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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

MATHEMATICAL MODELING OF THE SEALING PROCESS WITH SEALING ELEMENT

Specialty: 2002.01 - Mechanics of deformable solid

Field of science: Mathematics

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GENERAL DESCRIPTION OF WORK

Relevance and degree of study of the topic. Sealing systems are widely used in various industries, including the oil and gas sector. Sealing elements are used in hermetic systems and the operation of the equipment entirely depends on the performance of the sealer. The maximum working pressure of the equipment is determined by the condition of the sealing devices. Loss of tightness leads to a disruption of the process which can result in open fountains, fires, and environmental accidents in oil and gas industry. Therefore, comprehensive analysis, research, improvement of sealing elements design under different operating conditions, and on this basis, development of methods to determine its parameters have great scientific and practical significance.

Experience has shown that the sealing ability of sealing elements used in oil and gas production deteriorates over time until the end of their service life. Thus, due to the viscous-elastic properties of the sealing element, the previous deformation and contact stress relaxation occur and change over time. Despite the deformation remains stable, the physical and mechanical properties of the sealing element material change, the contact pressure decreases, and in some cases, even the sealing element loses its sealing ability. On the other hand, both the character and value of the pre-created contact pressure between the sealing element and the wall of the sealed body change. Hence, the equipment cannot perform its function due to hermeticity loss. It should be noted that a lot of work has been done both locally and abroad to study the sealing ability of the sealing elements, considering the heredity of its material. However, the sealing element configurations used in the petroleum industry, operating conditions, and sealing process requires the comprehensive study of their sealing ability, taking into account the actual working conditions. The above actualizes the problem of improving the sealing ability to seal elements applied in the oil and gas industry and the development of methods for solving sealing problems taking into account the real working conditions in practical and scientific terms. The dissertation work is devoted to the study of the ability to sealing elements of different shapes used in the oil and gas industry by constructing a mathematical solution model, taking into account edge effects and the physical and mechanical properties of the sealer.

The sealing ability of the sealing element depends on the value and distribution of the contact pressure between its surface and the sealed part of the body, and the value of the contact pressure which depends on the geometric dimensions and physical and mechanical properties of the sealing element. In all materials used for the sealing element, the modulus of elasticity is not a constant quantity and changes over time. As the material of the sealing element is viscoelastic, during long-term operation, as mentioned above, the contact pressure between the sealer and the sealing surface is relaxed, and both its value, and the character of distribution change. The practice of using sealing elements shows that the rate of external load applied to the sealing element to achieve tightness has a significant effect on its sealing ability. Failure to take into account the viscous-elasticity nature of the sealing element's material can lead to incorrect results. Also, achieving tightness with minimal external force leads to an improvement in the performance of the sealing element. Determining the effective parameters of the sealing element that ensure tightness with minimal external force has great scientific importance. Thus, achieving tightness with minimal external force improves working conditions, as well as prolongs the service life of the sealer. Experience in the application of sealing elements shows that edge effects significantly affect its sealing ability. Therefore, taking into account edge effects and the heredity of the sealing element's material, it is important both scientifically and practically to study its sealing ability and, on this basis, to determine the effective measures that improve its performance.

The works of many foreign and local authors devoted to the study and improvement of the sealing ability to sealing elements have been studied. However, the study and improvement of the sealing ability of the sealing elements used in the oil and gas production process, taking into account the physical and mechanical properties of the sealer’s material, as well as heredity and edge effects, remains little studied and requires further research, and the thesis is devoted to solving these problems.

Object and subject of research. Mathematical modeling and study the sealing process of sealing elements under different conditions, taking into account the edge effects and heredity.

The purpose and objectives of the study. The main purpose of the thesis is to study the sealing ability of sealing elements in different shapes used in oil and gas industry by creating a mathematical solution model of the process taking into account the physical and mechanical properties of the sealer, as well as heredity and edge effects and development of methods for solving obtained equations.

Research method. Fundamental methods of mathematical physics, theories of elasticity, and heredity have been applied to the solution of the problems. Differential equations obtained on the principle of variation were solved by Galerkin and Ritz methods.

The main provisions for the defense.

1. By constructig a mathematical model of the sealing process developed a method for determining the stress-strain state, taking into account the edge effects of cylindrical sealing elements, physical and mechanical properties, as well as heredity of the sealer and it was determined that the value of the axial compressive load, which ensures tightness, first decreases as the height of the sealer increases, and then stabilizes after a cetain value of height.
2. An analytical expression was obtained that allows to determine the distribution of the contact pressure between the inner surface of the sealing element and the wall of the cylinder depending on its physical and mechanical parameters and geometric dimensions. It is shown that the maximum contact pressure occurs in the area near the lower seat of the sealing element, and as the height of the sealer rises, the contact pressure decreases and practically disappears after a certain height value. It has been found that hermeticity can be created by compressing the sealing element on both sides at the same time with a relatively small axial compressive load.

3. It has been found that the hereditary nature of the sealer's material reduces the effect of the pre-applied axial load, which ensures tightness, several times in a short period of time. This, in turn, reduces the contact pressure between the inner surface of sealing element and the wall of the cylinder, and thus its sealing ability.

4. The effect of the rate of application of the axial force on the sealing ability of the sealing element was studied. It is shown that as the rate of relaxation of the contact pressure between the inner surface of the sealer and the cylinder wall also decreases and stabilizes over time.

5. Constructing a mathematical model of the sealing process of the cross-conical and annular sealing elements a method of determining the distribution of the contact pressure between its inner surface of the sealing element and the cylinder wall was developed, taking into account the heredity of the sealer and it has shown that as the rate of application of the axial load decreases, the rate of relaxation of the contact pressure decreases and stabilizes over time.

6. The method of determining the contact pressure between the semi-cylindrical surface with hole and the sealing element, taking into account the heredity, was developed and an analytical expression was obtained to determine the distribution character of the contact pressure. It has been shown that the contact pressure decreases at a relatively small rate over time, and then at a slightly greater rate, and stabilizes over time.

Scientific novelty of the research. Scientific novelty of the results of the thesis are:

- A mathematical model of the process was developed to determine the quantities that characterize the sealing ability of sealing elements of different shapes used in oil and gas extraction in the field of small deformations, taking into account the edge effects, physical and mechanical properties, as well as heredity of sealing elements and the method of analytical solution of the obtained differential equations is given.
- Using the principle of variation, an analytical expression was obtained that determines the minimum value of the axial

compressive force providing tightness depending on the geometric dimensions and physical and mechanical properties of the sealing element.

- Taking into account the edge effects, physical and mechanical properties, as well as heredity of sealing elements the character of the distribution of the contact pressure created during the sealing process has been determined.
- The effect of hereditary properties of the sealing element's material on its sealing ability was determined.
- A model of the hermetic process was established and the effect of heredity on the distribution character of the contact pressure in different cases of the deformation rate was determined.

The theoretical and practical significance of the study. The scientific results obtained are important for the research and development of the sealing ability of seals used in the oil and gas industry and can be used to determine the minimum value of the force that ensures the tightness and the effective dimensions of the sealing element.

Approbation and application of the work. The main provisions and results of the dissertation were presented at the I International Scientific Conference of Young Researchers dedicated to the 90th anniversary of the National Leader of the Azerbaijani people Heydar Aliyev (Baku-2013) [1], International Non-Newtonian Systems in Oil and Gas At the Scientific Conference (Baku-2013) [2], at the XVIII Republican Scientific Conference of Doctoral Students and Young Researchers (Baku-2013) [5], International Youth Scientific Conference "Severgeoecotech-2014" (Ukhta-2014) [6], Y.A. At the Republican Scientific Conference "Classical and modern problems of mechanics" dedicated to the 100th anniversary of Amanzade (Baku-2014) [7], at the VI Youth Scientific-Practical Conference "Modeling of gas and oil and gas condensate fields" (Moscow-2014) [8], Academician At the International Scientific Conference "Modern Problems of Innovative Technologies and Applied Mathematics in Oil and Gas Production" dedicated to the 90th anniversary of AXMirzajanzadeh (Baku-2018) [13] has been discussed.

Personal contribution of the author. The dissertation author belongs to the acquisition of expressions in the main solutions, solution of problems with application software and results, except for the setting of some issues in the dissertation work.

Author's publications. 11 (eleven) scientific articles and 7 (seven) conference materials were published on the basis of the dissertation work.

The name of the institution where the work was done. The dissertation work was carried out within the scientific plans of the "Mechanical Engineering" department of the Engineering faculty of Baku Engineering University.

The total volume of the thesis with an indication of the volume of the structural units of the thesis separately. The dissertation consists of an introduction, three chapters, a conclusion and a list of references, 155 pages. The total volume of the dissertation is 222917 characters (title page - 365 characters, content - 3293 characters, introduction - 25068 characters, the first chapter - 85581 characters, the second chapter - 60592 characters, the third chapter - 45708 characters, the result - 2310 characters). The dissertation contains 10 pictures, 44 graphics, 144 iterature titles.

CONTENT OF THE DISSERTATION

The first chapter is devoted to the study of the sealing ability of cylindrical sealing elements by constructing a mathematical solution model. The first chapter consists of five paragraphs.

Section 1.1 defines the stress-stain state of a sealing element in the form of a hollow cylinder caused by the action one-sided axial compressive force by assuming a cylindrical sealing element elastic body in the area of small deformations, the dependence of the character of the distribution of the contact pressure between the inner surface of the seal and the wall of the sealed cylinder on its physical and mechanical parameters and geometric dimensions have been theoretically studied [11]. Analytical expressions were obtained by applying the principle of minimum potential energy for the value of the contact pressure and the axial force required to achieve tightness between the inner surface of the sealing element and the wall of the sealed cylinder. The sealing process of sealing elements tightly put on the stock with a gap δ between its inner surface and the cylinder wall is considered. The tightness of the surface of the sealing element and the cylinder wall is achieved by one-sided axial compression. The solution of the problem is performed in two stages in the elastic statement. The first step was compression of the sealing element to the first contact of its outside surface with the cylinder wall; the second state was to achieve tightness.

Since the material of the sealing element is homogeneous, its deformation is assumed to be axisymmetric. Let us consider the hypothesis of flat sections, and assume, that the axial deformation of the sealing element depends only on the coordinate in the axial direction

$$w_1 = f_1(z), \qquad (1)$$

where $f_1(z)$ is an unknown function, depending on z and to be determined.

Taking the material of the sealing element to be incompressible, for the radial deformation of any point of the seal, we obtain the following equation

$$u_1(r,z) = \frac{1}{2}\left(\frac{R_2^2}{r} - r\right) f_1'(z) . \quad (2)$$

For the potential energy of the sealing element after its deformation, considering the axisymmetry and linearity, we have the equality

$$\Pi = 4\pi G \int_0^H \int_{R_1}^{R_2} \left(\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_z^2 + \frac{1}{2}\gamma_{rz}^2\right) r dr dz - \int_0^H Q \cdot f_1'(z) dz , \quad (3)$$

where G is the shear modulus of the sealing element, H is height, R_1, R_2 are inner and outer radii of the seal, ε_r, ε_θ, ε_z and γ_{rz} are respectively, radial, tangential, axial and shear deformations, Q is the axial compressive force.

The boundary conditions can be written as follows

$$\mu Q\big|_{z=H} = 2\pi G \int_{R_1}^{R_2} \gamma_{zr} r\, dr , \quad w_1\big|_{z=0} = 0, \quad u_1(r,z)\big|_{\substack{z=0 \\ r=R_1}} = -\delta , \quad (4)$$

where μ is the friction coefficient between the washer and the end of the seal.

Based on the Euler equation from the functional (3) we have

$$\varphi''(z) - k^2 \varphi(z) + A = 0 , \quad (5)$$

where $\varphi(z) = f_1'(z)$, k, A are constants.

Integrating the differential equation (5) within the boundary conditions(4), we obtain

$$f_1(z) = \frac{c_2}{k}\cosh kz + \frac{c_3}{k}\sinh kz + \frac{A}{k^2} z + c_4 , \quad (6)$$

where c_2, c_3 and c_4 are integration constants.

The following analytical expressions were obtained to determine the dependence of the value of the axial loads for compression of the sealing element to the first and complete contact of its inner surface with the cylinder wall Q and P on the physical and mechanical parameters and geometric dimensions of the seal

$$q=\sqrt[3]{-\frac{1}{2}\left(\frac{2a^3}{27}-\frac{ab}{3}+c\right)+\sqrt{\frac{1}{4}\left(\frac{2a^3}{27}-\frac{ab}{3}+c\right)^2+\left(-\frac{a^2}{9}+\frac{b}{3}\right)^3}}+$$

$$+\sqrt[3]{-\frac{1}{2}\left(\frac{2a^3}{27}-\frac{ab}{3}+c\right)-\sqrt{\frac{1}{4}\left(\frac{2a^3}{27}-\frac{ab}{3}+c\right)^2+\left(-\frac{a^2}{9}+\frac{b}{3}\right)^3}}-\frac{a}{3}, \quad (7)$$

$$p=2\frac{R(h)\delta(h)}{R_2^2-R^2(h)}\left(\frac{R_2^2}{R_0^2}+3\right), \quad (8)$$

where $q=\frac{Q}{\pi G\left(R_2^2-R_1^2\right)}$, $p=\frac{P}{\pi G\left(R_2^2-R_0^2\right)}$, a, b, c are constants.

The contact pressure between the inner surface of the sealing element and the cylinder wall after their full contact is found by analogy to the beam on an elastic foundation as follows

$$\sigma_r=\frac{\nu\sigma_0}{1-\nu}\exp\left(\frac{2\mu\nu\,(h-z)}{(1-\nu)(R_2-R_0)}\right)+k_0\cdot u_0(z). \quad (9)$$

The value σ_0 is determined from the tightness condition

$$\frac{\nu\,\sigma_0}{1-\nu}\exp\left(\frac{2\,\mu\,\nu}{(1-\nu)\,(R_2-R_0)}h\right)+k_0\cdot u_0(0)\ge P_{medium},$$

where P_{medium} is medium's pressure.

The formulas (7), (8) and (9) are numerically calculated for the values of the contact pressure and the axial compressive forces required deforming the sealing element until the first and complete contact of its inner surface with the cylinder wall for the values of parameters, which are showed in follows

$$R_0=0.073\,m,\ R_1=0.076\,m,\ R_2=0.1\,m,\ \delta=0.003\,m,$$

$$\nu=0.25,\ P_{medium}=2\cdot10^7\,Pa,\ G=1.3\cdot10^8\,Pa,$$

$$k_0=6.7\cdot10^9\,Pa/m,\ \mu=0.5.$$

The distribution of contact pressure between the outer surface of the sealing element and the cylinder wall depending on the coordinate z was depicted in Figure 1.

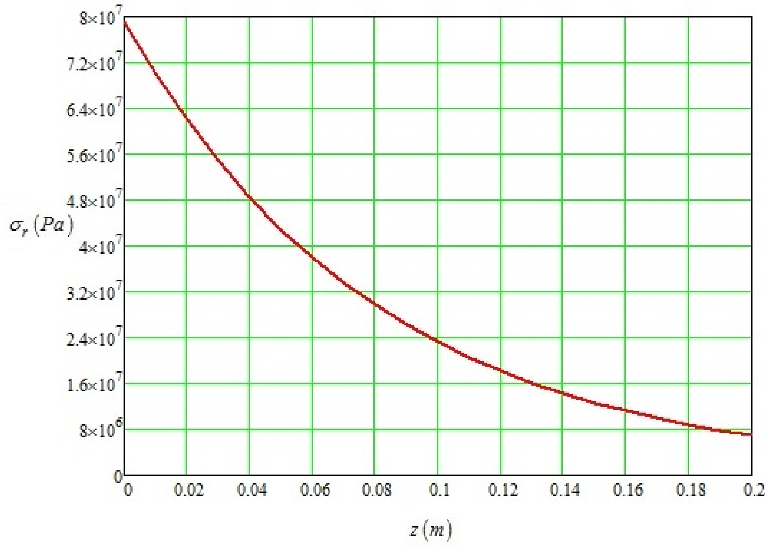


Figure 1. The graph of dependence of the character of contact pressure distribution depending on coordinate z ($H = 0.2\,m$)

As is seen from Figure 1 the greatest value of the contact pressure is achieved in the lower section of the sealing element. With increasing the value of z the contact pressure decreases and then after certain value of the height of the sealing element it disappears.

Section 1.2 considers the stress-strain state of a sealing element in the form of a hollow cylinder tightly put on the stock with a gap δ between its inner surface and the cylinder wall with regard to the viscous-elastic properties of its material.

The influence of the viscous-elastic properties of the sealing element's material on its sealing ability has been studied on the basis of theoretical studies [12]. The dependence between the stress tensor components and the strain tensor components for an arbitrary loading situation, which best describes the viscosity-elasticity property of the sealing element's material, is as follows

$$\dot{\sigma}+\lambda^{*}\sigma=E_M\left(\dot{\varepsilon}+\nu^{*}\varepsilon\right), \tag{10}$$

where $E_1=E_M$, $\lambda^{*}=\dfrac{E_1+E_2}{\eta}$, $\nu^{*}=\dfrac{E_2}{\eta}$, $n=\dfrac{1}{\nu^{*}}$ is the relaxation time, η is dynamical viscosity of the material of the sealing element, E_M is the instantaneous modulus of elasticity, E_2 is the elasticity modulus, σ is stress, ε is relative strain, $\dot{\sigma}$ and $\dot{\varepsilon}$ are the time derivative of the stress and strain components.

From the differential equation (10) $\sigma(t)$ stress and $\varepsilon(t)$ relative deformation can be found as follows:

$$\sigma(t)=E_M\left[\varepsilon(t)-\left(\lambda^{*}-\nu^{*}\right)\int_0^t \mathrm{e}^{-\lambda^{*}(t-\xi)}\,\varepsilon(\xi)d\xi\right], \tag{11}$$

$$\varepsilon(t)=\frac{1}{E_M}\left[\sigma(t)+\left(\lambda^{*}-\nu^{*}\right)\int_0^t \mathrm{e}^{-\nu^{*}(t-\xi)}\,\sigma(\xi)d\xi\right]. \tag{12}$$

The tightness of the inner surface of the sealing element and the cylinder wall is achieved by one-sided axial compression. Assume that the material of the sealing element is viscous-elastic and look at the instantaneous loading of the seal. The problem may be realized based on the hypothesis of elastic analogy by taking the process of deformation of the sealing element quasi-static. The dependence between the stress-strain components for an arbitrary case of loading in a model that describes best the viscous-elastic behavior of the material of a sealing element is of the form

$$\tau_{ij}=\left[2\varepsilon(\bar{x})+\delta_{ij}\,s(\bar{x})\right]G\left\{\mathrm{e}^{-\lambda^{*}t}+\int_0^t\left[(\varepsilon(\xi))_{,t}+\nu^{*}\varepsilon(\xi)\right]\mathrm{e}^{-\lambda^{*}(t-\xi)}\,d\xi\right\}, \tag{13}$$

where $\varepsilon(\bar{x})$, $\varepsilon(t)$ are relative strain components, respectively, depending on coordinate $\bar{x}$ and time t, G is the shear modulus of the sealing element, δ_{ij} is Kronecker’s symbol.

Introduce the following denotation

$$\overline{G} = G\left\{ \mathrm{e}^{-\lambda^* t} + \int_0^t \left[(\varepsilon(\xi))_{,t} + \nu^* \varepsilon(\xi) \right] \mathrm{e}^{-\lambda^*(t-\xi)} d\xi \right\}. \qquad (14)$$

We can represent expression (13) in the form looking like the Hooke law

$$\tau_{ij} = \overline{G} \left(2\varepsilon_{ij}\left(\overline{x}, t\right) + \delta_{\mathrm{ij}}\, s\left(\overline{x}, t\right) \right). \qquad (15)$$

Therefore, in the given case, after finding the solution of the problem in the elastic statement, using expressions (14) and (15) we can determine stress regarding hereditary properties of the sealing element.

Based on the elastic analogy, we accept the deformation of cross sections of the sealer in the form

$$\varepsilon_1(z, t) = \varepsilon_1\left(\overline{z}\right) \cdot \varepsilon_1(t), \qquad (16)$$
$$\varepsilon_1(t) = w_1(t) = 1.$$

Substituting expression (16) into formula (14) and then integrating it, we obtain the equality

$$\overline{G} = G\left[\left(1 - \frac{\nu^*}{\lambda^*} \right) \mathrm{e}^{-\lambda^* t} + \frac{\nu^*}{\lambda^*} \right]. \qquad (17)$$

Using expressions (7), (8), and (17) we can determine the axial compressive forces required deforming the sealing element until the first and complete contact of its inner surface with the cylinder wall Q and P with regard to hereditary properties of the sealing element.

Section 1.3 is devoted to the sealing problem of a sealing element in the form of a hollow cylinder tightly put on the stock with a gap δ between its inner surface and the cylinder wall with regard to viscous-elastic properties of its material in the case when the sealing element deforms uniformly [18].

Consider that a cylindrical sealing element is a viscous-elastic body and assume that the change external force and deformation at the boundary of the seal occurs at a slow rate. Then we can accept the method of quasi-static deformation of the sealing element and the hypothesis of elastic analogy to solve the problem.

Based on the elastic analogy, we can accept the deformation of cross-sections of the sealer in the form (Figure 2)

$$\varepsilon_1(z,t)=\varepsilon_1(\bar{z})\cdot\varepsilon_1(t), \tag{18}$$

$$\varepsilon_1(t)=w_1(t)=\frac{t}{T_1}\left[H(t)-H(t-T_1)\right]+H(t-T_1), \tag{19}$$

where $H(t)$ is the Heaviside function, T_1 is time of deformation of the upper section of the sealing element to first contact of the its inner surface with the cylinder wall.

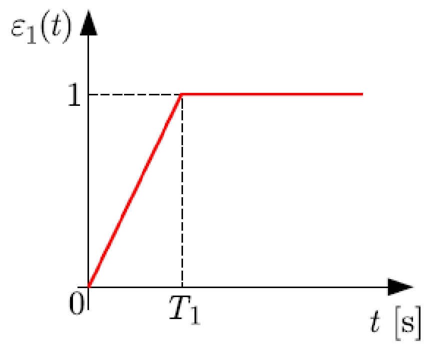


Figure 2. Graph of the time dependence of the relative axial deformation of the upper section

Substituting expression (18) and (19) into formula (14) and then integrating it, we obtain the equality

$$\overline{G}(t,T_1)=G\Big\{e^{-\lambda^* t}+\Big[\Big(\big(\nu^*-\lambda^*\big)\Big(1-e^{-\lambda^*(t-T_1)}\Big)-\nu^*\lambda^*(t-T_1)\Big)H(t-T_1)-$$

$$-\Big(\big(\nu^*-\lambda^*\big)\Big(1-e^{-\lambda^* t}\Big)-\nu^*\lambda^* t\Big)H(t)\Big]\Big\}. \tag{20}$$

Thus, in the case of a linear change the relative deformation rate over time under the external load, the value of the axial compressive forces Q and P required to deform the sealing element until first and full contact its inner surface with the cylinder wall can be found by the expressions (7), (8) and (20).

The numerical calculation is made for the following values of parameters

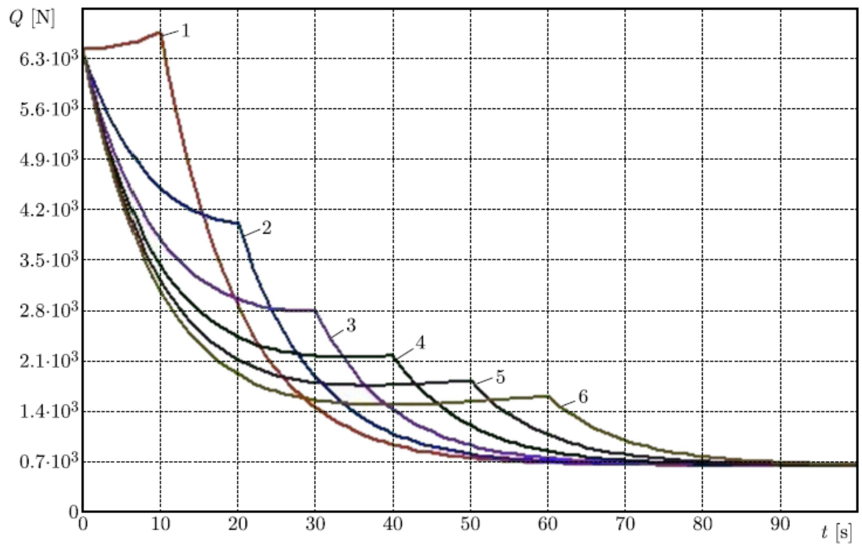
$$R_0 = 0.073\,m,\; R_1 = 0.076\;m,\; R_2 = 0.1\,m,\; \delta = 0.003\,m,$$

$$H = 0.02\;m,\; G = 1.3 \cdot 10^8\,Pa,\; k_0 = 6.7 \cdot 10^9\,Pa/m,$$

$$\mu = 0.5,\; \nu = 0.25,\; \nu^* = 0.01,\; \lambda^* = 0.1,$$

$$T_1 = 10,\; 15,\; 20,\; 25, 30,\; 35\; s.$$

The time dependence of the axial load Q for compression of the sealing element to the first contact of its inner surface with the cylinder wall is depicted in Figure 3. As can be seen in Figure 3, the value of external forces decreases and stabilizes after a while.



1- $T_1 = 10\,s$, ***2-*** $T_1 = 20\,s$, ***3-*** $T_1 = 30\,s$, ***4-*** $T_1 = 40\,s$,
5- $T_1 = 50\,s$, ***6-***$T_1 = 60\,s$

Figure 3. Time-dependent graph of the value of the axial force required to deform the inner surface of the sealing element until it first contacts with the cylinder wall, taking into account the heredity

Considering the heredity, the value of the axial force required to deform the sealing element until its inner surface first contact with the cylinder wall drops by up to nine times compared to the non-hereditary condition. This, in turn, leads to relaxation of the contact pressure between the inner surface of the sealing element and the cylinder wall, and thus to a deterioration of the sealing ability of the seal.

In **Section 1.4**, the focus is given to the study of the stress-strain state created during the process of sealing with the outer surface of the cylindrical sealing element based on theoretical investigation [17]. The influence of viscous-elastic properties of the material of the sealing element on its sealing ability is realized based on the hypothesis of elastic analogy.

The problem is solved in the case of instantaneous loading and linear change of the deformation rate of the sealing element over time under the influence of external force, analytical expressions were obtained for the value of compressive force ensuring tightness between sealed surfaces and the results of numerical calculations are represented in the form of graphs.

In **Section 1.5**, a mathematical model of the sealing process with a cylindrical sealing element was constructed by accepting an elastic body and the stress-stain state caused by the action of a bilateral axial compressive force of the seal was studied [3]. Analytical expressions were obtained by applying the principle of minimum potential energy for the value of the axial force required to achieve hermeticity by double-axial compression of the seal and the contact pressure between the inner surface of the sealing element and the sealed cylindrical wall. The effective height of the sealing element to achieve the required tightness between the sealing surfaces was determined by analytical calculations in the area of small deformations.

The second chapter is devoted to the mathematical modeling of the sealing process with annular and cross-conical sealing elements. On the basis of the principle of minimum potential energy in the field of small deformations, a mathematical model of the stress-stain state caused by the compressive force applied to the sealing element was developed and the problem was solved using the method of variation. The second chapter consists of five paragraphs.

In **Section 2.1**, the stress-stain state of the elastic annular sealing element under the influence of external load is studied, analytical expressions are obtained that determine the character of the distribution of contact stress between the annular seal and the smooth surface and the affective dimensions of the seal [4].

Consider an annular sealing element in the form of a hollow cylinder, the lower part of which is inserted into the seat of the rigid valve. The protruding part of the annular sealing element creates contact pressure, leaning on a smooth rigid surface. When the width of the annular sealing element is much smaller than its other dimensions, let us assume that the contact pressure, which is formed along the width, is systematically distributed, and its deformation condition is axially symmetric. Then, by accepting the hypothesis of plane sections the axial deformation of the protruding part of the sealing element can be obtained depending only on the coordinate z in the axial direction.

Using the principle of variation from the potential energy functional the following differential equation is obtained:

$$\psi'' - \frac{2\left[\dfrac{1}{1-\xi^2} - 3\left(1-\xi^2\right)\right]}{R_k^2\left[\dfrac{1-\xi^4}{4} + \left(1-\xi^2\right) + \ln\dfrac{1}{\xi}\right]}\psi = P_0, \qquad (21)$$

where $f'(z) = \psi(z)$, $\xi = \dfrac{R_1}{R_k}$, $P_0 = \dfrac{P}{\pi G R_k^4\left(\dfrac{1-\xi^4}{4} + 1 - \xi^2 - \ln\xi\right)}$.

The solution of the equation is found within the boundary conditions

$$\omega\Big|_{z=0} = 0, \qquad \tau_{rz}\Big|_{z=h} = \mu\sigma_z\Big|_{z=h}, \qquad u\Big|_{z=0} = 0.$$

$$f(z)=\frac{B}{k}(\operatorname{ch} kz-1)-\frac{P_0 R_k^2\left(\frac{1-\xi^4}{4}+1-\xi^2-\ln\xi\right)}{2\left[\frac{1}{1-\xi^2}-3\left(1-\xi^2\right)\right]}z\,, \qquad (22)$$

$$B=\frac{\mu P_0 R_k^2\left(\frac{1-\xi^4}{4}+1-\xi^2-\ln\xi\right)}{\left(2\mu \operatorname{sh} kh+kR_k \operatorname{ch} kh\right)\left[\frac{1}{1-\xi^2-3\left(1-\xi^2\right)}\right]}.$$

The following analytical expression is obtained for the value of the external compressive force necessary for achieving tightness

$$P=\frac{\pi G\Delta k R_k^2\left(2\mu \operatorname{sh} kh+k R_k \operatorname{ch} kh\right)\left[\frac{1}{1-\xi^2}-3\left(1-\xi^2\right)\right]}{\mu(\operatorname{ch} kh-1)+\frac{kh}{2}\left(2\mu \operatorname{sh} kh+kR_k \operatorname{ch} kh\right)}. \qquad (23)$$

The dependence of the value of the contact stress of the sealing element on its geometric dimensions, physical and mechanical properties and axial deformation is determined by the following expression

$$\sigma_k=\frac{G k\Delta\left[\frac{1}{1-\xi^2}-3\left(1-\xi^2\right)\right]\left(2\mu \operatorname{sh} kh-kR_k \operatorname{ch} kh\right)}{\left(1-\frac{R_1^2}{R_k^2}\right)\left(\mu(\operatorname{ch} kh-1)+\frac{kh}{2}\left(2\mu \operatorname{sh} kh+kR_k \operatorname{ch} kh\right)\right)}. \qquad (24)$$

Numerical calculation was made based on the formula (24) for the following values of the parameters of the sealing element:

$$R_k=5\cdot 10^{-2}\,m\,,\ \Delta=1,2\cdot 10^{-3}\,m\,,\ \mu=0.07\,,$$

$$\xi=0.8-0.95\,,\ h=1,2,5\cdot 10^{-3}\,m\,.$$

As can be seen from Figure 4, as the value of the parameter ξ increases, the contact pressure of the seal also increases, however, the increase to the value of $\xi = 0{,}87$ is linear, and the rate of growth accelerates in the interval $0{,}87 \le \xi \le 0{,}95$.

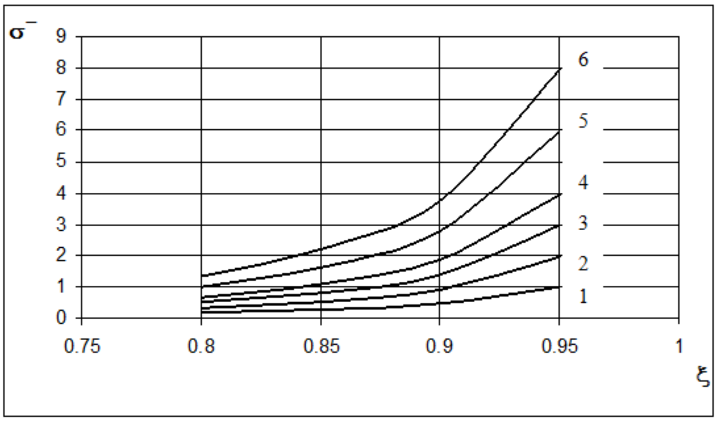


Figure 4. Graph of the ratio σ_k / G to the ratio $\xi = R_1 / R_k$ at different values of the quantities h and Δ ($1 - \Delta = 0{,}25 \cdot 10^{-3} m$, $2 - \Delta = 0{,}5 \cdot 10^{-3} m$, $3 - \Delta = 0{,}75 \cdot 10^{-3} m$, $4 - \Delta = \; 10^{-3} mm$, $5 - \Delta = 1{,}5 \cdot 10^{-3} m$, $6 - \Delta = 2 \cdot 10^{-3} m$, $h = 5 \cdot 10^{-3} m$)

Section 2.2 is devoted to the analysis of contact pressure relaxation by constructing a linear the viscous-elastic model of the annular sealing element. The effect of the viscous-elastic properties of the annular sealing element on its sealing ability is studied. The stress-stain state of the annular sealing element in the case of two different loads - instantaneous and gradual loading - has been determined.

Taking into account the viscous-elasticity of the material, the sealing process with an annular sealing element in the form of a hollow cylinder, the lower part of which is inserted into the seat of the rigid valve was considered. The results of numerical calculations were given in graphical form. Research shows that, viscous-elastic properties of seal's material greatly influence on its sealing ability. Because of heredity of the seal's material, the values of external forces in some cases drop about seven times.

In **Section 2.3**, the focus was given to the study of the sealing process with a cross-conical sealing element, tightly put on the stock with a gap δ between its inner surface and the cylinder wall [10]. The tightness between the inner surface of the conical sealing element and the cylinder wall is achieved by one-sided axial compression. The problem was solved for the elastic cross-conical sealing element and based on variational method non-homogeneous second order differential equation with non-constant coefficient was obtained from potential energy functional

$$\frac{1}{2}\left(\frac{1}{4}R^4(z)\ln\frac{R(z)}{R_1}-\frac{3}{16}R^4(z)+\frac{1}{4}R_1^2R^2(z)-\frac{1}{16}R_1^4\right)\varphi''(z)+\left(\frac{1}{2}R^3(z)\ln\frac{R(z)}{R_1}\operatorname{tg}\alpha-\right.$$

$$\left.-\frac{1}{4}R^3(z)\operatorname{tg}\alpha+\frac{1}{4}R_1^2R(z)\operatorname{tg}\alpha\right)\varphi'(z)+\left(-\frac{5}{4}R^2(z)\operatorname{tg}^2\alpha\ln\frac{R(z)}{R_1}-\frac{22+\operatorname{tg}^2\alpha}{8}R^2(z)+\right.$$

$$\left.+\frac{24+\operatorname{tg}^2\alpha}{8}R_1^2-\frac{1}{4}\frac{R^4(z)}{R_1^2}\right)\varphi(z)+\frac{1}{2}q\left(R_0^2-R_1^2\right)=0\,, \qquad (25)$$

where $\varphi_1(z)=f_1'(z)$, $q=\dfrac{Q}{\pi G\left(R_0^2-R_1^2\right)}$.

Border conditions are:

$$\mu Q\big|_{z=H}=2\pi G\int_{R_1}^{R_0}\gamma_{zr}\,r\,dr\,,\quad w_1\big|_{z=0}=0\,,\qquad u_1(r,z)\big|_{\substack{z=0\\ r=R_1}}=-\delta\,, \quad (26)$$

where μ is friction coefficient between the smooth surface and the sealer.

Let's solve the obtained differential equation (25) by the Ritz method. For this, the solution of the differential equation (25) that satisfies the boundary condition is chosen in the following form

$$f_1(z)=c_1\frac{z}{H}+c_2\left(\frac{z}{H}\right)^2+c_3\left(\frac{z}{H}\right)^3, \qquad (27)$$

where c_1, c_2 and c_3 are integration constants.

The following analytical expression is obtained, which determines the character of the contact pressure distribution between the inner surface of the conical sealing element and the wall of the sealed cylinder

$$\sigma_r(z) = \frac{\nu}{1-\nu}\sigma_0 \cdot \left(R(z) - R_3\right)^{\frac{2\mu\nu}{(1-\nu)tg\alpha}} \left(R^2(z) - R_3^2\right)^{\frac{\nu}{1-\nu}\cos\alpha} + k_0 \cdot u_0(z) . \quad (28)$$

In **Section 2.4 and 2.5**, the problem of the conical sealing element is solved in cases where the viscous-elastic property of the material is taken into account [14]. The influence of viscous-elastic properties of the material of the sealing element on its sealing ability was studied.

The third chapter is devoted to the study of the semi-cylindrical sealing element with a hole. A mathematical model of the sealing process was constructed and the distribution character of the contact pressure between the semi-cylindrical surface with a hole and the sealing element was studied (Figure 5). Analytical expressions were obtained using the variational method to determine the minimum value of the axial compressive force achieving tightness depending on the geometric dimensions and physical and mechanical properties of the seal, taking into account the existing hole. The influence of heredity on the distribution character of contact pressure in different cases of deformation rate has been determined.

Section 3.1 is considered a semi-cylindrical surface with a hole [9]. The seal is compressed to this surface by the clamp of the semi-cylindrical form (Figure 5).

In the case under consideration, the diameter of holes is many times small compared to the cylinder's diameter. Therefore, we accept the contour of the hole as a plane curve. We located the origin of the cylindrical coordinate system at the center of the cross-section of the cylinder, direct the coordinate axis r to the side of increase of radius as was shown in Figure 5. Besides this, we define the position of the points of the area of the hole by the coordinates φ and ρ (Figure 5).

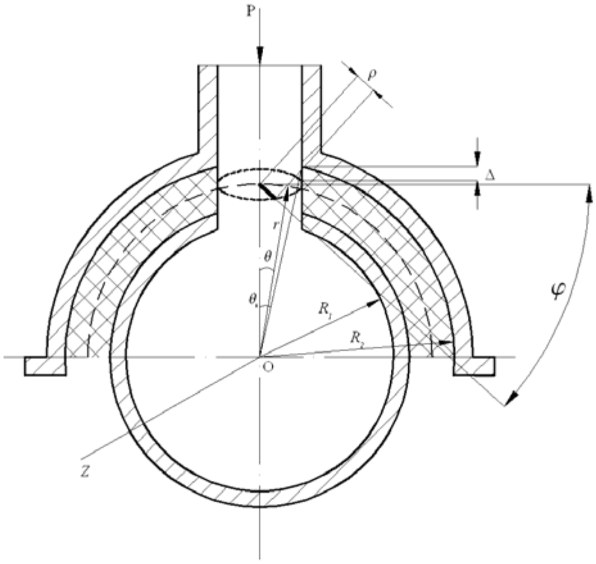


Figure 5. Design scheme

Considering the boundary conditions, let us assume the radial deformation of the sealing element in the following form:

$$u(r,\theta) = -\Delta f(r)\cos\theta\,, \tag{29}$$

where Δ is the deformation of sealing element, where $\theta = 0$, $f(r)$ is an unknown function, depending on r.

To calculate the potential energy of the sealing element, let us introduce a spherical coordinate system in the area of the hole (Figure 5). Then we can write for the potential energy of the sealing element, taking into account the hole in the cylinder

$$\Pi = G\int_{R_1}^{R_2}\int_{-\pi/2}^{\pi/2}\int_{-l}^{l}\left(\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_z^2 + \frac{1}{2}\gamma_{r\theta}^2 + \frac{1}{2}\gamma_{\theta z}^2 + \frac{1}{2}\gamma_{zr}^2\right) r\,dz\,d\theta\,dr -$$

$$- G\int_{R_1}^{R_2}\int_{0}^{2\pi}\int_{0}^{\theta_0}\left(\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_z^2 + \frac{1}{2}\gamma_{r\theta}^2 + \frac{1}{2}\gamma_{\theta z}^2 + \right.$$

$$+\frac{1}{2}\gamma_{zr}^2\Bigg)\, r^2 \sin\theta\, d\varphi\, d\theta\, dr - \int_{R_1}^{R_2} P\cdot u'_r\big|_{\theta=0}\, dr\,, \qquad (30)$$

where R_1 and R_2 in accordance, are inner and outer radii of sealing element, θ_0 is the angle between the radii passing through the center of the hole and the contour point; l is the half the length of sealing element, P is the external force compressing the sealing element in a radial direction on the cylinder wall, G is the shear modulus of the sealing element.

Using the Euler equation, a fourth order differential equation with nonconstant coefficient was obtained from the potential energy functional

$$\left(\frac{l^3}{12}r^3-\frac{1}{8}\left(\frac{2}{3}-\frac{1}{2}\cos\theta_0-\frac{1}{6}\cos 3\theta_0\right)r^6\right)\cdot f^{IV}(r)+\left(\frac{l^3}{6}r^2-\left(\frac{2}{3}-\frac{1}{2}\cos\theta_0-\frac{1}{6}\cos 3\theta_0\right)r^5\right)\cdot f'''(r)-$$

$$-\left(\frac{l^3}{3}r+4lr^3-\left(\frac{4}{3}-\frac{15}{16}\cos\theta_0-\frac{19}{48}\cos 3\theta_0\right)r^4\right)\cdot f''(r)+\left(\frac{l^3}{3}-4lr^2+\left(\frac{14}{3}-\frac{27}{8}\cos\theta_0-\frac{31}{24}\cos 3\theta_0\right)r^3\right)\cdot f'(r)+$$

$$+\left(\frac{17l}{4}r-\left(\frac{25}{6}-\frac{23}{4}\cos\theta_0+\frac{13}{12}\cos 3\theta_0\right)r^2\right)\cdot f(r)=0\,. \qquad (31)$$

Boundary conditions are

$$u(r,\theta)\big|_{\substack{r=R_2\\ \theta=0}}=-\Delta\,,\quad u(r,\theta)\big|_{\substack{r=R_2\\ \theta=\pm\pi/2}}=0\,,\quad \sigma_\theta\big|_{\theta=\frac{\pi}{2}}=0\,, \qquad (32)$$

where Δ is the radial deformation of the sealing element in the point $\theta=0$, $r=R_2$.

The obtained equation (31) is a fourth-order differential equation with non-constant coefficient. It is impossible to find an exact solution to this equation. Therefore, let us solve equation (31) approximately by applying Galerkin method. Given the boundary conditions (32), we can choose the solution of equation (31) as follows:

$$f(r)=A_1\frac{R_1-r}{R_1}+A_2\frac{(R_1-r)^2}{R_1^2}+A_3\frac{(R_1-r)^3}{R_1^3}+A_4\frac{(R_1-r)^4}{R_1^4}\,, \qquad (33)$$

where A_1, A_2, A_3 and A_4 are unknown coefficients.

Analytical expressions have been obtained the dependence of the value of the external compressive force creating tightness between the sealing surfaces and the distribution character of the contact pressure on the physical and mechanical parameters and geometrical dimensions.

Numerical calculations were made based on obtained expressions. Based on the numerical calculations, the graph of the dependence of the distribution character of the contact pressure between the inner surface of the sealing element and the cylinder wall on polar angle θ is given in Figure 6.

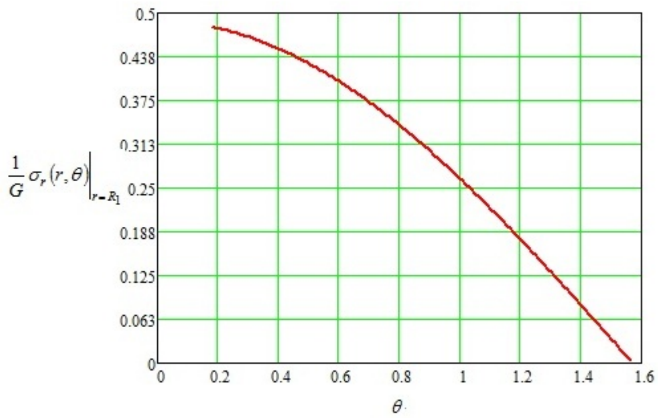


Figure 6. Graph of the polar angle θ dependence of the distribution of the contact pressure between the cylinder wall and the inner surface of the sealing element

Section 3.2 also examines a semi-cylindrical surface with a hole (Figure 5). Ignorance of the effect of axial deformation as the wall thickness of the sealing element increases may lead to incorrect conclusions. Therefore, in this section, the effect of taking into account the axial deformation on the distribution character of the contact pressure created on the semi-cylindrical surface with a hole is studied. In the case under consideration, the diameter of the holes is many times small compared to the cylinder's diameter. Therefore, we accept the contour of the hole as a plane curve. We located the origin of the cylindrical coordinate system at the center of the cross-section of the cylinder, direct the coordinate axis r to the side of increase of radius as was shown in Figure 5. Besides this, we define the position of the points of the area of the hole by the coordinates φ and ρ (Figure 5). Boundary conditions are as follows

$$u(r,\theta)\big|_{\theta=\pm\frac{\pi}{2}} = 0 , \qquad v(r,\theta)\big|_{\theta=0} = 0 .$$

Taking into account the boundary conditions, the radial and tangential deformation of the sealing element is assumed to be as follows

$$u(r,\theta) = -\Delta f(r)\cos\theta , \tag{34}$$

$$v(r,\theta) = f_1(r)\sin\theta , \tag{35}$$

where Δ is the deformation of sealing element, where $\theta = 0$, $f(r)$ is an unknown function, depending on r.

Analytical expressions were obtained expressing the dependence of the value of the external compressive force achieving the tightness between the sealing surfaces and the distribution character of the contact pressure on the physical and mechanical parameters and geometric dimensions of the seal.

In **Section 3.3 and 3.4**, the influence of the heredity of the sealing element on the sealing process of the semi-cylindrical surface with a hole was studied. In the case of two different loads - instantaneous and the time dependence of the axial deformation, the character of the contact pressure distribution was determined.

CONCLUSION

The following scientific results were obtained in the dissertation work:

1. By constructig a mathematical model of the sealing process developed a method for determining the stress-strain state, taking into account the edge effects of cylindrical sealing elements, physical and mechanical properties, as well as heredity of the sealer and it was determined that the value of the axial compressive load, which ensures tightness, first decreases as the height of the sealer increases, and then stabilizes after a cetain value of height.

2. An analytical expression was obtained that allows to determine the distribution of the contact pressure between the inner surface of the sealing element and the wall of the cylinder depending on its physical and mechanical parameters and geometric dimensions. It is shown that the maximum contact pressure occurs in the area near the lower seat of the sealing element, and as the height of the sealer rises, the contact pressure decreases and practically disappears after a certain height value. It has been found that hermeticity can be created by compressing the sealing element on both sides at the same time with a relatively small axial compressive load.

3. It has been found that the hereditary nature of the sealer's material reduces the effect of the pre-applied axial load, which ensures tightness, several times in a short period of time. This, in turn, reduces the contact pressure between the inner surface of sealing element and the wall of the cylinder, and thus its sealing ability.

4. The effect of the rate of application of the axial force on the sealing ability of the sealing element was studied. It is shown that as the rate of relaxation of the contact pressure between the inner surface of the sealer and the cylinder wall also decreases and stabilizes over time.

5. Constructing a mathematical model of the sealing process of the cross-conical and annular sealing elements a method of determining the distribution of the contact pressure between its inner surface of the sealing element and the cylinder wall was developed, taking into account the heredity of the sealer and it has shown that as the rate of

application of the axial load decreases, the rate of relaxation of the contact pressure decreases and stabilizes over time.

6. The method of determining the contact pressure between the semi-cylindrical surface with hole and the sealing element, taking into account the heredity, was developed and an analytical expression was obtained to determine the distribution character of the contact pressure. It has been shown that the contact pressure decreases at a relatively small rate over time, and then at a slightly greater rate, and stabilizes over time.

The main results of the thesis were published in the following works:

1. Rustamova, K.O. Determining the character of the distribution of contact pressure between the inner surface of the sealant and the wall of the cylinder // Materials of the I International Scientific Conference of Young Researchers dedicated to the 90th anniversary of the National Leader of the Azerbaijani people Heydar Aliyev, - Baku: Nurlar edition, - April 25-26, - 2013, - s.373.(in Azerbijani)

2. Abbasov, E.M., Rustamova, K.O. Determination of the character of the distribution of contact pressure between the inner surface of the sealant and the pipe wall during unilateral compression // Materials of the International Scientific Conference "Non-Newtonian systems in the oil and gas field" dedicated to the 85th anniversary of Academician AXMirzajanzade, - Baku: - November 21-22, - 2013 , - p.40-41. (in Azerbaijani)

3. Rustamova, K.O. Determination of the character of the distribution of contact pressure between the inner surface of the sealant and the cylinder wall during bilateral compression // Journal of Qafqaz University, - 2013. Vol.1, No 2, - p.113-122. (in Azerbaijani)

4. Abbasov, EM, Kahramanov, HT, Rustamova, KO Definition of contact pressure between the outer surface of the sealing ring and the shibra of the direct valve // SOCAR Proceedings, - 2013. № 3, - p. 57-59. (in Russian)

5. Rustamova, K.O. Determination of the character of the distribution of contact stress between the inner surface of the conical

sealant and the wall of the cylindrical tube // Materials of the XVIII Republican Scientific Conference of doctoral students and young researchers, - Baku: Translator, - December 19-20, - 2013, - p.14-17. (in Azerbaijani)

6. Rustamova, K.O. Mathematical modeling of the determination of the character of the distribution of contact pressure between the inner surface of the conical seal and the cylinder wall / International Youth Scientific Conference "Severgeoecotech-2014", - Ukhta: USTU Printing House, March 26-28, - 2014, - p. 225-228. (in Russian)

7. Abbasov, E.M., Rustamova, K.O. Determination of the nature of the distribution of contact pressure between the semi-cylindrical sealing element with a hole and the outer wall of the cylinder // Materials of the Republican Scientific Conference "Classical and modern problems of mechanics" dedicated to the 100th anniversary of YA Amanzade, - Baku: - May 22, - 2014, - p.9-12. (in Azerbaijani)

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12. Rustamova, K.O. Influence of heredity on the distribution of contact pressure between the inner surface of the seal and the cylinder

wall // Bulletin of Baku University, series of physical and mathematical sciences, - 2016. No. 3, - p. 116-124.(in Russian)

13. Rustamova, K.O. Mathematical simulation of heredity effect on character of contact pressure distribution between semi-cylindrical surface with a hole and sealing element // Proceedings of the International Conference “Modern Problems of Innovative Technologies in Oil and Gas Production and Applied Mathematics” dedicated to the 90th anniversary of Academician A.Kh.Mirzajanzade, – Baku, – 13-14 december, – 2018, – p.84-87.

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