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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

QUALITATIVE PROPERTIES OF THE SOLUTIONS TO THE BOUNDARY VALUE PROBLEMS FOR THE COMPOZITE AND MIXED TYPE EQUATIONS

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GENERAL CHARACTERISTICS OF THE WORK

Relevance of the topic and degree of study. It is known that problems for both mixed and composite type equations began to be considered at the beginning of the last century. The relevance of the problems considered for mixed type equations is that these types of equations are mainly related to the phenomenon of the velocity of the moving body exceeding the speed of sound. Thus, when the velocity is less than the speed of sound, the motion is given by the elliptic type equation, and when this velocity (body velocity) exceeds the speed of sound, then by a hyperbolic type equation. The Trikomi equation or the Lavrentyev-Bitsadze equation (this equation has a constant coefficient) explains such phenomena. Problems for mixed type equations were started by F.G.Trikomi [1] and continued by A.V.Bitsadze[2]. Later, these problems and problems for the composite type equations initiated by J.S.Adamar [3] were mainly studied by A.V.Bitsadze's students T.D.Curayev[4], A.M.Nakhusheva[5] and others. Problems for mixed and composite type equations were studied by M.S. Salahaddinov[6], T.D.Curayev[7] and others. Later, problems for such equations (within the framework of boundary conditions that hold non-local and global limits) were considered by N.A. Aliyev

[1] Trikomi, F. Ulteriori richerche sullequazione $yz_{xx} + z_{yy} = 0$, Rendiconti del Cirolo Mathematico di Palermo, S 2, –1928, –pp.63-90.

[2] Бицадзе А.В. К проблеме уравнений смешанного типа // Труды Математического института имени ВА Стеклова. – 1953. – Т. 41. – №. 0. – с.3-59.

[3] Hadamard, J. Propriétés d'une équation linéaire aux dérivées partielles du quatrième ordre //Tohoku Mathematical Journal, First Series. – 1933. – 37. – pp.133-150.

[4] Джураев, Т.Д. Об одной краевой задаче для уравнения составного типа // Док. АН Узб. ССР. –1962, –№4, –с.5-8.

[5] Нахушева, В.А. Краевые задачи для уравнения теплопроводности смешанного типа II Докл. Адыгской Международной академии наук, –2010, –т.12, –№2, –с.39-45.

[6] Салахитдинов, М.С. К вопросу о существование рещений краевых задач для уравнения смещанно-состовного типа // Изв, АН Узб.ССР, сер.физ.мат.наук – 1966, –№4, –с.17-20.

[7] Джураев, Т.Д. О некоторых краевых задачах для уравнения смешанно-составного типа //Сибирский математический журнал. – 1963. – Т. 4. – №. 4. – с. 775-787.

and M. Jahanshahi[8], A.M. Aliyev and A.Y. Garegeshlagi[9] and others.

A mixed-type equation is understood to be an equation that is elliptic in one part of a given domain, hyperbolic in the rest, and parabolic in their common boundary. A composite type equation is understood to be an equation that has both ellipticity and hyperbolicity properties at each point of the considered domain. In classical works, a composite type equation is an equation derived from the derivative of the Laplace equation. Thus, while the Laplace equation is of the elliptic type at each point, the derivative derived from the Laplace equation is of the hyperbolic type at each point. The representative of the mixed type is the Tricomi or Lavrentiev-Bitsadze equation.

Some practical problems are described by boundary value problems for differential equations of the third order of mixed and composite type of the third order, obtained by taking derivatives of the Cauchy-Riemann equation. Of particular interest for such problems is the case when the boundary conditions contain non-local and global terms.

The dissertation work is devoted to investigation of such problems. The study of such problems does not allow us to apply some classical methods. When studying these problems, we use the methods developed by N.A. Aliyev and others. Thus, unlike classical works, not the Green's function, but the fundamental solution of the equation or the adjoint equation was used. With the help of the fundamental solution, the necessary conditions were investigated. These are conditions that an arbitrary solution of the equation under consideration satisfies. The singularities that are not subject to the general situation in these necessary conditions are regularized in a unique way with the help of the given boundary conditions.

Object and subject of the research. The study of solutions of

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[8] Aliev, N., Jahanshahi, M. Sufficient conditions for reduction of the BVP including a mixed PDE with non-local boundary conditions to Fredholm integral equations //International Journal of Mathematical Education in Science and Technology. – 1997. – 28(3). – pp. 419-425.

[9] Gharehgheshlaghi, A.Y.D., Aliyev, N.A. A Problem for a Composite Type Equation of Third Order with General Linear Boundary Conditions //Transact. NAS Azerbaijan. Baku, – 2009. №4, – 29. – pp. 36-46.

nonlocal and global boundary condition problems for mixed, composite, mixed and composite type equations with global and non-local boundary conditions is a rather urgent problem in the theory of partial differential equations. When the elliptic part of the considered problem is of the first order, even the formulation of the problem differs from the boundary value problems posed in classical works. For this reason, it is necessary to seriously change many classical statements. Therefore, the study of boundary value problems for Cauchy-Riemann type elliptic equations, as well as composite and mixed type third-order partial differential equations with their second-order pure or mixed derivatives, is of particular interest.

Aims and objectives of the research. The main aim of the dissertation work is to study the boundary problems with boundary conditions containing non-local and global terms for some mixed and composite type equations. This includes studying the solution of the problem within the non-local boundary condition for the first-order mixed type equation, and boundary conditions for the third-order joint type equation obtained from pure and mixed derivatives of the Cauchy-Riemann equation, which contain non-local and global limits. In this case, one of the main problems is to obtain necessary conditions for the solutions of the above mentioned problems and regularize the singularities that are not subject to the general case under these necessary conditions, and to investigate the problems posed by reducing them into the system of Fredholm-type integral equations of the second kind with a regular kernel.

Research methods. The methods of the theory of differential and integral equations, the theory of functions of complex variables, and functional analysis were used in the dissertation work.

Main results presented for the defense. The following main propositions are put forward for the defense:

1. Study of problems with non-local boundary conditions, not problems considered within local boundary conditions for mixed and composite type equations.

2. Investigation of solutions of boundary problems with boundary conditions containing non-local and global terms for the first-order mixed type equations and Tricomi equations.

3. Investigation of solutions of problems with non-local and global boundary conditions for the third-order composite type equations obtained from the second-order pure derivative and mixed derivative of the Cauchy-Riemann equation.

4. Investigation of the solution of the problem with non-local boundary condition for the third-order partial derivative equation belonging to both mixed and composite types.

5. Finding the basic relations and necessary conditions that express an arbitrary solution of the considered equation and its derivatives in terms of boundary values. Regularization of singularities in necessary conditions and finding sufficient conditions for ensuring the Fredholm property of the problem using the obtained regular expressions and boundary conditions.

Scientific novelty of the research: The following main results were obtained in the dissertation work.

1. The solutions of the problems with boundary conditions containing nonlocal and global termsts for the first-order mixed-type equation and the Tricomi equation were investigated. The basic relations expressing the arbitrary solution of each equation and its derivatives by boundary values were established, and the necessary conditions were derived from the basic relations. The singular expressions in the necessary conditions were regularized.

2. The solutions of nonlocal and global boundary value problems for the third-order composite type equations obtained from the second-order pure derivative and mixed derivative of the Cauchy-Riemann equation were investigated. The basic relations and necessary conditions were derived, and the singularities in these conditions were regularized.

3. The solution of the nonlocal boundary value problem for the third-order partial derivative equation belonging to both mixed and composite types was investigated.

4. Sufficient conditions for the Fredholm property of the considered boundary value problems were obtained.

Theoretical and practical significance of the research. The results of the thesis, which is of a theoretical nature, can also be used to obtain approximate solutions. In addition, the results obtained in the thesis can be used in some applied problems related to the phenomenon of the speed of a moving object exceeding the speed of sound.

Approval and implementation. The main results of the thesis were presented at the seminars of the Department of “Mathematics and Informatics” of Lankaran State University; Republican scientific conference “Problems of development of natural and humanitarian sciences” dedicated to the 94th anniversary of the national leader Heydar Aliyev (Lankaran, 2017); international scientific conference “Theoretical and applied problems of mathematics” (Sumgayit, 2017); Republican scientific young researchers conference “Modernizing Azerbaijan: A new stage of growth” (Lankaran, 2017); Republican scientific conference “Integration and current problems of science in the modern world” (Lankaran, 2017); International conference on “Sustainable development and actual problems of humanitarian sciences" dedicated to the 95th anniversary of the National Leader Haydar Aliyev (Bakı, 2018); XXXI International Conference “Problems of decision making under uncertainties” PDMU-2018 (Baku, Lankaran, 2018); Republican scientific young researchers conference “Tasks facing Azerbaijani science in an integration environment” (Lankaran, 2018); University scientific conference “A new stage in the development of mathematical science” dedicated to the 80th anniversary of prof. Nihan Aliyev (Lankaran, 2018); Republican scientific-practical conference on “Ways of applying scientific innovations in the educational process” dedicated to the 96th anniversary of the national leader Heydar Aliyev (Lankaran, 2019); Republican scientific-practical conference of young researchers “The impact of the application of modern educational technologies on the quality of education” (Lankaran, 2019). Also, reports were made abroad at international scientific conferences: XXIX International Conference “Problems of decision making under uncertainties” PDMU-2017 (Mukachevo, Kyiv, Ukraine, 2017); XXXVI International Conference Problems of Decision Making under Uncertainties PDMU-2021 (Skhidnytsia, Kyiv, Ukraine, 2021).

Personal contribution of the author. All obtained in the thesis results and propositions belong to the author.

Author's publications. 8 research papers (2 of them are included in the Zentralblatt MATH database, 1 published abroad), 12 conference materials (20 works in total) were published in the journals recommended by the Higher Attestation Commission under the President of the Republic of Azerbaijan. The list of works is given at the end of the abstract.

Name of the organization where the dissertation work was carried out. The dissertation work was carried out at the Department of Mathematics and Informatics of the Lankaran State University.

Total volume of the thesis in characters, indicating the volume of the structural sections of the thesis separately. The thesis consists of an introduction, three chapters, a conclusion and a list of 141 references. The total volume of the work: 221961 characters (title page – 346 characters, table of contents – 1499 characters, introduction – 42683 characters, Chapter I – 68199 characters, Chapter II – 46298 characters, Chapter III – 61834 characters, Conclusion – 1102 characters).

CONTENT OF THE THESIS

The introduction substantiates the relevance of the dissertation topic, indicates the purpose of the work, scientific novelty, theoretical and practical significance, and identifies the main results for defense. It also provides information on the structure of the thesis.

The first chapter, entitled "Investigating the solution of problems within non-local and global boundary conditions for mixed type equations", is devoted to the study of solving general linear boundary value problems for such types of equations.

In the first paragraph, entitled "Investigation of the solution of a problem with boundary conditions containing non-local and global terms for the first-order mixed-type equation in an arbitrary domain", the solution of the problem within the general linear boundary condition for this type of first-order in an arbitrary domain is investigated.

The following boundary value problem is considered

$$\frac{\partial u_s(x)}{\partial x_2}+\sqrt{(-1)^s}\,\frac{\partial u_s(x)}{\partial x_1}= f_s(x),\ x=(x_1,x_2)\in D_s\subset R^2,\quad s=1,2; \tag{1}$$

$$\sum_{s=0}^{1}\{[\alpha_{k1}^{(s)}(x_1)u_1(x_1,s\gamma_1(x_1))+\alpha_{k2}^{(s)}(x_1)u_2(s\sigma(x_1)+$$

$$+(1-s)x_1,s\gamma_2(\sigma(x_1))]+\int_{a_1}^{b_1}[\beta_{k1}^{(s)}(x_1,t)u_1(t,s\gamma_1(t))+$$

$$+\beta_{k2}^{(s)}(x_1,t)u_2(t,s\gamma_2(t))]dt+\int_D K_{ks}(x_1,\xi)u_s(\xi)d\xi\}=\alpha_k(x_1)\ k=1,2;$$

$$x_1\in[a_1,b_1]. \tag{2}$$

Here $D=D_1\cup D_2$ is a bounded and convex in the direction x_2 domain, the boundary $\Gamma=\Gamma_1\cup\Gamma_2$ is a Lyapunov line and the curve Γ_k is defined by the equation $x_2=\gamma_k(x_1)$, $a_1\le x_1\le b_1, k=1,2$. The considered equaton is the first order, elliptic equation (Cauchy-Riemann equation) in the domain D_1 and is the first oreder hyperbolic equation in the domain D_2 . Boundary conditions (2) are not linearly dependent. The right hand side of the equation $f_s(x)$, $x\in D_s$, $s=1,2$ is a given continuous function. The functions $\alpha_{kj}^{(s)}(x_1), k=1,2;\ j=1,2;\ s=\overline{0,2};$ $\beta_{kj}^{(s)}(x_1,t)$, $k=1,2;$ $j=1,2;$ $s=\overline{0,2};$ $K_{kj}(x_1,\xi)$, $k=1,2;$ $j=1,2;$ və $\alpha_k(x_1)$, $k=1,2;$ in the boundary conditions for $x_1\in[a_1,b_1]$, $t\in[a_1,b_1]$ and $\xi\in D$ are given continuous functions.

We used the fundamental solutions

$$U_1(x-\xi)=\frac{1}{2\pi}\cdot\frac{1}{x_2-\xi_2+i(x_1-\xi_i)}$$

and

$$U_2(x-\xi)=e(x_2-\xi_2)\delta(x_1-\xi_1-(x_2-\xi_2))$$

of equation (1), where

$$e(t)=\begin{cases}\frac{1}{2} & t>0,\\ 0 & t=0,\\ -\frac{1}{2} & t<0,\end{cases}$$

is Heviside's symmetric unit function and $\delta(t)$ is Dirac's "delta" function.

By the help of these fundamental solutions we obtain the following main relations

$$\int_{\partial D_1} u_1(x) U_1(x-\xi)[\cos(\nu_1, x_2) + i\cos(\nu_1, x_1)]dx -$$

$$- \int_{D_1} f_1(x) U_1(x-\xi) dx = \begin{cases} u_1(\xi), \xi \in D_1 \\ \frac{1}{2} u_1(\xi), \xi \in \partial D_1 \end{cases} \tag{3}$$

and

$$\int_{\partial D_2} u_2(x) U_2(x-\xi)[\cos(\nu_2, x_2) + i\cos(\nu_2, x_1)]dx -$$

$$- \int_{D_2} f_2(x) U_2(x-\xi) dx = \begin{cases} u_2(\xi), \xi \in D_2 \\ \frac{1}{2} u_2(\xi), \xi \in \partial D_2 \end{cases}. \tag{4}$$

By ν_s we define the outward normal to the boundary ∂D_s $s = 1,2;$ of the domain D_s. From (3) and (4) we obtain the necessary conditions as follows

$$u_1(\xi_1, \gamma_1(\xi_1)) = \frac{i}{\pi} \int_{a_1}^{b_1} \frac{u_1(x_1, \gamma_1(x_1))}{x_1 - \xi_1} dx_1 - \ldots \tag{5}$$

$$u_1(\xi_1, 0) = -\frac{i}{\pi} \int_{a_1}^{b_1} \frac{u_1(x_1, 0)}{x_1 - \xi_1} dx_1 + \ldots \tag{6}$$

$$u_2(\xi_1, 0) = u_2(\sigma(\xi_1), \gamma_2(\sigma(\xi_1))) - \int_{a_1}^{b_1} f_2(x_1, x_1 - \xi_1) dx_1, \tag{7}$$

$$u_2(\xi_1, \gamma_2(\xi_1)) = u_2(\xi_1 - \gamma_2(\xi_1), 0) -$$

$$- \int_{a_1}^{b_1} f_2(x_1, x_1 - \xi_1 + \gamma_2(\xi_1)) e(x_1 - \xi_1) dx_1, \tag{8}$$

here and further by (...) is defined the sum of the non singular terms and by $\gamma_k(x_1)$ is defined the parts $k = 1,2$ of the boundary D_k (lower boundary of D_1 and upper boundary of D_2). By $x_1 = \sigma(\xi_1)$ we define the solution of the equation $x_1 - \xi_1 - \gamma_2(x_1) = 0,$

$x_1 \in [a_1, b_1]$, $\xi_1 \in [a_1, b_1]$. We introduce the following function:

$$\alpha(x_1) = \left[\alpha_1(x_1) - \alpha_{12}^{(2)}(x_1) \int_{a_1}^{b_1} f_2(\eta_1, \eta_1 - x_1) d\eta_1 - \right.$$

$$\left. - \int_{a_1}^{b_1} \beta_{12}^{(2)}(x_1, \sigma(\tau)) \sigma'(\tau) d\tau \int_{a_1}^{b_1} f_2(\eta_1, \eta_1 - \tau) d\eta_1 \right] \times$$

$$\times \left[\alpha_{22}^{(0)}(x_1) + \alpha_{22}^{(2)}(x_1) \right] - \left[\alpha_2(x_1) - \alpha_{22}^{(2)}(x_1) \times \right.$$

$$\times \int_{a_1}^{b_1} f_2(\eta_1, \eta_1 - x_1) d\eta_1 - \int_{a_1}^{b_1} \beta_{22}^{(2)}(x_1, \sigma(\tau)) \sigma'(\tau) d\tau \times$$

$$\times \int_{a_1}^{b_1} f_2(\eta_1, \eta_1 - \tau_1) d\eta_1 \left[\alpha_{12}^{(0)}(x_1) + \alpha_{12}^{(2)}(x_1) \right].$$

This paragraph also constructs the following linear combination with the help of singular necessary conditions:

$$\alpha_1(\xi_1) u_1(\xi_1, \gamma_1(\xi_1)) - \alpha_0(\xi_1) u_1(\xi_1, 0) = \frac{i}{\pi} \int_{a_1}^{b_1} \frac{\alpha(x_1) dx_1}{x_1 - \xi_1} + \ldots . \qquad (9)$$

The singular terms at the right hand of the obtained expression become regular after changing the order of the integrals. Since the rest of the first trems does not include an unknown function, these integrals exist in the main sense of Cauchy.

Using conditions (9) and boundary conditions (2), we obtain that $\alpha_s(x_1) \neq 0,\ s = 0,1;$ is from the Holder class with positive index. If the conditions $\alpha(x_1) \in C^{(1)}[a_1, b_1],\ \alpha(a_1) = \alpha(b_1) = 0$ hold true then the Fredholm type of problem (1)-(2) is provided.

More precisely, the following theorem is true:

Theorem 1. *If $D = D_1 \cup D_2$ is a plane convex bounded domain in the direction x_2, the boundary $\Gamma = \Gamma_1 \cup \Gamma_2$ is a Lyapunov line, the functions $f_s(x),\ x \in D_s\ s = 1,2;$ are continuous, all data of boundary condition* (2) *are continuous, $\alpha_s(x_1) \neq 0,\ s = 0,1;$ are from Holder class with positive index, the conditions $\alpha(x_1) \in C^{(1)}[a_1, b_1],$*

$\alpha(a_1)=\alpha(b_1)=0$ *hold true, then problem* (1)-(2) *is of Fredholm type.*

In the second section of this chapter, the following boundary value problem for a homogeneous equation of mixed type of the first order is considered in the combination of a parallelogram and a rectangle:

$$\frac{\partial u_s(x)}{\partial x_2}+i\sqrt{(-1)^s}\,\frac{\partial u_s(x)}{\partial x_1}=0,\ x=(x_1,x_2)\in D_s,\ s=1,2; \tag{10}$$

$$\alpha_{k,-1}^{(1)}(t)u_1(t+1,-1)+\alpha_{k,0}^{(1)}(t)u_1(t,0)+$$
$$+\alpha_{k,0}^{(2)}(t)u_2(t,0)+\alpha_{k,1}^{(2)}(t)u_2(t,1)=\alpha_k(t),\quad k=1,2;\ t\in[-1;1], \tag{11}$$

$$\beta_{11}(t)u_1(-t,t-1)+\beta_{12}(t)u_1(2-t,t-1)=\beta_1(t),\ t\in[0,1], \tag{12}$$

$$\beta_{11}(t)u_2(-1,t-1)+\beta_{12}(t)u_2(1,t-1)=\beta_1(t),\ t\in[1,2], \tag{13}$$

where $i=\sqrt{-1}$, conditions (11) are linearly independent, and all functions involved in (12), (13) are continuous functions.

It is known that if take $s=2$ in (10) then the fundamental solution of the Cauchy-Riemann equation is

$$U_2(x-\xi)=\frac{1}{2\pi}\cdot\frac{1}{x_2-\xi_2+i(x_1-\xi_i)},$$

for $s=1$ (in the direction x_2) is

$$U_1(x-\xi)=\theta(x_2-\xi_2)\delta(x_1-\xi_1+x_2-\xi_2).$$

Here $\theta(t)$ is Hevisides's unit function and $\delta(t)$ is Dirac's "delta" function.

It is easy to see that (as in the second Green's formula) the following main relations are obtained by using the these fundamental solutions and equations (10):

$$-\int_0^2 u_1(t,-1)\theta(-1-\xi_2)\delta(t-\xi_1-1-\xi_2)dt+$$
$$+\int_{-1}^1 u_1(t,0)\theta(-\xi_2)\delta(t-\xi_1-\xi_2)dt=\begin{cases}u_1(\xi), & \xi\in D_1,\\ \frac{1}{2}u_1(\xi), & \xi\in\partial D_1,\end{cases}$$

$$-\frac{1}{2\pi}\int_{-1}^{1}\frac{u_2(t,0)}{-\xi_2+i(t-\xi_1)}dt+\frac{1}{2\pi}\int_{-1}^{1}\frac{u_2(t,1)}{1-\xi_2+i(t-\xi_1)}dt-$$
$$-\frac{i}{2\pi}\int_{0}^{1}\frac{u_2(-1,t)}{t-\xi_2+i(-1-\xi_1)}dt+\frac{i}{2\pi}\int_{0}^{1}\frac{u_2(1,t)}{t-\xi_2+i(1-\xi_1)}dt$$
$$=\begin{cases}u_2(\xi), & \xi\in D_2,\\ \frac{1}{2}u_2(\xi), & \xi\in\partial D_2,\end{cases}$$

The second parts of the obtained main relations (the parts belonging to the boundary) are necessary conditions.

Thus from basing relation we obtain

$$u_1(\xi)=u_1(\xi_1+\xi_2,0),\quad \xi\in D,$$
$$u_1(\tau,-1)=u_1(\tau-1,0),\quad \tau\in[0,2],$$
$$u_1(2(k-1)-\tau,\tau-1)=u_1((-1)^k,0)=const,\quad \tau\in[0,1],\ k=1,2$$
$$u_2(\tau,k)=\frac{(-1)^k i}{\pi}\int_{-1}^{1}\frac{u_2(t,k)}{t-\tau}dt+\ldots\ \tau\in[-1,1],\ k=0,1;$$
$$u_2((-1)^k,\tau)=\frac{(-1)^k i}{\pi}\int_{0}^{1}\frac{u_2((-1)^k,t)}{t-\tau}dt+\ldots,\ \tau\in[0,1],\ k=0,1;$$

In the necessary conditions derived from the elliptical part, singularities are already separated. These singularities, which are not in general condition, are regulated in a special way.

Let

$$\Delta(\tau)=\begin{vmatrix}\Delta_{11}(\tau) & \Delta_{12}(\tau) & \Delta_{13}(\tau)\\ \Delta_{21}(\tau) & \Delta_{22}(\tau) & \Delta_{23}(\tau)\\ \Delta_{31}(\tau) & \Delta_{32}(\tau) & \Delta_{33}(\tau)\end{vmatrix}$$

where

$$\Delta_{k1}(\tau)=\alpha_{k,-1}^{(1)}(\tau)+\alpha_{k,0}^{(1)}(\tau),\ k=1,2;\ \Delta_{kj}(\tau)=\alpha_{kj-2}^{(2)}(\tau),\ k=1,2;\ j=2,3;$$
$$\Delta_{3j}(\tau)=(-1)^j\alpha_{1,j-2}^{(2)}(\tau)\left[\alpha_{2,0}^{(1)}(\tau)+\alpha_{2,-1}^{(1)}(\tau)\right]-\alpha_{2,j-2}^{(2)}(\tau)\left[\alpha_{1,0}^{(1)}(\tau)+\alpha_{1,-1}^{(1)}(\tau)\right],$$
$$j=2,3;\ \Delta_{31}(\tau)=0\,.$$

Theorem 2. *If in equation* (10) $i=\sqrt{-1}$, *all data of conditions* (11)-(13) *are continuous functions and satisfy*

$$\alpha_k(t) \in C^{(1)}[-1,1], \alpha_k(-1) = \alpha_k(1) = 0, k = 1,2, \sum_{j=1}^{2}\sum_{k=-1}^{0}\alpha_{j,k}^{(1)}(t) = 0,$$

$$\alpha_{k,0}^{(1)}(t) + \alpha_{k,-1}^{(1)} \in H^{(\mu)}[-1,1], \mu \in (0,1), \beta_1(t) \in C^{(1)}[1,2],$$

$$\beta_1(1) = \beta_1(2) = 0 \ ,$$

$\beta_{11}(t)$ *and* $\beta_{12}(t)$ *for* $t \in [1,2]$ *belong to the Holder class, the conditions*

$$\Delta(\tau) = \begin{vmatrix} \Delta_{11}(\tau) & \Delta_{12}(\tau) & \Delta_{13}(\tau) \\ \Delta_{21}(\tau) & \Delta_{22}(\tau) & \Delta_{23}(\tau) \\ \Delta_{31}(\tau) & \Delta_{32}(\tau) & \Delta_{33}(\tau) \end{vmatrix} \neq 0,$$

$$\Delta_1(\tau) = \begin{vmatrix} \beta_1(\tau+1) & \beta_2(\tau+1) \\ -\beta_1(\tau+1) & \beta_2(\tau+1) \end{vmatrix} = 2\beta_1(\tau+1)\beta_2(\tau+1) \neq 0, \ \tau \in [0,1],$$

are satisfied, then the boundary value problem (10)-(13) *is Fredholm property.*

The third paragraph of the first chapter is devoted to the study of the solution of the boundary value problem for Tricomi equation with non-local and global boundary conditions. Let's consider the following problem

$$x_2 \frac{\partial^2 u(x)}{\partial x_1^2} + \frac{\partial^2 u(x)}{\partial x_2^2} = 0 \ \ x = (x_1, x_2) \in D \subset R^2, \tag{14}$$

$$\sum_{k=1}^{2}\left\{\sum_{j=1}^{2}\alpha_{ij}^{(k)}(x_1)\frac{\partial u(x)}{\partial x_j}\bigg|_{x_2=\gamma_k(x_1)} + \alpha_{i0}^{(k)}(x_1)u(x)\bigg|_{x_2=\gamma_k(x_1)} + \right.$$

$$+\sum_{j=1}^{2}\int_{a_1}^{b_1} K_{ij}^{(k)}(x_1,t_1)\frac{\partial u(t)}{\partial t_j}\bigg|_{t_2=\gamma_k(t_1)} dt_1 +$$

$$\left. + \int_{a_1}^{b_1} K_{i0}(x_1,t_1)u(t)\big|_{t_2=\gamma_k(t_1)} dt_1 \right\} = \alpha_i(x_1), \ x_1 \in [a_1, b_1], \ i = 1,2, \tag{15}$$

Here D is convex in the direction x_2 bounded domain, with Lyapunov line boundary $\Gamma = \partial D$, the data of condition (15) are continuous functions. Equations of the parts where the boundary is divided during the projection are

$$x_2=\gamma_k(x_1),\ k=1,2;\ x_1\in[a_1,b_1],\ 0<\gamma_1(x_1)<\gamma_2(x_1).$$

In this paragraph, using the first main relationships obtained from the fundamental solution

$$U(x-\xi)=\frac{1}{2\pi}\ln\sqrt{(x_1-\xi_1)^2+\frac{4}{9}(x_2-\xi_2)^3}.$$

of the Tricomi equation and the second Green formula, the necessary conditions

$$u(\xi_1,\gamma_1(\xi_1))=\frac{1}{\pi}\int_{a_1}^{b_1}\gamma_1(x_1)\gamma_1'(x_1)u(x_1,\gamma_1(x_1))\times$$

$$\times\frac{1}{1+\frac{4}{9}\gamma_1'^3(\sigma_1)(x_1-\xi_1)}\cdot\frac{dx_1}{x_1-\xi_1}+\ldots,$$

$$u(\xi_1,\gamma_2(\xi_1))=-\frac{1}{\pi}\int_{a_1}^{b_1}\gamma_2(x_1)\gamma_2'(x_1)u(x_1,\gamma_2(x_1))\times$$

$$\times\frac{1}{1+\frac{4}{9}\gamma_2'^3(\sigma_2)(x_1-\xi_1)}\cdot\frac{dx_1}{x_1-\xi_1}+\ldots,$$

are established. Then, the following necessary conditions are obtained from the basic relations obtained with the help of the derivative of the fundamental solution

$$\left.\frac{\partial u(\xi)}{\partial\xi_p}\right|_{\xi_2=\gamma_q(\xi_1)}=\frac{-(-1)^p}{\pi}\int_{a_1}^{b_1}\gamma_q(x_1)\left.\frac{\partial u(x)}{\partial x_p}\right|_{x_2=\gamma_q(x_1)}\times$$

$$\times\frac{\gamma_q'(x_1)}{(x_1-\xi_1)+\frac{4}{9}\gamma_q'^3(\sigma_1)(x_1-\xi_1)^2}dx_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_{3-p}}\right|_{x_2=\gamma_q(x_1)}\frac{1}{(x_1-\xi_1)+\frac{4}{9}\gamma_q'^3(\sigma_1)(x_1-\xi_1)^2}dx_1+\ldots,\ p,q=1,2,$$

are founded.

In this paragraph, under certain conditions, the Fredholm property of problem (14)-(15) is also proved.

The last, fourth paragraph of the first chapter deals with the following boundary value problem:

$$\frac{\partial u_1(x)}{\partial x_2}+\frac{\partial u_1(x)}{\partial x_1}=0,\ x_1<0,\ x_2\in(0,1), \tag{16}$$

$$\frac{\partial u_2(x)}{\partial x_2}+i\,\frac{\partial u_2(x)}{\partial x_1}=0,\ x_1>0,\ x_2\in(0,1), \tag{17}$$

$$u_1(x_1,1)+\alpha_1 u_1(x_1,0)=\varphi_1(x_1),\ x_1\le 0, \tag{18}$$

$$u_2(x_1,1)+\alpha_2 u_2(x_1,0)=\varphi_2(x_1),\ x_1\ge 0, \tag{19}$$

$$u_1(0,x_2)=u_2(0,x_2),\ x_2\in[0,1], \tag{20}$$

$$u_1(-\infty,x_2)+\alpha_0 u_2(\infty,x_2)=\varphi_0(x_2),\ x_2\in[0,1], \tag{21}$$

where

$$i=\sqrt{-1},\ D_1=\{(x_1,x_2):x_1<0,x_2\in(0,1)\},$$
$$D_2=\{(x_1,x_2):x_1>0,x_2\in(0,1)\},$$

$\alpha_k,\ k=\overline{0,2}$ – are given constant numbers, $\varphi_1(x_1)$, $\varphi_2(x_1)$ and $\varphi_0(x_1)$ are given continuous functions (correspondingly for $x_1\le 0$, $x_1\ge 0$ and $x_2\in[0,1]$).

In this paragraph by means of fundamental solutions of equations (16) and (17) in the direction x_2

$$U_1(x-\xi)=\theta(x_2-\xi_2)\delta(x_1-\xi_1-(x_2-\xi_2)),$$
$$x_1,\xi_1<0;\ x_2,\xi_2\in(0,1),$$
$$U_2(x-\xi)=\theta(x_2-\xi_2)\delta(x_1-\xi_1-i(x_2-\xi_2)),$$
$$x_1,\xi_1>0;\ x_2,\xi_2\in(0,1),$$

the main relations for problem (16)-(21) are obtained, where $\theta(t)$ is Heviside's unit function and $\delta(t)$ is Dirac's "delta" function. From the main relations the necessary conditions were found and the singularities in the same conditions were regularized.

In the second chapter of the dissertation, solutions of boundary value problem with non-local and global conditions for the third-order composite type equations, deriveds from pure and mixed derivatives of the Cauchy-Riemann equation are investigated.

The first paragraph, entitled “Investigation of the solution of the equation obtained from the second order Caucgy-Riemann equation by taking derivative with non-local and global boundary conditions” deals with the following problem

$$\frac{\partial^3 u(x)}{\partial x_2^3} + i\frac{\partial^3 u(x)}{\partial x_1 \partial x_2^2} = 0, \quad x \in D \subset R^2, \tag{22}$$

$$\sum_{s=1}^{2} \sum_{\substack{p+q\le 2\\ p<2}} \alpha_{kpq}^{(s)}(x_1) \frac{\partial^p \partial^q u(x)}{\partial x_1^p \partial x_2^q}\bigg|_{x_2=\gamma_s(x_1)} +$$

$$+\sum_{s=1}^{2} \sum_{\substack{p+q\le 2\\ p<2}} \int_{a_1}^{b_1} \left[\left(K_{kpq}^{(s)}(x_1,t\right) \frac{\partial^p \partial^q u(y)}{\partial y_1^p \partial y_2^q}\bigg|_{\substack{y_1=t\\ y_2=\gamma_s(t)}} dt = \alpha_k(x_1)$$

$$p,q = 1,2, \quad k = \overline{1,3}; \; x_1 \in [a_1, b_1], \tag{23}$$

where conditions (23) are linearly independend with continuous functions. Using the fundamental solution of problem (22)

$$U(x-\xi) = \frac{1}{2\pi}\left[x_2 - \xi_2 + i(x_1 - \xi_1)\right]\left\{\ln\left[x_2 - \xi_2 + i(x_1 - \xi_1)\right] - 1\right\},$$

by the help of the second Green’s formula and its analogues for the functions

$$u(\xi), \; \frac{\partial u(\xi)}{\partial \xi_1}, \; \frac{\partial u(\xi)}{\partial \xi_2}, \; \frac{\partial^2 u(\xi)}{\partial \xi_2^2} \text{ and } \frac{\partial^2 u(\xi)}{\partial \xi_1 \partial \xi_2}.$$

we obtain the basic relations and corresponding necessary conditions as below

$$\frac{\partial u(\xi)}{\partial \xi_p}\bigg|_{\xi_2=\gamma_q(\xi_1)} = \frac{(-i)^{p-1}(-1)^q}{\pi} \int_{a_1}^{b_1} \frac{\partial u(x)}{\partial x_2}\bigg|_{x_2=\gamma_q(x_1)} \frac{dx_1}{x_1 - \xi_1} + \ldots,$$

$$p,q = 1,2, \tag{24}$$

$$\frac{\partial^2 u(\xi)}{\partial \xi_p \partial \xi_2}\bigg|_{\xi_2=\gamma_q(\xi_1)} = \frac{(-i)^{p-1}(-1)^q}{\pi} \int_{a_1}^{b_1} \frac{\partial^2 u(x)}{\partial x_2^2}\bigg|_{x_2=\gamma_q(x_1)} \frac{dx_1}{x_1 - \xi_1} + \ldots,$$

$$p,q = 1,2, \tag{25}$$

where (...) is the sum of non-singular terms. The singularities in necessary conditions (24) and (25) given above are regulated using boundary conditions (23). Note that there is no singularity in the necessary conditions for the boundary values of $u(\xi)$.

Thus, we obtain the following statement

Theorem 3. *If* D *is convex in the direction* x_2 *bounded plane domain, the boundary* $\Gamma = \partial D$ *is a Lyapunov line, all the functions in* (23) *are continuous functions then the obtained regular relations and boundary conditions* (23) *are reduced to the system of* 10 *non continuing singularies in the kernels Fredholm equations for the functions*

$$u(\xi),\ \frac{\partial u(\xi)}{\partial \xi_1},\ \frac{\partial u(\xi)}{\partial \xi_2},\ \frac{\partial^2 u(\xi)}{\partial \xi_2^2} \text{ and } \frac{\partial^2 u(\xi)}{\partial \xi_1 \partial \xi_2}.$$

Finally, a sufficient condition is found for the normalization of the system of integral equations stated in the theorem.

In the second paragraph of this chapter, the solution of the problem within the non-local and global boundary conditions for the third-order composite-type equation derived from the mixed derivative of the Cauchy-Riemann equation is investigated.

The following boundary value problem is considered here

$$\frac{\partial^3 u(x)}{\partial x_1 \partial x_2^2} + i\frac{\partial^3 u(x)}{\partial x_1^2 \partial x_2} = f(x),,\ \ x \in D \subset R^2, \tag{26}$$

$$\sum_{s=1}^{2}\Big[\alpha_k^{(s)}(x_1)\frac{\partial^2 u(x)}{\partial x_1 \partial x_2} + \alpha_{k,1}^{(s)}(x_1)\frac{\partial u(x)}{\partial x_1} + \alpha_{k,2}^{(s)}(x_1)\frac{\partial u(x)}{\partial x_2}\times$$

$$\times \alpha_{k,0}^{(s)}(x_1)u(x)\Big]_{x_2=\gamma_s(x_1)} + \sum_{s=1}^{2}\int_{a_1}^{b_1}\Big[\beta_k^{(s)}(x_1,t)\frac{\partial^2 u(t,y_2)}{\partial t \partial y_2} +$$

$$+\beta_{k,1}^{(s)}(x_1,t)\frac{\partial u(t,y_2)}{\partial t} + \beta_{k,2}^{(s)}(x_1,t)\frac{\partial u(t,y_2)}{\partial y_2} +$$

$$+\beta_{k,0}^{(s)}(x_1,t)u(t,y_2)\Big]_{y_2=\gamma_s(t)}dt + \int_D\Big[K(x,y)\frac{\partial^2 u(y)}{\partial y_1 \partial y_2} + K_1(x_1,y)\frac{\partial u(y)}{\partial y_1} +$$

$$+K_2(x_1,y)\frac{\partial u(y)}{\partial y_2}+K_0(x_1,y)u(y)\Big]dy=\alpha_k(x_1).\ \ k=\overline{1,3};\ x_1\in[a_1,b_1],$$

(27)

where D is convex in the direction x_2 bounded plane domain, $\Gamma=\partial D$ is a Lyapunov line. All the data of the boundary conditions (coefficients, right-hand sides and kernels of integrals) and of linearly independent conditions (27) are continuous functions.

As in the first chapter here also using the fundamental solution of composite type equation (26) we derive the main relations for the functions

$$u(\xi),\ \ \frac{\partial u(\xi)}{\partial \xi_1},\ \ \frac{\partial u(\xi)}{\partial \xi_2},\ \ \frac{\partial^2 u(\xi)}{\partial \xi_1 \partial \xi_2}.$$

Necessary conditions are obtained from these main relations. Below are regularized forms of these basic conditions that contain singularity.

$$u(\xi_1,\gamma_1(\xi_1))=\frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))}{x_1-\xi_1}dx_1+\ldots, \tag{28}$$

$$u(\xi_1,\gamma_2(\xi_1))=-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_2(x_1))}{x_1-\xi_1}dx_1+\ldots, \tag{29}$$

$$\left.\frac{\partial^2 u(\xi)}{\partial \xi_1 \partial \xi_2}\right|_{\xi_2=\gamma_1(\xi_1)}=\frac{i}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial^2 u(x)}{\partial x_1 \partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\xi_1}+\ldots, \tag{30}$$

$$\left.\frac{\partial^2 u(\xi)}{\partial \xi_1 \partial \xi_2}\right|_{\xi_2=\gamma_2(\xi_1)}=-\frac{i}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial^2 u(x)}{\partial x_1 \partial x_2}\right|_{x_2=\gamma_2(x_1)}\frac{dx_1}{x_1-\xi_1}+\ldots, \tag{31}$$

thus we obtain the following theorem

Theorem 4. *If D is a convex in the direction x_2 bounded plane domain, $\Gamma=\partial D$ is a Lyapunov line, $f(x)$ is a continuous function, the any solution of equation* (26) *in satisfies singular necessary conditions* (28)-(31) *and non-singular conditions for the boundary values of the functions* $\frac{\partial u(\xi)}{\partial \xi_1},\ \frac{\partial u(\xi)}{\partial \xi_2}$.

After the singularities obtained here are regularized with the help of boundary conditions (27), sufficient conditions for the Fredholm property of boundary problem (26), (27) are obtained.

The third chapter, entitled “Investigation of the solution of the boundary value problem for mixed and composite type equations”, consists of eight parts dedicated to the following problem

$$\frac{\partial^3 u_s(x)}{\partial x_1 \partial x_2^2} + i^s \frac{\partial^3 u_s(x)}{\partial x_1^2 \partial x_2} = 0,\ x \in D \subset R^2, s = 1{,}2; \tag{32}$$

$$\sum_{m=1}^{2} \sum_{0 \le p+q \le 2} \left\{ \alpha_{k,p,q}^{(m)}(x_1) \left. \frac{\partial^{p+q} u_m(x)}{\partial x_1^p \partial x_2^q} \right|_{x_2=\gamma_m(x_1)} + \right.$$

$$\left. + \beta_{k,p,q}^{(m)}(x_1) \left. \frac{\partial^{p+q} u_m(x)}{\partial x_1^p \partial x_2^q} \right|_{x_2=0} \right\} = \alpha_k(x_1)\ k = \overline{1,6};\ x_1 \in [a_1, b_1], \tag{33}$$

where $i = \sqrt{-1}$, all data of linearly independend boundary conditions (33) are continuous functions, the domains $D_1 = \{x;\ x_1 \in (a_1, b_1),$ $x_2 \in (\gamma_1(x_1), 0)\},$ $D_2 = \{x;\ x_1 \in (a_1, b_1),\ x_2 \in (0, \gamma_2(x_1),)\}$ are convex in the direction x_2 bounded plane domains $\gamma_m(a_1) = \gamma_m(b_1) = 0,\ m = 1{,}2;$ $\gamma_1(x_1) < 0,\ \gamma_2(x_1) > 0,\ x_1 \in (a_1, b_1),$ the boundary $\overline{\Gamma} = \overline{\Gamma_1} \cup \overline{\Gamma_2}$ of the domain $\overline{D} = \overline{D_1} \cup \overline{D_2}$ is a Lyapunov line.

Using the fundamental solution

$$U_1(x - \xi) = \frac{i}{2\pi} [x_2 - \xi_2 + i(x_1 - \xi_1)] \{\ln[x_2 - \xi_2 + i(x_1 - \xi_1)] - 1\}$$

or

$$U_2(x - \xi) = \int_{x_1 - \xi_1}^{\frac{1}{2}[x_2 - \xi_2 + x_1 - \xi_1]} \theta(t) \theta(x_2 - \xi_2 + x_1 - \xi_1 - t) dt,$$

of problem (32) for the functions

$$u_1(\xi),\ \frac{\partial u_1(\xi)}{\partial \xi_1},\ \frac{\partial u_1(\xi)}{\partial \xi_2},\ \frac{\partial^2 u_1(\xi)}{\partial \xi_1 \partial \xi_2},\ \frac{\partial^2 u_1(\xi)}{\partial \xi_1^2},\ \frac{\partial^2 u_1(\xi)}{\partial \xi_2^2},$$

$$u_2(\xi),\ \frac{\partial u_2(\xi)}{\partial \xi_1},\ \frac{\partial u_2(\xi)}{\partial \xi_2},\ \frac{\partial^2 u_2(\xi)}{\partial \xi_1 \partial \xi_2},\ \frac{\partial^2 u_2(\xi)}{\partial \xi_1^2},\ \frac{\partial^2 u_2(\xi)}{\partial \xi_2^2},$$

we obtain the main relations and corresponding necessary conditions. The necessary conditions obtained for this case were studied separately for the elliptic and hyperbolic parts. After regularization of the singularities in the necessary conditions corresponding to the elliptic part, the sufficient conditions are obtained for the Fredholm property of boundary value problem (32), (33), within with the regular necessary conditions obtained for the hyperbolic part and the given boundary conditions. In this chapter it is proved that if $D = D_1 \cup D_2$ is a convex in the direction x_2 bounded plane domain, the boundary $\Gamma = \Gamma_1 \cup \Gamma_2$ is a Lyapunov line, all functions involved in the linearly independend boundary conditions (33) are continuous, then the analytical expression for the solution of the boundary value problem (32), (33) is obtained from the main relations defined for the functions $u_1(x)$ and $u_2(x)$.

CONCLUSION

The dissertation is devoted to the study of boundary value problem with non-local and global conditions for some mixed and composite type differential equations. The main results of the dissertation are as follows:

1. The solutions of the problems with boundary conditions containing nonlocal and global termsts for the first-order mixed-type equation and the Tricomi equation were investigated. The basic relations expressing the arbitrary solution of each equation and its derivatives by boundary values were established, and the necessary conditions were derived from the basic relations. The singular expressions in the necessary conditions were regularized.

2. The solutions of nonlocal and global boundary value problems for the third-order composite type equations obtained from the second-order pure derivative and mixed derivative of the Cauchy-Riemann equation were investigated. The basic relations and necessary conditions were derived, and the singularities in these conditions were regularized.

3. The solution of the nonlocal boundary value problem for the third-order partial derivative equation belonging to both mixed and composite types was investigated.

4. Sufficient conditions for the Fredholm property of the considered boundary value problems were obtained.

The main results of the thesis are published in the following works of the author:

1. Əliyev, N.Ə., İbrahimov, N.S., Niftullayeva, Ş.A., Qarışıq tip tənlik üçün qeyri-lokal və qlobal hədli bir sərhəd məsələsinin həllinin araşdırılması // – Lənkəran: Elmi Xəbərlər, Təbiyyat elmləri seriyası, Lənkəran Dövlət Universiteti, – 2016. – s.3-13.
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