv \{\tau_k\}, k = \overline{1, l}$. Assume

$$\beta_k = -\frac{p}{2\pi} \sum_{i=1}^{r} h_i \chi_{T_k}(s_i) + \sum_{i=1}^{m} \alpha_i \chi_{T_k}(t_i), \quad k = \overline{1, l}\ . \tag{18}$$

Let us assume that the inequalities

$$-1<\beta_k<\frac{p}{q},\ k=\overline{1,l}\,, \tag{19}$$

are satisfied. It is easy to show that when inequalities (19) are satisfied, relations (15) hold and, as a result, Theorem 15 is valid.

In **4.3** we consider a nonhomogeneous problem with a right-hand side from the weighted Lebesgue space and study its solvability in weighted Smirnov classes under the same assumptions.

Power weight. Consider the non-homogeneous Riemann boundary value problem

$$F^+\left(z(s)\right)-D(s)F^-\left(z(s)\right)=g\left(z(s)\right),s\in\left(0,S\right), \tag{20}$$

where $g\in L_{p,\rho}\left(\Gamma\right)-$ is a given function. By the solution to problem (20) is understood as a pair of functions

$$\left(F^+(z);F^-(z)\right)\in E_{p,\rho}\left(D^+\right)\times_m E_{p,\rho}\left(D^-\right),$$

whose boundary values $F^{\pm}$ on Γ are a.e. satisfy relation (20).

The following theorem is true.

Theorem 16. *Let the complex-valued functions* $A(\xi),B(\xi)$ *and the curve* Γ *satisfy conditions i)-iii). Let us take a weight* ρ *of the form (17), and determine the numbers* $\left\{\beta_k\right\}_1^l$ *using expressions (18). Let inequalities (19) be satisfied and*

$$\alpha_k<\frac{q}{p},k=\overline{1,m}\ ,$$

holds. Then the general solution to problem (20) in classes $E_{p,\rho}\left(D^+\right)\times_m E_{p,\rho}\left(D^-\right)$ *has the form*

$$F(z)=F_0(z)+F_1(z)$$

where $F_0(z)$ *is the general solution of the corresponding homogeneous problem,* $F_1(z)$ *is expressed by the formula*

$$F_1(z) \equiv \frac{Z_\Omega(z)}{2\pi i} \int_\Gamma \frac{g(\xi)d\xi}{Z^+(\xi)(\xi - z)},$$

in which $Z_\Omega(z)$ *is the canonical solution and* $m \geq 0$ *is an integer.*

General weight. The nonhomogeneous Riemann problem of the theory of analytic functions with a piecewise Hölder coefficient in weighted Smirnov classes with a common weight is considered. Sufficient conditions are found for the coefficient of the problem and for the weight function so that the considered Cauchy-type integral is a particular solution to this problem.

In **4.4**, the obtained results are applied to establishing the basicity of double systems from generalized Faber polynomials in weighted Lebesgue spaces. This section establishes the basicity of generalized Faber polynomials in weighted Smirnov spaces when the weight and boundary values of the conformally mapping function belong to certain Muckenhoupt classes.

Let $\rho : \Gamma \to R_+ -$ be some weight function and

$$\varphi : D^- \to O_1^-(0) : \varphi(\infty) = \infty, \ \varphi'(\infty) = \gamma > 0,$$

and

$$\psi : D^+ \to O_1^-(0) : \psi(0) = \infty, \lim_{z \to 0} z\psi(z) = \alpha > 0,$$

are conformal mappings of the domains $D^- = ext\,\Gamma$ and $D^+ = \operatorname{int}\Gamma$ onto the exterior of the unit circle $O_1^-(0)$. Let us denote by $z = \varphi_{-1}(w)$ ($\psi_{-1}(w)$) the function inverse to $w = \varphi(z)$ ($w = \psi(z)$). Suppose

$$\rho_+(w) = \rho[\varphi_{-1}(w)]; \ \rho_-(w) = \rho[\psi_{-1}(w)], \ w \in \partial\omega. \quad (21)$$

The following theorem is proved.

Theorem 17. *Let* Γ *be a regular curve and* $0 \in \operatorname{int} \Gamma$. *If* $\rho_\pm(\cdot) \in A_p(\partial\omega)$, $\rho(\cdot) \in A_p(\Gamma)$, $1 < p < +\infty$, *then systems from generalized* p*-Faber polynomials form bases for the weighted Smirnov spaces* $E_{p,\rho}(D^+)$ *and* ${}_{-1}E_{p,\rho}(D^-)$, *respectively.*

Consider a double system from p-Faber polynomials

$$\left\{A(\xi)F^{+}_{p,n}(\xi);B(\xi)F^{-}_{p,k}(\xi)\right\}_{n\geq 0,\,k\geq 1}, \tag{22}$$

with complex-valued coefficients $A(\cdot)$ and $B(\cdot)$.

The following theorem is also true.

Theorem 18. *Let the curve* Γ *belong to the class* $\mathscr{LR}$, $0\in \operatorname{int}\Gamma$, *the functions* $A(\cdot)$, $B(\cdot)$ *satisfy the conditions i)-iii) and* $\rho:\Gamma\to R_{+}$ *– is a weight functions. Let the weight functions* $\rho_{\pm}(\cdot)$ *on be* $\partial\omega$ *defined by expressions (21). If conditions (15) and*

$$\rho_{\pm}(\cdot)\in A_{p}(\partial\omega);\ \rho(\cdot)\in A_{p}(\Gamma),\ 1<p<+\infty,$$

are satisfied with respect to the weights $\rho(\cdot)$ *and* $\rho_{\pm}(\cdot)$, *then the double system of generalized* p *-Faber polynomials (22) forms a basis for* $L_{p;\rho}(\Gamma)$, $1<p<+\infty$.

Let

$$F_{p,n}(\xi)=\begin{cases}F^{+}_{p,n}(\xi), n\in Z_{+},\\ \\ F^{-}_{p,n}(\xi),\ -n\in N,\end{cases}$$

and consider the system

$$\left\{e^{i\alpha\arg\xi\, sign\, n}\ F_{p,n}(\xi)\right\}_{n\in Z}\ , \tag{23}$$

where $\alpha\in R-$ is a real parameter. From Theorem 18 it directly follows

Corollary 4. *Let the curve* Γ *belongs to the class* $\mathscr{LR}$; $0\in \operatorname{int}\Gamma$ *and the weight functions* $\rho(\cdot)$; $\rho_{\pm}(\cdot)$ *satisfy all conditions of Theorem 18. If the inequalities*

$$-\frac{1}{2q}<\alpha<\frac{1}{2p},$$

are valid, then the system (23) forms a basis for $L_{p;\rho}(\Gamma)$.

Chapter V is devoted to some related issues in the theory of statistical convergence and approximation in Lebesgue spaces of functions.

Section **5.1** considers measurable space with a measure on the real axis. The concept of μ-statistical limit of a measurable function at a point is introduced. The corresponding concept of statistical fundamentality at a point is also introduced and their equivalence is proved. This concept generalizes the similar concept of statistical convergence of a sequence.

It should be noted that the concept of statistical convergence was introduced by Hardy and Littlewood in connection with the point convergence of Fourier series, more precisely in connection with the Luzin hypothesis, and it was called almost convergence. In this regard, one can see, for example, the monograph [[12]].

***μ* - statistical convergence.** Let $\mathscr{B}$ be σ-algebra of Borel subsets of segment $E \equiv [a, +\infty)$ and $(\mathscr{B}; \mu)$ be a measurable space with σ-finite measure $\mu : \mu(E) = +\infty$. Consider a $\mathscr{B}$-measurable function $f : E \to R$. The measure of a set $e \in \mathscr{B}$ will be denoted by $|e|$, i.e. $|e| = \mu(e)$. Let $M \in \mathscr{B}$ be some set

Definition 5. *We will say that the point ∞ is a point of μ-statistical (μ-stat) density for the set M, if*

$$\lim_{x\to\infty} \frac{|M \cap I_x|}{I_x} = 1, \text{ where } I_x \equiv [a, x].$$

Let $\varepsilon > 0 -$ be an arbitrary number and assume

$$A_\varepsilon^f \equiv \{x \in E : |f(x) - A| \geq \varepsilon\},$$

where $A \in R -$ is come number.

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[12] А.Зигмунд. Тригонометрические ряды. Москва, т. 2, 1965

Definition 6. *We will say that* f *has a* μ *-st (* μ *-statistical) limit* A *at infinity, if*

$$\lim_{x\to\infty}\frac{\left|A_{\varepsilon}^{f}\cap I_{x}\right|}{\left|I_{x}\right|}=0,\ \ \forall\varepsilon>0,$$

and we will denote this limit as μ *-st* $\lim\limits_{x\to\infty} f(x)=A$.

$\mathscr{B}$ -measurable functions having μ -*st* lim at infinity form a linear space, and we denote this space by $\mathscr{B}_{st}$. The set of all subsets E for which infinity is the μ *-stat* density point will be denoted by E_{st}^{∞} .

μ **-statistical fundamentality.** Let us define the concept of μ -statistical fundamentality at infinity.

Definition 7. *We will say that the function* $f: E\to R$ *is* μ *-stat fundamental at infinity if for* $\forall\varepsilon>0,\ \exists x_{\varepsilon}\in E$ *:*

$$\lim_{x\to\infty}\frac{\left|X_{\varepsilon}\cap I_{x}\right|}{\left|I_{x}\right|}=0,$$

where $X_{\varepsilon}\equiv\left\{x\in E:\left|f(x)-f(x_{\varepsilon})\right|\geq\varepsilon\right\}$.

Let us also accept the following

Definition 8. *Functions* $f;g: E\to R$ *will be called* μ *-stat equivalent at infinity if* $E_{f,g}\in E_{st}^{\infty}$ *,where* $E_{f,g}=\{x\in E: f(x)=g(x)\}$, *and we denote this by* $f\overset{st}{\sim}g$.

The following theorem is true.

Theorem 19. *Let* $\mu(E)=+\infty$ *. Then for a* $\mathscr{B}$ *- measurable function* $f: E\to R$ *the following properties are equivalent:*

a) $\exists\mu$ *-st* $\lim\limits_{x\to\infty} f(x)$;

b) f *is* μ *-stat fundamental at infinity;*

v) $\exists g: f\overset{st}{\sim}g$ *and* $\exists\lim\limits_{x\to\infty} g(x)$.

Based on this concept, the concept of μ-stat continuity at a point x_0 is defined and the facts related to this concept are established.

In **5.2** the concept of μ-strongly Cesaro summability for locally integrable functions at infinity is introduced. The concept of $\mu-$statistical convergence at infinity is also considered and the relationships between these concepts are established. This approach is applied to the study of the convergence of the Fourier-Stieltjes transform.

Let $(I;\mathscr{B};\mu)$ be a measurable space, $I=[a,+\infty)$, where $\mu:\mathscr{B}\to R_+ -$ σ-finite measure on I. By $L_p(\mu), 0<p<+\infty$, as usual we will denote the space of measurable (in the sense $(\mathscr{B};\mu)$) functions $f:I\to R,$ for which $\|f\|_p<+\infty$, where

$$\|f\|_p=\begin{cases}\int_I |f(t)|^p d\mu(t)\ , & 0<p<1,\\ \left(\int_I |f(t)|^p d\mu\right)^{\frac{1}{p}}, & 1\le p<+\infty.\end{cases}$$

Assume $I_x=[a,x],\ \forall x\ge a$. Let us accept the following

Definition 9. *Let* $|f|^p,\ 0<p<+\infty$, *be a locally integrable function on* $[a,+\infty)$. *We will say that a function* f *has a* $\mu[p]$*-strong Cesaro limit at infinity, equal to the number* A, *if*

$$\lim_{x\to\infty}\frac{1}{\mu(I_x)}\int_{I_x}|f(t)-A|^p d\mu=0.$$

This limit will be denoted by

$$\mu[p]\text{-}\lim_{x\to\infty} f(x)=A.$$

It is easy to see that this limiting operation is linear.

It should be noted that in the discrete case p-Cesaro summability takes the form

$$\lim_{n\to\infty}\frac{1}{n+1}\sum_{k=0}^{n}|x_k-\xi|^p=0,\ 0<p<+\infty.$$

The following theorem is the μ-analog of the discrete case

Theorem 20. *i) Let the function f $\mu[p]$- Cesaro converge to a number A for some $p\in(0,+\infty)$, at infinity. Then $\exists$ μ-st $\lim_{x\to\infty} f(x)$ μ-st $\lim_{x\to\infty} f(x)=A$; ii) If $\exists$ μ-st $\lim_{x\to\infty} f=A$ and f is μ-a.e. bounded, then $\exists$ $\mu[p]$-$\lim_{x\to\infty} f(x)$ and this limit is equal to A.*

Section **5.3** suggests one method to form frames. Namely, double and unary systems of functions are considered, interconnected by certain relationships. These relations to some extent generalize similar relations that occur between systems of exponents, sines and cosines. Connections are established between atomic decompositions of double and unary systems in Lebesgue spaces.

Let us denote the sequence $\vec{y}\equiv\{y_n\}_{n\in N}$ by $\vec{y}$. Let $L_p(a,b)$ be the usual Lebesgue space of functions on (a,b) with norm

$$\|f\|_{L_p(a,b)}\equiv\left(\int_a^b|f(t)|^p\,dt\right)^{\frac{1}{p}}.$$

Throughout what follows we will assume that $q-$is conjugate of p: $\frac{1}{p}+\frac{1}{q}=1$.

We will consider unary systems of the form

$$x_n^{\pm}(t)\equiv\varphi_n(t)\pm\psi_n(t),\ \ n\in N\ ,$$

where $\varphi_n;\psi_n:[0,a]\to C-$ are complex-valued functions. Using the functions φ_n and ψ_n we will define a new system on the segment $[-a,a]$. Then we will establish connections between atomic decompositions and the frameness of these systems.

$$\Phi_n(t) \equiv \begin{cases} \varphi_n(t), t \in [0,a], \\ \psi_n(-t), t \in [-a,0), \end{cases}$$

and assume

$$\Psi_n(t) = \Phi_n(-t), \ \forall t \in [-a,a].$$

Let $\{\vartheta_n^{\pm}\} \subset L_q(0,a)$ – be some system. In a similar way we define

$$\Omega_k^{\pm}(t) \equiv \begin{cases} \vartheta_k^{\pm}(t), t \in (0,a), \\ \pm \vartheta_k^{\pm}(-t), t \in (-a,0), \end{cases}$$

and assume

$$h_k^{\pm}(t) \equiv \frac{1}{2}\left[\Omega_k^{+}(t) \pm \Omega_k^{-}(t)\right], \ \forall t \in (-a,a) \ .$$

It is easy to see that the following relations hold:

$$h_k^{+}(-t) = h_k^{-}(t), \ h_k^{-}(-t) = h_k^{+}(t) \ , \ \forall t \in (-a,a) \ ,$$

which we will use to obtain the main results.

We will consider a double system $\{\vec{\Phi};\vec{\Psi}\}$ and establish connections between the frame properties of this system and unary systems $\{\vec{x}^{\pm}\}$ in Lebesgue spaces L_p .

Assume that $\left(\left(\vec{h}^{+};\vec{h}^{-}\right);\left(\vec{\Phi};\vec{\Psi}\right)\right)$ is an atomic decomposition of $L_p(-a,a)$ with respect to $\mathscr{K}$. Assume

$$\vartheta_n^{\pm}(t) \equiv \frac{1}{2}\left(h_n^{+}(t) \pm h_n^{-}(t) \pm h_n^{+}(-t) + h_n^{-}(-t)\right) . \qquad (24)$$

The following theorem is true.

Theorem 21. *Let* $\left(\left(\vec{h}^{+};\vec{h}^{-}\right);\left(\vec{\Phi};\vec{\Psi}\right)\right)$ *be an atomic decomposition of* $L_p(-a,a)$ *with respect to* $\mathscr{K}$ *. Then the system* $\vec{\vartheta}^{\pm}$ *is* $\mathscr{K}$ *-Bessel system in* $L_p(0,a)$ *and an arbitrary function from* $L_p(0,a)$ *is expanded in terms of the pair* $\left(\vec{\vartheta}^{\pm};\vec{x}^{\pm}\right)$*, where* $\vec{\vartheta}^{\pm}$ *is* *determined by expression (24). Moreover, if the relations* $h_n^{\pm}(f) = \vartheta_n^{+}(f^{+}) \pm \vartheta_n^{-}(f^{-})$, $\forall n \in N$ *hold, then the pair* $(\vec{\vartheta}^{\pm}; \vec{x}^{\pm})$ *is an atomic decomposition of* $L_p(0,a)$ *with respect to* $\mathscr{K}$.

The author expresses deep gratitude to his scientific consultant, corresponding member of NAS of Azerbaijan, Professor B.T. Bilalov for valuable advice, constant attention to the work and providing comprehensive support during the completion of the dissertation work.

CONCLUSIONS

The dissertation work is devoted to the study of frame properties (basis properties, an atomic decomposition, frameness) of systems of elements and the acquisition of new methods of summation and convergence in various linear topological spaces. The following main results are obtained in the dissertation work.

The following main results are obtained in the dissertation work.

1. an abstract generalization of double bases is given and an abstract analogue of the well-known "$\frac{1}{4}$-Kadets" theorem is obtained;

2. based on the abstract space $L_p(X)$ the Hardy vector classes $H_p(X)$ are determined and their main properties are studied;

3. the concept of space of coefficients of systems of elements is defined in linear topological spaces in two cases - sequential and general cases, its properties are studied;

4. the basicity of half of degenerate systems of exponents in Hardy classes are proved when the degeneracies satisfy the Muckenhoupt condition;

5. atomic decompositions in degenerate systems of exponents of Hardy classes are studied when the degeneracies do not satisfy the Muckenhoupt condition;

6. the solvability of Riemann boundary value problems in weighted Smirnov classes is studied;

7. the basicity of systems from Faber polynomials in weighted Smirnov classes are proved when the weights satisfy the Muckenhoupt condition;

8. conditions for the basicity of a double system from Faber polynomials in Lebesgue spaces on Lyapunov or Radon curves are found;

9. a continuous analogue of the concept of statistical convergence is found and the main provisions of this theory are transferred to this case;

10. concept of μ-strong Cesaro summability is introduced, its properties and applications to Fourier transforms are given;

11.connections are established between atomic decompositions of double and unary systems connected with each other by certain relations in Lebesgue spaces.

The main results of the dissertation are published in the following works:

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The defense will be held on **22 December 2023** at **14^{00}** at the meeting of the Dissertation council ED 1.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics of the Ministry of Science and Education of Azerbaijan Republic.

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Dissertation is accessible Institute of Mathematics and Mechanics of the Ministry of Science and Education of Azerbaijan Republic. Library.

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Abstract was sent to the required addresses on **13 November 2023**.

Signed for print: 06.11.2023
Paper format: 60x84 1/16
Volume: 76842
Number of hard copies: 20