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**QUALITATIVE PROPERTIES OF SOLUTIONS OF HIGH-  
ORDER DEGENERATE PARABOLIC EQUATIONS**

Specialty: 1211.01- Differential equations

Field of science: Mathematics

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**ABSTRACT**

of the dissertation for the degree of Doctor of Philosophy

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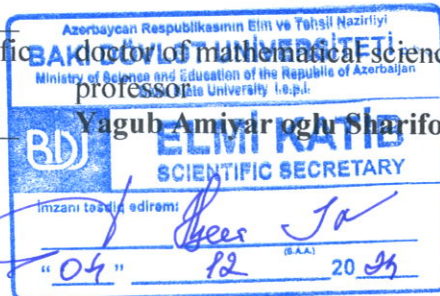
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## **GENERAL CHARACTERISTICS OF THE DISSERTATION**

### **Relevance of the topic and degree of elaboration.**

Nonlinear differential equations arise in many fields of modern science, including physics, mechanics, biophysics, biology, ecology, and biochemistry, and are prevalent in various issues. Their investigation is particularly significant in the contemporary era where numerous processes occur at high temperatures, under significant stress, and involve large deformations.

The theory of second-order, both limited and unbounded, and any desired order nonlinear elliptic and parabolic equations with truble geometric structure is one of the relatively advanced directions of the modern theory of partial differential equations.. The study of second-order nonlinear elliptic and parabolic partial differential equations in domains with truble geometry has a long history, and the main directions of research are as follows: Regularity of the solution, solvability of the border issue. The research of S.N.Bernstein, Lere, Schauder, C.Morrey, E.Georgie, J.Nash, M.Miranda, Dzh.Serrina, E.Custi, G.Stampakkia, V.G.Moser, O.Ladizhenskaya, N.N.Uraltseva and other authors contributed not only to the solution of the Hilbert problem but also to the development of numerous methods that play a fundamental role in the theory of differential equations and other areas of mathematics<sup>1</sup>.

However, the rich experience in the study of second-order equations became less significant during the research of arbitrary higher-order nonlinear equations.

The point is that new effects were discovered for high-order equations and systems by M.Miranda, E.Custi, V.G.Moser and I.V.Skrypnik.

When these equations  $n > 2$  are satisfied, the problem may have nontrivial generalized solutions even when the data is nontrivial. The idea of the existence of the special solution with zero measure for a system of high-order nonlinear elliptic-type equations and fast functions

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<sup>1</sup> Ladyzhenskaya O.A., Uraltseva N.N. Linear and quasi-linear equations of the elliptic type. M.: Nauka, 1973, 576 p.

came into play in connection with the multi-dimensional Plateau problem. Physical experiments with soap films, which are "generalized solutions" of the Plateau problem, indicated that such solutions have a characteristic point or even a line.

This led to the necessity of creating new methods for arbitrary higher-order nonlinear equations.

The purpose of the dissertation is to investigate the solution of the initial boundary value problem for high-order nonlinear parabolic equations in nonregular domains, as well as to find conditions for removing compactness in the context of high-order nonlinear equations in the  $C_{\omega}^{\lambda}(D)$  phase.

This issue has been investigated by many researchers. The appropriate result for the Laplace equation has been obtained by L. Carleson. When it comes to second-order elliptic equations with oscillating structures, we can refer to the works of Agranovic, Vishikin, Avantaggiati and Troisi in this direction<sup>2</sup>. We should also mention the works of Volkov, Vishik, and Wigley, where the condition for removing compactness in the context of cut-off functions phase was found. It is also worth noting the works of Azerbaijani scientists E.Novruzov and I.Mammadov in this field. The dissertation work is dedicated to the study of the regularity and self-adjointness of the solution of arbitrary higher-order nonlinear parabolic equations in domains with complex geometric structure and in non-trivial boundary conditions, both in domains with limited complexity and unbounded domains with unbounded complexity. For this, Saint-Venant inequalities, which are an integral analog of E.M.Landis' "Lemma o vozrastanyi", known in qualitative theory, were obtained. Using Saint-Venant inequalities and a number of auxiliary lemmas, estimations around the boundary point were obtained in bounded nonsmooth regions, Phragmen-Lindelof type theorems were obtained for the solution in unbounded regions with non-compact boundaries, and elimination of properties of the solution was studied.

Therefore, it is possible to consider the topic of the dissertation

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<sup>2</sup>Agranovich M.S., Vishik M.I. Elliptic problems with parameters and parabolic problems of the general type. // UMN, 1964, T. 19, No.3, p.53-161

work as relevant<sup>3</sup>.

### **The object and subject of the research.**

In the dissertation, the generalized solutions of mixed problems for degenerate divergent parabolic equations in a wide class of both non-smooth bounded and unbounded domains are studied. Various methods are presented for obtaining energy a priori estimates of solutions depending on the geometry of the region, similar to the Saint-Venant principle, well known in mechanics. Based on these estimates, a study of the behavior of solutions at infinity was carried out, and estimates were established for the behavior of the energy integral in the vicinity of the boundary points of the region for solutions with both a bounded energy integral and an unbounded solutions.

**The purpose and objectives of the research.** The initial boundary problem set for higher order nonlinear parabolic equations is to study the solution of the problem in irregular domains and to find the conditions for eliminating the compactness for the high-order nonlinear equation in  $C_{\omega}^{\lambda}(D)$  space.

**Research Methodology.** Functional analysis, methods from the theory of partial differential equations, as well as methods from the theory of special singular equations, have been used in obtaining the main results.

### **Main conclusion presented for defense.**

- A priori estimates of energy integrals in different classes of bounded non-smooth domains analogous to the Saint-Venant principle.
- Evaluations of the behavior of the solutions around the boundary point.
- Examples of various areas are given and accurate calculations are made for those areas.

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<sup>3</sup> 1. Landis E.M. Necessary and sufficient conditions for the regularity of the boundary point for the Dirichlet problem for the heat conduction equation. // ДАН СССР, 1969, Т. 185, No.3, p. 517-520.

2. Landis E.M. On the behavior of solutions of elliptic equations of high order in unbounded domains. // Trudy MMO, 1974, No.31, p.35-58.

- Obtaining evaluations in different classes of unbounded domains with non-compact boundaries.
- Derivation of Fragment-Lindelöf type theorems in unbounded domains.
- According to Saint-Venant type energy calculations, the property elimination conditions for the solution are obtained.
- Examples of the correctness of the assessment of the behavior of solutions in classes of different areas are shown.
- Behavior of generalized solutions of mixed boundary value problems for parabolic equations.
- The obtained results are also updated for the class of linear equations.

**Scientific innovation.** The following main results have been obtained in the dissertation:

- A priori estimations for the solution of high-order nonlinear parabolic equations in different classes of bounded non-smooth domains, analogous to the Saint-Venant principle;
- Evaluations on the behavior of solutions around the boundary point;
- Examples of oblasts where the behavior of the solutions are accurate in the estimations are given;
- Saint-Venant-type inequalities are obtained, on the basis of which the conditions for elimination of properties of the boundary solutions are obtained;
- Saint-Venant-type inequalities for unbounded regions are obtained;
- Fragment-Lindelöf type theorems are obtained;
- Examples of the correctness of the results are shown in various areas;
- The mixed boundary value problem has been studied.

**Theoretical and practical significance of research.** The results carry theoretical characteristics. They can be applied in the theory of singular differential equations, the theory of partial differential equations, and the theory of qualitative differential equations.

**Approbation and application of the dissertation.** The main

results of the dissertation work: Functional Analysis" (head, Prof. H.I. Aslanov), "Differential Equations" (head, Prof. A.B. Aliyev) departments of Institute of Mathematics and Mechanics of Ministry of Science and Education of Azerbaijan Republic, at the department of Mathematical Analysis of Nakhchivan State University (head Ph.D., associate professor E.Agayev), the IV International Scientific conference (Baku, 2011), the 70th anniversary of the establishment of the Georgian Academy of Sciences and Academician Nikolas Mukeleshvili International conference dedicated to the 120th anniversary of (Batumi, 2011), International Scientific conference (Mersin, 2012), International conference dedicated to the 110th anniversary of Academician Viktor Kupradzen (Batumi, 2013) scientific conference held by SSU (Sumgayıt, 2017), presented at COIA-2024- 9th Conference on Control and Optimization with Industrial Applications (Turkey 2024).

**Applicant's personal contribution.** All the main scientific results obtained in the dissertation are the result of the activity of the applicant personally as a result of application of the ideas of the supervisor in specific direction, formulation of the problems and development of the solution methods.

**Publications of the author.** The main results of the dissertation were published in 18 scientific works of the author. The list of these works is given at the end of the dissertation.

**The name of the institution where the dissertation was performed.** The dissertation work has been carried out at the Department of "General mathematics" of Nakhchivan State University.

**The total volume of the dissertation with indicating the volume of the structural sections of the dissertation separately.** The dissertation consists of an introduction, table of contents, three chapters, conclusion and a list of 101 cited references. The total volume of the dissertation is 224290 characters (title page - 334 characters, table of contents - 2040 characters, introduction - 58114 characters, first chapter 42153 characters, second chapter 46211 characters, third chapter 44220 characters). The list of used literature consists of 101 cited references.

## Contents of the Dissertation

The dissertation consists of an introduction, three chapters, and a bibliography

The first chapter is dedicated to the study of the generalized solution of non-regular domains for divergent parabolic-type equations with high-order degenerate and non-smooth boundaries.

In the 1.1 subchapter the self-implementation of the initial-boundary value problem has been studied for divergent quasi-elliptic parabolic-type equations swirling around the boundary.

Let's assume that there is an  $Q = \Omega \times (0, T)$ ,  $\Omega \subset \mathbb{R}^n, n \geq 2$ ,  $\partial Q = \Gamma_0 \cup \Gamma_T \cup \Gamma$ ,  $\Gamma_0 = \partial Q \cap \{(x, t) : t = 0\}$ ,  $\Gamma_T = \partial Q \cap \{(x, t) : t = T\}$ ,  $\Omega$  -dimensional non-convex domain.

The space  $L_p(0, T, W_{q, \omega}^m(\Omega'_t))$  is defined as  $\left\{ u(x, t) : \int_0^T \left( \|u\|_{W_{q, \omega}^m(\Omega'_t)} \right)^p dt < \infty \right\}$ ,

where  $Q'$  is the limited subdomain of  $Q$ ,  $\Omega'_t = Q' \cap \{(x, t) : t = \tau\}$ .

Functions from  $W_{q, \omega}^m(\Omega'_t) \subset C^m(\Omega'_t)$ ,

$$\|u\|_{W_{p, \omega}^m(\Omega)} = \left( \int_{\Omega} \omega(x) \sum_{|\alpha| \leq m} |D^\alpha u|^p dx dt \right)^{\frac{1}{p}} \text{ in the norm.}$$

$$\frac{\partial u}{\partial t} - \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, t, u, \nabla u, \dots, \nabla^m u) =$$

$$= \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha F_\alpha(x, t)$$

$$u|_{t=0} = 0 \quad D^\alpha u|_{\Gamma} = 0, \quad |\alpha| \leq m-1 \quad (1)$$

We will look at the generalized solution of the (1) problem in the cylindrical bounded domain from the  $L_p(0, T; W_{p, \omega}^m(\Omega)) \cap W_2^1(0, T; L_2(\Omega))$  space, where

$$D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \dots \partial x_n^{\alpha_n}}, \quad |\alpha| = \alpha_1 + \alpha_2 + \dots + \alpha_n, \quad m \geq 1.$$



Let's assume that the coefficient of  $A_\alpha(x, \xi)$  are measured based on  $x \in \overline{\Omega}$ ,  $\xi \in R^m$  is unavoidable ( $m$  is the number of different multiindexes whose length does not exceed  $m$ ) and satisfies the following conditions:

$$\begin{aligned} \sum_{|\alpha|=m} A_\alpha(x, t, \xi) \xi_\alpha^m &\geq \omega(x) |\xi^m|^p - c_1 \omega(x) \sum_{i=1}^{m-1} |\xi_i|^p - f_1(x, t), \\ |A_\alpha(x, t, \xi)| &\leq c_2 \omega(x) \sum_{i=0}^m |\xi^i|^{p-1} + f_2(x, t), \end{aligned} \quad (2)$$

here

$$\begin{aligned} \xi &= (\xi^0, \dots, \xi^m), \quad \xi^i = (\xi_\alpha^i), \quad |\alpha| = i, \quad c_1, c_2 > 0, \quad p > 1, \\ f_1(x, t) &\in L_p(0, T; L_{p,loc}(\Omega_t)), \quad f_2(x, t) \in L_{1,loc}(Q), \quad \Omega_\tau = Q \cap \{(x, t) : t = \tau\}, \\ F_\alpha(x, t) &\in L_{p',loc}(Q), \quad p' = \frac{p}{p-1}. \end{aligned}$$

Let's assume that  $\omega(x), x \in \Omega -$  is a measurable, non-negative function satisfying condition  $\omega \in L_{1,loc}(\Omega)$  and for each  $\rho > 0$  and certain  $\sigma > 1$

$$\int_{\Omega_\rho} \omega^{-1/(\sigma-1)} dx < \infty, \quad \text{ess sup}_{x \in \Omega_\rho} \omega(x) \leq c_3 \rho^{n(\sigma-1)} \left( \int_{\Omega_\rho} \omega^{\frac{1}{(\sigma-1)}} dx \right)^{1-\sigma} \quad (3)$$

Here,  $\Omega_\rho = \Omega \cap B_\rho$ ,  $B_\rho = \{x : |x| < \rho\}$ ,  $c_i$  - are only positive constants depending on the given parameters of the problem.

Let's assume that for arbitrary  $s \geq h > 0$

$$\frac{\omega(\Omega_s)}{\omega(\Omega_h)} \leq c_6 \left( \frac{s}{h} \right)^{n\mu}, \quad (4)$$

$$\mu < 1 + p/n, \quad \text{here } \omega(\Omega_s) = \int_{\Omega_s} \omega(x) dx.$$

Let's assume that  $0 \in \partial\Omega$ ,  $S_\rho = \Omega \cap \partial\Omega_\rho$ .  $K(\rho_1, \rho_2) \equiv \Omega_{\rho_2} \setminus \Omega_{\rho_1}$   
 $Q(\tau) = Q \cap \{B_\tau \times (0, T)\}$ .

Our main goal is to obtain an estimate depending on the geometrical structure of the  $\Omega$  domain around the point 0 for the behavior of the  $I_\rho = \int_{\Omega_\rho} \omega(x) |\nabla^m u|^p dx$  energy integral at small values of  $\rho$ . We will

characterize the  $\partial\Omega$  boundary in the nonlinear fundamental frequency-  $\lambda_p^\rho(r)$  of the  $S_r$  section

$$\lambda_p^\rho(r) = \inf \left( \int_{S_r} |\nabla_s v|^p ds \right) \left( \int_{S_r} |v|^p ds \right)^{-1},$$

where that

$\inf \partial\Omega$  is taken over functions that are infinitely differentiable around a certain  $S_r$ , which becomes zero on  $s$ .

$\nabla_s v(x)$  – is the projection of the vector  $\nabla v(x)$  on the tangent plane touching the surface  $S_r$  at point  $x$ . If the arbitrary function

$$\varphi \in L_p \left( 0, T; W_{p, \omega}^m(\Omega'_t) \right) \cap L_2(Q')$$

$$\begin{aligned} & \int_Q \frac{\partial u}{\partial t} \varphi dx dt + \int_Q \sum_{|\alpha| \leq m} A_\alpha(x, t, u, \dots, D^m u) D^\alpha \varphi dx dt = \\ & = \int_Q \sum_{|\alpha| \leq m} F_\alpha(x, t) D^\alpha \varphi dx dt \end{aligned}$$

integral identity, then we will call the function  $u(x)$

$$u(x, t) \in L_p \left( 0, T; W_{p, \omega, loc}^m(\Omega'_t) \right) \cap W_2^1 \left( 0, T; L_{2, loc}(\Omega'_t) \right) \quad (5)$$

the general solution of the equation (1).

We will consider the class of domains for which the following estimation is satisfied:

$$\int_{\sigma(r, \tau)} \omega(x) |u|^p dx dt \leq \lambda_p^{-p}(r, \tau) \int_{\sigma(r, \tau)} \omega(x) |\nabla u|^p dx dt \quad (6)$$

If the estimation (6) is correct, the necessary and sufficient conditions on the domain are given in V.Q. Mazya works.

In the 1.1 subchapter apriori estimations for energy integrals

have been obtained for various classes of domains. We can divide the considered domains into two classes. The first class consists of "narrow" domains, specifically, their complements around the 0 point are sufficiently large.

In the language of frequently used terms, this class of domains satisfies the condition

$$r\lambda_p(r) > d_1 > 0, \quad \forall r \in (0, r_0), \quad r_0 > 0, \quad (\text{A})$$

The second class consists of "broad" domains - more precisely, these domains have "internal sharpness" at the 0 point.

In the language of frequently used terms, this class of domains satisfies the condition

$$r\lambda_p(r) < d_2 < \infty, \quad \forall r \in (0, r_0), \quad (\text{B})$$

Let's determine the  $\psi(r)$  function expressed by the following inequalities in the  $(0, r_0)$  interval

$$\inf_{r\psi(r) < |x| < r} \lambda_p(|x|)(r - r\psi(r))\omega(x) \geq \mu > 0, \quad (\text{7})$$

Here,  $\mu$  is chosen in such a way that the function  $0 < 1 - c_0 < \psi(r) < 1$ .  $\lambda_p(r)$  satisfies the following (7) inequalities when it is monotonically decreasing:

$$\begin{aligned} r\lambda_p(r)(1 - \psi(r))\omega(x) &\geq \mu, \\ \varphi(r) \equiv 1 - \psi(r) &\geq \mu\omega^{-1}(x)(r\lambda_p(r))^{-1}. \end{aligned}$$

Let's accept the following notation.

$$\begin{aligned} J(r) &= \int_{\Omega} \omega(x) |D^m u|^p dx dt, \\ G(r) &= \int_{\Omega} \left( \sum_{|\alpha| \leq m} \omega(x) (|F_\alpha| + |f_2|)^{\frac{p}{p-1}} \lambda_p^{-\frac{m-|\alpha|}{p-1}} (|x| + |f_1|) \right) dx dt. \end{aligned}$$

The following theorem has been proven.

**Theorem 1.**

Suppose that  $u(x, t) \in L_p \left( 0, T; \overset{\circ}{W}_{p, \omega, loc}^m(\Omega_t) \right) \cap W_2^1(0, T; L_2(\Omega_t))$  is

the generalized solution of equation (1).

Let's assume that the coefficients of the equation satisfy the conditions of (2), the domain  $\Omega$  satisfies the condition of (A), and the gravitational function satisfies the conditions of (3), (4).

Suppose that  $\bar{\psi}$  is an arbitrarily continuous, non-increasing function in the interval  $(0, r_0)$  satisfying the inequality  $0 < 1 - c_0 < \bar{\psi}(r) \leq \psi(r) < 1$ , where  $\psi(r)$  is determined from inequality (7) and suppose that when  $c_{13} > 0, \theta < 1 - c_0$ ,  $\beta = \text{const} < 1$  condition

$$G \left( r \exp \left( - \frac{1 - \bar{\psi}(r)}{1 - c_0 - \theta} \right) \right) < c_{13} \exp \left( - \theta \ln \beta^{-1} \int_r^{r_0} \frac{d\tau}{\tau(1 - \bar{\psi}(\tau))} \right) G(r_0)$$

is satisfied.

Then for  $J(r)$  when  $\forall \nu > 0$  evaluation of

$$J \left( r \exp \left( - \frac{1 - \bar{\psi}(r)}{1 - c_0 - \theta} \right) \right) < c_{14}(c_{13}, \nu) \exp \left( - \theta \ln(\beta + \nu)^{-1} \int_r^{r_0} \frac{d\tau}{\tau(1 - \bar{\psi}(\tau))} \right) (J(r_0) + G(r_0))$$

is correct.

Let's assume that  $\psi(r) \equiv d$ ,  $0 < d < 1$ .  $\tilde{\lambda}_p(r) = \inf_{dr < \tau < r} \lambda_p(\tau)$  and denote it.  $\lambda_p(r)$  is a non-increasing function, and thus,  $\tilde{\lambda}_p(r) = \lambda_p(r)$ .

The following theorem has been proven.

**Theorem 2.** Let's assume that,  $u(x, t) \in L_p \left( 0, T; W_{p, \text{loc}}^m(\Omega_t) \right) \cap W_2^1(0, T; L_2(\Omega_t))$

it is a generalized solution for the equation (1). Let's assume that the coefficients of the equation satisfy the conditions of (2), the domain  $\Omega$  satisfies the condition of (B), and the gravitational function satisfies the conditions of (3), (4)

Let's suppose that  $\bar{\varphi}(r)$  is a function defined on the interval  $(0, r_0)$  which is non-decreasing and satisfies the inequality

$\bar{\varphi}(r) < \varphi(r) \equiv r\tilde{\lambda}_p(r)$  and let's assume that the following condition is satisfied::

$$G(r) < c_{15} \exp\left(-\frac{(1-d)^m}{2c_{14} \ln d^{-1}} \int_r^{r_0} \frac{\bar{\varphi}^m(\tau) d\tau}{\tau}\right) G(r_0).$$

Then, for  $J(r)$

$$J(r) < c_{16} \exp\left(-\frac{(1-\nu)(1-d)^m}{2c_{14} \ln d^{-1}} \int_r^{r_0} \frac{\bar{\varphi}^m(\tau) d\tau}{\tau}\right) (J(r_0) + G(r_0)), \quad \forall \nu > 0$$

the evaluation is correct.

Some examples related to the evaluation of solutions have been provided in the **1.3 subchapter**.

Specifically, the initial boundary value problem is considered for the equation

$$\frac{\partial u}{\partial t} - \sum_{|\alpha| \leq m} a_\alpha(x, t) D^\alpha u = \sum_{|\alpha| \leq m} D^\alpha F_\alpha(x, t),$$

$$c_1 \omega(x) |\xi|^{2m} < \sum_{|\alpha|=m} a_\alpha \xi^\alpha < c_2 \omega(x) |\xi|^{2m}, \quad \forall x \in \Omega, \quad \forall \sigma \in R^n, \quad c_1, c_2 > 0$$

here, when  $F_\alpha(x, t) \in L_{2, \omega}(\Omega)$ ,  $a_\alpha(x) \in C^{|\alpha|-m}(\bar{\Omega})$ ,  $|\alpha| > m$ .  $|\alpha| \leq m$   $a_i(x)$  – are measured limited functions. The general solution is from the space  $L_p\left(0, T; \overset{\circ}{W}_{2, \omega}^m(\Omega_t)\right)$ . Appropriate evaluations are established.

In the **1.4 subchapter** the evaluation of the solution around the boundary point is taken.

**Theorem 3.** Let's assume that  $u(x, t) \in L_p\left(0, T; \overset{\circ}{W}_{p, \omega, loc}^m(\Omega_t)\right) \cap W_2^1(0, T; L_2(\Omega_t))$

is a generalized solution to the boundary value problem (1) and  $0 \in \partial\Omega$ .  $\Omega$  region has such a boundary around point 0 that any

$r \in (0, r_0)$ ,  $\lambda^{(0)} > 0$  for  $\lambda_p(r) > \lambda^{(0)} r^{-1}$  compensates for inequality.

So, there is a  $\gamma_0 > 0$  such that if for the evaluation of A=B is true,

$$G(r) < Ar^{\gamma_0 + \varepsilon} G(r_0), \quad \forall r \in (0, r_0), \quad A > 0, \quad (8)$$

evaluation is true, then for sufficiently small  $\varepsilon > 0$  and arbitrary

$x \in \Omega_{\bar{r}_0}$  ,  $x \in \Omega_{\bar{r}_0}$  ,  $\bar{r}_0 < r_0$  when  $\delta > 0$  the following evaluation is correct:

$$\omega(x) |D^j u(x)| < C |x|^{m - \frac{n}{p} - j + \gamma_0} (J(r) + G(r))^{\frac{1}{p}} \quad j=0,1,\dots, \left[ m - \frac{n}{p} \right] \quad (9)$$

here  $\delta = m - \frac{n}{p} - \left[ m - \frac{n}{p} \right]$  ,  $m - \frac{n}{p} \geq 0$  ,  $C$  - is a constant independent of  $u(x)$  ,  $\Omega_R = \Omega$  . In case of  $\delta = 0$  , the accuracy of the evaluation (9) is only proven when  $j = 0, 1, \dots, \left[ m - \frac{n}{p} \right] - 1$  .

The main purpose of the second chapter is to study the uniqueness of the solution for non-linear parabolic type on the boundary of the domain. For this purpose, the method of obtaining apriori energetic estimates is applied to study the increasing generalized solution around the boundary point.

In the **2.1 subchapter** some assistant suggestions have been introduced. Along with the conditions of (2), the focus has been on the (1) equation In the 2.2 subchapter, various domain classes have been selected, and apriori evaluations of the solution have been taken. Let's look at the distance function with  $\partial\Omega$  or :  $g(x) = \rho(x, \partial\Omega)$ . It is known that, there exists  $\exists \delta > 0$  , when  $\Gamma_\delta = \{x : 0 < \rho(x, \partial\Omega) < \delta\}$  ,  $g(x) \in C^m$  ,  $|\nabla g(x)| = 1$  .

Let's denote  $\Omega_r = \Omega \cap \{x : g(x) < r\}$  .

Let  $\overset{\circ}{W}_{p,\omega}^m(\Omega, \Gamma)$  denote the closure of the set of functions from  $C^\infty(\Omega)$  space, which become zero near  $\partial\Omega \setminus \Gamma$  according to the norm of space, for arbitrary  $\Gamma \subset \partial\Omega$  .

If  $\Gamma \cap \partial\Omega' = \emptyset$  is  $u(x, t) \in \overset{\circ}{W}_{p,\omega}^m(\Omega', \partial\Omega' \setminus \partial\Omega)$  for any  $\Omega' \subset \Omega$  subdomain,

we say  $u(x) \in \overset{\circ}{W}_{p,\omega,loc}^m(\Omega, \Gamma)$  .

In the **2.2 subchapter**, apriori estimates have been taken for energy integrals. Let's denote  $I(r) \equiv \int_{\Omega \setminus \Omega_r} \omega(x) |D^m u|^p dx dt$ , accept

$$\psi(r) \equiv d \quad , \quad 0 < d < 1 \quad \forall \vartheta \quad \bar{\lambda}_p(r) = \inf_{dr < \tau < r} \lambda_p(\tau) \quad \text{indications.}$$

The following theorem is true.

**Theorem 4.** Let  $u(x,t) \in L_p \left( 0, T; W_{p, \vartheta, loc}^m(\Omega, \Gamma) \right) \cap W_2^1(0, T; L_2(\Omega))$  is the general solution to the Dirichlet problem for the equation (1). Assume that the coefficients of the equation satisfy condition (2), and the domain  $\Omega$  satisfies condition (B). Let's assume that the coefficients of the equation satisfy condition (2) and domain satisfies condition (B). Let's assume that  $\bar{\varphi}(r)$  is an arbitrary non-decreasing function in the interval  $(0, r_0)$  that satisfies the inequality  $\bar{\varphi}(r) \leq \varphi(r) \equiv r \bar{\lambda}_p(r)$

Then the following alternative is true for  $I(r)$  :

1. For a given sequence of  $r_i \rightarrow 0$ , either the inequality  $I(r_i) < c_1(1 + G(r_i))$  holds, where  $c_1 < \infty$  – is a constant,
2. or when  $r \rightarrow 0$ ,  $I(r)$  increases rapidly, meaning

$$I(r) > c_2(\gamma) \exp \left( - \frac{(1-d)^m (1-\gamma)}{\ln d^{-1}} \int_r^{r_0} \frac{\bar{\varphi}^m(\tau) \tau^{-1} d\tau}{A_2 + (1-d)^m \bar{\varphi}(\tau)} \right), \quad \forall \gamma > 0 \quad (10)$$

is satisfied and furthermore, when  $r \rightarrow 0$ , and the evaluation of (10) transitions to the evaluation of

$$I(r) > c_2(\gamma) \exp \left( - \frac{m(1-e^{-1})(1-\gamma)}{A_2} \int_r^{r_0} \frac{\bar{\varphi}(\tau)}{\tau} d\tau \right), \quad \forall r < r_0(\gamma), \quad (11)$$

In the **2.4 subchapter** we obtain the elimination of singularity in the solution. The following theorems have been proven.

**Theorem 5.** Let  $u(x,t) \in L_p \left( 0, T; W_{p, \vartheta, loc}^m(\Omega, \Gamma) \right) \cap W_2^1(0, T; L_2(\Omega))$  is the general solution to the Dirichlet problem for the equation (1).. Let's assume that the coefficients of the equation satisfy condition (2), and the domain  $\Omega$  satisfies condition (A).  $G(r)$  is a bounded function, and

for certain  $\gamma > 0$ , within the conditions of Theorem 4, the evaluation

$$I(r) < c \exp \left( c_0 \nu \ln(k_0 + \gamma)^{-1} \int_r^{r_0} \frac{d\tau}{\tau(1-\psi(\tau))} \right), \forall r < r_0. \quad (12)$$

holds true. Then the uniqueness of the solution  $u(x)$  is eliminated for the specific multiplicity  $\Gamma$ , more precisely  $u(x) \in \overset{\circ}{W}_{p,\omega}^m(\Omega)$ .

Similarly, the following theorem is also true.

**Theorem 6** .Let  $u(x,t) \in L_p \left( 0,T; \overset{\circ}{W}_{p,\omega,loc}^m(\Omega,\Gamma) \right) \cap W_2^1(0,T;L_2(\Omega))$  is the general solution to the Dirichlet problem for the equation (1). Let's assume that the coefficients of the equation satisfy condition (2), and the domain  $\Omega$  satisfies condition (B).  $G(r)$  is a bounded function and for certain  $\gamma > 0$  within the conditions of Theorem 5, the evaluation

$$I(r) < c_8(\gamma) \exp \left( - \frac{(1-d)^m(1-\gamma)}{\ln d^{-1}} \int_r^{r_0} \frac{\bar{\varphi}^m(\tau)\tau^{-1}d\tau}{A_2 + (1-d)^m\bar{\varphi}(\tau)} \right), \forall r < r_0.$$

holds true.

Then the uniqueness of the solution  $u(x)$  is eliminated for the specific multiplicity  $\Gamma$ , more precisely  $u(x) \in \overset{\circ}{W}_{p,\omega}^m(\Omega)$ .

In **Chapter III**, using the geometric structure of the domain in non-compact bordered unbounded regions, a priori estimates have been obtained for the generalized solution of the boundary problem for a parabolic-type equation with non-smooth coefficients that degenerates along the non-characteristic boundary depending on the spatial organization of the domain. Based on the obtained estimates, alternative theorems of Phragmen-Lindelof type have been derived, regarding the self-realization of the solution possessing a non-finite energy integral in unbounded domains.

In the **3.1 subchapter**, the generalized solution  $u(x,t) \in L_p \left( 0,T; \overset{\circ}{W}_{p,\omega,loc}^m(\Omega,\Gamma) \right) \cap W_2^1(0,T;L_2(\Omega))$  has been considered for the equation (1) in a non-compact  $\partial\Omega$  boundary an unbounded



domain.

In the 3.1 subchapter, the following results have been obtained. As in the case of a bounded domain, we can divide the domains satisfying isoperimetric conditions into two classes in the future.  $\lambda_p(r)$ ,  $r \rightarrow \infty$  based on the behavior of the function, we will divide the unbounded domains into two classes. The first class - "narrow regions" and in the language of common terms, these classes satisfy the following condition

$$r\lambda_p(r) > C > 0, \quad \forall r > r_0 > 0. \quad (\text{A}_1)$$

The second class - "wide areas" and in the language of common terms, these classes satisfy the following condition

$$r\lambda_p(r) \leq C_1 < \infty, \quad \forall r > r_0 > 0. \quad (\text{B}_1)$$

Let's define the function  $\psi(r)$  and the constant  $h_0 > 0$  with the following inequalities:

$$\inf_{r < \tau < r\psi(r)} r\lambda_p(\tau)(\psi(r)-1) \geq h_0, \quad \psi(r) > 1, \quad \forall r > r_0 \quad (13)$$

For any  $h_0 < C$ , when  $\lambda_p(r)$  is a non-monotonic decreasing function, we can take

$$\psi(r) \geq h_0 (r\lambda_p(r))^{-1} + 1, \quad (14)$$

If the  $r\lambda_p(r)$  are non-decreasing while  $\psi(r)$  satisfies the inequality

$$\psi(r) \geq 1 + h_0 (r\lambda_p(r) - h_0)^{-1}, \quad h_0 < C, \quad (15)$$

it is sufficient. Let's assume

$$J(r) = \int_{\Omega_r} \omega(x) |D^m u|^p dx, \quad G(r) = \int \left[ \sum_{|\alpha| \leq m} (|F_\alpha| + |f_2|)^{\frac{p}{p-1}} \lambda_p^{\frac{m-|\alpha|}{p-1}} (g(x) + |f_1|) \right] dx.$$

The following theorem has been proven.

**Theorem 7.** Suppose  $u(x,t) \in L_p \left( 0, T; \dot{W}_{p, \omega, loc}^m(\Omega, \Gamma) \right) \cap W_2^1(0, T; L_2(\Omega, ))$  is a generalized solution for equation (1). The weight function  $\omega(x)$

satisfies conditions (3)-(4).

Assume that the coefficients of the equation satisfy the condition (15), and the domain  $\Omega$  satisfies the isoperimetric conditions and furthermore the domain  $\Omega$  is narrow enough in the sense  $\lambda_p(r) > \delta^{-1} > \delta_0^{-1}$  for arbitrary  $r \in (0, \infty)$ . Suppose that  $\psi(r)$  is an arbitrary function satisfying (13).

Then there is an  $0 < \theta_0(h_0) < \theta < 1$  constant depending on certain constants for  $I(r)$  for which the following alternatives are correct

1. either

$$\underline{\lim}_{r \rightarrow \infty} (I(r)G^{-1}(r)) < \infty; \quad (16)$$

2. Or  $\forall \nu \in (0, 1)$  and  $\forall r > r_0$ , for a sufficiently large  $r_0$

$$I\left(r \exp\left(\frac{\varphi_0(r)}{1-\nu}\right)\right) \geq \theta \exp\left(\nu \ln \theta^{-1} \int_{r_0}^r \frac{d\tau}{\tau \varphi_0(\tau)}\right) I(r_0), \quad (17)$$

evaluation is true.

If in addition condition (16) is satisfied for the function  $\varphi_0(r)$ , then (17) is equal to the estimate

$$I(r) \geq \theta \exp\left(\frac{\gamma}{1+\gamma} \ln \theta^{-1} \int_{r_0}^r \frac{d\tau}{\tau \varphi_0(\tau)}\right) I(r_0), \quad (18)$$

assessment is also correct (for a certain  $\gamma > 0$  constant).

In the **3.1 subchapter**, some examples have been included for apriori valuation assessments in various class domains.

**Theorem 8.** Let  $u(x, t) \in L_p\left(0, T; W_{p, \omega, loc}^m(\Omega, \Gamma)\right) \cap W_2^1(0, T; L_2(\Omega_t))$  is the generalized solution to the mixed boundary value problem for the equation (1).  $\omega(x)$  weight function satisfies conditions (3)-(4). Assume that the coefficients of the equation satisfy the condition (2), and the domain  $\Omega$  is isoperimetric and satisfy the condition (A1). Additionally, the condition .

$$A_\alpha(x, \xi) = 0, \quad |\alpha| < m; C_2 = 0, \quad (19)$$

for equation (1). Suppose that the function  $\psi(r)$ ,  $h_0 > 0$  is determined

by condition (13).

In that case, there exists a constant  $0 < \theta_0 < 1$  for any  $r_0 > 0$ ,  $0 < \nu < 1$  for which the evaluation of

$$I\left(r \exp\left(\frac{\varphi_0(r)}{1-\nu}\right)\right) \geq \theta_0 \exp\left(\nu \ln \theta_0^{-1} \int_{r_0}^r \frac{d\tau}{\tau \varphi_0(\tau)}\right) I(r_0), \quad \forall r > r_0, \quad (20)$$

is correct.

This theorem shows the correctness of energy evaluations of the type (17), (18) for a wider class of regions by placing additional restrictions on the function formed from the equation (1).

Suppose that  $d$  is any arbitrary number.

Suppose that  $d -$  is any arbitrary number  $0 < d < 1$ . Let's define the function  $0 < \varphi(r) < C_1$  based on the given equalities:

$$\varphi(r) = r \tilde{\lambda}_p(r), \quad \tilde{\lambda}_p(r) = \inf_{dr < \tau < r} \lambda_p(\tau), \quad (21)$$

where  $C_1$  is from condition (B1). In the domains from the class (B1) in the appendices, is usually a non-decreasing function, and as a result.

In addition,  $\lambda_p(r) -$  is usually a non-decreasing function on domains of class (B1), resulting in  $\tilde{\lambda}_p(r) = \lambda_p(r)$ .

**Theorem 9.** Suppose that  $u(x,t) \in L_p\left(0,T; \overset{\circ}{W}_{p,\omega}^m(\Omega_t)\right) \cap W_2^1\left(0,T; L_p(\Omega_t)\right)$  has a generalized solution for the mixed boundary problem for equation (1). Assume that the coefficients of the equation satisfy the condition (2), and the domain  $\Omega$  is isoperimetric and satisfy the condition (B1).

Additionally, condition (19) is satisfied, and let's assume that there exists a non-decreasing function that fulfills the condition  $\bar{\varphi}(r) - \bar{\varphi}(r) \leq \varphi(r)$  for  $\forall r > r_0$ , where  $\varphi(r)$  is the function determined by the condition (21). Then there exists a constant  $\omega = \omega(d) \geq 0$  for which the following estimates is correct for  $I(r)$

$$I(r) \geq C \exp\left(\frac{(1-d)^m}{\ln d^{-1}} \int_{r_0}^r \frac{\bar{\varphi}^m(\tau) \tau^{-1} d\tau}{(1-d)^m \bar{\varphi}^m(\tau) + \omega(d)}\right) I(r_0), \quad (22)$$

Here,  $C > 0$  is a determined constant.

### 3.4 subchapter Fraqmen-Lindelyof type teorems holds.

The border in subsection 3.5  $\partial\Omega = \Gamma_1 \cup \Gamma_2$  which is  $\Omega \subset \mathbb{R}^n$ ,  $n \geq 2$ , in the area

$$\sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, \dots, D^m u) = \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha F_\alpha(x), \quad (23)$$

the boundary problem is considered for (23). On  $\Gamma_1$  given Dirixle conditions, on  $\Gamma_2$  given Newman conditions.

Let's denote that  $0 \in \overline{\Gamma_1} \cap \Gamma_2$ .  $\Omega_R = \Omega \cap B_R(0)$ , In this case  $B_R(0) = \{x : |x| \leq R\}$ ;  $S(R) = \Omega \cap \partial\Omega_R$ .

Let's introduce the concept of main intersection  $S(R)$ , which characterizes the geometric structure of the boundary  $\partial\Omega$ :

$$\lambda_p^p(R) = \inf \left( \int_{S(R)} |\nabla_S \mathcal{G}|^p ds \right) \left( \int_{S(R)} |\mathcal{G}|^p ds \right)^{-1}, \quad (24)$$

here, the lower bound is taken for all functions that are continuously differentiable around a certain  $S(R)$  and become zero on  $\overline{S(R)} \cap \Gamma_1$ ;

Given the coefficients, we will assume that the following conditions are satisfied:  $x \in \Omega$  for  $\xi = \{\xi_\alpha : |\alpha| \leq m\} \in \mathbb{R}^m$ ,  $m$  is the number of multiindexes whose length does not exceed  $m$ , Functions  $A_\alpha(x, \xi)$  are as if discontinuous on  $\xi$  for all  $x \in \overline{\Omega}$ , measurable on  $x$  for all  $\xi$ , and satisfy the following inequality

$$\sum_{|\alpha|=m} A_\alpha(x, \xi) \xi_\alpha > C_1 \omega(x) \sum_{|\alpha|=m} |\xi_\alpha|^p - C_2 \omega(x) \sum_{|\alpha|=m} |\xi_\alpha|^p - f_1(x), \quad (25)$$

$$|A_\alpha(x, \xi)| \leq C_3 \omega(x) \sum_{|\alpha| \leq m} |\xi_\alpha|^{p-1} + f_2(x).$$

Let's determine the function  $\psi(r)$  expressed with the following inequalities in the interval  $(0, r_0)$ :

$$\inf_{r\psi(r) < |x| < r} \lambda_p(\rho)(r - r\psi(r)) \geq \mu > 0, \quad (26)$$

Here,  $\mu$  is such that  $0 < 1 - C_0 < \psi(r) < 1 \quad \forall \vartheta \quad s = \|x\|$

The following theorem has been proven.

**Theorem 10.** Let's assume that

$u(x, t) \in L_p \left( 0, T; \overset{\circ}{W}_p^m(\Omega_t) \right) \cap W_2^1(0, T; L_p(\Omega_t))$  is the general solution of the

mixed boundary value problem for equation (23) Assume that the coefficients of the equation satisfy conditions (25), the domain  $\Omega$  satisfy isoperimetric condition, and condition (A).

The weight function  $\omega(x)$  satisfies conditions (3), (4). Suppose that the  $\bar{\psi}$  is continuous, non-decreasing in the interval  $(0, r_0)$  and satisfying the inequality  $0 < 1 - C_0 < \bar{\psi}(r) \leq \psi(r) < 1$ , where the function  $\psi(r)$  is determined from the inequality (26) and suppose that

$$G \left( r \exp \left( -\frac{1 - \bar{\psi}(r)}{1 - C_0 - \theta} \right) \right) < C_1 \exp \left( -\theta \ln \omega^{-1} \int_r^{r_0} \frac{d\tau}{\tau(1 - \bar{\psi}(\tau))} \right) G(r_0), \quad (27)$$

is true, here  $C_1 > 0$ ,  $\theta < 1 - C_0$ ,  $\omega = \text{const} < 1$ . In that case, the following evaluation is correct for  $J(r)$ :

$$\begin{aligned} & J \left( r \exp \left( -\frac{1 - \bar{\psi}(r)}{1 - C_0 - \theta} \right) \right) < C_2(C_1, \nu) \times \\ & \times \exp \left( -\theta \ln(\omega + \nu)^{-1} \int_r^{r_0} \frac{d\tau}{\tau(1 - \bar{\psi}(\tau))} \right) (J(r_0) + G(r_0)), \quad \forall \nu > 0. \end{aligned} \quad (28)$$

Similarly, the following theorem is also true.

**Theorem 11.** Suppose that  $u(x, t) \in L_p \left( 0, T; \overset{\circ}{W}_{p, \omega}^m(\Omega_t) \right) \cap W_2^1(0, T; L_p(\Omega_t))$  is

the generalized solution of the boundary value problem for equation (23).

Assume that the coefficients of the equation satisfy conditions (25), the domain  $\Omega$  satisfy isoperimetric condition, and condition (B).

Suppose  $\bar{\varphi}(r)$  is non-decreasing in the interval  $(0, r_0)$  and satisfies the inequality  $\bar{\varphi}(r) < \varphi(r) \equiv r \tilde{\lambda}_p(r)$ . Let's assume that the

inequality

$$G(r) < C_1 \exp\left(-\frac{(1-d)^m}{2C_4 \ln d^{-1}} \int_r^{r_0} \frac{\bar{\varphi}^m(\tau) d\tau}{\tau}\right) G(r_0), \quad (29)$$

is satisfied.

In that case, the following evaluation is correct for  $J(r)$  :

$$J(r) < C_5 \exp\left(-\frac{(1-\nu)(1-d)^m}{2C_4 \ln d^{-1}} \int_r^{r_0} \frac{\bar{\varphi}^m(\tau) d\tau}{\tau}\right) (J(r_0) + G(r_0)), \quad \forall \nu > 0. \quad (30)$$

Based on the obtained a priori evaluations, it is possible to obtain an estimate for solving around the boundary point.

**Theorem 12.**

Suppose that  $u(x, t) \in L_p\left(0, T; \overset{\circ}{W}_{p, \omega}^m(\Omega_t)\right) \cap W_2^1(0, T; L_p(\Omega_t))$  is the generalized solution of the mixed boundary value problem for equation (23) and The domain  $0 \in \bar{\Gamma}_1 \cap \Gamma_2$ .  $\Omega$  satisfies the isoperimetric conditions furthermore, around the boundary of the origin point 0, the border is such that the inequality  $\forall r \in (0, r_0)$ ,  $\lambda^{(0)} > 0$  is satisfied for  $\lambda_p(r) > \lambda^{(0)} r^{-1}$ .

Then there exists  $\gamma_0 > 0$  such that if the estimate

$$G(r) < Ar^{\gamma_0 + \varepsilon} G(r_0), \quad \forall r \in (0, r_0), \quad A > 0, \quad (31)$$

is true for  $\varepsilon > 0$  small enough for  $r$ , then the following estimate is true if  $\forall x \in \Omega_{\bar{r}_0}$ ,  $\bar{r}_0 < r_0$ ,  $\delta > 0$  for  $G(r)$ .

$$|D^j u(x)| < C |x|^{m - \frac{n}{p} - j + \gamma_0} (J(R) + G(R))^{\frac{1}{p}}, \quad j = 0, 1, \dots, \left[m - \frac{n}{p}\right], \quad (32)$$

where  $\delta = m - \frac{n}{p} - \left[m - \frac{n}{p}\right]$ ,  $m - \frac{n}{p} \geq 0$ , The constant  $C$  – is independent of  $u(x)$ , In case of  $\delta = 0$ , the correctness of estimation (32) is proved only for  $j = 0, 1, \dots, \left[m - \frac{n}{p}\right] - 1$

## CONCLUSION

The following main results were obtained in the dissertation:

- A priori estimates of energy integrals in different classes of bounded non-smooth domains analogous to Saint-Venant's principle.
- Evaluations of the behavior of solutions around a boundary point.
- Examples of various areas are given and the accuracy of calculations for those areas is shown.
- Valuations on co-authorship classes of unbounded domains with non-compact domains.
- Phragmen-Lindelof type theorems in unbounded domains. According to Saint-Venant-type energy evaluations, the property elimination conditions for the solution.
- Examples of the accuracy of the assessment of the behavior of solutions in different classes of regions.
- Behavior of generalized solutions of mixed boundary value problems for parabolic equations.
- The obtained results are also new for the class of linear equations.

**The main content of the dissertation has been published in the following works:**

1. Gadjiev T.S., Sadigova N.R., Mamedova K.N. Behaviour of solution degenerate elliptic equations. / The 4<sup>th</sup> Congress of the Turkic World Mathematical Society (TÜMS) Baku, Azerbaijan, 1-3 July, 2011, p.56.
2. Gadjiev T.S., Mamedova K.N. On behavior of solutions of higher order degenerate parabolic equations // Visnyk of the Lviv Univ. Series Mech. Math. 2011. Issue 74. p.41-46.
3. Gadjiev T.S., Mamedova K. Behaviour of solution to Degenerate parabolic equations / Dedicated to the 70 th Anniversary of the Georgian National Academy of Sciences the 120 th Birthday of its First President Academician Nikoloz (Niko) Muskhelishvili, September 15-19, 2011, Batumi, Georgia, p.95
4. Gadjiev T.S., Mamedova K.N. On behavior of solutions degenerate parabolic equations. // Transactions of NAS of Azer-baijan, 2012, vol. XXXII, No 4, pp. 43-50.
5. Mamedova K.N. On behavior of solutions of higer order degenerate parabolic equations in nonsmooth bounded domains. // Proceedings of IMM of NAS of Azerbaijan, 2012, vol. XXXVI (XLIV), pp. 69-74.
6. Gadjiev T.S., Sadigova N.R., Mamedova K.N. Behavior of solution degenerate elliptic and parabolic equations. /International Conference Mathematical Analysis Diferensial Equations and their Applications, 04-09 September, 2012,p.48, Mersin, Turkey.
7. Gadjiev T.S., Sadigova N.R., Mamedova K.N. The remo-vable of solution degenerate elliptic and parabolic equations. / IV annual conference of the Georgian Mathematical Union Dedicated to academician Victor Kupradze on his 110-th birthday anniversary and Georgian Mathematical Union on his 90-th year from founding September 9-15, 2013, in Tbilisi and Batumi, Georgia, p. 96.



8. Mamedova K.N. Removable singularity of solution degenerate nonlinear parabolic equations.// Proceedings of IMM of NAS of Azerbaijan, 2013, vol. XXXVIII (XLVI), pp. 99-102.
9. Mamedova K.N., Zulfəliyeva G. Elliptik parabolic tənliklərin həllərinin keyfiyyət xüsusiyyətləri. / Riyaziyyat Mexanika İnstitutu və SDU-nun keçirdiyi elmi konfrans. 2017. s.68
10. Məmmədova K. Qeyri xətti elliptik-parabolik tip tənliklərin həllərinin təbiiqi. Naxçıvan Dövlət Universiteti. Elmi əsərlər, № 8 (89), səh.29-31, 2017
11. Məmmədova K. Sərhəd nöqtəsi ətrafında qeyri-xətti parabolik tənliyin həllinin tədqiqi. Naxçıvan Dövlət Universiteti. Elmi əsərlər, № 4(93), səh 62-64, 2018
12. Мамедова К. Поведение решений задачи Дирихле для некоторого класса недивергентных линейных уравнений. Нах-чыванский Государственный университет. Научные труды, № 8(97), стр. 20-25, 2018
13. Мамедова К. Теоремы типа Фрагмена-Линдельёфа для решений нелинейных параболических уравнений в неограниченных областях. Bakı Universitetinin xəbərləri (fizika-riyaziyyat elmləri seriyası, № 4 səh 71-75, 2018
14. Mamedova K. Uniqueness classes of generalized solutions for degenerate parabolic equations in unbounded non-cylindrical domains. Trans. Nat. Acad. Sci. Azerb. Ser. Phys-Tech, Math Sci, Issue mech, 39 (7), pp. 18-22, 2019
15. Mamedova K. The behaviour of generalized solutions degenerate parabolic equations in nonregular domains. International Conference Modern Problems of Mathematics and Mechanics, 23-25 October 2019, Baku
16. Mamedova K. T.Gadjiev. The behavior of solutions of nonlinear degenerate parabolic equations in nonregular domains and removability of singularity on boundary. Trans. Nat. Acad. Sci. Azerb. Ser. Phys-Tech, Math Sci, Issue math, 40 (4), pp. 1-9, 2020

17. T.S.Gadjiev, G.M.Zulfaliyeva, K.N.Mamedova. The solvability degenerate elliptic-parabolic problem with nonlinear boundary conditions. Trans. Nat. Acad. Sci. Azerb. Ser. Phys-Tech, Math Sci, Issue math, 42 (1), 1-7 (2022).
18. Mamedova K. Removable of Compacts Sets of the solition of Nonlinear parabolic equations. // The 9th International Conference on Control and Optimization with Industrial Applications, 27-29 August 2024, İstanbul, Türkiye.

A handwritten signature in blue ink, appearing to be 'M. Mamedova', is written over the text of the second reference.

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