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# ABSTRACT

of the dissertation for the degree of
Doctor of Science

# NONLOCAL INVERSE BOUNDARY VALUE PROBLEMS FOR SOME CLASSES OF PARTIAL DIFFERENTIAL EQUATIONS

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## GENERAL CHARACTERISTICS OF THE WORK

**Rationale of the topic and the degree of development.** In many cases, during experiments and research in the field of mechanics, physics, geology, astronomy, medical biology and other natural sciences, such problems arise that mathematical modeling of the solution of these problems leads to the solution of various linear and nonlinear inverse problems for the equations of mathematical physics. The inverse problem for partial differential equations (PDEs) is understood as such a problem in which, together with the solution, it is required to determine the right side or (and) one or another coefficient (coefficients) of the equation itself. In other words, the problem of determining the right side of a differential equation or (and) recovering unknown coefficients from some data is called an inverse boundary value problem in the theory of equations of mathematical physics.

The history of theory of inverse problems starts with the 17th century. These problems involves the problems of finding the figure of equilibrium rotating fluid, the kinematic problem in seismology, the inverse Sturm-Liouville problem, etc. Newton’s problem of determining the forces as a result of which the planets move in accordance with Kepler’s laws was one of the first inverse problems of the dynamics of mechanical systems, solved earlier. Inverse problems of potential theory, where it is necessary to determine the position of the body, its shape and density based on the known value of its potential are of geophysical origin. The inverse problems of electromagnetic exploration were caused by the need to develop a theory and methodology for the use of electromagnetic fields in the study of the internal structure of the earth’s crust.

Note that from the classical point of view, inverse boundary value problems for partial differential equations are, generally speaking, ill-posed problems. The general approach and modern theory of ill-posed and conditionally well-posed problems is related to the application of various regularization methods to such problems of mathematical physics. In many cases, these problems include auxiliary information about the structure of the main differential equation, the type of its coefficients, and some other parameters. Frequently, the unique solvability of the inverse problem is provided by an excess of information of this kind. A certain structure of the coefficients of the differential equation leads to the fact that the inverse problem becomes well-posed from a general point of view.

The foundations of the theory and practice of studying inverse problems were established and developed in the fundamental works of A.N. Tikhonov, M.M. Lavrentiev, V.K. Ivanov, A.I. Prilepko and others. Sufficiently complete bibliographies of recent works related to the study of inverse problems for partial differential equations are reflected in many monographs and articles.

The study of various aspects of inverse problems of recovering the coefficients of partial differential equations, as well as the study of inverse problems by reducing them to variational formulations is considered in the works of A.Y.Akhundov, O.M.Alifanov, A.Kh. Amirov, A.M.Denisov, Y.S.Gasimov, H.F.Guliyev, A.Hazanee, V.M. Isakov, A.D.Iskenderov, N.Sh.Iskenderov, A.I.Ismailov M.I.Ismailov, N.I.Ivanchov, S.I.Kabanikhin, F.Kanca, N.B.Kerimov, K.I.Khudaverdiyev, D.Lesnic, Y.T.Mehraliyev, G.K.Namazov, E.S. Panahov, Y.A.Sharifov, R.K.Tagiyev, I.Tekin, and others. In the works of K.R.Aida-zade, C.Ashyralyyev, D.Baleanu, M.Dehghan, M.Huntul, A.B.Rahimov, and many others studied the numerical aspects of inverse boundary value problems for partial differential equations in various formulations.

In the literature, one can find numerous works on the study of the existence and uniqueness of the solution of inverse boundary value problems for various types of partial differential equations. Interest in these problems is primarily due to the fact that many mathematical models of physical phenomena can be described by partial differential equations. Inverse problems for second-order PDEs arise in the study of such physical processes as heat conduction, diffusion, propagation of electromagnetic fields in conducting media, the movement of a viscous fluid in the case when the area of physical characteristics of the medium under consideration is not available for direct measurements, but, at the same time, it is possible to obtain additional information about the characteristics of the process itself.

Note that among the inverse problems devoted to PDEs, a special place is occupied by inverse boundary value problems for equations of parabolic type. The solvability of inverse problems for the parabolic equations were considered in the works of A.I.Prilepko, Yu.Ya.Belov, V.M.Isakov, I.A.Vasin, A.I. Kozhanov, Yu.E. Anikonov, J.R.Cannon, M.Yamamoto, N.I.Ivanchov, G.Eskin, G. Nakamura, A.Yu.Scheglov, S.G.Pyatkov, A.Lorenzi, D.Lesnic, A.B.Kostin, V.V.Solovyov, D.S.Tkachenko, V.L.Kamynin, G.A.Kirillov, I.V. Frolenkov, M.V.Neshadim, Y.P.Lin, M.S.Hussein, M.V.Klibanov, A.R. Zainullov and many others.

One of the classes of qualitatively new problems are problems with nonlocal conditions. The term “nonlocal conditions” was first introduced by A.A.Dezin. Nonlocal boundary-value problems are usually called problems in which, by together specifying the values of the solution or its derivatives on a fixed part of the boundary, a relationship is established between these values and the values of the same functions on other internal or boundary manifolds. The theory of nonlocal boundary value problems is important as a branch of the general theory of boundary value problems for PDEs, as well as a branch of mathematics that has numerous applications in mechanics, physics, biology and other natural science disciplines. The beginning of systematic studies of nonlocal problems for PDEs was laid by the article by A.V.Bitsadze and A.A.Samarsky, in which the problem of finding a solution to an elliptic equation (and a system of equations) of the second order is posed, the values of which at the points of some part of the boundary of the considered region are equal to its values in the images of these points for a given diffeomorphism. Such problems can be considered as a generalization of classical boundary value problems. However, when studying them can be arise additional difficulties. Subsequently, this problem was developed in the works of the authors: V.A.Ilyin, A.A.Dezin, E.I.Moiseev, A.K. Gushin, V.N.Vragov, L.I.Kamynin, A.L.Skubachevsky, A.M.Nakhushev, T.Sh.Kalmenov, N.I.Ionkin, I.S.Lomov, A.I.Kozhanov, O.A. Repin, L.Byszewski, J.R.Cannon, and others. In contrast to the above works, in this thesis, inverse boundary value problems with nonlocal boundary conditions are investigated for both time and space variables for one-dimensional and two-dimensional parabolic equations and some numerical examples are given.

Nonlocal conditions may arise in a situation where the boundary of the domain of a real process is not available for direct measurements, but it is possible to obtain some additional information about the phenomenon under study at the internal points of the domain. Often such information comes in the form of some average values of the desired solution. In mathematical modeling, it is convenient to represent such information in the form of an integral. Note that problems with nonlocal integral conditions arise in the study of processes occurring in turbulent plasma, heat propagation processes and in the theory of diffusion processes, in modeling some technological processes.

In the theory of inverse problems, the study of inverse boundary value problems for equations of hyperbolic type is also of great importance from the theoretical and practical points of view. Inverse and ill-posed problems associated with a hyperbolic type equation arise in various fields of human activity, such as acoustics, seismology, electromagnetism, hydrodynamics, remote sensing, and many other areas. Nonlinear inverse problems for hyperbolic equations in various formulations were studied in the works of M.M.Lavrentiev, V.G.Romanov, Yu.E.Anikonov, B.A.Bubnov, A.I.Prilepko, A.Kh. Amirov, A.M.Denisov, M.Grasseli, M.Klibanov, M.Yamamoto, and many others.But problems with nonlocal conditions with respect to the time variable are still remain little studied. It should be noted that problems of this type are known as ill-posed problem in Hadamard sense. In the second chapter of the presented dissertation, the questions of existence and uniqueness of the classical solution of inverse boundary value problems with nonlocal boundary conditions and various additional conditions in time and space variables for one-dimensional and two-dimensional hyperbolic equations are studied, and various suitable numerical examples are solved.

As is known from the bibliography, many physical processes are described by pseudohyperbolic equations, such as unsteady flow of a viscous gas, convective diffusion of salts in a porous medium, propagation of initial densifications in a viscous gas, etc. It should be noted that the inverse problems for pseudohyperbolic equations are less developed than for second-order equations. This is explained by the fact that the study of inverse problems for differential equations is closely related to the investigating of the corresponding direct problem. And the theory of direct problems for pseudohyperbolic equations is still incomplete. Direct problems for pseudohyperbolic equations were studied by S.Ya.Kirichenko, V.A.Vodakhov, I.V.Suveika, G.B.Whitham, K.Longren, H.Ikezi, and others. A number of results on inverse problems for pseudohyperbolic equations were obtained by A.I.Kozhanov, B.S.Ablabekov, A.R.Asanov, E.R.Atamanov, S.A. Ablakimov, A.Lorenzi, E.Paparoni, T.K.Yuldashev, A.O.Bulov, G.V. Namsaraeva, A.I.Grigorieva, I.Tekin, etc.

It is known that the study of the mathematical model of the motion of an elastic beam is one of the most important issues in mechanics. Moreover, the vibrations and wave movements of an elastic beam on an elastic base were investigated by Yu.A.Mitropolsky and B.I.Moseenkov, V.Z.Vlasov, N.N.Leont'ev, J.M.T.Thompson, H.B.Stewart, B.S.Bardin, S.D.Furta, G.S.Pisarenko and et al. The simplest nonlinear mathematical model of the motion of a homogeneous beam is described by the equation

$$\frac{\partial^2 w}{\partial t^2}+\frac{\partial^4 w}{\partial x^4}+k\frac{\partial^2 w}{\partial x^2}+\alpha w+w^3=0,$$

where $w$ is beam deflection.

Motivated by these works, in the last chapter of the dissertation, we study the inverse boundary value problem for the linearized equation of motion of a homogeneous beam with nonlocal boundary conditions. However, unlike previous works, in the inverse boundary value problems considered in the fourth chapter of the present dissertation, the boundary conditions include non-local conditions due to both temporal and spatial variables. On the other hand, the research methods used here are quite different from the methods used in other works.

**Object and subject of the research.** The main object of the dissertation work is inverse boundary value problems for parabolic and hyperbolic equations of different dimensions, as well as equations of motion of longitudinal waves and homogeneous beams. And the subject of research is the study of the classical solvability of nonlinear inverse boundary value problems for parabolic and hyperbolic equations, as well as for equations of longitudinal wave propogation and motion of homogeneous beams of various dimensions.

**Purpose and objectives of the study.** The main purpose and objectives of the work is to study the solvability of nonlinear inverse boundary value problems for parabolic and hyperbolic equations of the second order, as well as study of the existence and uniqueness of the classical solution of the longitudinal wave propagation equation and for the equation of motion of a homogeneous beam in bounded domain with additional conditions and with unknown time depended coefficients.

**General research methodology.** The main research methodologies are the methods of the theory of differential equations, functional analysis, spectral theory and computational mathematics. The main research tools are the Fourier method, the contraction mapping principle, the finite difference method and the Tikhonov regularization method.

**Basic provisions of dissertation.**

1. Investigation of the existence and uniqueness of the classical solution of a self-adjoint inverse boundary value problem for a one-dimensional parabolic equation with a nonlocal boundary condition and an additional integral condition.
2. Study of the classical solvability of a non-self-adjoint inverse boundary value problem for one-dimensional parabolic equation with a nonlocal boundary condition.
3. Study of the existence and uniqueness of the classical solution of an inverse boundary value problem for a two-dimensional parabolic equation with a nonlocal boundary condition.
4. Investigation of the existence and uniqueness of a classical solution of a self-adjoint inverse boundary value problem with a nonlocal boundary condition and an additional integral condition for a one-dimensional hyperbolic equation in a rectangular domain.

5. Study of the classical solvability of a non-self-adjoint inverse boundary value problem for a one-dimensional hyperbolic equation with a non-local boundary condition.
6. Investigation of the existence and uniqueness of a classical solution of an inverse boundary value problem for a two-dimensional hyperbolic equation with a nonlocal boundary condition.
7. Recovery of unknown coefficients and determination of the right side of the equation in the inverse boundary value problem for the one-dimensional longitudinal wave propagation equation.
8. Study of the classical solvability of a non-self-adjoint inverse boundary value problem with a non-local boundary condition for the one-dimensional longitudinal wave propagation equation.
9. Investigation of the existence and uniqueness of the classical solution of the inverse boundary value problem with a nonlocal boundary condition for the three-dimensional equation of propagation of longitudinal wave.
10. Recovery of unknown coefficients and determination of the right side of the equation using various additional conditions in the inverse boundary value problem for the one-dimensional linearized equation of motion of a homogeneous beam.
11. Study of the existence and uniqueness of the classical solution of the inverse boundary value problem for the one-dimensional equation of motion of a homogeneous beam with a nonlocal boundary condition.
12. Study of the classical solvability of a nonlinear inverse boundary value problem for a two-dimensional linearized equation of motion of a homogeneous beam with a nonlocal boundary condition.

**Scientific novelty of the research.** In this dissertation, the following main results are obtained:

1. The classical solvability of the self-adjoint and non-self-adjoint nonlinear inverse boundary value problems with nonlocal boundary conditions (for both time and space variables) and with various additional conditions for the one-dimensional and two-dimensional parabolic equations is studied and some numerical approaches are established.

2. The unique classical solvability of the self-adjoint and non-self-adjoint nonlinear inverse boundary value problems with nonlocal boundary conditions (for both time and space variables) and with various additional conditions for the one-dimensional and two-dimensional hyperbolic equations is investigated and some corresponding numerical examples are solved.
3. The results on the existence and uniqueness of the classical solution of some self-adjoint nonlinear inverse boundary value problems with a nonlocal boundary conditions (with respect to time and space variables) and various additional conditions for the one-dimensional longitudinal wave propagation equation are obtained.
4. Theorems on the existence and uniqueness of the inverse boundary value problem with a nonlocal boundary condition for the three-dimensional equation for the longitudinal wave propagation equation are proved.
5. The simultaneous recovery of unknown coefficients and the determination of the right side in nonlinear inverse boundary value problems with nonlocal boundary data (with respect to time and space variables) and various additional conditions for the one-dimensional linearized equation of motion of a homogeneous beam are studied.
6. The classical solvability of a nonlinear inverse boundary value problem with a nonlocal boundary condition for the two-dimensional equation of motion of a homogeneous beam is studied.

**Theoretical and practical value of the research.** The dissertation work is mainly of a theoretical character. However, the results obtained can be used in development of the theory of nonlocal inverse problems, as well as in the study of the source of many practical problems of natural science (for example, in the theory of heat conduction, in processes characterized by wave equations, in the propagation of longitudinal waves, in the movement of a homogeneous beam, in the study of phenomena occurring in plasma, in the study of moisture permeability in capillary-porous media, in mathematical modeling of the process of chemical cleaning of silicon coatings from impurities, in mathematical biology, in demography, in diffusion processes, etc.).

**Approbation and application.** The results of the dissertation were presented at the seminar of the faculty of Mechanics and Mathematics of Baku State University (prof. Z.S.Aliyev), at the seminar of the faculty of Mechanics and Mathematics of Baku State University (prof. N.Sh.Iskenderov), at the seminars of the department Differential and integral equations of Baku State University (prof. Y.T.Mehraliyev), at the seminar of the department Equations of Mathematical Physics of Baku State University (Acad. Yu.A.Mamedov), at the general institute seminar of Institute of Mathematics and Mechanics of Azerbaijan (Corresponding Member of Azerbaijan National Academy of Sciences, prof. M.J.Mardanov), at the seminar of the department of Non-harmonic analysis of Institute of Mathematics and Mechanics (Corresponding Member of Azerbaijan National Academy of Sciences, prof. B.T.Bilalov), at the seminar of the department of Mathematics of Cumhuriyet University (Turkey) (prof. R.Kh.Amirov), at the seminar of the department of Mathematics of Gebze Technical University (Turkey) (prof. M.I.Ismailov), and also at the "XIII International Scientific Conference devoted to the honor of acad. M.Kravchuk" (Kyiv, 2010), at the "International Conference on Differential Equations and Dynamical Systems" (Moscow, 2010), at the "3rd International Conference for Young Mathematicians on Differential Equations and Applications dedicated to Yaroslav Lopatynsky" (Lviv, 2010), at the "XV International Conference on Dynamical System Modeling and Stability Investigation" (Kyiv, 2011), at the International Conference "Differential Equations and Their Applications" (Samara, 2011), at the International Conference dedicated to the 120th anniversary of Stefan Banach (Lviv, 2012), at the International Conference "Operators, Functions, and Systems of Mathematical Physics", dedicated to the 70th anniversary of prof. H.A.Isaxanli (Baku, 2018), at the International Conference "Operators, Functions, and Systems of Mathematical Physics" (Baku, 2019), at the International Workshop "Spectral Theory and its Applications" dedicated to the 80th anniversary of Academician M.G.Gasymov (Baku, 2019), at the "2nd IEEE International Conference on Science and Technology” (Thanyaburi, Thailand, 2017), at the “XIII International Scientific Conference “Differential Equations and their Applications in Mathematical Modeling” (Saransk, 2017), at the International Conference “The actual problems of theoretical and applied mathematics dedicated to the 100th anniversary of academician M.L.Rasulov” (Sheki, 2016), at the VIII International Scientific and Practical Conference “Problems and perspectives of modern science” (Moscow, 2016), at the XVIII International Conference “Dynamical System Modeling and Stability Investigations” (Kyiv, 2017), at the “8th International Eurasian Conference on Mathematical Sciences and Applications” (Baku, 2019), at the International Conference “Modern Problems of Mathematics and Mechanics” dedicated to the 60th anniversary of the Institute of Mathematics and Mechanics, (Baku, 2019), at the Fourteenth International Conference of “Analytical and Numerical Methods of Modeling of Natural Science and Social Problems” (Penza, 2019), at the conference dedicated to the 97th anniversary of the national leader of the Azerbaijani people H.A.Aliyev (Baku, 2020), at the “$7^{th}$ International Conference on Control and Optimization with Industrial Applications” (Baku, 2020), at the “$4^{th}$ International E-Conference on Mathematical Advances and Applications” (Istanbul, 2021), at the “Thirteenth Conference of the Euro-American Consortium for Promoting the Application of Mathematics in Technical and Natural Sciences” (Albena, 2021), at the “8th International Conference on Control and Optimization with Industrial Applications” (Baku, 2022), at the conference dedicated to the 99th anniversary of the national leader of the Azerbaijani people H.A.Aliyev (Baku, 2022).

**The personal contribution of the author** lies in the formulation of the goal and the choice of the direction of the study. In addition, all conclusions and results obtained, as well as research methods, belong personally to the author.

**Publications of the author.** The main results of the dissertation were published in the form of 24 articles, 13 conference materials and 11 abstracts in publishing houses recommended by the Supreme Attestation Commission under the President of the Republic of Azerbaijan.

**The name of the institution where the dissertation was completed.** The work was completed at the department of Differential and integral equations of the Baku State University.

**The structure and volume of the dissertation (in characters, indicating the volume of each structural unit separately).** The total volume of the dissertation work is 476730 characters (title page - 455 characters, content - 2275 characters, introduction - 68000 characters, first chapter - 118000 characters, second chapter - 104000 characters, third chapter - 84000 characters, fourth chapter - 100000 characters/signs). The list of used literature consists of 223 items.

## THE CONTENT OF THE DISSERTATION

The dissertation consists of an introduction, four chapters, conclusions, and a list of references. Each chapter is divided into sections.

The introduction substantiates the relevance of the dissertation topic, formulates its purpose and gives a brief overview of the work related to the dissertation topic, and presents the main results of the work.

In the first chapter of the dissertation, which consists of four sections, the existence and uniqueness of the classical solution of one-dimensional and two-dimensional nonlinear inverse problems for a second-order parabolic equation are studied and some numerical examples are presented.

In Section 1.1, we study the solvability of nonlinear inverse boundary value problems for finding, together with the solution $u(x,t)$, also unknown factors $a(t)$ and $b(t)$ in a rectangular domain $D_T = \{(x,t): 0 \le x \le 1,\ 0 \le t \le T\}$ for the equation:

$$c(t)u_t(x,t) = u_{xx}(x,t) + a(t)u(x,t) + b(t)g(x,t) + f(x,t) \ \ (x,t) \in D_T, \quad (1)$$

under the conditions

$$u(x,0) + \delta\, u(x,T) + \int_0^T p(t)u(x,t)dt = \varphi(x),\ \ 0 \le x \le 1, \quad (2)$$

$$u(0,t) = u(1,t),\ \ 0 \le t \le T, \quad (3)$$

$$\int_0^1 u(x,t)\,dx = 0,\ \ 0 \le t \le T, \tag{4}$$

$$u(x_i, t) = h_i(t),\ \ i = 1,2;\ \ 0 \le t \le T, \tag{5}$$

where $T > 0$ is any fixed time moment, $\delta \ge 0$, $x_i \in (0,1)$, $i = 1, 2$, $x_1 \ne x_2$ are known numbers, $c(t) > 0$, $f(x,t)$, $g(x,t)$, $p(t) \ge 0$, $\varphi(x)$, $h_i(t)$, $i = 1, 2$ are given functions, and in turn, $u(x,t)$, $a(t)$, and $b(t)$ are desired functions.

**Definition 1.1.** The triple $\{u(x,t), a(t), b(t)\}$ is said to be a classical solution to problem (1)-(5), if the functions $u(x,t)$, $a(t)$, and $b(t)$ satisfy the following conditions:

i) the function $u(x,t)$ and its derivatives $u_t(x,t)$, $u_x(x,t)$, and $u_{xx}(x,t)$ are continuous in the domain $D_T$;

ii) the functions $a(t)$ and $b(t)$ are continuous on the interval $[0,T]$;

iii) equation (1) and conditions (2)-(5) are satisfied in the classical (usual) sense.

Note that the definitions of the classical solution of all the studied problems in the dissertation work are introduced similarly to Definition 1.1 taking into account the order of the equations and the corresponding given conditions of the problems.

The following assertion is proved.

**Theorem 1.1.** Assume that $0 < c(t) \in C[0,T]$, $0 \le p(t) \in C[0,T]$, $f(x,t), g(x,t) \in C(D_T)$, $\int_0^1 f(x,t)\,dx = \int_0^1 g(x,t)\,dx = 0$, $\varphi(x) \in C[0,1]$, $h(t) \equiv h_1(t) g(x_2,t) - h_2(t) g(x_1,t) \ne 0$, $h_i(t) \in C^1[0,T]$, $i = 1, 2$, and the compatibility conditions

$$\int_0^1 \varphi(x)\,dx = 0, \tag{6}$$

$$h_i(0) + \delta\, h_i(T) + \int_0^T p(t) h_i(t) dt = \varphi(x_i),\ \ i = 1, 2, \tag{7}$$

hold. Then the problem of finding a classical solution of (1)-(5) is equivalent to the problem of determining the functions

$u(x,t) \in C^{2,1}(D_T)$, $a(t) \in C[0,T]$, and $b(t) \in C[0,T]$ satisfying (1)-(3), and the conditions

$$u_x(0,t) = u_x(1,t),\ \ 0 \le t \le T, \tag{8}$$

$$c(t)h_i'(t) = u_{xx}(x_i,t) + a(t)h_i(t) + \\ + b(t)g(x_i,t) + f(x_i,t),\ \ i = 1,2,\ \ 0 \le t \le T. \tag{9}$$

Let $B_{2,T}^3$ denotes the set of all functions of the form

$$u(x,t) = \sum_{k=0}^{\infty} u_{1k}(t)\cos\lambda_k x + \sum_{k=1}^{\infty} u_{2k}(t)\sin\lambda_k x,\ \ \lambda_k = 2k\pi\,,$$

considered in domain $D_T$, where the functions $u_{1k}(t)$ $(k = 0,1,...)$ and $u_{2k}(t)$ $(k = 1,2,...)$ are continuous on the interval $[0,T]$ and satisfy the following condition:

$$\left\|u_{10}(t)\right\|_{C[0,T]} + \left(\sum_{k=1}^{\infty}\left(\lambda_k^3 \left\| u_{1k}(t) \right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}} + \left(\sum_{k=1}^{\infty}\left(\lambda_k^3 \left\| u_{2k}(t) \right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}} < +\infty.$$

The norm in this set is defined as follows:

$$\left\| u(x,t) \right\|_{B_{2,T}^3} = \left\|u_{10}(t)\right\|_{C[0,T]} + \left(\sum_{k=1}^{\infty}\left(\lambda_k^3 \left\| u_{1k}(t) \right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}} + \\ + \left(\sum_{k=1}^{\infty}\left(\lambda_k^3 \left\| u_{2k}(t) \right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}.$$

Let $E_T^3$ denote the space consisting of the topological product $B_{2,T}^3 \times C[0,T] \times C[0,T]$, which is the norm of the element $z(x,t) = \{u(x,t), a(t), b(t)\}$ described by the formula

$$\left\|z(x,t)\right\|_{E_{2,T}^3} = \left\|u(x,t)\right\|_{B_{2,T}^3} + \left\|a(t)\right\|_{C[0,T]} + \left\|b(t)\right\|_{C[0,T]}.$$

It is known that $B_{2,T}^3$ and $E_T^3$ are Banach spaces.

Although the designations of the introduced spaces in different sections of the dissertation look the same, in fact they have a different structure.

Suppose that the data of the problem (1)-(3), (8), (9) satisfy the following conditions:

1.1. $\varphi(x)\in C^2[0,1]$, $\varphi'''(x)\in L_2(0,1)$, $\varphi(0)=\varphi(1)$,
$\varphi'(0)=\varphi'(1)$, $\varphi''(0)=\varphi''(1)$;

1.2. $f(x,t), f_x(x,t), f_{xx}(x,t)\in C^2[0,1]$, $f_{xxx}(x,t)\in L_2(D_T)$,
$f(0,t)=f(1,t)$, $f_x(0,t)=f_x(1,t)$, $f_{xx}(0,t)=f_{xx}(1,t)=0$, $0\le t\le T$;

1.3. $g(x,t), g_x(x,t), g_{xx}(x,t)\in C^2[0,1]$, $g_{xxx}(x,t)\in L_2(D_T)$,
$g(0,t)=g(1,t)$, $g_x(0,t)=f_x(1,t)$, $g_{xx}(0,t)=g_{xx}(1,t)=0$, $0\le t\le T$;

1.4. $p(t)\in C[0,T]$, $h_1(t),h_2(t)\in C^1[0,T]$,
$h_1(t)g(x_2,t)-h_2(t)g(x_1,t)\ne 0$, $0\le t\le T$.

**Theorem 1.2.** Let the statements 1.1-1.4 and the condition

$$(B(T)(A(T)+2)+C(T)+D(T))(A(T)+2)<1, \tag{10}$$

hold, then problem (1)-(3), (8), (9) has a unique solution in the ball $K=K_R\subset E_T^3$, $R=A(T)+2$.

Note that the expressions $A(T)$, $B(T)$, $C(T)$ and $D(T)$ are given in section 1.1 of the dissertation.

**Theorem 1.3.** Assume that all assumptions of Theorem 1.2, and the compatibility conditions (6), (7) hold. If

$$\int_0^1 f(x,t)\,dx=\int_0^1 g(x,t)\,dx=0,\ \ 0\le t\le T,$$

then problem (1)-(5) has a unique classical solution in the ball $K\subset E_T^3$.

In Section 1.2, we investigate the unique solvability of the parabolic equation

$$c(t)u_t(x,t)=u_{xx}(x,t)+a(t)u(x,t)+b(t)g(x,t)+f(x,t)\ \ (x,t)\in D_T\ , \tag{11}$$

with the following conditions

$$u(x,0)+\delta u(x,T)+\int_0^T p(t)u(x,t)\,dt=\varphi(x),\ \ 0\le x\le 1, \tag{12}$$

$$u(0,t)=\beta u(1,t),\ \ 0\le t\le T, \tag{13}$$

$$\int_0^1 u(x,t)\,dx=0,\ \ 0\le t\le T, \tag{14}$$

$$u(x_i,t)=h_i(t),\ \ i=1,2;\ \ 0\le t\le T, \tag{15}$$

where $T, \beta > 0$, $\delta \ge 0$, $x_i \in (0,1)$ $(i=1, 2; x_1 \ne x_2)$ are given numbers, $D_T = \{(x,t) : 0 \le x \le 1, 0 \le t \le T\}$ is certain rectangular domain, $c(t) > 0$, $g(x,t)$, $f(x,t)$, $\varphi(x)$, $p(t) \ge 0$, $h_1(t), h_2(t)$ are known functions, and $u(x,t)$, $a(t)$, $b(t)$ are unknown functions.

**Theorem 1.4.** Suppose that $0 < c(t) \in C[0,T]$, $0 \le p(t) \in C[0,T]$, $f(x,t), g(x,t) \in C(D_T)$, $\int_0^1 f(x,t)\,dx = \int_0^1 g(x,t)\,dx = 0$, $\varphi(x) \in C[0,1]$, $h_i(t) \in C^1[0,T]$, $i=1,2$, $h(t) \equiv h_1(t)g(x_2,t) - h_2(t)g(x_1,t) \ne 0$, $t \in [0,T]$, and the compatibility conditions

$$\int_0^1 \varphi(x)\,dx = 0, \tag{16}$$

$$h_i(0) + \delta h_i(T) + \int_0^T p(t)h_i(t)dt = \varphi(x_i),\ \ i=1,2. \tag{17}$$

Then the problem of finding a classical solution of (11)-(15) is equivalent to the problem of determining the functions $u(x,t) \in C^{2,1}(D_T)$, $a(t) \in C[0,T]$, and $b(t) \in C[0,T]$ satisfying (11)-(13), and the

$$u_x(0,t) = u_x(1,t),\ \ 0 \le t \le T, \tag{18}$$

$$\begin{gathered} c(t)h_i'(t) = u_{xx}(x_i,t) + a(t)h_i(t) + \\ + b(t)\,g(x_i,t) + f(x_i,t),\ \ i=1,2;\ \ 0 \le t \le T. \end{gathered} \tag{19}$$

Assume that the data of the problem (11)-(13), (18), (19) satisfy the following conditions:

1.5. $\varphi(x) \in C^2[0,1]$, $\varphi'''(x) \in L_2(0,1)$, $\varphi(0) = \beta\varphi(1)$, $\varphi'(0) = \varphi'(1)$,
$\varphi''(0) = \beta\varphi''(1)$, $\beta \ne \pm 1$;

1.6. $f(x,t) \in C^{2,0}_{x,t}(D_T)$, $f_{xxx}(x,t) \in L_2(D_T)$, $f(0,t) = \beta f(1,t)$,
$f_x(0,t) = f_x(1,t)$, $f_{xx}(0,t) = \beta f_{xx}(1,t)$, $\beta \ne \pm 1$, $0 \le t \le T$;

1.7. $g(x,t) \in C^{2,0}_{x,t}(D_T)$, $g_{xxx}(x,t) \in L_2(D_T)$, $g(0,t) = \beta\, g(1,t)$,
$g_x(0,t) = f_x(1,t)$, $g_{xx}(0,t) = \beta\, g_{xx}(1,t) = 0$, $\beta \ne \pm 1$, $0 \le t \le T$;

1.8. $c(t), p(t) \in C[0,T]$, $h_i(t) \in C^1[0,T]$, $i=1,2$,
$h(t) = h_1(t)g(x_2,t) - h_2(t)\,g(x_1,t) \ne 0$, $0 \le t \le T$.

**Theorem 1.5.** Let the conditions 1.5-1.8, the inequality

$$R\,(B(T)\,R + C(T) + D(T)) < 1,$$

and the compatibility conditions

$$\int_0^1 \varphi(x)dx = 0,\;\; h_i(0) + \delta\, h_i(T) + \int_0^T p\,(t)\, h_i(t)\, dt = \varphi(x_i),\;\; i = 1, 2$$

be fulfilled. Then, problem (11)-(15) has a unique classical solution in the ball $K \subset E_T^3$, where $R = A(T) + 2$, and the expressions of $A(T)$, $B(T)$, $C(T)$, $D(T)$, and the structure of the space $E_T^3$ were introduced in section 1.2 of the dissertation.

Section 1.3 is devoted to studying the existence and uniqueness of the classical solution of the following inverse problem:

$$a_1(t)\, u_t(x,t) + a_0(t)\, u(x,t) = u_{xx}(x,t) + f(x,t) \;\; (x,t) \in D_T, \quad (20)$$

$$u(x,0) = \varphi(x),\;\; 0 \le x \le 1, \quad (21)$$

$$u(1,t) = 0,\;\; 0 \le t \le T, \quad (22)$$

$$\int_0^1 u(x,t)\, dx = 0,\;\; 0 \le t \le T, \quad (23)$$

$$u(0,t) = h(t),\;\; 0 \le t \le T, \quad (24)$$

where $D_T = \{(x,t) : 0 \le x \le 1, 0 \le t \le T\}$ represent rectangular domain, $a_0(t)$, $\varphi(x)$, $f(x,t)$, $h(t)$ are given functions, $u(x,t)$ and $a_1(t)$ are unknown functions. The main results of this section consist of the existence and uniqueness theorems for the classical solution of the inverse problem (20)-(24). In this section the conditions for the existence and uniqueness of the classical solution to the inverse problem (20)-(24) are established by reducing it to an equivalent boundary value problem.

Section 1.4 is devoted to the inverse boundary value problem of determining the functions $u(x,y,t) \in C^{2,2,1}(D_T)$ and $a(t)$, $b(t) \in C[0,T]$ satisfying the following two-dimensional parabolic equation

$$u_t(x,y,t) - c(t)(u_{xx}(x,y,t) + u_{yy}(x,y,t)) =$$

$$= a(t)u(x,y,t) + b(t)g(x,y,t) + f(x,y,t) \;\; (x,y,t) \in D_T, \quad (25)$$

with the non-local condition

$$u(x,y,0)+\delta u(x,y,T)+\int_0^T p(t)u(x,y,t)dt=\varphi(x,y)\ \ (x,y)\in\overline{Q}_{xy}, \quad (26)$$

the boundary conditions

$$u(0,y,t)=u_x(1,y,t)=0,\ \ 0\le y\le 1,\ \ 0\le t\le T, \quad (27)$$

$$u_y(x,0,t)=u(x,1,t)=0,\ \ 0\le x\le 1,\ \ 0\le t\le T, \quad (28)$$

and over-specification conditions

$$u(1,0,t)=h_1(t),\ \ 0\le t\le T, \quad (29)$$

$$u(x_0,y_0,t)=h_2(t),\ \ 0\le t\le T,\ \ x_0,y_0\in Q_{xy}, \quad (30)$$

where $T>0$ is an arbitrary fixed time moment, $\delta\ge 0$ is given number, $D_T=\overline{Q}_{xy}\times[0,T]$, $Q_{xy}=\{(x,y): 0<x<1,\ 0<y<1\}$, $c(t)>0$, $p(t)\ge 0$, $f(x,y,t)$, $g(x,y,t)$, $\varphi(x,y)$, $h_1(t)$ and $h_2(t)$ are known functions.

**Theorem 1.6.** Assume that $\delta\ge 0$, $c(t),p(t)\in C[0,T]$, $f(x,y,t),g(x,y,t)\in C(D_T)$, $\varphi(x,y)\in C(\overline{Q}_{xy})$, $h_1(t),h_2(t)\in C^1[0,T]$, $h(t)\equiv h_1(t)g(x_0,y_0,t)-h_2(t)g(1,0,t)\ne 0,\ 0\le t\le T$, and the compatibility conditions

$$\begin{aligned} &h_1(0)+\delta h_1(T)+\int_0^T p(t)h_1(t)dt=\varphi(1,0),\\ &h_2(0)+\delta h_2(T)+\int_0^T p(t)h_2(t)dt=\varphi(x_0,y_0). \end{aligned} \quad (31)$$

Then the problem of finding a classical solution of (25)-(30) is equivalent to the problem of determining the functions $u(x,y,t)\in C^{2,2,1}(D_T)$, $a(t)\in C[0,T]$ and $b(t)\in C[0,T]$ satisfying (25)-(28), and the conditions

$$\begin{aligned} &h_1'(t)-c(t)(u_{xx}(1,0,t)+u_{yy}(1,0,t))=\\ &=a(t)h_1(t)+b(t)g(1,0,t)+f(1,0,t),\ \ 0\le t\le T, \end{aligned} \quad (32)$$

$$\begin{aligned} &h_2'(t)-c(t)(u_{xx}(x_0,y_0,t)+u_{yy}(x_0,y_0,t))=\\ &=a(t)h_2(t)+b(t)g(x_0,y_0,t)+f(x_0,y_0,t),\ \ 0\le t\le T. \end{aligned} \quad (33)$$

Let $B^3_{2,T}$ denote the set of all functions of the form

$$u(x,y,t)=\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}u_{k,n}(t)\sin\lambda_k x\cos\gamma_n y\,,$$

$$\lambda_k=\frac{\pi}{2}(2k-1),\ \gamma_n=\frac{\pi}{2}(2n-1),\ k,n=1,2,\ldots,$$

considered in domain $D_T$. Moreover, the functions $u_{k,n}(t)$ $(k,n=1,2,\ldots)$ contained in last double sum are continuously differentiable on $[0,T]$, and the norm in the space $B_{2,T}^3$ is defined as follows

$$\|u(x,y,t)\|_{B_{2,T}^3}=\left\{\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}(\mu_{k,n}^3\|u_{k,n}(t)\|_{C[0,T]})^2\right\}^{\frac{1}{2}}<+\infty.$$

Let $E_T^3$ denote the space consisting of the topological product $B_{2,T}^3\times C[0,T]\times C[0,T]$, which is the norm of the element $z=\{u,a,b\}$, described by the formula

$$\|z\|_{E_T^3}=\|u(x,y,t)\|_{B_{2,T}^3}+\|a(t)\|_{C[0,T]}+\|b(t)\|_{C[0,T]}.$$

It is known that the spaces $B_{2,T}^3$ and $E_T^3$ are Banach spaces.

Assume that the data for the problem (25)-(28), (32), (33) satisfy the following conditions:

1.9. $\varphi(x,y),\varphi_x(x,y),\varphi_{xx}(x,y),\varphi_y(x,y),\varphi_{xy}(x,y),\varphi_{yy}(x,y)\in C(\overline{Q}_{xy})$,
$\varphi_{xxy}(x,y),\varphi_{xyy}(x,y),\varphi_{xxx}(x,y),\varphi_{yyy}(x,y)\in L_2(Q_{xy})$,
$\varphi(0,y)=\varphi_x(1,y)=\varphi_{xx}(0,y)=0,\ 0\le y\le 1$,
$\varphi_y(x,0)=\varphi(x,1)=\varphi_{yy}(x,1)=0,\ 0\le x\le 1$;

1.10. $f(x,y,t),f_x(x,y,t),f_{xx}(x,y,t),f_y(x,y,t),f_{xy}(x,y,t),f_{yy}(x,y,t)\in C(D_T)$,
$f_{xxy}(x,y,t),f_{xyy}(x,y,t),f_{xxx}(x,y,t),f_{yyy}(x,y,t)\in L_2(D_T)$,
$f(0,y,t)=f_x(1,y,t)=f_{xx}(0,y,t)=0,\ 0\le y\le 1,\ 0\le t\le T$,
$f_y(x,0,t)=f(x,1,t)=f_{yy}(x,1,t)=0,\ 0\le x\le 1,\ 0\le t\le T$;

1.11. $g(x,y,t),g_x(x,y,t),g_{xx}(x,y,t),g_y(x,y,t),g_{xy}(x,y,t),g_{yy}(x,y,t)\in C(D_T)$,
$g_{xxy}(x,y,t),g_{xyy}(x,y,t),g_{xxx}(x,y,t),g_{yyy}(x,y,t)\in L_2(D_T)$,
$g(0,y,t)=g_x(1,y,t)=g_{xx}(0,y,t)=0,\ 0\le y\le 1,\ 0\le t\le T$,

$$g_y(x,0,t) = g(x,1,t) = g_{yy}(x,1,t) = 0, \ \ 0 \le x \le 1, \ \ 0 \le t \le T;$$

1.12. $\delta \ge 0, p(t) \in C[0,T], \ 0 < c(t) \in C[0,T], \ h_i(t) \in C^1[0,T] \ (i=1,2),$

$$h(t) \equiv h_1(t) g(x_0, y_0, t) - h_2(t) g(1,0,t) \ne 0, \ \ 0 \le t \le T.$$

**Theorem 1.7.** Let the conditions 1.9-1.12, the inequality

$$(B(T)(A(T)+2) + C(T) + D(T))(A(T)+2) < 1,$$

and compatibility conditions

$$h_1(0) + \delta h_1(T) + \int_0^T p(t) h_1(t) dt = \varphi(1,0),$$

$$h_2(0) + \delta h_2(T) + \int_0^T p(t) h_2(t) dt = \varphi(x_0, y_0),$$

hold. Then, problem (25)-(30) has a unique classical solution in the ball $K = K_R \subset E_T^3$, where $R$ defined by $A(T)+2$, the expressions $A(T)$, $B(T)$, $C(T)$, and $D(T)$ are introduced in section 1.4 of the dissertation.

At the end of the chapter some numerical examples are given that correspond to the considered inverse problems, and their numerical solutions and graphs are demonstrated using the MatLab mathematical software package.

In the second chapter of this dissertation, the existence and uniqueness of the classical solution of self-adjoint and non-self-adjoint nonlinear inverse boundary value problems with non-local data (both with respect to time and space variables) and with various overdetermination conditions for a second-order hyperbolic equation are investigated.

Section 2.1 is devoted to the study of solvability of the inverse boundary value problem with an unknown time-dependent coefficient for the hyperbolic equation

$$u_{tt}(x,t) - u_{xx}(x,t) = a(t) u(x,t) + b(t) g(x,t) + f(x,t) \ \ (x,t) \in D_T, \quad (34)$$

with the conditions

$$u(x,0)=\int_0^T P_1(t)u(x,t)dt+\varphi(x),$$
$$u_t(x,0)=\int_0^T P_2(t)u(x,t)dt+\psi(x),\ \ 0\le x\le 1, \tag{35}$$
$$u(0,t)=u(1,t),\ \ 0\le t\le T, \tag{36}$$
$$\int_0^1 u(x,t)dx=0,\ \ 0\le t\le T, \tag{37}$$
$$u(x_i,t)=h_i(t),\ \ i=1,2;\ \ 0\le t\le T, \tag{38}$$

where $D_T=\{(x,t):0\le x\le 1,\ 0\le t\le T\}$ is a certain rectangular region, $x_i\in(0,1)$, $i=1,2$; $x_1\ne x_2$ are known fixed numbers, $f(x,t)$, $g(x,t)$, $P_1(t)$, $P_2(t)$, $h_1(t)$, $h_2(t)$ are given functions, $u(x,t)$, $a(t)$ and $b(t)$ are desired functions.

**Definition 2.1.** The triple $\{u(x,t),a(t),b(t)\}$ is said to be a classical solution to the problem (34)-(38), if the functions $u(x,t)\in C^2(D_T)$ and $a(t),b(t)\in C[0,T]$ satisfy equation (34) in $D_T$ and the conditions (35)-(38) in the classical (usual) sense.

To study problem (34)-(38), we consider the following auxiliary inverse boundary value problem: it is required to find the functions $u(x,t)\in C^2(D_T)$, $a(t)$, $b(t)\in C[0,T]$, from the relations (34)-(36) and

$$u_x(0,t)=u_x(1,t),\ \ 0\le t\le T, \tag{39}$$
$$h''_i(t)-u_{xx}(x_i,t)=a(t)h_i(t)+f(x_i,t)+b(t)g(x_i,t),\ \ i=1,2;\ \ 0\le t\le T. \tag{40}$$

The following theorem is valid.

**Theorem 2.1.** Let $\varphi(x),\psi(x)\in C[0,1]$, $P_i(t)\in C[0,T]$, $h_i(t)\in C^2[0,T]$, $i=1,2$, $f(x,t),g(x,t)\in C(D_T)$, $\int_0^1 f(x,t)dx=\int_0^1 g(x,t)dx=0$, $h(t)\equiv h_1(t)g(x_2,t)-h_2(t)g(x_1,t)\ne 0$, $0\le t\le T$, and the following consistency conditions be fulfilled:

$$\int_0^1\varphi(x)dx=\int_0^1\psi(x)dx=0, \tag{41}$$

$$h_i(0)=\int_0^T p_1(t)h_i(t)dt+\varphi(x_i),\ h_i'(0)=\int_0^T p_2(t)h_i(t)dt+\psi(x_i),\ i=1,2. \quad (42)$$

Then the following assertions are true:

i) each classical solution $\{u(x,t),a(t),b(t)\}$ to the problem (34)-(38) is a solution of problem (34)-(36), (39), (40), as well as;

ii) each solution $\{u(x,t),a(t),b(t)\}$ to the problem (34)-(36), (39), (40) satisfying the condition

$$\left(T\|P_2(t)\|_{C[0,T]}+\|P_1(t)\|_{C[0,T]}+\frac{T}{2}\|a(t)\|_{C[0,T]}\right)T<1, \quad (43)$$

is a classical solution of problem (34)-(38).

Let $B_{2,T}^{\alpha}$ denotes a set of all functions of the form

$$u(x,t)=\sum_{k=0}^{\infty}u_{1k}(t)\cos\lambda_k x+\sum_{k=1}^{\infty}u_{2k}(t)\sin\lambda_k x,\ \lambda_k=2\pi k,$$

considered in $D_T$. Moreover, the functions $u_k(t)$ $(k=0,1,2,...)$ are continuously differentiable on $[0,T]$ and

$$J(u)\equiv\|u_0(t)\|_{C[0,T]}+\left\{\sum_{k=1}^{\infty}\left(\lambda_k^{\alpha}\|u_{1k}(t)\|_{C[0,T]}\right)^2\right\}^{\frac{1}{2}}+$$

$$+\left\{\sum_{k=1}^{\infty}\left(\lambda_k^{\alpha}\|u_{2k}(t)\|_{C[0,T]}\right)^2\right\}^{\frac{1}{2}}<+\infty,$$

where $\alpha$ is some non-negative number.

The norm on the set $J(u)$ is established as follows:

$$\|u(x,t)\|_{B_{2,T}^{\alpha}}=J(u).$$

Let $E_T^{\alpha}$ denote the space consisting of the topological product $B_{2,T}^{\alpha}\times C[0,T]\times C[0,T]$, which is the norm of the element $z=\{u,a,b\}$ defined by the formula

$$\|z(x,t)\|_{E_T^{\alpha}}=\|u(x,t)\|_{B_{2,T}^{\alpha}}+\|a(t)\|_{C[0,T]}+\|b(t)\|_{C[0,T]}.$$

It is known that the spaces $B_{2,T}^{\alpha}$ and $E_T^{\alpha}$ are Banach spaces.

It is assumed that the data of problem (34)-(36), (39), (40) satisfy the following conditions:

2.1. $\varphi(x)\in C^2[0,1], \varphi'''(x)\in L_2(0,1), \varphi(0)=\varphi(1),$
$\varphi'(0)=\varphi'(1),\ \varphi''(0)=\varphi''(1);$

2.2. $\psi(x)\in C^2[0,1],\ \psi''(x)\in L_2(0,1),\ \psi(0)=\psi(1),\ \psi'(0)=\psi'(1);$

2.3. $f(x,t), f_x(x,t)\in C(D_T),\ \ f_{xx}(x,t)\in L_2(D_T),$
$f(0,t)=f(1,t),\ f_x(0,t)=f_x(1,t),\ \ 0\le t\le T;$

2.4. $g(x,t), g_x(x,t)\in C(D_T),\ \ g_{xx}(x,t)\in L_2(D_T),$
$g(0,t)=g(1,t),\ \ g_x(0,t)=g_x(1,t),\ \ 0\le t\le T;$

2.5. $h_i(t)\in C^2[0,T],\ \ i=1,2,\ \ h(t)\equiv h_1(t)g(x_2,t)-h_2(t)g(x_1,t)\ne 0.$

**Theorem 2.2.** Let the conditions 2.1-2.5, the inequality

$$(A(T)+2)(B(T)(A(T)+2)+C(T)+D(T))<1, \tag{44}$$

and the conditions

$$\int_0^1 \varphi(x)dx=\int_0^1 \psi(x)dx=0,\ \ \int_0^1 f(x,t)dx=\int_0^1 g(x,t)dx=0,\ \ 0\le t\le T,$$

$$h_i(0)=\int_0^T p_1(t)h_i(t)dt+\varphi(x_i),\ \ h_i'(0)=\int_0^T p_2(t)h_i(t)dt+\psi(x_i),\ \ i=1,2,$$

$$\left(T\|P_2(t)\|_{C[0,T]}+\|P_1(t)\|_{C[0,T]}+\frac{T}{2}(A(T)+2)\right)T<1$$

be fulfilled. Then, problem (34)-(38) has a unique classical solution in the ball $K=K_R\left(\|z\|_{E_T^3}\le R\le A(T)+2\right)$ of the space $E_T^\alpha$ .

Note that the expressions $A(T), B(T), C(T)$ and $D(T)$ are given in section 2.1 of the dissertation.

Section 2.2 considers the following inverse boundary value problem:

$$u_{tt}(x,t)-u_{xx}(x,t)=a(t)\,u(x,t)+f(x,t)\ \ (x,t)\in D_T, \tag{45}$$

$$u(x,0)+\delta_1 u(x,T)=\varphi(x),\ \ u_t(x,0)+\delta_2 u_t(x,T)=\psi(x),\ \ 0\le x\le 1, \tag{46}$$

$$u_x(0,t)=u(1,t)=0,\ \ 0\le t\le T, \tag{47}$$

$$\int_0^1 w(x)u(x,t)dx=H(t),\ \ 0\le t\le T, \tag{48}$$

where $D_T = \{(x,t) : 0 \le x \le 1,\ 0 \le t \le T\}$ is a rectangular region, $\delta_1, \delta_2 \ge 0$, $0 < T < +\infty$ are given numbers, and $f(x,t)$, $\varphi(x)$, $\psi(x)$, $w(x)$, $H(t)$ are known functions, $u(x,t)$ and $a(t)$ are desired functions of $x \in [0,1]$ and $t \in [0,T]$. First, the original problem is reduced to an equivalent problem, then the existence and uniqueness theorem is proved for the reduced problem using the contraction mapping principle. Further, on the basis equivalence the existence and uniqueness of the classical solution of the original problem is proved.

In the third section of the second chapter, we study the following inverse boundary value problem:

$$u_{tt}(x,t) - u_{xx}(x,t) = a(t)u(x,t) + f(x,t) \quad (x,t) \in D_T, \tag{49}$$

$$u(x,0) + \delta_1 u(x,T) = \varphi(x), \quad u_t(x,0) + \delta_2 u_t(x,T) = \psi(x), \quad 0 \le x \le 1, \tag{50}$$

$$u(0,t) = \beta u(1,t), \quad 0 \le t \le T, \tag{51}$$

$$\int_0^1 u(x,t)dx = 0, \quad 0 \le t \le T, \tag{52}$$

$$u\left(\frac{1}{2}, t\right) = h(t), \quad 0 \le t \le T, \tag{53}$$

in which $D_T = \{(x,t) : 0 \le x \le 1,\ 0 \le t \le T\}$ is a rectangular domain, $\delta_1, \delta_2 \ge 0$, $\beta \ne \pm 1$ are given numbers, $f(x,t)$, $\varphi(x)$, $\psi(x)$, and $h(t)$ are known functions, $u(x,t)$ and $a(t)$ are desired functions of $x \in [0,1]$ and $t \in [0,T]$. The goal of the problem is to simultaneously determine the solution to the problem and the unknown coefficient. To study of the original problem, we first consider an auxiliary inverse boundary value problem and prove its equivalence (in a certain sense) to the original problem. The existence and uniqueness of a solution to an auxiliary problem are proved with the help of contracted mappings principle. Next, on the basis equivalence the existence and uniqueness of the classical solution of the original problem is proved.

Section 2.4 is devoted to the study of the unique classical solvability of a nonlinear inverse boundary value problem with nonlocal conditions for a two-dimensional second-order hyperbolic equation.

The aim of the problem is to find the functions $u(x,y,t) \in C^2(D_T)$ and $a(t), b(t) \in C[0,T]$ satisfying the equation:

$$u_{tt}(x,y,t) = u_{xx}(x,y,t) + u_{yy}(x,y,t) + a(t)u(x,y,t) +$$
$$+ b(t)u_t(x,y,t) + f(x,y,t) \;\; (x,y,t) \in D_T, \tag{54}$$

with the conditions

$$u(x,y,0) = \varphi(x,y), \;\; u_t(x,y,0) = \psi(x,y), \;\; 0 \le x, y \le 1, \tag{55}$$

$$u_x(0,y,t) = u(1,y,t) = 0, \;\; 0 \le y \le 1, \;\; 0 \le t \le T, \tag{56}$$

$$u(x,0,t) = u_y(x,1,t) = 0, \;\; 0 \le y \le 1, \;\; 0 \le t \le T, \tag{57}$$

$$u(0,1,t) = h_1(t), \;\; 0 \le t \le T, \tag{58}$$

$$\int_0^1\int_0^1 u(x,y,t)dxdy = h_2(t), \;\; 0 \le t \le T, \tag{59}$$

where $D_T = \overline{Q}_{xy} \times \{0 \le t \le T\}$ and $Q_{xy} = \{(x,\ y) : 0 < x < 1, 0 < y < 1\}$, $T > 0$ is given number, and $f(x,y,t)$, $\varphi(x,y)$, $\psi(x,y)$, and $h_i(t)$ $(i = 1,2)$ are known functions of $x, y \in [0,1]$ and $t \in [0,T]$.

**Theorem 2.3.** Assume that $\varphi(x,y), \psi(x,y) \in C(Q_{xy})$, $f(x,y,t) \in C(D_T)$, $h_1(t), h_2(t) \in C^2[0,T]$, $h(t) \equiv h_1(t)h_2'(t) - h_2(t)h_1'(t) \ne 0$, $0 \le t \le T$ and the compatibility conditions

$$\varphi(0,1) = h_1(0), \;\; \psi(0,1) = h_1'(0), \tag{60}$$

$$\int_0^1\int_0^1 \varphi(x,y)dxdy = h_2(0), \;\; \int_0^1\int_0^1 \psi(x,y)dxdy = h_2'(0), \tag{61}$$

hold. Then the problem of finding a classical solution of (54)-(59) is equivalent to the problem of determining the functions $u(x,y,t) \in C^2(D_T)$, $a(t), b(t) \in C[0,T]$ satisfying (54)-(57), and

$$h_1''(t) = u_{xx}(0,1,t) + u_{yy}(0,1,t) + a(t)h_1(t) +$$
$$+ b(t)h_1'(t) + f(0,1,t), \;\; 0 \le t \le T, \tag{62}$$

$$h_2''(t) = \int_0^1 u_x(1,y,t)dy - \int_0^1 u_y(x,0,t)dx +$$
$$+ a(t)h_2(t) + b(t)h_2'(t) + \int_0^1\int_0^1 f(x,y,t)dxdy, \;\; 0 \le t \le T. \tag{63}$$

Let us consider the functional space $B_{2,T}^{3,2}$, where $B_{2,T}^{3,2}$ denotes a set of all functions of the form

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}u_{k,n}(t)\cos\lambda_k x\sin\gamma_n y,$$

considered in $D_T$. Moreover, the functions $u_{k,n}(t)\ (k,n=1,2,...)$ contained in last sum are continuously differentiable on $[0,T]$ and

$$\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\mu_{k,n}^3\|u_{k,n}(t)\|_{C[0,T]})^2\right\}^{\frac{1}{2}}+\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\mu_{k,n}^2\|u'_{k,n}(t)\|_{C[0,T]})^2\right\}^{\frac{1}{2}}<+\infty.$$

The norm on this set is defined as follows:

$$\|u(x,y,t)\|_{B_{2,T}^{3,2}}=\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\mu_{k,n}^3\|u_{k,n}(t)\|_{C[0,T]})^2\right\}^{\frac{1}{2}}+$$

$$+\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\mu_{k,n}^2\|u'_{k,n}(t)\|_{C[0,T]})^2\right\}^{\frac{1}{2}}.$$

Let $E_T^{3,2}$ denote the space consisting of the topological product $B_{2,T}^{3,2}\times C[0,T]\times C[0,T]$, which is the norm of the element $z=\{u,a,b\}$ defined by the formula

$$\|z\|_{E_T^{3,2}}=\|u(x,y,t)\|_{B_{2,T}^{3,2}}+\|a(t)\|_{C[0,T]}+\|b(t)\|_{C[0,T]}.$$

It is known that the spaces $B_{2,T}^{3,2}$ and $E_T^{3,2}$ are Banach spaces.

We impose the following conditions on the data of problem (54)-(57), (62), (63):

2.6. $\varphi(x,y),\varphi_x(x,y),\varphi_{xx}(x,y),\varphi_y(x,y),\varphi_{xy}(x,y),\varphi_{yy}(x,y)\in C(\overline{Q}_{xy})$,
$\varphi_{xxy}(x,y),\varphi_{xyy}(x,y),\varphi_{xxx}(x,y),\varphi_{yyy}(x,y)\in L_2(Q_{xy})$,
$\varphi_x(0,y)=\varphi(1,y)=\varphi_{xx}(1,y)=0,\ \ 0\le y\le 1$,
$\varphi(x,0)=\varphi_y(x,1)=\varphi_{yy}(x,0)=0,\ \ 0\le x\le 1$.

2.7. $\psi(x,y),\psi_x(x,y),\psi_y(x,y)\in C(\overline{Q}_{xy}),\ \ \psi_{xx}(x,y),\psi_{yy}(x,y)\in L_2(Q_{xy})$,
$\psi_x(0,y)=\psi(1,y)=0,\ \psi(x,0)=\psi_y(x,1)=0,\ \ 0\le x,y\le 1$.

2.8. $f(x,y,t)\in C(D_T), f_x(x,y,t), f_y(x,y,t)\in L_2(D_T)$,

$$f_x(0,y,t)=f(1,y,t)=0,\ \ 0\le y\le 1,\ \ 0\le t\le T,$$
$$f(x,0,t)=f_y(x,1,t)=0,\ \ 0\le x\le 1,\ \ 0\le t\le T.$$

2.9. $h_1(t), h_2(t)\in C^2[0,T],\ \ h(t)\equiv h_1(t)h_2'(t)-h_2(t)h_1'(t)\neq 0,\ \ 0\le t\le T.$

**Theorem 2.4.** Suppose that conditions 2.6.-2.9 and the inequality

$$(A(T)+2)^2 B(T)<1,$$

holds. Then, problem (54)-(57), (62), (63) has a unique solution in the ball $K=K_R$ of the space $E_T^{3,2}$.

Note that the expressions of $A(T)$ and $B(T)$ are introduced in section 2.4 of the dissertation.

**Theorem 2.4.** Assume that all the conditions of Theorem 2.4 and the compatibility conditions (60) and (61) are satisfied. Then problem (54)-(59) has a unique classical solution in the ball $K=K_R\left(\|z\|_{E_T^{3,2}}\le R=A(T)+2\right)$ of the space $E_T^{3,2}$.

At the end of section considered some numerical examples corresponding to the considered inverse problems, as well as their numerical solutions and graphs are demonstrated using the MatLab mathematical software package.

The third chapter is devoted to the study of the solvability of the inverse boundary value problem for the longitudinal wave propagation equation with self-adjoint and non-self-adjoint boundary conditions. The main purpose of our study is to prove the existence and uniqueness of the classical solution of the considered inverse boundary value problems.

In the first section of this chapter, we consider a time-nonlocal inverse boundary value problem for determining the coefficients in the one-dimensional longitudinal wave propagation equation.

The precise statement of the problem is in the following form: Find $u(x,t)$ and $a(t)$, satisfying the equation:

$$u_{tt}(x,t)-u_{ttxx}(x,t)-u_{xx}(x,t)=a(t)u(x,t)+f(x,t)\ \ (x,t)\in D_T, \quad (64)$$

with the conditions

$$u(x,0)=\int_0^T P_1(x,t)u(x,t)dt+\varphi(x),\ \ 0\le x\le 1, \tag{65}$$

$$u_t(x,0)=\int_0^T P_2(x,t)u(x,t)dt+\psi(x),\ \ 0\le x\le 1, \tag{66}$$

$$u_x(0,t)=u(1,t)=0,\ \ 0\le t\le T, \tag{67}$$

$$u(0,t)=h(t),\ \ 0\le t\le T, \tag{68}$$

where $D_T=\{(x,t): 0\le x\le 1, 0\le t\le T\}$ is a rectangular domain, $f(x,t)$, $\varphi(x)$, $\psi(x)$, $P_1(x,t)$, $P_2(x,t)$ and $h(t)$ are known functions.

The pair $\{u(x,t), a(t)\}$ is said to be a classical solution to the problem (64)-(68), if the functions $u(x,t)\in \tilde{C}^{2,2}(D_T)$ and $a(t)\in C[0,T]$ satisfies the relations (64)-(68) in the usual sense, where

$$\tilde{C}^{2,2}(D_T)=\{u(x,t): u(x,t)\in C^2(D_T), u_{ttxx}(x,t)\in C(D_T)\}.$$

To investigate problem (64)-(68), we first consider the following auxiliary problem: It is required to determine a pair of functions $\{u(x,t), a(t)\}$ such that $u(x,t)\in \tilde{C}^{2,2}(D_T)$, $a(t)\in C[0,T]$, from relations (64)-(67) and

$$h''(t)-u_{ttxx}(0,t)-u_{xx}(0,t)=a(t)h(t)+f(0,t),\ \ 0\le t\le T. \tag{69}$$

**Theorem 3.1.** Suppose that $\varphi(x), \psi(x)\in C[0,1]$, $h(t)\in C^2[0,T]$, $h(t)\ne 0$, $0\le t\le T$, $f(x,t)$, $P_1(x,t)$, $P_2(x,t)\in C(D_T)$ and the compatibility conditions

$$h(0)=\int_0^T P_1(0,t)h(t)dt+\varphi(0),\ \ \ h'(0)=\int_0^T P_2(0,t)h(t)dt+\psi(0),$$

hold. Then the following statements are valid:

i) each classical solution $\{u(x,t), a(t)\}$ of problem (64)-(68) is the solution of problem (64)-(67), (69) as well;

ii) each solution $\{u(x,t), a(t)\}$ of problem (64)-(67),(69) is a classical solution of the problem (64)-(68), if

$$\left(T\|P_2(0,t)\|_{C(D_T)}+\|P_1(0,t)\|_{C(D_T)}+\frac{T}{2}\|a(t)\|_{C(D_T)}\right)T<1.$$

We impose the following conditions on the functions $\varphi$, $\psi$, $f$,

$P_1$, $P_2$, and $h$:

3.1. $\varphi(x) \in C^2[0,1], \varphi'''(x) \in L_2(0,1)$ and $\varphi'(0) = \varphi(1) = \varphi''(1) = 0$;

3.2. $\psi(x) \in C^2[0,1], \psi'''(x) \in L_2(0,1)$ and $\psi'(0) = \psi(1) = \psi''(1) = 0$;

3.3. $f(x,t) \in C(D_T),\ f_x(x,t) \in L_2(D_T),\ f(1,t) = 0,\ \ 0 \le t \le T$;

3.4. $P_1(x,t), P_{1x}(x,t), P_{1xx}(x,t), P_{1xxx}(x,t) \in C(D_T)$,
$P_{1x}(0,t) = P_{1x}(1,t) = 0,\ \ 0 \le t \le T$;

3.5. $P_2(x,t), P_{2x}(x,t), P_{2xx}(x,t), P_{2xxx}(x,t) \in C(D_T)$,
$P_{2x}(0,t) = P_{2x}(1,t) = 0,\ \ 0 \le t \le T$;

3.6. $h(t) \in C^2[0,T],\ \ h(t) \ne 0,\ \ 0 \le t \le T$.

**Theorem 3.2.** If conditions 3.1-3.6, the inequalities

$$(B(T)(A(T)+2) + C(T))(A(T)+2) < 1,$$

$$\left( T \| P_2(0,t) \|_{C(D_T)} + \| P_1(0,t) \|_{C(D_T)} + \frac{T}{2}(A(T)+2) \right) T < 1,$$

and the compatibility conditions

$$h(0) = \int_0^T P_1(0,t) h(t)\,dt + \varphi(0),\ \ h'(0) = \int_0^T P_2(0,t) h(t)\,dt + \psi(0),$$

hold, then problem (64)-(68) has a unique classical solution in the ball $K = K_R \subset E_T^3$, $R = A(T) + 2$ of the space $E_T^3$.

Note that the expressions of $A(T)$, $B(T)$, $C(T)$, and the structure of the space $E_T^3$ were introduced in section 3.1 of the dissertation.

In Section 3.2, in the rectangular domain $D_T$, defined by the inequalities $0 \le x \le 1$ and $0 \le t \le T$ the following inverse boundary value problem is studied:

$$u_{tt}(x,t) - u_{ttxx}(x,t) - u_{xx}(x,t) = q(t)u(x,t) + f(x,t)\ \ (x,t) \in D_T, \quad (70)$$

$$\begin{gathered} u(x,0) = \int_0^T P_1(t) u(x,t)\,dt + \varphi(x), \\ u_t(x,0) = \int_0^T P_2(t) u(x,t)\,dt + \psi(x),\ \ 0 \le x \le 1, \end{gathered} \quad (71)$$

$$u(0,t) = \beta u(1,t),\ \ u_x(0,t) = u_x(1,t),\ \ 0 \le t \le T, \quad (72)$$

$$u\left(\frac{1}{2}, t\right) = h(t), \ \ 0 \le t \le T, \tag{73}$$

where $T, \beta$ are positive integers, $f(x,t)$, $\varphi(x)$, $\psi(x)$, $g(x)$, $h(t)$ are known functions, and $u(x,t)$ and $q(t)$ are desired functions. The object of the section is the study of a nonlinear inverse coefficient problem for the equation for the propagation of longitudinal waves of the pseudohyperbolic type. More precisely, the problem of determining, together with the solution of the corresponding equation, as well as, an unknown coefficient in the solution or in the derivative of the solution with respect to the time variable is studied. A distinctive feature of the considered problem is that the unknown coefficient is a function that depends only on time variable. Note that in the considered problem, the final overdetermination condition (observation) is used as an additional condition. In order to investigate the solvability of the considered inverse problem, first, the original problem is reduced to the auxiliary problem within trivial data. Then, applying the Fourier method and contraction mappings principle, the existence and uniqueness of the solution of the obtained equivalent problem is proved. Furthermore, using the equivalence, the unique solvability of the appropriate auxiliary inverse problem is shown.

In the third section of chapter 3, we study the unique solvability of a nonlocal inverse boundary value problem for determining a pair of functions $\{u(x,t), a(t)\}$ satisfying the following equation:

$$u_{tt}(x,t) - u_{ttxx}(x,t) - u_{xx}(x,t) = a(t)\, u(x,t) + f(x,t) \ \ (x,t) \in D_T, \tag{74}$$

with the conditions

$$u(x,0) = \varphi(x), \ \ \ u_t(x,0) = \psi(x), \ \ 0 \le x \le 1, \tag{75}$$

$$u_x(0,t) = 0, \ \ 0 \le t \le T, \tag{76}$$

$$qu(1,t) + \int_0^1 u(x,t)dx = 0, \ \ 0 \le t \le T, \tag{77}$$

$$u(0,t) = h(t), \ \ 0 \le t \le T, \tag{78}$$

where $D_T = \{(x,t) : 0 \le x \le 1, 0 \le t \le T\}$ is a given rectangular domain, $T, q > 0$ are given fixed numbers, $f(x,t)$, $\varphi(x)$, $\psi(x)$, $h(t)$ are known functions, while the functions $u(x,t)$ and $a(t)$ are unknowns.

Section 3.4 presents the main results concerning the unique solvability of the three-dimensional equation for the propagation of longitudinal waves

$$u_{tt}(x,y,z,t)-\Delta u_{tt}(x,y,z,t)-\Delta u(x,y,z,t)=$$
$$=a(t)u(x,y,z,t)+f(x,y,z,t) \ \ (x,y,z,t)\in D_T, \qquad (79)$$

under the following conditions

$$u(x,y,z,0)+\delta_1 u(x,y,z,T)=\varphi(x,y,z),$$
$$u_t(x,y,z,0)+\delta_2 u_t(x,y,z,T)=\psi(x,y,z), \ \ 0\le x,y,z\le 1, \qquad (80)$$

$$u_x(0,y,z,t)=u(1,y,z,t)=0, \ \ 0\le y,z\le 1, \ \ 0\le t\le T, \qquad (81)$$

$$u(x,0,z,t)=u_y(x,1,z,t)=0, \ \ 0\le x,z\le 1, \ \ 0\le t\le T, \qquad (82)$$

$$u(x,y,0,t)=u(x,y,1,t)=0, \ \ 0\le x,y\le 1, \ \ 0\le t\le T, \qquad (83)$$

$$\int_0^1\int_0^1\int_0^1 w(x,y,z)u(x,y,z,t)dxdydz=h(t), \ \ 0\le t\le T, \qquad (84)$$

where $\delta_1,\delta_2\ge 0$ are known fixed numbers, $D_T=\overline{Q}_{xyz}\times\{0\le t\le T\}$, $Q_{xyz}=\{(x,y,z):0<x<1,0<y<1,0<z<1\}$, $f(x,y,z,t)$, $\varphi(x,y,z)$, $\psi(x,y,z)$, $w(x,y,z)$, $h(t)$ are known functions, $u(x,y,z,t)$ and $a(t)$ are desired functions, and $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

The classical solution of the inverse boundary value problem (79)-(84) is understood as a pair of functions $u(x,y,z,t)\in\widetilde{C}^2(D_T)$ and $a(t)\in C[0,T]$ satisfying relations (79)-(84) in the usual sense, where

$$\widetilde{C}^2(D_T)=\{u(x,y,z,t):u(x,y,z,t)\in C^2(D_T),$$
$$u_{ttxx}(x,y,z,t),u_{ttyy}(x,y,z,t),u_{ttzz}(x,y,z,t)\in C(D_T)\}.$$

To study the problem (79)-(84), we consider the following auxiliary inverse boundary value problem: it is required to find a pair of functions $u(x,y,z,t)\in\widetilde{C}^2(D_T)$ and $a(t)\in C[0,T]$, from (79)-(83) and

$$h''(t)-\int_0^1\int_0^1\int_0^1 v(x,y,z)\Delta u_{tt}(x,y,z,t)dxdydz+$$

$$+\int_0^1\int_0^1\int_0^1 v(x,y,z)\Delta u(x,y,z,t)dxdydz=$$

$$=h(t)a(t)+\int_0^1\int_0^1\int_0^1 v(x,y,z)f(x,y,z,t)dxdydz,\ 0\le t\le T. \qquad (85)$$

The following theorem is valid.

**Theorem 3.3.** Assume that $\varphi(x,y,z),\psi(x,y,z)\in C(\overline{Q}_{xyz})$, $h(t)\in C^2[0,T]$, $h(t)\ne 0$ $(0\le t\le T)$, $f(x,y,z,t)\in C(D_T)$, and the following compatibility conditions are fulfilled

$$\begin{aligned}&\int_0^1\int_0^1\int_0^1 \varphi(x,y,z)dxdydz=h(0)+\delta_1 h(T),\\ &\int_0^1\int_0^1\int_0^1 \psi(x,y,z)dxdydz=h'(0)+\delta_2 h'(T).\end{aligned} \qquad (86)$$

Then the following statements are true:

i) each classical solution $\{u(x,y,z,t),a(t)\}$ of problem (79)-(84) is a solution of problem (79)-(83), (85), as well;

ii) each solution $\{u(x,y,z,t),a(t)\}$ of problem (79)-(83), (85) under the circumstance

$$\frac{(1+2\delta_1+3\delta_2+\delta_1\delta_2)T^2}{2(1+\delta_1)(1+\delta_2)}\|a(t)\|_{C[0,T]}<1 \qquad (87)$$

is a classical solution of problem (79)-(84).

Suppose that the data for problem (79)-(83), (85) satisfy the following conditions:

3.7. $\delta_1,\delta_2\ge 0,\ 1+\delta_1\delta_2\ge\delta_1+\delta_2$;

3.8. $\varphi(x,y,z)\in C^2[\overline{Q}_{xyz}],\ \varphi_{xxx}(x,y,z),\varphi_{xxy}(x,y,z),\varphi_{xxz}(x,y,z),\ \varphi_{xyy}(x,y,z),$
$\varphi_{yyy}(x,y,z),\varphi_{yyz}(x,y,z),\varphi_{xzz}(x,y,z),\varphi_{yzz}(x,y,z),\varphi_{zzz}(x,y,z)\in L_2(0,1),$
$\varphi_x(0,y,z)=\varphi(1,y,z)=\varphi_{xx}(1,y,z)=0,\ 0\le y,z\le 1,$
$\varphi(x,0,z)=\varphi_y(x,1,z)=\varphi_{yy}(x,0,z)=0,\ 0\le x,z\le 1,$
$\varphi(x,y,0)=\varphi(x,y,1)=\varphi_{zz}(x,y,0)=\varphi_{zz}(x,y,1)=0,\ 0\le x,y\le 1;$

3.9. $\psi(x,y,z)\in C^2[\overline{Q}_{xyz}],\ \psi_{xxx}(x,y,z),\psi_{xxy}(x,y,z),\psi_{xxz}(x,y,z),\psi_{xyy}(x,y,z),$

$$\psi_{yyy}(x,y,z), \psi_{yyz}(x,y,z), \psi_{xzz}(x,y,z), \psi_{yzz}(x,y,z), \psi_{zzz}(x,y,z) \in L_2(Q_{xyz}),$$
$$\psi_x(0,y,z) = \psi(1,y,z) = \psi_{xx}(1,y,z) = 0, \ \ 0 \le y,z \le 1,$$
$$\psi(x,0,z) = \psi_y(x,1,z) = \psi_{yy}(x,0,z) = 0, \ \ 0 \le x,z \le 1,$$
$$\psi(x,y,0) = \psi(x,y,1) = \psi_{zz}(x,y,0) = \psi_{zz}(x,y,1) = 0, \ \ 0 \le x,y \le 1;$$

3.10. $f(x,y,z,t) \in C(D_T), f_x(x,y,z,t), f_y(x,y,z,t), f_z(x,y,z,t) \in L_2(D_T),$
$$f(1,y,z,t) = 0, \ \ 0 \le y,z \le 1, \ \ 0 \le t \le T,$$
$$f(x,0,z,t) = 0, \ \ 0 \le x,z \le 1, \ \ 0 \le t \le T,$$
$$f(x,y,0,t) = f(x,y,1,t) = 0, \ \ 0 \le x,y \le 1, \ \ 0 \le t \le T;$$

3.11. $w(x,y,z) \in L_2(Q_{xyz}), \ \ h(t) \in C^2[0,T], \ \ h(t) \ne 0, \ \ 0 \le t \le T.$

Let $B_{2,T}^3$ denotes a set of all functions of the form
$$u(x,y,z,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} u_{k,n,m}(t)\cos\lambda_k x \sin\gamma_n y \sin\eta_m z,$$
considered in $D_T$ and let the functions $u_{k,n,m}(t) \ \ (k,n,m = 1,2,\ldots)$ are continuous on $[0,T]$, and
$$J_T(u) \equiv \left\{ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \left( \mu_{k,n,m}^3 \left\| u_{k,n,m}(t) \right\|_{C[0,T]} \right)^2 \right\}^{\frac{1}{2}} < +\infty .$$

The norm on the set $J(u)$ is established as follows:
$$\left\| u(x,y,z,t) \right\|_{B_{2,T}^3} = J_T(u).$$

Let $E_T^3$ be the space consisting of the topological product $B_{2,T}^3 \times C[0,T]$, which is the norm of the element $z = \{u, a\}$ defined by the formula
$$\|z\|_{E_T^3} = \left\| u(x,y,z,t) \right\|_{B_{2,T}^3} + \|a(t)\|_{C[0,T]}.$$

It is known that the spaces $B_{2,T}^3$ and $E_T^3$ are Banach spaces.

**Theorem 3.4.** Suppose that the assumptions 3.7 - 3.11 and the condition
$$(A(T)+2)^2 B(T) < 1,$$

are satisfied. Then problem (79)-(81), (85) has a unique solution in the ball $K = K_R$ of the space $E_T^3$.

Note that the expressions $A(T)$ and $B(T)$ are given in section 3.4 of the dissertation.

**Theorem 3.5.** Assume that all assumptions of Theorem 3.4, the inequality

$$\frac{(1+2\delta_1+3\delta_2+\delta_1\delta_2)T^2(A(T)+2)}{2(1+\delta_1)(1+\delta_2)}<1,$$

and the compatibility condition (86) holds. Then problem (79)-(84) has a unique classical solution in the ball $K = K_R(\|z\|_{E_T^3} \le A(T)+2)$ of the space $E_T^3$.

In the fourth chapter, we study the solvability of one-dimensional and two-dimensional inverse boundary value problems for the equation of motion of a homogeneous beam.

In the first section of chapter 4, the inverse boundary value problem for linearized equation of motion of a homogeneous elastic beam is studied; i.e. it is required to find a pair of functions $u(x,t)$ and $p(t)$, satisfying the following problem:

$$u_{tt}(x,t)+u_{xxxx}(x,t)+\beta u_{xx}(x,t)=p(t)u(x,t)+f(x,t) \quad (x,t)\in D_T, \tag{88}$$

$$u(x,0)=\varphi(x), \quad u_t(x,0)=\psi(x), \quad 0\le x\le 1, \tag{89}$$

$$u(0,t)=u(1,t)=u_{xx}(0,t)=u_{xx}(1,t)=0, \quad 0\le t\le T, \tag{90}$$

$$\int_0^1 g(x)u(x,t)dx = h(t), \quad 0\le t\le T, \tag{91}$$

where $\beta>0$ is positive integer, $f(x,t), \varphi(x), \psi(x), g(x),$ and $h(t)$ are known functions.

The pair $\{u(x,t), p(t)\}$ is said to be a classical solution of the problem (88)-(91), if the functions $u(x,t)\in \tilde{C}^{2,4}(D_T)$ and $p(t)\in C[0,T]$ satisfies Eq. (88) in $D_T$, condition (89) on $[0,1]$, and the statements (90)-(91) on the interval $[0,T]$.

**Theorem 4.1.** Suppose that $f(x,t)\in C(D_T)$, $\varphi(x), \psi(x)\in C[0,1]$, $g(x)\in L_2(0,1)$, $h(t)\in C^2[0,T]$ and the compatibility conditions

$$\int_0^1 g(x)\varphi(x)dx = h(0), \ \int_0^1 g(x)\psi(x)dx = h'(0),$$

hold. Then the problem of finding a classical solution of (88)-(91) is equivalent to the problem of determining the functions $u(x,t)\in\tilde{C}^{2,4}(D_T)$ and $p(t)\in C[0,T]$ from the (88)-(90), and satisfying the condition

$$h''(t) + \int_0^1 g(x)u_{xxxx}(x,t)dx + \beta\int_0^1 g(x)u_{xx}(x,t)dx =$$

$$= p(t)h(t) + \int_0^1 g(x)f(x,t)dx, \ \ 0\le t\le T. \qquad (92)$$

Assume that the data of problem (88)-(90), (92) satisfy the conditions:

4.1. $\varphi(x)\in C^4[0,1], \ \varphi^{(5)}(x)\in L_2(0,1),$

$\varphi(0)=\varphi(1)=\varphi''(0)=\varphi''(1)=\varphi^{(4)}(0)=\varphi^{(4)}(1)=0;$

4.2. $\varphi(x)\in C^4[0,1], \psi'''(x)\in L_2(0,1), \ \psi(0)=\psi(1)=\psi''(0)=\psi''(1)=0;$

4.3. $f(x,t), \ f_x(x,t), \ f_{xx}(x,t)\in C(D_T), \ f_{xxx}(x,t)\in L_2(D_T),$ and

$f(0,t)=f(1,t)=f_{xx}(0,t)=f_{xx}(1,t)=0, \ \ 0\le t\le T;$

4.4. $0<\beta<\dfrac{\pi^2}{2}$, $g(x)\in L_2(0,1)$, $0\ne h(t)\in C^2[0,T]$, $0\le t\le T$.

**Theorem 4.2.** If conditions 4.1-4.4, the inequality

$$B(T)(A(T)+2)^2<1,$$

and the compatibility conditions

$$\int_0^1 g(x)\varphi(x)dx = h(0), \ \int_0^1 g(x)\psi(x)dx = h'(0),$$

holds, then problem (88)-(91) has a unique classical solution in the ball $K=K_R\subset E_T^3$, $R=A(T)+2$ of the space $E_T^3$.

Note that the expressions of $A(T)$, $B(T)$, and the structure of the space $E_T^3$ are introduced in section 4.1 of the dissertation.

In Section 4.2, we study the one-dimensional inverse boundary value problem for equation

$$u_{tt}(x,t)+u_{xxxx}(x,t)+\beta u_{xx}(x,t)=$$

$$=a(t)u(x,t)+b(t)u_t(x,t)+c(t)g(x,t)+f(x,t) \quad (x,t)\in D_T, \tag{93}$$
with the conditions
$$u(x,0)=\varphi(x), \;\; u_t(x,0)=\psi(x) \;\; (0\le x\le 1), \tag{94}$$
$$u(0,t)=u(1,t), u_x(0,t)=u_x(1,t), u_{xx}(0,t)=u_{xx}(1,t) \;\; (0\le t\le T), \tag{95}$$
$$\int_0^1 u(x,t)dx=0 \;\; (0\le t\le T), \tag{96}$$
$$U_i(u)\equiv u(x_i,t)+\int_0^1 \omega_i(x)u(x,t)dx=h_i(t) \;\; (i=1,2,3;\; 0\le t\le T), \tag{97}$$
where $x_i\in(0,1)$ $(i=1,2,3;\; x_1\ne x_2\ne x_3)$ and $\beta>0$ are given numbers, $D_T=\{(x,t):0\le x\le 1,\; 0\le t\le T\}$ is certain rectangular domain, $f(x,t)$, $g(x,t)$, $\varphi(x)$, $\psi(x)$, $\omega_i(x)$, $h_i(t)$ $(i=1,2,3)$ are known functions, and $u(x,t)$, $a(t)$, $b(t)$, $c(t)$ are desired functions. In the current section an inverse boundary value problem for an equation of motion of a homogeneous beam is considered in which both boundary conditions are integral. Conditions for the existence and uniqueness of the classical solution of this problem are established by reducing it to an equivalent boundary value problem for the considered equation.

In the third section of chapter 4, we study the solvability of nonlinear inverse problem for determining the solution $u(x,t)$ and the coefficient $p(t)$ of the equation of motion of a homogeneous beam:
$$u_{tt}(x,t)+u_{xxxx}(x,t)+\beta u_{xx}(x,t)+\alpha u(x,t)+u^3(x,t)=$$
$$=p(t)g(x,t)+f(x,t) \quad (x,t)\in D_T, \tag{98}$$
in the domain $D_T=\{(x,t):0\le x\le 1,\; 0\le t\le T\}$ with the conditions
$$u(x,0)+\delta u(x,T)=\varphi(x),\; u_t(x,0)+\delta u_t(x,T)=\psi(x), \;\; 0\le x\le 1, \tag{99}$$
$$u_x(0,t)=u_x(1,t)=u_{xxx}(0,t)=u_{xxx}(1,t)=0, \;\; 0\le t\le T, \tag{100}$$
$$u(0,t)=h(t), \;\; 0\le t\le T, \tag{101}$$
where $\delta\ge 0$, $\alpha>0$ and $\beta>0$ are given numbers satisfying the condition $\beta<4\alpha$ and $f(x,t)$, $g(x,t)$, $\varphi(x)$, $\psi(x)$, $h(t)$ are given functions.

Section 4.4 presents the main results concerning the unique solvability of the two-dimensional equation:

$$u_{tt}(x,y,t)+\Delta^2 u(x,y,t)+\beta\Delta u(x,y,t)=$$
$$=a(t)u(x,y,t)+b(t)g(x,y,t)+f(x,y,t)\ \ (x,y,t)\in D_T\,, \qquad (102)$$

with the conditions

$$u(x,y,0)=\int_0^T p_1(t)u(x,y,t)dt+\varphi(x,y),$$
$$u_t(x,y,0)=\int_0^T p_2(t)u(x,y,t)dt+\psi(x,y)\ \ (0\le x,y\le 1), \qquad (103)$$

$$u_x(0,y,t)=u(1,y,t)=$$
$$=u_{xxx}(0,y,t)=u_{xx}(1,y,t)=0\ \ (0\le y\le 1, 0\le t\le T), \qquad (104)$$
$$u(x,0,t)=u_y(x,1,t)=$$
$$=u_{yy}(x,0,t)=u_{yyy}(x,1,t)=0\ \ (0\le x\le 1,\ 0\le t\le T), \qquad (105)$$
$$u(0,1,t)=h_1(t)\ \ (0\le t\le T), \qquad (106)$$
$$\int_0^1\int_0^1 u(x,y,t)dxdy=h_2(t)\ \ (0\le t\le T), \qquad (107)$$

where $Q_{xy}=\{(x,y):0<x<1,\ 0<y<1\}$ is rectangular domain, $\beta>0$ is known fixed numbers and $\Delta=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$.

To study the inverse boundary value problem (102)-(107), we first consider the following auxiliary inverse boundary value problem: it is required to find the functions $u(x,y,t)\in C^{2,4,4}(D_T)$ and $a(t),\ b(t)\in C[0,T]$ from (102)-(104) and

$$h_1''(t)+u_{xxxx}(0,1,t)+2u_{xxyy}(0,1,t)+u_{yyyy}(0,1,t)+\beta\Big(u_{xx}(0,1,t)+u_{yy}(0,1,t)\Big)=$$
$$=a(t)h_1(t)+b(t)g(0,1,t)+f(0,1,t)\ \ (0\le t\le T), \qquad (108)$$
$$h_2''(t)+\int_0^1 u_{xxx}(1,y,t)dy-\int_0^1 u_{yyy}(x,0,t)dx-$$

$$-2u_{xy}(1,0,t)+\beta\left(\int_0^1 u_x(1,y,t)dy-\int_0^1 u_y(x,0,t)dx\right)=$$

$$=a(t)h_2(t)+b(t)\int_0^1\int_0^1 g(x,y,t)dxdy+\int_0^1\int_0^1 f(x,y,t)dxdy \ \ (0\le t\le T)\ .\qquad (109)$$

**Theorem 4.3.** Let $\varphi(x,y),\psi(x,y)\in C(\overline{Q}_{xy})$, $f(x,y,t)$, $g(x,y,t)\in C(\overline{D}_T)$, $h_1(t),h_2(t)\in C^2[0,T]$,

$$h(t)\equiv h_1(t)\int_0^1\int_0^1 g(x,y,t)dxdy-h_2(t)g(0,1,t)\neq 0 \ \ (0\le t\le T)$$

and let the following consistency conditions be fulfilled:

$$\varphi(0,1)=h_1(0)-\int_0^T p_1(t)h_1(t)dt,\ \psi(0,1)=h_1'(0)-\int_0^T p_2(t)h_1(t)dt,\qquad (110)$$

$$\begin{aligned}&\int_0^1\int_0^1 \varphi(x,y)dxdy=h_2(0)-\int_0^T p_1(t)h_2(t)dt,\\&\int_0^1\int_0^1 \psi(x,y)dxdy=h_2'(0)-\int_0^T p_2(t)h_2(t)dt.\end{aligned}\qquad (111)$$

Then the following assertions are true:

i) each classical solution $\{u(x,y,t),a(t),b(t)\}$ to the problem (102)-(107) is a solution of problem (102)-(105), (108), (109), as well as;

ii) each solution $\{u(x,y,t),a(t),b(t)\}$ to the problem (102)-(105), (108), (109) satisfying the condition

$$\left(T\|p_2(t)\|_{C[0,T]}+\|p_1(t)\|_{C[0,T]}+\frac{T}{2}\|a(t)\|_{C[0,T]}\right)T<1,\qquad (112)$$

is a classical solution of problem (102)-(107).

Let $B_{2,T}^5$ denotes a set of all functions of the form

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}u_{k,n}(t)\cos\lambda_k x\sin\gamma_n y,$$

considered in $D_T$. Moreover, the functions $u_{k,n}(t)$ $(k,n=1,2,...)$ are continuously on $[0,T]$ and

$$\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\mu_{k,n}^{5}\left\|u_{k,n}(t)\right\|_{C[0,T]}\right)^{2}\right\}^{\frac{1}{2}}<+\infty ,$$

where $\mu_{k,n}^{5}=(\lambda_k^4+\gamma_n^4)\sqrt{\lambda_k^2+\gamma_n^2}$ .

The norm on this set is established as follows:

$$\left\|u(x,y,t)\right\|_{B_{2,T}^{5}}=\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\mu_{k,n}^{5}\left\|u_{k,n}(t)\right\|_{C[0,T]}\right)^{2}\right\}^{\frac{1}{2}}.$$

Let $E_T^5$ denote the space consisting of the topological product $B_{2,T}^5\times C[0,T]\times C[0,T]$, which is the norm of the element $z=\{u,a,b\}$ defined by the formula

$$\left\|z\right\|_{E_T^5}=\left\|u(x,y,t)\right\|_{B_{2,T}^5}+\left\|a(t)\right\|_{C[0,T]}+\left\|b(t)\right\|_{C[0,T]}.$$

It is known that the spaces $B_{2,T}^5$ and $E_T^5$ are Banach spaces.

Assume that the data of problem (102)-(105), (108), and (109) satisfy the following conditions

4.5. $\varphi(x,y),\varphi_x(x,y),\varphi_{xx}(x,y),\varphi_y(x,y),\varphi_{xy}(x,y),\varphi_{yy}(x,y),\varphi_{xxx}(x,y),$
$\varphi_{xxy}(x,y),\varphi_{xyy}(x,y),\varphi_{yyy}(x,y),\varphi_{xxxx}(x,y),$
$\varphi_{xxxy}(x,y),\varphi_{xxyy}(x,y),\varphi_{xyyy}(x,y),\varphi_{yyyy}(x,y)\in C(\overline{Q}_{xy})$ ,
$\varphi_{xxxxy}(x,y),\varphi_{xyyyy}(x,y),\varphi_{xxxxx}(x,y),\varphi_{yyyyy}(x,y)\in L_2(Q_{xy}),$
$\varphi_x(0,y)=\varphi(1,y)=\varphi_{xx}(1,y)=\varphi_{xxx}(0,y)=\varphi_{xxxx}(1,y)=0\ \ (0\le y\le 1),$
$\varphi(x,0)=\varphi_y(x,1)=\varphi_{yy}(x,0)=\varphi_{yyy}(x,1)=\varphi_{yyyy}(x,0)=0\ \ (0\le x\le 1).$

4.6. $\psi(x,y),\psi_x(x,y),\psi_y(x,y),\psi_{xx}(x,y),\psi_{xy}(x,y),\psi_{yy}(x,y)\in C(\overline{Q}_{xy}),$
$\psi_{xxx}(x,y),\psi_{xyy}(x,y),\psi_{xxy}(x,y),\psi_{yyy}(x,y)\in L_2(Q_{xy}),$
$\psi_x(0,y)=\psi(1,y)=\psi_{xx}(1,y)=0\ \ (0\le y\le 1),$
$\psi(x,0)=\psi_y(x,1)=\psi_{yy}(x,0)=0\ \ (0\le x\le 1).$

4.7. $f(x,y,t),f_x(x,y,t),f_y(x,y,t),f_{xx}(x,y,t),f_{xy}(x,y,t),f_{yy}(x,y,t)\in C(D_T),$
$f_{xxx}(x,y,t),f_{xxy}(x,y,t),f_{xyy}(x,y,t),f_{yyy}(x,y,t)\in L_2(D_T),$
$f_x(0,y,t)=f(1,y,t)=f_{xx}(1,y,t)=0\ \ (0\le y\le 1,\ \ 0\le t\le T),$

$$f(x,0,t) = f_y(x,1,t) = f_{yy}(x,0,t) = 0 \ \ (0 \le x \le 1, \ \ 0 \le t \le T).$$

4.8. $g(x,y,t), g_x(x,y,t), g_y(x,y,t), g_{xx}(x,y,t), g_{xy}(x,y,t), g_{yy}(x,y,t) \in C(D_T),$

$g_{xxx}(x,y,t), g_{xxy}(x,y,t), g_{xyy}(x,y,t), g_{yyy}(x,y,t) \in L_2(D_T),$

$g_x(0,y,t) = g(1,y,t) = g_{xx}(1,y,t) = 0 \ \ (0 \le y \le 1, \ \ 0 \le t \le T),$

$g(x,0,t) = g_y(x,1,t) = g_{yy}(x,0,t) = 0 \ \ (0 \le x \le 1, \ \ 0 \le t \le T).$

4.9. $0 < \beta < \dfrac{\pi^2}{4}$, $h_i(t) \in C^2[0,T]$ $(i = 1,2)$, and

$$h(t) \equiv h_1(t) \int_0^1 \int_0^1 g(x,y,t)dxdy - h_2(t) g(0,1,t) \ne 0 \ \ (0 \le t \le T).$$

**Theorem 4.4.** Suppose that the assumptions 4.5-4.9 and the condition

$$(A(T)+2)(B(T)(A(T)+2)+C(T)+D(T)) < 1,$$

are satisfied. Then problem (102) - (105), (108), (109) has a unique solution in the ball $K = K_R$ of the space $E_T^3$.

Note that the expressions of $A(T)$, $B(T)$, $C(T)$, and $D(T)$ are given in section 4.4 of the dissertation.

**Theorem 4.5.** Assume that all assumptions of Theorem 4.4, the inequality

$$\left( T\|p_2(t)\|_{C[0,T]} + \|p_1(t)\|_{C[0,T]} + \frac{T}{2}(A(T)+2) \right) T < 1,$$

and the compatibility conditions (110), (111) holds. Then problem (102)-(107) has a unique classical solution in the ball $K = K_R(\|z\|_{E_T^5} \le R = A(T)+2)$ of the space $E_T^3$.

## CONCLUSIONS

The following results are obtained in the dissertation work:

1. The existence and uniqueness of the classical solution of self-adjoint and non-self-adjoint nonlinear inverse boundary value problems with non-local boundary (for both time and spatial variables) and with various overdetermination conditions for a one-dimensional parabolic equations are shown.

2. The classical solvability of a nonlinear inverse boundary value problem with the nonlocal boundary conditions (in time and spatial variables) for a two-dimensional parabolic equation is studied and some numerical approaches are presented.
3. The existence and uniqueness of the classical solution of self-adjoint and non-self-adjoint nonlinear inverse boundary value problems with non-local boundary (with respect to time and spatial variables) and with various overdetermination conditions for a one-dimensional hyperbolic equations are studied.
4. The unique classical solvability of a nonlinear inverse boundary value problem with the nonlocal boundary conditions (for both time and spatial variables) for a two-dimensional hyperbolic equation is studied and some numerical approaches are presented.
5. Results are obtained on the existence and uniqueness of the classical solution of some self-adjoint nonlinear inverse boundary value problems with the nonlocal boundary (with respect to time and space variables) and various additional conditions for the one-dimensional equation of longitudinal wave propagation.
6. Theorems on the existence and uniqueness of an inverse boundary value problem with a nonlocal boundary condition for the three-dimensional equation for the longitudinal wave propagation are proved.
7. The simultaneous recovery of unknown coefficients and the determination of the right-hand side in nonlinear inverse boundary value problems with nonlocal boundary (with respect to time and space variables) and with various additional conditions for the equation one-dimensional linearized equation of motion of a homogeneous beam are studied.
8. The classical solvability of a nonlinear inverse boundary value problem with the nonlocal boundary conditions for the two-dimensional equation of motion of a homogeneous beam is studied.

## The main results of the dissertation were published in the following works:


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