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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

INVESTIGATION OF SOME INVERSE PROBLEMS BY OPTIMIZATION METHODS FOR THE SECOND ORDER HYPERBOLIC EQUATIONS AND ALGORITHMS FOR THEIR SOLUTION

Specialty: 1203.01-Computer sciences

Field of science: Mathematics

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Baku-2021

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GENERAL CHARACTERISTICS OF THE WORK

Rationale of the theme. Since the middle of the 20-th century the inverse problems became on object of systematic study and application in physics, geophysics, astronomy, medicine, biology, ecology, economy and in general in all fields of science where mathematical methods are applicable. With appearance of powerful computers, the field of applications of inverse problems covered many scientific directions. At present, the inverse problems turned to be a rapidly developing field of knowledge influencing practically into all spheres of mathematics, especially in differential equations, mathematical physics, computational mathematics, etc. Note that anyway every inverse problem may be formulated as inverse to some direct well-posed problem.

In mathematical physics, under direct problems one understands the problem of simulation of any processes, phenomena, etc. Therefore, in direct problems it is requaired to find the functions describing various physical phenomena. The data for solving the direct problems are the domains where the process is studied, the coefficients and the right hand side of the equation, boundary conditions, initial conditions (if the process in nonstationary). However, in many cases, the right hand side of the medium under investigation (the coefficients of the equations), initial state of the process (initial conditions), boundary mode (boundary conditions) are unknown. Then there arise inverse problems where by the information about the solution of the direct problem it is required to determine these unknown quantities. Thus, in the inverse problem, in addition to the solution of the boundary value problem some functions included in the direct problem are unknown.

Note that inverse and ill-posed problems were studied by A.S. Alexeev, Yu.E. Anikonov, V.Ya. Arsenin, A.Ya. Akhundov, M.I. Belishev, A.S. Blagoveshenskiy, F.P. Vasil'ev, V.V. Vasin, V.K. Ivanov, K.I. Iskakov, A.D. Iskenderov, S.I. Kabanikhin, A.I .Kozhanov, M.M. Lavrentyev, R. Lattece, J.L. Lions, V.A. Morozov, V.G. Romanov, A.N. Tikhonov, A.G. Yagola, V.G. Yakhno, A. Hasanov and others.In great majority of cases, the inverse problems are ill-posed and there exist various methods for solving them: the regilarization method, the quasi-inversion method, gradient methods, etc.

Recently, some inverse problems of definition of the right hand sides, initial and boundary functions, the coefficients of partial equations are reduced to optimal control problems and the obtained problems are studied by the methods of optimal control theory. Note that there is a close tie between the solutions of inverse problems for mathematical physics equations and optimal control problems for such equations. The sought - for right hand sides, the coefficients of equations play as a control and quality criterion (functional) is composed by means of additional information. This functional is called a residual functional or objective functional if the minimal value of the objective functional equals zero, then the additional condition in the inverse problem is fulfilled. Such an approach to inverse problems is called variational or optimizational method for solving inverse problems. In their papers V.M.Abdullayev, K.R.Ayda-zadeh, O.M.Alifanov, E.A.Artyukhin, K.T.Iskakov, A.D.Iskenderov, S.I.Kabanikhin, A.L.Karchevskiy, H.F.Kuliyev, S.V.Rumyantsev, R.K.Tagiyev, G.Yu.Yagubov and others have studied optimizational statements of some inverse problems for different partial equations. This method was most developed for parabolic equations in the works of O.M.Alifanov, E.A.Artyukhin, S.V.Rumyantsev, A.P.Iskenderov, R.K.Tagiyev. As is known the inverse problems for hyperbolic equations were not studied enough by this method.

Finally, note that K.R.Ayda-zadeh, J.L.R.Arman, S.S.Akhiev, K.T.Akhmedov, F.A.Aliyev A.G.Butkovskiy, F.P.Vasilev, A.I.Egorov, Yu.V.Egorov, A.Z.Ishmukhametov, K.K.Gasanov, Y.S.Gasimov, A.D.Iskenderov, V.Komkov, H.F.Kuliyev, J.L.Lions, K.A.Lourie, A.Y.Mamedov, K.B.Mansimov, M.J.Mardanov, T.K.Melikov, A.A.Niftiyev, V.I.Plotnikov, U.E.Raytum, M.A.Sadygov, S.Ya.Serovoyskiy, T.K.Sirazetdinov, V.I.Sumin, M.I.Sumin, R.K.Tagiyev, M.A.Yagubov, G.A.Yagubov, Ya.A.Sharifov, Sh.Sh.Yusubov, Z.I.Khalilov, M.V.Suryanarayana, J.Sokolowski, T.Zolezze and others investigated different optimal control problems for the partial equations. In the present dissertation work some inverse problems for defining the right hand sides, initial and boundary functions, coefficients for second order hyperbolic equations are reduced to optimal control problems and the obtained problems are studied by the methods of optimal control theory. Proceeding from what has been said above, we think the topic of the dissertation work is urgent.

Object and subject of research. The research object of the presented dissertation is boundary value problems, inverse problems and optimal control problems for two-order hyperbolic equations. The subject of the research is the approaches based on bringing the right side of the equations, the initial, boundary functions, coefficients to the problem of optimal control and methods of solving optimal control problems.

Research goals and problems. Reducing various inverse problems of definition of the right hand side, initial and boundary functions, coefficients of second order hyperbolic equations to the appropriate optimal control problems, studying the obtained problems with the methods of optimal control theory, deriving optimality conditions and developing the solution algorithms based on the obtained optimality conditions.

Research methodology. In the dissertation work we use the methods of mathematical theory of optimal control and optimization, of mathematical physics and functional analysis.

Main results to be defended:

-reducing the right hand side of second order hyperbolic type eguations, an inverse problem on definition of initial and boundary functions to optimal control problem;

-reducing an inverse problem on definition of the coefficients of second order hyperbolic type partial differential eguations to an optimal control problem;

-the study of the obtained optimal control problems by menus of the methods of optimal control theory;

-obtaining the optimality condition for the obtained optimal control problem;

-compiling an algoritm for the atated problems by means of the optimality condition .

Scientific results. Different inverse problems on the definition of the right hand side, initial and boundary condition, coefficients of second order hyperbolic equations are reduced to the optimal control problems;

- The obtained optimal control problems are investigated;
- Differentiability of objective functionals are proved and the expressions for their gradients are found;
- Variational inequalities type optimality conditions are established;
- Based on the derived optimality conditions, algorithms for solving the optimal control problems are developed;
- Numerical experiments were carried out in some cases.

Theoretical and practical importance. The results obtained in the work are mainly of theoretical character. The methods of the present work may be used when studying close inverse problems for the systems described by other partial equations. The practical value of the work is that the obtained results may be used in approximate solution of various inverse problems in vibrational and wave processes.

Approbation of the work. The results of the dissertation work were reported at the following scientific seminars and conferences: At the seminars of the chair "Informatics" (the head prof. V.A.Mustafayev) SSU, "Differential equations and optimization" (the head prof. F.G.Feyziyev) SSU, "Mathematical methods of control theory" (the head prof. M.A.Yagubov) BSU, at the XX Republican scientific conference of doctoral student and young scientists (Baku,2014), at the scientific conference "Functional analysis and its applications" dedicated to 100 years of A.Sh.Habibzadeh (Baku,2016), at the III Republican scientific conference on "Applied problems of mathematics and information technology" (Sumgayit 2016), at the conference "Fundamental and applied problems of mathematics and informatics", at the XII International scientific conference dedicated to 85 years of prof. M.G.Alishaev (Mahachkala 2017), at the International scientific conference "On theoretical and applied problems of Mathematics" devoted to 55 years of Sumgayit

State University (Sumgayit 2018), at the Republican scientific conference "Problems of development of natural and humanitarian science" devoted to 94-th years of the national leader Heydar Aliyev (Lenkeran 2017), at the Republican scientific conference "Actual problems of mathematics and mechanics" devoted to 100-th jubilee of corr. member of ANAS, the outstanding scientifist and famous mathematician, doctor of physico-mathematical sciences, prof. Goshgar Teymur oglu Ahmedov (Baku 2017), at the International conference "Modern problems of mathematics and mechanics" devoted to the 80-th anniversary of acad. Akif Gadjiev (Baku 2018), at the IV International scientific conference "Actual problems of applied mathematics" (Nalchik 2018), "Fundamental and applied problems of mathematics and informatics" Proceeding of the XIII International conference dedicated to the 55 years of the faculty of mathematics and computer sciences (Mahachkala 2019), “Fundamental problems of mathematics and application of intellectual texnologies in education”, Republican Scientific Conference (Sumgayit 2020).

Personal contribution of the author. All conclusions and obtained results belong to the author.

Publications. The main results of the work were published in 22 papers the list of which is given at the end of the abstract.

Structure and volume of the dissertation (in signs with incating the volume of each structural subsection separately). The dissertation work consists of 240000 signs (the title page-486 signs, the content 7009 signs, introduction-24079 signs, chapter I-79200 signs, chapter II-104026 signs, chapter III-25200 signs). The list of references consists of 103 names.

THE CONTENT OF THE WORK

The work consists of introduction, three chapters, conclusion, list of references and appendix.

In introduction the rationale of the work is substantiated, the brief content of the dissertation work is stated.

Chapter I consisting of four sections was devoted to the optimal control problems to which the inverse problems of definition of the right hand side, initial and boundary condition of a second order hyperbolic equation were reduced.

Insection 1.1 In domain $Q=\{(x,t)|0<x<\ell, 0<t<T\}$ we consider the following boundary value problem

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=\upsilon(x,t), \qquad (1)$$

$$u(x,0)=u_0(x), \frac{\partial u(x,0)}{\partial t}=u_1(x), 0\le x\le \ell, \qquad (2)$$

$$u(0,t)=0, u(\ell,t)=0, 0\le t\le T. \qquad (3)$$

Here $u_0\in \overset{0}{W}{}_2^1(0,\ell), u_1\in L_2(0,\ell)$ are the given functions, a $\upsilon\in L_2(Q)$ is an unknown function. To determine $\upsilon(x,t)$ we use the additional information

$$u(d(t),t)=f(t),\ t\in(0,T), 0<d(t)<\ell, \qquad (4)$$

where $f(t)\in L_2(0,T)$ is a given function, $x=d(t), t\in[0,T]$ is a given piecewise smooth function.

This problem is reduced to the following optimal control problem: on the solutions of problem (1)-(3) to minimize the functional

$$J(\upsilon)=\frac{1}{2}\int_0^T[u(d(t),t;\upsilon)-f(t)]^2\,dt, \qquad (5)$$

where the function $\upsilon(x,t)$ is called a control, $u(x,t;\upsilon)$ is the solution from $W_{2,0}^1(Q)$ of problem (1)-(3), corresponding[1] to $\upsilon(x,t)$.

If a control giving zero value to functional (5) is found, then additional condition (4) is fulfilled.

In the work, at first it is proved that $\inf_{\upsilon\in L_2(Q)} J(\upsilon)=0$. Then a problem of minimization of the functional

[1] Ладыженская, О.А. Краевые задачи математической физики / О.А. Ладыженская. - Москва: Наука, -1973. -408 с.

$$J_\beta(\upsilon) = J(\upsilon) + \frac{\beta}{2}\int_Q \upsilon^2(x,t)dxdt, \quad (\beta = const > 0) \tag{6}$$

on the convex closed set $V \in L_2(Q)$ under constraints (1)-(3) is considered. We prove the following theorem.

Theorem 1. Let the above supposed conditions on data of problem (6), (1)-(3) be fulfilled. Then functional (6) is continuously Frechet differentiable on V and its differential at the point $\upsilon \in V$ at the increment $\delta\upsilon \in L_2(Q), \upsilon + \delta\upsilon \in V$ is determined by the expression

$$\langle J'(\upsilon, \delta\upsilon)\rangle = \int_Q (\psi(x,t;\upsilon) + \beta\upsilon)\delta\upsilon dxdt,$$

where $\psi = \psi(x,t;\upsilon)$ is the solution from $W_{2,0}^1(Q)$ of the following adjoint problem

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial x^2} = 0, (x,t) \in Q,$$

$$\psi(x,T) = 0, \frac{\partial\psi(x,T)}{\partial t} = 0, 0 \le x \le \ell,$$

$$\psi(0,t) = 0, \psi(\ell,t) = 0,\ 0 \le t \le T,$$

$$[\psi]_\Gamma = 0, \left[\frac{\partial\psi}{\partial x}\right]_\Gamma = u(d(t),t;\upsilon) - f(t),\ 0 \le t \le T,$$

where Γ is a line $x = d(t), t \in [0,T]$, dividing Q into two domains Q_1, Q_2, the symbol $[\omega]_\Gamma$ means difference between parallel values $\omega(x,t)$ in the sense L_2 on Γ calculated when approaching to Γ from the side of domain Q_1 and Q_2.

The following theorem establishes optimality condition for solving problem (6), (1)-(3).

Theorem 2. Let above supposed conditions on the data of problem (6), (1)-(3) be fulfilled. Then for the control $\upsilon_* = \upsilon_*(x,t) \in V$ be optimal, it is necessary and sufficient that in problem (6), (1)-(3) the following inequality be fulfilled

$$\int_Q (\psi_*(x,t) + \beta\upsilon_*(x,t))(\upsilon(x,t) - \upsilon_*(x,t))dxdt \ge 0 \quad \forall \upsilon \in V,$$

where $\psi_* = \psi_*(x,t) = \psi(x,t;\upsilon_*)$ is the solution of the adjoint problem for $\upsilon = \upsilon_*$.

At the end of the section we offer an algorithm for finding minimizing sequencies in problem (6), (1)-(3).

In section 1.2 we consider a problem of definition of the pair $(u(x,t),\upsilon(x)) \in W_{2,0}^1(Q)\times L_2(\Omega)$ from the following relations

$$\frac{\partial^2 u}{\partial t^2} + Lu = f(x,t), (x,t) \in Q = \Omega \times (0,T), \tag{7}$$

$$u(x,0) = 0, \frac{\partial u(x,0)}{\partial t} = \upsilon(x), x \in \Omega, u|_S = 0, \tag{8}$$

$$u(x,T) = \varphi(x), x \in \Omega, \tag{9}$$

where $\Omega \subset R^n$ is a bounded domain with a smooth boundary Γ, Q is a cylinder, $S = \Gamma \times (0,T)$ is a lateral surface of the cylinder Q, $f \in L_2(Q)$, $u_0 \in \overset{0}{W}{}_2^1(\Omega), \varphi \in L_2(\Omega)$ are the given functions,

$$Lu = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right) + a_0(x,t)u,$$

is a differential expression with smooth coefficients, that satisfy the ellipticity condition. Problem (7)-(9) is reduced to the following optimal control problem: to minimize the functional

$$J_0(\upsilon) = \frac{1}{2}\int_\Omega [u(x,T;\upsilon) - \varphi(x)]^2 dx$$

on the solutions of problem (7), (8), where $\upsilon(x)$ is a control function, $u(x,t;\upsilon)$ is the solution of problem (7), (8). At first we prove that $\inf_{\upsilon \in L_2(Q)} J(\upsilon) = 0.$

Then we consider a problem of minimization of the functional

$$J_\beta(\upsilon) = J_0(\upsilon) + \frac{\beta}{2}\int_\Omega \upsilon^2(x)dx \tag{10}$$

in the convex closed set $V \subset L_2(\Omega)$ under constraints (7), (8). For problem (10), (7), (8) we introduce the conjugate problem

$$\frac{\partial^2 \psi}{\partial t^2} + L\psi = 0, (x,t) \in Q, \tag{11}$$

$$\psi(x,T)=0, \frac{\partial\psi(x,T)}{\partial t}=u(x,T;\upsilon)-\varphi(x), x\in\Omega, \psi|_S=0. \qquad (12)$$

Theorem 3. Let the conditions in the statement of problem (10), (7), (8) be fulfilled. Then the functional (10) is continuously Frechet differentiable on V and its functional at the point $\upsilon\in V$ at the increment $\delta\upsilon\in L_2(\Omega), \upsilon+\delta\upsilon\in V$ is determined by the expression

$$\left\langle J'_\beta(\upsilon),\delta\upsilon\right\rangle_{L_2(\Omega)}=\int_\Omega[-\psi(x,0;\upsilon)+\beta\upsilon(x)]\delta\upsilon(x)dx.$$

Theorem 4. Let the conditions of theorem 3 be fulfilled. Then for the control $\upsilon_*=\upsilon_*(x)\in V$ be optimal in problem (10), (7), (8) it is necessarty and sufficient that the following inequality be fulfilled

$$\int_\Omega[-\psi_*(x,0)+\beta\upsilon_*(x)](\upsilon(x)-\upsilon_*(x))dx\geq 0 \quad \forall\upsilon\in V,$$

where $\psi_*=\psi_*(x,t)=\psi(x,t;\upsilon_*)$ is the solution of the adjoint problem (11), (12) for $\upsilon=\upsilon_*(x)$.

At the end of the section we given an algorithm for finding minimizing sequence in problem (10), (7), (8).

In section 1.3 we consider a Neumann problem for a wave equation in the two-dimensional case:

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \quad (x,y,t)\in Q=\Omega\times(0,T), \qquad (13)$$

$$\frac{\partial u}{\partial t}\Big|_{t=0}=f_1(x,y), \frac{\partial u}{\partial t}\Big|_{t=T}=f_2(x,y), (x,y)\in\Omega, \qquad (14)$$

$$\frac{\partial u}{\partial x}\Big|_{x=0}=\frac{\partial u}{\partial x}\Big|_{x=\ell}=0, \ (y,t)\in(0,\ell)\times(0,T), \qquad (15)$$

$$\frac{\partial u}{\partial y}\Big|_{y=0}=\frac{\partial u}{\partial y}\Big|_{y=\ell}=0, \ (x,t)\in(0,\ell)\times(0,T), \qquad (16)$$

where $\Omega=(0,\ell)\times(0,\ell), \ell>0, T>0$ are the given members, $f_1, f_2\in W_2^1(\Omega)$ are the given functions. It is known that the Neumann problem for a hyperbolic equation is ill-posed[2]. The inverse problem

[2] Kabanikhin, S.I. An optimizition method in the Dirichlet problem for the wave equation/ Bektemesov M.A., Nurseitov D.B., Krivorotko O.I. and Alimova A.N. // Journal Inverse III-Posed Problem, de Gruyter , 20(2012), -pp. 193-211.

is: to determine the pair of function $u = u(x,y,t), \upsilon = \upsilon(x,y)$ from relations (13), (15), (16)

$$u\big|_{t=0} = \upsilon(x,y), \frac{\partial u}{\partial t}\big|_{t=0} = f_1(x,y), \qquad (17)$$
$$\frac{\partial u}{\partial t}\big|_{t=T} = f_2(x,y), (x,y) \in \Omega$$

for the given functions $f_1(x,y)$, $f_2(x,y)$. Instead of this problem we consider the following problem: to find such a function from the class

$$V = \left\{ \begin{array}{l} \upsilon(x,y) \big| \ \upsilon \in W_2^2(\Omega), \\ \left.\frac{\partial \upsilon}{\partial x}\right|_{x=0} = \frac{\partial \upsilon}{\partial x}\big|_{x=\ell} = 0, \ \left.\frac{\partial \upsilon}{\partial y}\right|_{y=0} = \left.\frac{\partial \upsilon}{\partial y}\right|_{y=\ell} = 0, \|\upsilon\|_{W_2^2(\Omega)} \le M \end{array} \right\}, \qquad (18)$$

that together with the solution of problem (13), (15), (16), (17) gives minimum to the functional

$$J(\upsilon) = \frac{1}{2}\int_{\Omega}\left[\frac{\partial u(x,y,T;\upsilon)}{\partial t} - f_2(x,y)\right]^2 dxdy. \qquad (19)$$

We prove the uniqueness of the solution of the inverse problem in the case when $\ell = \pi$, and then prove a theorem on solvability of problem (13), (15)-(18), (19).

Then we prove differentiability of functional (19).

Theorem 5. Let the conditions on the data of problem (13), (15)-(18), (19) be fulfilled. Then functional (19) is continuously Frechet differentiable on V and its differential at the point $\upsilon \in V$ at the increament $\delta\upsilon \in W_2^2(\Omega), \upsilon + \delta\upsilon \in V$ is determined by the expression

$$\langle J'(\upsilon), \delta\upsilon \rangle = -\int_{\Omega} \frac{\partial \psi(x,y,0;\upsilon)}{\partial t} \delta\upsilon(x,y) dxdy,$$

where $\psi(x,y,t;\upsilon)$ is the solution from $W_2^1(Q)$ of the adjoint problem

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}, (x,y,t) \in Q,$$

$$\psi\big|_{t=0} = \frac{\partial u(x,y,T;\upsilon)}{\partial t} - f_2(x,y), \ \frac{\partial \psi}{\partial t}\big|_{t=T} = 0, (x,y) \in \Omega,$$

$$\frac{\partial \psi}{\partial x}\big|_{x=0} = \frac{\partial \psi}{\partial x}\big|_{x=\pi} = 0, \ (y,t) \in (0,\pi) \times (0,T),$$

$$\frac{\partial \psi}{\partial y}\Big|_{y=0} = \frac{\partial \psi}{\partial y}\Big|_{y=\pi} = 0,\ (x,t)\in(0,\pi)\times(0,T).$$

Theorem 6. Let the conditions of theorem 5 be fulfilled. Then for the control $\upsilon_* = \upsilon_*(x,y)\in V$ to be optimal in problem (13), (15)-(18), (19) it is necessary and sufficient that the following inequality be fulfilled

$$\int_\Omega \frac{\partial \psi_*(x,y,0)}{\partial t}(\upsilon(x,y)-\upsilon_*(x,y))dxdy \le 0\ \forall \upsilon\in V,$$

where $\psi_*(x,y,t)=\psi(x,y,t;\upsilon_*)$ is the solution of the conjugate problem for $\upsilon=\upsilon_*(x,y)$.

In section 1.4 we consider a problem of definition of the pair of functional $(u(x_1,x_2,t),\upsilon(x_2,t))\in W_2^1(Q)\times L_2((0,\ell_2)\times(0,T))$ from the relation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + f(x_1,x_2,t),\quad (x_1,x_2,t)\in Q, \tag{20}$$

$$u(x_1,x_2,0)=\varphi_0(x_1,x_2),\ \frac{\partial u}{\partial t}\Big|_{t=0}=\varphi_1(x_1,x_2),\ (x_1,x_2)\in\Omega, \tag{21}$$

$$\frac{\partial u}{\partial x_1}\Big|_{x_1=0}=0,\ \frac{\partial u}{\partial x_1}\Big|_{x_1=\ell_1}=\upsilon(x_2,t),\ (x_2,t)\in(0,\ell_2)\times(0,T), \tag{22}$$

$$\frac{\partial u}{\partial x_2}\Big|_{x_2=0}=0, \frac{\partial u}{\partial x_2}\Big|_{x_2=\ell_2}=0,\ (x_1,t)\in(0,\ell_1)\times(0,T), \tag{23}$$

$$u\big|_{x_1=0}=a(x_2,t),\ (x_2,t)\in(0,\ell_2)\times(0,T), \tag{24}$$

where $Q=\Omega\times(0,T)$ is a parallelepiped, $\Omega=\{0<x_1<\ell_1, 0<x_2<\ell_2\}$ is a rectangle $\Omega=\{0<x_1<\ell_1, 0<x_2<\ell_2\}$, ℓ_1, ℓ_2, T are the given positive numbers, $f\in L_2(Q), \varphi_0\in W_2^1(\Omega)$, $\varphi_1\in L_2(\Omega)$, $a\in L_2((0,\ell_2)\times(0,T))$ are the given functions.

We reduce this problem to the following optimal control problem: to minimize the functional

$$J_0(\upsilon)=\frac{1}{2}\int_0^{\ell_2}\int_0^T [u(0,x_2,t;\upsilon)-a(x_2,t)]^2 dtdx_2$$

given (20)-(23).

We prove that $\inf J_0(\upsilon)=0, \quad \upsilon \in L_2((0,\ell)\times(0,T))$.

Then we consider a problem of minimization of the functional

$$J_\beta(\upsilon)=J_0(\upsilon)+\frac{\beta}{2}\int_0^{\ell_2}\int_0^T \upsilon^2 dx_2 dt \tag{25}$$

and prove differentiability of functional (25).

Theorem 7. Let above suppose conditions on the data of problem (20)- (23), (25) be fulfilled.

Then for the control $\upsilon_*(x_2,t)\in V$ be optimal in problem (20)-(23), (25) it is necessary and sufficient that the following inequality be fulfilled

$$\int_0^{\ell_2}\int_0^T [\psi_*(\ell_1,x_2,t)+\beta\upsilon_*(x_2,t)](\upsilon(x_2,t)-\upsilon_*(x_2,t))dx_2 dt \geq 0 \quad \forall \upsilon \in V,$$

where $\psi_*(x_1,x_2,t)=\psi(x_1,x_2,t;\upsilon_*)$ is the solution of the following adjoint problem for $\upsilon=\upsilon_*(x_2,t)$:

$$\frac{\partial^2\psi}{\partial t^2}=\frac{\partial^2\psi}{\partial x_1^2}+\frac{\partial^2\psi}{\partial x_2^2},\ (x_1,x_2,t)\in Q,$$

$$\psi\big|_{t=T}=0,\ \frac{\partial\psi}{\partial t}\Big|_{t=T}=0,\ (x,x_2)\in\Omega,$$

$$\frac{\partial\psi}{\partial x_1}\Big|_{x_1=0}=-[u(0,x_2,t;\upsilon)-a(x_2,t)],\ \frac{\partial\psi}{\partial x_1}\Big|_{x_1=\ell_1}=0,(x_2,t)\in(0,\ell_2)\times(0,T),$$

$$\frac{\partial\psi}{\partial x_2}\Big|_{x_2=0}=0,\ \frac{\partial\psi}{\partial x_2}\Big|_{x_2=\ell_2}=0,\ (x_1,t)\in(0,\ell_1)\times(0,T).$$

Chapter II consists of 6 sections and is devoted to problems of definition of the coefficients of a second order hyperbolic equation.

In section 2.1 we consider a problem of definition of a leading coefficient of a string vibration equation with an additional integral condition: it is required to find a pair of functions $\{u(x,t),v(x,t)\}$ from the following conditions

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial}{\partial x}\left(\upsilon(x,t)\frac{\partial u}{\partial x}\right)=f(x,t),(x,t)\in Q=\{(x,t):0<x<\ell,0,<t<T\}, \tag{26}$$

$$u(x,0)=\varphi_0(x),\ \frac{\partial u(x,0)}{\partial t}=\varphi_1(x),\quad 0\leq x\leq\ell, \tag{27}$$

$$u(0,t)=0,\ u(\ell,t)=0,\quad 0\le t\le T, \tag{28}$$

$$\int_0^\ell K(x,t)u(x,t)dx=\chi(t),\ 0\le t\le T, \tag{29}$$

$$\upsilon=\upsilon(x,t)\in V=\left\{\begin{array}{l}\upsilon=\upsilon(x,t)\in W_p^1(Q):\ 0<\nu\le\upsilon(x,t)\le\mu,\\ \left|\dfrac{\partial\upsilon(x,t)}{\partial x}\right|\le\mu_1,\ \left|\dfrac{\partial\upsilon(x,t)}{\partial t}\right|\le\mu_2\\ a.e. on Q,\infty>p>2\end{array}\right\} \tag{30}$$

where $f\in L_2(Q),\ \varphi_0\in \overset{0}{W}{}_2^1(0,\ell),\varphi_1\in L_2(0,\ell),\ K\in L_\infty(Q),\chi\in L_2(0,T)$ are the given functions.

To problem (26)-(30) we associate the following optimal control problem: to find the minimum of the functional

$$J(\upsilon)=\frac{1}{2}\int_0^T\left[\int_0^\ell K(x,t)u(x,t;\upsilon)dx-\chi(t)\right]^2dt \tag{31}$$

given (26)-(28), (30).

We prove

Theorem 8. Let the conditions in the statement of problem (26)-(30) be fulfilled. Then in problem (26)-(28), (30), (31) the set $V_*=\{\upsilon_*\in V: J(\upsilon_*)=\inf_{\upsilon\in V} J(\upsilon)\}\neq\emptyset$ in weakly compact in the space $W_p^1(Q)$ and any minimizing sequence $\{\upsilon^{(n)}\}\subset V$ of functional (31) weakly converges to the set V_* in the space $W_p^1(Q)$.

We introduce the following adjoint problem

$$\frac{\partial^2\psi}{\partial t^2}-\frac{\partial}{\partial x}\left(\upsilon(x,t)\frac{\partial\psi}{\partial x}\right)=-K(x,t)\left[\int_0^\ell K(x,t)u(x,t;\upsilon)dx-\chi(t)\right],(x,t)\in Q, \tag{32}$$

$$\psi(x,T)=0,\ \frac{\partial\psi(x,T)}{\partial t}=0,\quad 0\le x\le\ell, \tag{33}$$

$$\psi(0,t)=0,\ \psi(\ell,t)=0,0\le t\le T. \tag{34}$$

Theorem 9. Let the conditions of theorem 8 be fulfilled $\dfrac{\partial^2u}{\partial x^2},\dfrac{\partial^2\psi}{\partial x^2}\in L_2(Q)$. Then functional (31) is continuously Frechet differentiable on V and its differential at the point $\upsilon \in V$ with the increament $\delta\upsilon \in W_\infty^1(Q), \upsilon + \delta\upsilon \in V$ is determined by the expression

$$\langle J'(\upsilon), \delta\upsilon \rangle = \int_Q \frac{\partial u}{\partial x} \frac{\partial \psi}{\partial x} dxdt.$$

Based on this formula the prove:

Theorem 10. Let the conditions of theorem 9 be fulfilled. Then for the control $\upsilon_* = \upsilon_*(x,t) \in V$ be optimal, in problem (26)-(28), (30), (31) it is necessary that the following inequality be fulfilled

$$\int_Q \frac{\partial u_*}{\partial x} \frac{\partial \psi_*}{\partial x} (\upsilon(x,t) - \upsilon_*(x,t)) dxdt \geq 0 \quad \forall \upsilon \in V,$$

where $u_* = u(x,t;\upsilon_*)$, $\psi_* = \psi(x,t;\upsilon_*)$ is the solution of problem (26)-(28) and (32)-(34), respectively for $\upsilon = \upsilon_*(x,t)$.

Then we calculate the gradient of the functional (31) in the form $J'(\upsilon) = \omega(x,t;\upsilon)$, where $\omega(x,t;\upsilon)$ is the solution of the following boundary value problem

$$-\frac{\partial^2 \omega}{\partial x^2} - \frac{\partial^2 \omega}{\partial t^2} + \omega = \frac{\partial u}{\partial x} \frac{\partial \psi}{\partial x}, \ (x,t) \in Q,$$

$$\left.\frac{\partial \omega}{\partial x}\right|_{x=0} = \left.\frac{\partial \omega}{\partial x}\right|_{x=\ell} = 0, \ 0 \leq t \leq T, \ \left.\frac{\partial \omega}{\partial t}\right|_{t=0} = \left.\frac{\partial \omega}{\partial t}\right|_{t=T} = 0, \ 0 \leq x \leq \ell.$$

By means of the gradient of the functional (31) we derive necessary condition for optimality in the form of variational inequality.

In section 2.2 we consider a problem of definition of a leading coefficient of a string vibration equation, when an additional condition is given on some line. The similar results in 2.1 are obtained.

In section 2.3 a problem of definition of the coefficinets of a minor term in the Cauchy problem for a string vibration equation is reduced to an optimal control problem. A theorem on the existence and uniqueness of an optimal control in the reduced problem is proved and necessary condition for optimality in the form of variational inequality is derived.

In section 2.4 we consider problem: to find the function $u(x,t)$ and $\upsilon(t)$, connected with the equation

$$\frac{\partial^2 u}{\partial t^2} - \Delta u + \upsilon(t)\frac{\partial u}{\partial t} = f(x,t), \quad (x,t) \in Q, \tag{35}$$

with the condition

$$u|_{t=0} = u_0(x), \ \frac{\partial u}{\partial t}|_{t=0} = u_1(x), x \in \Omega, \quad u|_S = 0, \tag{36}$$

and with overdetermination condition

$$\int_\Omega K(x,t)u(x,t)dx = g(t), \quad t \in [0,T], \tag{37}$$

where $f(x,t), u_0(x), u_1(x), K(x,t), g(t)$ are the given functions, Δ is a Laplace operator with respect to x, Ω is a bounded domain in R^n with a smooth boundary Γ, $Q = \Omega \times (0,T)$ is a cylinder, $S = \Gamma \times (0,T)$ is a lateral surface of the cylinder Q.

The considered problem is reduced to the optimal control problem: in the class

$$V = \begin{cases} \upsilon = \upsilon(t) : \upsilon \in W_2^1[0,T], |\upsilon(t)| \le \mu_1, |\upsilon'(t)| \le \mu_2 \\ a.e. on \ [0,T] \end{cases}, \tag{38}$$

where $\mu_1 > 0, \mu_2 > 0$ are the given numbers, to minimize the functional

$$J(\upsilon) = \frac{1}{2}\int_0^T \left(\int_\Omega K(x,t)u(x,t;\upsilon)dx - g(t) \right)^2 dt. \tag{39}$$

In this section at first we prove a theorem on the existence of an optimal control in problem (35)-(36), (38), (39). Then we study differentiability of functional (39) and prove the necessary optimality condition

$$\int_0^T \left[\psi_{1*}(t)(\upsilon(t) - \upsilon_*(t)) + \frac{d\psi_{1*}(t)}{dt}\left(\frac{d\upsilon(t)}{dt} - \frac{d\upsilon_*(t)}{dt} \right) \right] dt \ge 0 \ \ \forall \upsilon \in V,$$

where $\psi_{1*}(t) = \psi_1(t;\upsilon_*)$ is the solution of the boundary value problem

$$-\frac{d^2\psi_1}{dt^2}+\psi_1=\int_\Omega \frac{\partial u}{\partial t}\psi dx,\ 0\le t\le T,$$

$$\frac{d\psi_1}{dt}\Big|_{t=0}=\frac{d\psi_1}{dt}\Big|_{t=T}=0$$

for $\upsilon=\upsilon_*(t)$, while $\psi=\psi(x,t;\upsilon)$ is the solution of the following adjpint problem

$$\frac{\partial^2\psi}{\partial t^2}-\Delta\psi-\frac{\partial}{\partial t}(\upsilon\psi)=-K(x,t)\left(\int_\Omega K(x,\tau)u(x,\tau;\upsilon)dx-g(t)\right),(x,t)\in Q,$$

$$\psi(x,T)=0,\frac{\partial\psi(x,T)}{\partial t}=0,x\in\Omega,\ \psi|_S=0.$$

In section 2.5 we consider a problem of definition of a pair of functions $(u(x,t),\upsilon(x))$ from the following relations

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}+\upsilon(x)\frac{\partial u}{\partial x}=f(x,t),(x,t)\in Q\equiv(0,\ell)\times(0,T), \tag{40}$$

$$u(x,0)=u_0(x),\frac{\partial u(x,0)}{\partial t}=u_1(x),0\le x\le\ell, \tag{41}$$

$$\frac{\partial u}{\partial x}\Big|_{x=0}=0,\frac{\partial u}{\partial x}\Big|_{x=\ell}=0,0\le t\le T, \tag{42}$$

$$u(x,T)=g(x),0\le x\le\ell. \tag{43}$$

This problem is reduced to an optimal control problem to find minimum of the functional

$$J(\upsilon)=\frac{1}{2}\int_0^\ell[u(x,T;\upsilon)-g(x)]^2dx \tag{44}$$

in the class

$$V=\left\{\begin{matrix}\upsilon(x)\in \overset{0}{W}{}_2^1[0,\ell]:|\upsilon(x)|\le\mu_1,|\upsilon'(x)|\le\mu_2\\ a.e.on[0,\ell]\end{matrix}\right\}, \tag{45}$$

under the constaints (40)-(42), where $f\in L_2(Q)$, $u_0\in W_2^1[0,\ell]$, $u_1\in L_2(0,\ell)$, $g\in W_2^1[0,\ell]$ are the given functions, $\mu_1>0$, $\mu_2>0$ are the given numbers.

In the work at first we prove a theorem on the existence of an optimal control. Then instead of the functional (44) we take the functional

$$I(\upsilon) = J(\upsilon) + \alpha \|\upsilon - \omega\|^2_{W_2^1[0,\ell]}, \quad \alpha > 0,$$

and prove a theorem on the existence and uniqueness of optimal control.

Then we prove theorem

Theorem 11. Let the above supposed conditions on the data of problem (40)-(42), (44), (45) be fulfilled. Then functional (44) is continuously Frechet differentiable on V and its differential at the point $\upsilon \in V$ for the increament $\delta\upsilon \in \overset{0}{W}{}_2^1[0,\ell]$ is determined by the expressions

$$\langle J'(\upsilon), \delta\upsilon \rangle = \int_Q \frac{\partial u}{\partial x} \psi \delta\upsilon dxdt.$$

Theorem 12. Let the conditions of theorem 11 be fulfilled. Then for the control $\upsilon_*(x) \in V$ be optimal in the problem (40)-(42), (44), (45) the following inequality be fulfilled

$$\int_Q \frac{\partial u_*(x,t)}{\partial x} \psi_*(x,t)(\upsilon(x) - \upsilon_*(x)) dxdt \geq 0 \quad \forall \upsilon \in V,$$

where $u_*(x,t) = u(x,t;\upsilon_*)$, of problem (40)-(42) for $\upsilon = \upsilon_*(x)$, while $\psi_* = \psi_*(x,t) = \psi(x,t;\upsilon_*)$ is the solution of the adjoint problem

$$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial}{\partial x}(\upsilon\psi) = 0, \; (x,t) \in Q,$$

$$\delta\psi|_{t=T} = 0, \frac{\partial \psi}{\partial t}|_{t=T} = u(x,T;\upsilon) - g(x), \; 0 \leq x \leq \ell,$$

$$\frac{\partial \psi}{\partial x}|_{x=0} = 0, \frac{\partial \psi}{\partial x}|_{x=\ell} = 0, \; 0 \leq t \leq T$$

for $\upsilon = \upsilon_*(x)$.

Finally, we calculate the gradient of functional (44) in the form $J'(\upsilon) = \psi_1(x;\upsilon)$, where $\psi_1(x;\upsilon)$ is the solution of the following boundary value problem

$$-\frac{d^2\psi_1}{dt^2}+\psi_1=\int_0^T\frac{\partial u}{\partial t}\psi dx, \tag{46}$$

$$\frac{d\psi_1}{dt}\Big|_{x=0}=\frac{d\psi_1}{dt}\Big|_{x=\ell}=0. \tag{47}$$

We also prove a necessary condition below.

Theorem 13. Let the conditions of theorem 11 be fulfilled. Then for the control $\upsilon_*(x)\in V$ be optimal,in problem (40)-(42), (44), (45) it is necessary that the following inequality be fulfilled

$$\int_0^\ell\left[\psi_{1*}(x)(\upsilon(x)-\upsilon_*(x))+\frac{d\psi_{1*}(x)}{dx}\left(\frac{d\upsilon(x)}{dx}-\frac{d\upsilon_*(x)}{dx}\right)\right]dx\ge 0$$

$\forall\upsilon\in V,$ where $\psi_{1*}(x)=\psi_1(x;\upsilon_*)$ is the solution of problem (46), (47) for $\upsilon=\upsilon_*(x)$.

Then we given an algorithm for finding the approximate solution of problem (40)-(42), (44),(45), using the gradient projection method.

In section 2.6 we consider a problem of definition of a pair of functions $\{u(x,t),\upsilon(x)\}$ from the condition

$$\frac{\partial^2 u}{\partial t^2}-\sum_{i=1}^n\frac{\partial}{\partial x_i}\left(\upsilon(x)\frac{\partial u}{\partial x_i}\right)+a_0(x)u=f(x,t),(x,t)\in Q, \tag{48}$$

$$u(x,0)=u_0(x),\frac{\partial u(x,0)}{\partial t}=u_1(x),x\in\Omega\quad u|_S=0, \tag{49}$$

$$\int_0^T K(x,t)u(x,t)dt=\varphi(x),x\in\Omega, \tag{50}$$

$$\upsilon=\upsilon(x)\in V=\left\{\begin{array}{l}\upsilon(x)\in W_2^1(\Omega):\nu_0\le\upsilon(x)\le\mu_0,\\ \left|\frac{\partial\upsilon}{\partial x_i}\right|\le\mu_i, i=1,...,n\ a.e.on\ \Omega\end{array}\right\}- \tag{51}$$

where

$a_0\in L_\infty(\Omega),\ f\in L_2(Q),u_0\in W_2^1(\Omega),u_1\in L_2(\Omega),K\in L_\infty(Q),\varphi\in L_2(\Omega)-$ are the given functions $\nu_0,\mu_0,\mu_1,...,\mu_n$ are the given positive numbers.

To problem (48)-(51) we associate an optimal control problem: it is required to minimiza the functional

$$J(\upsilon)=\frac{1}{2}\int_{\Omega}\left[\int_0^T K(x,t)u(x,t;\upsilon)dt-\varphi(x)\right]^2 dx \tag{52}$$

under the conditions (48), (49), (51).

At first we prove a theorem on the existence of an optimal control in problem (48), (49), (51), (52).

We introduce a adjoint problem

$$\frac{\partial^2\psi}{\partial t^2}-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\upsilon(x)\frac{\partial\psi}{\partial x_i}\right)+a_0\psi=$$

$$=-K(x,t)\left[\int_0^T K(x,\tau)u(x,\tau;\upsilon)d\tau-\varphi(x)\right],(x,t)\in Q, \tag{53}$$

$$\psi(x,T)=0,\ \ \frac{\partial\psi(x,T)}{\partial t}=0,\ \ x\in\Omega,\ \ \psi|_S=0. \tag{54}$$

Theorem 14. Let the above conditions on the data of problem (48), (49), (51), (52) and $\frac{\partial^2 u}{\partial x^2_i},\frac{\partial^2\psi}{\partial x^2_i}\in L_2(Q),\ i=1,...,n$ be fulfilled. Then the functional is continuously Frechet differentiable on V and its gradient at the point $\upsilon\in V$ at the increament $\delta\upsilon\in W^1_\infty(\Omega)$ is determined by the expression

$$\langle J'(\upsilon),\delta\upsilon\rangle=\int_{\Omega}\left[\int_0^T\sum_{i=1}^{n}\frac{\partial u}{\partial x_i}\frac{\partial\psi}{\partial x_i}dt\right]\delta\upsilon(x)dx.$$

Theorem 15. Let the conditions of theorem 14 be fulfilled. Then for the control $\upsilon_*=\upsilon_*(x)\in V$ be optimal, in problem (48), (49), (51), (52) it is necessary that the following inequality be fulfilled

$$\int_{\Omega}\left[\int_0^T\sum_{i=1}^{n}\frac{\partial u_*}{\partial x_i}\frac{\partial\psi_*}{\partial x_i}dt\right](\upsilon(x)-\upsilon_*(x))dx\geq 0\quad\forall\upsilon\in V,$$

where $u_*=u(x,t;\upsilon_*)$ and $\psi_*=\psi(x,t;\upsilon_*)$ is the solution of problems (48), (49) and (53), (54) respectively, for $\upsilon=\upsilon_*(x)$.

Chapter III of the dissertation consisting of three sections, is devoted to numerical solution of some model problems studied in chapters 1 and 2.

In section 3.1 we consider a numerical solution of the problem of definition of the pair $(u(x,t),\upsilon(t))$ from the relations[3]

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+\upsilon(t),\ (x,t)\in Q=(0,\ell)\times(0,T), \qquad (55)$$

$$u(x,0)=u_0(x),\ \frac{\partial u(x,0)}{\partial t}=u_1(x),\ 0\le x\le \ell, \qquad (56)$$

$$u(0,t)=u(\ell,t)=0,\quad 0\le t\le T, \qquad (57)$$

$u(x_0,t)=p(t),\ 0\le t\le T,\ 0<x_0<\ell,$ where $u_0(x), u_1(x), p(t)-$ are the given functions. This problem is reduced to the optimal control problem: to find the minimum of the functional

$$J_\beta(\upsilon)=\frac{1}{2}\int_0^T[u(x_0,t;\upsilon)-p(t)]^2+\frac{\beta}{2}\int_0^T[\upsilon(t)-\omega(t)]^2\,dt$$

under conditions (55)-(57) in the class

$$V=\left\{\upsilon(t)/\int_0^T\upsilon^2(t)dt\le r^2\right\},$$

where $\omega(t)$ is a given function, $r>0$ is a given number.

Applying the gradient projection method, this problem is solved numerically. In the work, the results of numerical experiments, graphs and tables are reduced.

In section 3.2 we solve numerically the following problem of definition of the initial function in a mixed problem for a string vibration equation:

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+a_0(x,t)u+f(x,t);(x,t)\in Q\equiv(0,\ell)\times(0,T), \qquad (58)$$

$$u(x,0)=u_0(x),\ \frac{\partial u(x,0)}{\partial t}=\upsilon(x),\ 0\le x\le \ell, \qquad (59)$$

[3] Кулиев, Г.Ф., Насибзаде, В.Н. Обратная задача об определении правой части уравнения колебаний струны // -Bakı: Bakı Universitetinin xəbərləri, Fizika-riyaziyyat elmləri seriyası, -2016. №2, -s.19-28.

$$u(0,t)=0,\ u(\ell,t)=0, \quad 0\le t\le T, \tag{60}$$
$$u(x,T)=\varphi(x), \quad 0\le x\le \ell,$$

This problem is reduced to a problem of finding minimum of the functional

$$J_\beta(\upsilon)=\frac{1}{2}\int_0^\ell [u(x,T;\upsilon)-\varphi(x)]^2dx+\frac{\beta}{2}\int_0^\ell [\upsilon(x)-\omega(x)]^2dx$$

under constraints (58)-(60) in the class $V=\left\{\upsilon(x)/\int_0^\ell \upsilon^2(x)dx\le r^2\right\}$ or in the class $V=\{\upsilon(x)/|\upsilon(x)|\le M\}$, where $\omega(x)$ is a given function, r, M are the given positive numbers.

Applying the gradient projection method, we solve the problem numerically. In the dissertation work we have the results of numerical experiments[4].

At last, **in section 3.3** we consider a numerical solution of the problem of definition of the coefficient in the inverse acoustic problem[5].

We search the pair $(u(x,t),\upsilon(x))$ from the relations

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}+\upsilon(x)\frac{\partial u}{\partial x}=f(x,t),\ (x,t)\in Q=(0,\ell)\times(0,T), \tag{61}$$

$$u(x,0)=u_0(x),\ \frac{\partial u(x,0)}{\partial t}=u_1(x),\ 0\le x\le \ell, \tag{62}$$

$$\frac{\partial u}{\partial x}\Big|_{x=0}=0, \frac{\partial u}{\partial x}\Big|_{x=\ell}=0, 0\le t\le T, \tag{63}$$
$$u(x,T)=g(x),\ 0\le x\le \ell,$$

where $f(x,t), u_0(x), u_1(x), g(x)$ are the given functions.

We derive the functional

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[4] Самарский, А.А., Лазаров, Р.Д., Макаров, В.Л. Разностные схемы для дифференциальных уравнений с обобщенными решениями / А.А. Самарский, Р.Д. Лазаров, В.Л. Макаров, - Москва: Высшая школа, -1987, -296 с.

[5] Кулиев, Г.Ф., Насибзаде, В.Н. Приведение обратной задачи акустики к задаче оптимального управления и её исследование // Вестник Томского Государственного Университета, математика и механика, -2018. №54, -с.5-16.

$$J(\upsilon)=\frac{1}{2}\int_0^\ell [u(x,T;\upsilon)-g(x)]^2 dx$$

and search its minimum in the class

$$V=\left\{\upsilon(x)\in \overset{0}{W}{}_2^1(0,\ell)\middle|\ |\upsilon(x)|\le M\right\},$$

where M is a positive number.

Again, applying the gradient projection method, we solve this problem numerically. Then we derive the functional

$$J_\beta(\upsilon)=J(\upsilon)+\frac{\beta}{2}\int_0^\ell [\upsilon(x)-\omega(x)]^2 dx,$$

and consider a problem of finding minimum of this functional in the class V under constraints (61)-(63). The last problem is also solved numerically. The results of numerical experiments are given in the dissertation work.

In conclusion, I express my deep gratitude to my supervisor doctor of physico-mathematical sciences, prof. H.F. Guliyev for the problem statement and his constant attention to the work.

CONCLUSIONS

In the thesis some inverse certain inverse problems of determining right-hand sides, initial and boundary functions, coefficients for second-order hyperbolic equations are reduced to problems of optimal control and the problems obtained are investigated by methods of optimal control theory.

The following main results are obtained:

- various inverse problems on the definition of the right-hand side, initial and boundary functions, coefficients of second-order hyperbolic equations to optimal control problems are formulated;
- the obtained optimal control problems are investigated;
- differentiability of the objective functionals are proved and expressions for their gradients are found;

- optimality conditions for the type of the variational inequality is obtained;
- based on the derived optimality conditions, algorithms for solving the obtained optimal control problems are developed;
- numerical experiments are carried out in some cases.

The list of the works published on the dissertation subject

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3. Кулиев, Г.Ф., Насибзаде, В.Н. Обратная задача нахождения коэффициента уравнения колебаний струны //Sumqayıt: Sumqayıt Dövlət Universiteti, Elmi xəbərlər,Təbiət və texniki elmlər bölməsi, -2016. Cild 16, №4, -s.14-21.
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The defense will be held on 30 June 2021 at 14^{00} at the meeting of the Dissertation Council ED 1.19 of the Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Address: AZ1141, Baku, B. Vahabzade str, 68.

Dissertation is accessible at the Library of the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Electronic versions of dissertation and its abstract is available on the official website (http://www.isi.az) of the Institute of Control Systems of Azerbaijan National Academy of Sciences.

Abstract was sent to the required addresses on 27 May 2021.

Signed for print: 25.05.2021

Paper format: A5

Volume: 42730

Number of hard copies: 20