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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

ASYMPTOTICS OF THE SOLUTIONS OF SOME BOUNDARY VALUE PROBLEMS FOR NONCLASSIC TYPE SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS

Speciality: 1211.01-Differential equations

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GENERAL CHARACTERISTICS OF THE WORK

Ratioanale of the topic and development degree. Passing from one physical characteristics to other ones, a great majority of nonsmooth real processes are reduced to studying boundary value problems for differential equations with a small parameter in front of the higher order derivatives. These problems are called singularly perturbed problems. One of the first studies in this field belongs to A.N. Tikhonov [1][1]. For studying dependence of the solutions of singularly perturbed differential equations different asymptotic methods were worked out by M.I. Vishik, L.A. Lusternik, A.B. Vasileva, V.F. Butuzov, A.M. Il'in and others. The asymptotic method elaborated by M.I. Vishik, L.A. Lusternik in the sixties of the XX century and whose content was reflected in the papers [2][2], [3][3] is the best of the existing asymptotic methods by its application area and from point of view of exact mathematical justification.

One work of M.I. Vishik and L.A. Lusternik, with the exaption of some papers of M.M. Sabzaliev and I.M. Sabzalieva, all the studies carried out for singularly perturbed partial differential equations relate to differential equations belonging to one of classic types. But mathematical models of some applied problems are described by singularly perturbed differential equations belonging to none of the classic types. Therefore, study of boundary value problems stated for noncalssic type singularly perturbed differential equations is of

[1] Тихонов А.Н. О зависимости решений дифференциальных уравнений от малого параметра // Математический сборник, - Москва:-1948, №2, вып.22(64), -с.193-204.

[2] Вишик М.И., Люстерник Л.А. Регулярное вырождение и пограничный слой для линейных дифференциальных уравнений смалым параметром // Успехи математических наук, -Москва:-1957, вып.5(77), т.12, -с.3-122

[3] Вишик М.И., Люстерник Л.А. Решение некоторых задач о возмущении в случае матриц и самосопряженных и несамосопряженных дифференциальных уравнений // Успехи математических наук, - Москва:-1960, вып.3(93), -с.3-80.

theoretical and applied importance. In their monograph [4][4], M.M. Sabzaliev and I.M. Sabzalieva worked out general approach methods for studying dependence of the solutions of boundary value problems stated for nonclassic type singularly perturbed differential equations on a small parameter.

In the work complete asymptotics of the solutions of boundary value problems stated in bounded and unbounded domains for a third order nonclassic type differential equation degenerated into a second order parabolic equation was constructed. In the work, complete asymptotics of the solution of a boundary value problem stated for a nonclassic type third order differential equation degenerated into a quasilinear hyperbolic equation was constructed, using minimal conditions imposed on the data and internal layer type functions the first terms of the asymptotic expansion of the solution was constructed and complete asymptotics of a boundary value problem stated for this equation in an infinite strip, was structured.

The object and subject of the study. The object and subject of the study are boundary value problems for non-classic type third order linear differential equations degenerated into a second order parabolic equation, and non-classic type third order quasilinear and linear differential equations degenerated into first order linear partial differential equations.

Goals and duties of the study. To construct asymptotic expansion with respect to a small parameter of the solutions of some boundary value problems stated in bounded and unbounded domains for two classes of nonclassic type perturbed differential equations degenerated into a linear parabolic or quasilinear hyperbolic equation.

Methods of the study. In the work, theory of differential equations, the methods of theory of series and “Vishik-Lusternik” method were used.

The basic defended statements. The following statements are defended:

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[4] Səbzəliyev M.M., Səbzəliyeva İ.M. Klassik tiplərə aid olmayan sinqulyar həyəcanlanmış diferensial tənliklər nəzəriyyəsinə giriş. Monoqrafiya./ -Bakı: Elm nəşriyyatı, -2018, -200 s.

- Constructing complete asymptotics of the solution of a boundary value problem stated in a rectangular domain for a third order, nonclassic type singularly perturbed differential equation degenerated into a second order linear parabolic equation with respect to a small parameter, and estimating the obtained residual term.
- Constructing complete assymptotics of the solution of a boundary value problem stated in an infinite semistrip for a nonclassic type singularly perturbed differential equation degenerated into a second order linear parabolic equation with respect to a small parameter and estimation of the obtained residual term.
- Constructing complete asymptotics of the solution of a boundary value problem stated in an infinite strip for a third order nonclassic type singularly perturbed differential equation degenerated into a second order linear parabolic equation with respect to a small parameter, and estimation of the obtained residual term.
- Constructing complete asymptotics of the solution of a boundary value problem stated on a rectangular domain for a third order nonclassic type singularly perturbed differential equation degenerated into a quasilinear hyperbolic equation with respect to a small parameter, and estimation of the residual term.
- Constructing initial terms of asymptotic expansion of the solution of a bisingular boundary value problem stated in a rectangular domain for a nonclassic type differential equation degenerated into a quasilinear hyperbolic equation with respect to a small parameter by internal layer type functions and estimation of the residual term.
- Constructing asymptotic expansion of a boundary value problem stated in an infinite strip for a third order nonclassic type singularly perturbed differential equation degenerated into a linear hyperbolic equation with respect to a small parameter and estimation of the residual term.

Scientific novelty of the study. The following scientific novelties of the work are the followings:

- Asymptotic expansion of the solution of a boundary value problem stated in a rectangular domain for a third order nonclassic type singularly perturbed differential equation degenerated into a second order linear parabolic equation was constructed to within to any positive power of a small parameter and the residual term was estimated.

- Asymptotic expnasion of the solution of a boundary value problem stated in an infinite semi-strip for a nonclassic type singularly perturbed differential equation degenerated into a second order linear parabolic equation was constructed to within any positive power of a small parameter and the obtained residual term was estimated.
- Asymptotic expansion of the solution of a boundary value problem stated in an infinite strip for a third order nonclassic type singularly perturbed differential equation degenerated into a second order linear parabolic equation was structured to within any positive power of a small parameter and the resudual term was estimated.
- Asymptotic expansion of the solution of a boundary value problem stated on a rectangular domain for a nonclassic type singularly perturbed differential equation degenerated into a quasilinear hyperbolic equation was structured to within any positive power of a small paramter and the obtained residual term was estimated.
- The first terms of the asymptotic expansion of the solution of a boundary value problem stated in a rectangular domain for a bisingular boundary probem for a third order nonclassic type differential equation degenerated into a quasilinear hyperbolic equation with respect to a small parameter were structured by means of internal layer type functions and the residual term was estimated.
- The asymptotic expansion of a boundary value problem stated in an infinite semi-strip for a third order nonclassic type singularly perturbed differential equation degenerated into a linear hyperbolic equation was structured to within any positive power of a small parameter and the obtained residual term was estimated.

Theoretical and practical importance of the work. The dissertation work is of theoretical character. The results obtined in the work enrich the theory of nonclassic type singularly perturbed differential equations, has a great theoretical importance and can be effectively applied in many practical works.

Approbation and application. The results obtained in the work were reported at the International and Republican Conferences. The international Conference dedicated to 85-th jubilee of acad. A.Kh. Mirzajanzade (Baku, 2013); The international Azerbaijan-Turkey-Ukraine Conference, MADEA7 (Baku- 2015); The international Conference “Non-harmonic Analysis and Differential Operators” (Baku-2016); The XX Republican Conference of doctoral students and young researchers (Baku-2016); The international Conference dedicated to 55 years of Sumgayit State University held by SSU and IMM ANAS (Baku-2017); International Conference dedicated to 80-th jubilee of acad. Akif Gadjiev (Baku-2017); The V International Conference dedicated to 95 years of corr. member of Russian Academy of Sciences L.D. Kudryavtsev (Moscow-2018); International Conference dedicated to 90-th anniversary of acad. A.Kh. Mirzajanzade (Baku- 2018); International Conference dedicated to 95-th anniversary of Heydar Aliyev held at Business University (Baku- 2018); International Conference dedicated to 96-th anniversary of Heydar Aliyev at Business University (Baku- 2019); The 7th International conference on Control and Optimization with industrial applications (Baku- 2020).

Author’s publications. The results of the work were published in 19 scientific papers. They are given at the end of the thesis.

The organization where the work was executed. The work was executed at the chair of “Universal and applied mathematics” of Azerbaijan State Oil and Industry University.

Total volume of the dissertation. The total volume of the work is 239808 signs (title page- 369 signs, content- 184000 signs, introduction- 53350 signs, chapter I- 92000 signs, chapter II- 92000 signs, result- 2458 signs). The list of referencess 122 titles.

THE CONTENT OF THE DISSERTATION WORK

The dissertation work consists of introduction, 2 chapters, result and list of references. Each chapter consists of 3 sections.

Rationale of the work, short abstract of results related to the topic of the dissertation work and basic results obtained in the dissertation work were given in introduction.

Now we comment the basic results.

In chapter complete asymptotics of the solution of a boundary value problem stated in a rectangular domain, infinite semi-strip and infinite strip for a third order nonclassic type partial differential equation degenerated into a second order linear parabolic equation was structured with respect to a small parameter and the obtained residual terms are estimated.

In section 1.1 in the rectangular domain $D=\{(t,x)|0\le t\le T,\ 0\le x\le 1\}$ we consider the following boundary value problem:

$$L_\varepsilon u \equiv \varepsilon^2 \frac{\partial}{\partial t}(\Delta u) - \varepsilon \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + au = f(t,x), \qquad (1)$$

$$u|_{t=0} = u|_{t=T} = 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=T} = 0, \quad (0\le x\le 1), \qquad (2)$$

$$u|_{x=0} = u|_{x=1} = 0, \quad (0\le t\le T). \qquad (3)$$

Here $\varepsilon>0$ is a small parametr, $\Delta \equiv \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}$, $a>0$ is a constant, $f(t,x)$ is the given function.

Our global is to construct the asymptotics of the solution of boundary value problem (1)-(3) with respect to a small parameter. For that we carry out iterative processes

In the first iterative process, the approximate solution of differential equation (1) is sought in the form $W=\sum_{i=0}^{n}\varepsilon^i W_i$. For the unknown function $W_i(t,x);\ \ i=0,1,\ldots,n$ we get the following boundary value problems:

$$\frac{\partial W_0}{\partial t}-\frac{\partial^2 W_0}{\partial x^2}+aW_0 = f(t,x),\ \ W_0\big|_{t=0}=0,\ \ W_0\big|_{x=0}=W_0\big|_{x=1}=0, \quad (4)$$

$$\frac{\partial W_i}{\partial t}-\frac{\partial^2 W_i}{\partial x^2}+aW_i = f_i(t,x);\ W_i\big|_{t=0}=0,\ \ W_i\big|_{x=0}=W_i\big|_{x=1}=0. \quad (5)$$

Here the functions $f_i(t,x)$ are the known functions dependent on the functions $W_0,W_1,...,W_{i-1};\ \ i=1,2,\dots,n$.

In order to determine the functions $W_0,W_1,...,W_n$ from the problems (4), (5) we prove the following statement.

Lemma 1. Assume that the function $f(t,x)$ on *D* with respect to *t* has *p-1*-th order, with respect to *x* has *p+2*-th order continuous derivatives and satisfies the following condition:

$$\frac{\partial^{2s} f(t,0)}{\partial x^{2s}}=\frac{\partial^{2s} f(t,1)}{\partial x^{2s}}=0;\quad s=0,1,\dots,p.$$

Then the solution of boundary value problem on *D* with respect to *t* has *p*-th order, with respect to *x* has *2p*-th order continuous derivative and this solution satisfies the condition (4)

$$\frac{\partial^{i_1+2i_2} W_0(t,0)}{\partial t^{i_1}\partial x^{2i_2}}=\frac{\partial^{i_1+2i_2} W_0(t,1)}{\partial t^{i_1}\partial x^{2i_2}}=0;\qquad i_1+i_2\le p$$

(*p* is an arbitrary natural number).

For *p=n+3* the functions $W_1,W_2,...,W_n$ also are defined from boundary value problem (5). The function $W=\sum_{i=0}^{n}\varepsilon^i W_i$ structured in the iterative process satisfies the boundary conditions

$$W\big|_{t=0}=0,\ \ W\big|_{x=0}=W\big|_{x=1}=0\,. \quad (6)$$

But this function need not satisfy the boundary conditions in (2) on *t=T*. In order to provide satisfaction of the lost boundary conditions we carry out the second iterative process and construct a boundary layer type functions near the boundary *t=T* of domain *D*.

In order to carry out the second iterative process, at first near the boundary *t=T* we make subsitution $T-t=\varepsilon\tau,\ x=x$ and write a new expansion L_ε of the operator $L_{\varepsilon,1}$ with respect to the powers of the small parameter. The boundary layer type function near $t=T$ is sought in the form $V=\sum_{j=0}^{n+1}\varepsilon^{j}V_{j}$ as an approximate solution of the equation $L_{\varepsilon,1}V=0$. As a result, to determine the unknown functions $V_j(\tau,x);\ j=0,1,...,n+1$ we obtain the following ordinary differential equation:

$$\frac{\partial^3 V_0}{\partial\tau^3}+\frac{\partial^2 V_0}{\partial\tau^2}+\frac{\partial V_0}{\partial\tau}=0, \tag{7}$$

$$\frac{\partial^3 V_j}{\partial\tau^3}+\frac{\partial^2 V_j}{\partial\tau^2}+\frac{\partial V_j}{\partial\tau}=h_j\,. \tag{8}$$

Here the functions h_j are the known functions dependent on the functions $V_0, V_1,...,V_{j-1};\ j=1,2,...,n+1$. The boundary conditions for differenial equations (7), (8) are found from the equalities

$$\left(W+V\right)\Big|_{t=T}=0,\qquad \frac{\partial}{\partial t}\left(W+V\right)\Big|_{t=T}=0 \tag{9}$$

and are in the following form:

$$V_0\Big|_{\tau=0}=-W_0(T,x),\ \ \frac{\partial V_0}{\partial\tau}\Big|_{\tau=0}=0\,; \tag{10}$$

$$V_i\Big|_{\tau=0}=-W_i(T,x)\,;\ \ i=1,2,\dots,n;\ \ V_{n+1}\Big|_{\tau=0}=0;$$

$$\frac{\partial V_j}{\partial\tau}\Big|_{\tau=0}=\frac{\partial W_{j-1}}{\partial t}\Big|_{t=T};\ \ j=1,2,\dots,n+1\,. \tag{11}$$

The boundary layer type solution of the boundary value problem (7), (10) is determined by the formula

$$V_0(\tau,x)=\frac{W_0(T,x)}{\lambda_1-\lambda_2}\left(\lambda_2 e^{\lambda_1\tau}-\lambda_1 e^{\lambda_2\tau}\right). \tag{12}$$

Here $\lambda_{1,2}=-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$ are negative roots of a characteristic equation corresponding to the ordinary differential equation (7) with negative real part. To determine the boundary layer type solutions of the problems (8), (11) the following statement is proved.

Lemma 2. The boundary layer type solutions of differential equations (8) satisfying appropriate boundary conditions in (11) are determined by the formulas

$$V_j(\tau,x)=\sum_{k=0}^{j}\left\{\left[b_{jk}^{(1)}(x)\tau^k\right]e^{\lambda_1\tau}+\left[b_{jk}^{(2)}(x)\tau^k\right]e^{\lambda_2\tau}\right\};\ j=1,2,\ldots,n+1. \quad (13)$$

Here the functions $b_{js}^{(k)}(x)$; $k=1,2$; $s=1,2,...,j$ are the known functions expressed by the values of the functions $W_0(T,x),W_1(T,x),...,W_j(T,x)$, their first order derivatives with respect to *t* and only even order derivatives with respect to *x*.

Multiplying all the functions V_j by the smoothing functions, we retain for the obtained new functions the previous denotation V_j; $j=0,1,...,n+1$. Hence, from (12), lemma 2 and (6) we get that the structured sum $W+V$ in addition to (9) satisfies the following boundary conditions as well:

$$\left.(W+V)\right|_{t=0}=0, \qquad \left.(W+V)\right|_{x=0}=\left.(W+V)\right|_{x=1}=0.$$

Theorem 1. Assume that for $p=n+3$ the conditions of lemma 1 are satisfied. Then for the solution of the boundary value problem (1)-(3) the following asymptotic expansion is valid

$$u=\sum_{i=0}^{n}\varepsilon^i W_i+\sum_{j=0}^{n+1}\varepsilon^j V_j+\varepsilon^{n+1}z\ . \quad (14)$$

Here the functions W_i are determined in the first iterative process, the functions V_j are boundary layer type functions near $t=T$ and are determined in the second iterative process, $\varepsilon^{n+1}z$ is a residual term, for the function z the estimation

$$\varepsilon^2\left\|\left.\frac{\partial z}{\partial t}\right|_{t=0}\right\|_{L_2(0,1)}^2+\varepsilon\left\|\frac{\partial z}{\partial t}\right\|_{L_2(D)}^2+\left\|\frac{\partial z}{\partial x}\right\|_{L_2(D)}^2+C_1\|z\|_{L_2(D)}^2\le C_2$$

is valid. The constants $C_1>0,\ C_2>0$ are independent of ε.

The results obtained in this section were published in the author's works [1], [3].

In section 1.2 in the infinite semi-strip

$P_+ = \{(t,x) | 0 \le t \le 1,\ 0 \le x < +\infty\}$ for the differential equation (1) we give the following boundary conditions

$$u\big|_{t=0} = u\big|_{t=1} = 0, \quad \frac{\partial u}{\partial t}\bigg|_{t=1} = 0, \quad (0 \le x < +\infty), \qquad (15)$$

$$u\big|_{x=0} = 0, \ \lim_{x \to +\infty} u = 0, \quad (0 \le t \le 1). \qquad (16)$$

In this section the asymptotic expansion of boundary value problem (1), (15), (16) is sought in the form of (14). But here for the unknown functions $W_i(t,x);\ i = 0,1,...,n$ the following boundary conditions are obtained:

$$W_i\big|_{t=0} = 0,\ W_i\big|_{x=0} = 0, \lim_{x \to +\infty} W_i = 0; i = 0,1,...,n. \qquad (17)$$

We prove the following statement.

Lemma 3. Assume that the function $f(t,x)$ on P_+ has $n+3$-th order continuous derivatives with respect to t, and $n+2$-th order continuous derivatives with respect to x, and satisfies the condition

$$\left|\frac{\partial^k f(t,x)}{\partial t^{k_1} \partial x^{k_2}}\right| \le c_1 \exp(-c_2 x); \quad c_1 > 0,\ c_2 > 0; \ \ k = k_1 + k_2;$$

$$k_1 = 0,1,\ldots,n+3; \quad k_2 = 0,1,\ldots,n+2.$$

The degenerating boundary value problem has a unique solution and this solution satisfies the condition

$$\left|\frac{\partial^k W_0(t,x)}{\partial t^{k_1} \partial x^{k_2}}\right| \le c_3 \exp(-c_2 x); \ c_3 > 0, \ \ k = k_1 + k_2;$$

$$k_1 = 0,1,\ldots,n+4; \quad k_2 = 0,1,\ldots,n+2. \qquad (18)$$

The functions $W_1, W_2, \ldots, W_n$ are determined by Lemma 3.

The function $W = \sum_{i=0}^{n} \varepsilon^i W_i$ satisfies the boundary condition

$$W\big|_{t=0} = 0,\ W\big|_{x=0} = 0, \quad \lim_{x \to +\infty} W = 0. \qquad (19)$$

But the function W need not satisfy in (15) the boundary condition on *t=1*. In order to provide the satisfaction of these lost boundary conditions we construct boundary layer type functions near *t=1*. Construction of boundary layer type functions near $t=1$ is carried out similar to construction of boundary layer type functions near $t=T$ in section 1.1.

According to the formula for determined functions V_j, we obtain that the $V=\sum_{j=0}^{n+1}\varepsilon^j V_j$ structured in the second iterative process satisfies the following boundary conditions:

$$V\big|_{x=0}=0,\quad \lim_{x\to+\infty}V=0,\quad (0\le\tau<+\infty). \tag{20}$$

The results obtained in section 1.2 are expressed in the form of the following theorem.

Theorem 2. Assume that the function $f(t,x)$ satisfies the conditions of lemma 3. Then for solving the boundary value problem (1), (15), (16) the asymptotic expansion (14) is valid. Here the functions W_i are determined from first iterative process. The functions V_j are boundary layer type functions near $t=1$, $\varepsilon^{n+1}z$ is a residual term, and for the function z the following estimation

$$\varepsilon^2\left\|\frac{\partial z}{\partial t}\bigg|_{t=0}\right\|^2_{L_2(0,+\infty)}+\varepsilon\left\|\frac{\partial z}{\partial t}\right\|^2_{L_2(P_+)}+\left\|\frac{\partial z}{\partial x}\right\|^2_{L_2(P_+)}+C_1\|z\|^2_{L_2(P_+)}\le C_2$$

is valid, the constants $c_1>0,\ c_2>0$ are independent of ε.

The results obtained in section 1.2 are in the author's works [4], [5], [6].

In section 1.3, in the infinite strip $P=\{(t,x)\,|\,0\le t\le 1,-\infty<x<+\infty\}$ for the differential equation (1) the following conditions are given:

$$u\big|_{t=0}=u\big|_{t=1}=0,\quad \frac{\partial u}{\partial t}\bigg|_{t=1}=0,\quad (-\infty<x<+\infty), \tag{21}$$

$$\lim_{|x|\to+\infty}u=0,\ (0\le t\le 1). \tag{22}$$

In this section for the functions $W_i;\ i=0,1,...,n$ constructed in the first iterative process the following boundary value problems are obtained:

$$\frac{\partial W_0}{\partial t}-\frac{\partial^2 W_0}{\partial x^2}+aW_0 = f(t,x),\ W_0\big|_{t=0}=0,\ \lim_{|x|\to+\infty} W_0=0. \qquad (23)$$

$$\frac{\partial W_i}{\partial t}-\frac{\partial^2 W_i}{\partial x^2}+aW_i = f_i(t,x);\ W_i\big|_{t=0}=0,$$

$$\lim_{|x|\to+\infty} W_i=0,\quad i=1,2,\ldots,n. \qquad (24)$$

The following statement on boundary value problem (23) is proved.

Lemma 4. Assume that on P the function $f(t,x)$ has $n+3$-th order continuous derivatives with respect to the variable t, is infinitely differentiable with respect to the variable x and satisfies the following condition:

$$\sup_x\left(1+|x|^l\right)\left|\frac{\partial^k f(t,x)}{\partial t^{k_1}\partial x^{k_2}}\right| = C^{(1)}_{l_{k_1k_2}} < +\infty.$$

Here l is a non-negative number, $k=k_1+k_2, k_1\le n+3,\ k_2$ is an arbitrary non-negative integer, and $C_{lk_1k_2}=const>0$. Then boundary value problem (23) has a unique solution and on P this solution has *n+4*-th order conitnuous derivatives with respect to *t*, is infinitely differentiable with respect to x and satisfies the following condition:

$$\sup_x\left(1+|x|^l\right)\left|\frac{\partial^k W_0(t,x)}{\partial t^{k_1}\partial x^{k_2}}\right| = C^{(2)}_{lk_1k_2} < +\infty. \qquad (25)$$

Here $k_1\le n+4,\quad C^{(2)}_{lk_1k_2}=const>0.$

Lemma 4 is also used in determining the functions $W_1,W_2,...,W_n$ as the solution of boundary value problems (24).

The function $W=\sum_{i=0}^{n}\varepsilon^i W_i$ structured in such a way satisfies the following conditions:

$$W\big|_{t=0}=0,\quad (-\infty<x<+\infty);\ \lim_{|x|\to+\infty} W=0,\quad (0\le t\le 1). \qquad (26)$$

As in section 1.2, we construct such a boundary layer type function $V=\sum_{j=0}^{n+1}\varepsilon^j V_j$ that for the sum of $W+V$ the boundary conditions are satisfied for considered problem.

The results obtained in this section were published in the author's works [2], [10].

Theorem 3. Assume that the function $f(t,x)$ satisfies the condition of lemma 4. Then for solving the boundary value problem (1), (21), (22) the asymptotic expansion (14) is valid. Here the functions W_i are determined from the boundary conditions (23), (24), the functions V_j are boundary layer type functions near $t=1$ and are determined in the second iterative process, $\varepsilon^{n+1}z$ is a residual term, and for the function z the following estimation is valid:

$$\varepsilon^2 \left\| \frac{\partial z}{\partial t}\bigg|_{t=0} \right\|^2_{L_2(-\infty,+\infty)} + \varepsilon \left\| \frac{\partial z}{\partial t} \right\|^2_{L_2(P)} + \left\| \frac{\partial z}{\partial x} \right\|^2_{L_2(P)} + C_1 \|z\|^2_{L_2(P)} \le C_2$$

the constants, $c_1 > 0,\ c_2 > 0$ are independent of ε.

In chapter II for nonclassic type third order partial differential equation degenerating into a quasilinear hyperbolic equation we consider boundary value problems on a rectangular domain and infinite semi-strip. This chapter consists of 3 sections.

In section 2.1 on the rectangular domain $D = \{(x,y) \,|\, 0 \le x \le 1, 0 \le y \le 1\}$ we consider the following boundary value problem:

$$L_\varepsilon u \equiv \varepsilon^2 \frac{\partial}{\partial x}(\Delta u) - \varepsilon \Delta u + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + F(x,y,u) = 0, \quad (27)$$

$$u\big|_{x=0} = u\big|_{x=1} = 0, \frac{\partial u}{\partial x}\bigg|_{x=1} = 0, \ (0 \le y \le 1), \quad (28)$$

$$u\big|_{y=0} = u\big|_{y=1} = 0, \ (0 \le x \le 1). \quad (29)$$

Here $F(x,y,u)$ is a given function.

In the case when the function $F(x,y,u)$ is linearly dependent on the variable u, more exactly when $F(x,y,u) = u - f(x,y)$, by imposing very hard conditions on the function $f(x,y)$, only initial terms of the asymptotic expansion of the solution of boundary value problem (27)-(29) was found by Vishik M.I. and Lusternik L.A. in

[2][2]. By rejecting to impose hard conditions on the function $f(x,y)$, using inner boundary layer type functions, in [5][5] M.N. Javadov and M.M. Sabzaliev have constructed initial terms of the asymptotic expansion of the solution of this linear boundary value problem. In the paper [5][5] complete asymptotics of the solution of the boundary value problem stated for this linear equation in an infinite strip was also constructed. In this section, the complete asymptotics of the solution of boundary value problem (27)-(29) stated on a rectangular domain for a quasiliner equation was structured.

For constructing asymptotic expansion of the solution of boundary value problem (27)-(29) in the carried out iterative process the approximate solution of the equation (27) was sought in the form of $W=\sum_{i=0}^{n}\varepsilon^i W_i$ and for the functions $W_i(x,y)$ the following differential equations were obtained:

$$\frac{\partial W_0}{\partial x}+\frac{\partial W_0}{\partial y}+F(x,y,W_0)=0, \qquad (30)$$

$$\frac{\partial W_j}{\partial x}+\frac{\partial W_j}{\partial y}+\frac{\partial F(t,x,W_0)}{\partial W_0}W_j = f_j;\ j=1,2,...,n. \qquad (31)$$

Here $f_j(W_0,W_1,...,W_{j-1})$ denote known functions dependent on the functions $W_0,W_1,...,W_{j-1}$. We look for the solutions of differential equations (30), (31) satisfying the boundary conditions

$$W_0\big|_{x=0}=0;\ W_0\big|_{y=0}=0, \qquad (32)$$

$$W_j\big|_{x=0}=0,\ \ W_j\big|_{y=0}=0; j=1,2,...,n. \qquad (33)$$

The solution of boundary value problem (30), (32) has a singularity on a bisector of the first coordinate quarter. To eliminate this singularity, when the function $F(x,y,u)$ is linearly dependent on the variable u, on the function $f(x,y)\in C^{2n+3}(D)$ we impose the condition

[5] Джавадов М.Г., Сабзалиев М.М. Об одной краевой задаче для однохарактеристического уравнения, вырождающегося в однохарактеристическое//ДАН СССР,-Москва:1979, №5,т.247,-с.1041-1046

$$\frac{\partial^i f(x,y)}{\partial x^{i_1}\partial y^{i_2}}\Big|_{y=x} = 0;\ i = i_1 + i_2;\ i = 0,1,...,2n+3;\ (0 \le x \le 1) \qquad (34)$$

when the function $F(x,y,u) \in C^{2n+3}\left(D \times \left(-\infty,+\infty\right)\right)$ is linearly independent of the variable u the condition

$$F(x,y,u)\Big|_{y=x} = 0;\ \frac{\partial^i F(x,y,o)}{\partial x^{i_1}\partial y^{i_2}\partial u^{i_3}}\Big|_{y=x} = 0; i = i_1 + i_2 + i_3;$$

$$i = 1,2,...,2n+3;\ (0 \le x \le 1). \qquad (35)$$

In this case the function $W_0(x,y)$ enters the space $C^{2n+3}(D)$ and satisfies the following condition:

$$\frac{\partial^i W_0(x,y)}{\partial x^{i_1}\partial y^{i_2}}\Big|_{y=x} = 0;\ i = i_1 + i_2;\ i = 0,1,...,2n+3;\ (0 \le x \le 1). \qquad (36)$$

Thus, in the first iterative process we construct such a function $W = \sum_{i=0}^{n} \varepsilon^i W_i$ that this function satisfies the boundary conditions

$$W\Big|_{x=0} = 0, \quad W\Big|_{y=0} = 0. \qquad (37)$$

In order to satisfy the boundary conditions near *x=1* and *y=1,* we construct a boundary layer type functions.

Near *x=1* the boundary layer type function is sought in the form $V = \sum_{j=0}^{n+1} \varepsilon^j V_j$ as the solution of the equation

$$L_{\varepsilon,1}(W+V) - L_{\varepsilon,1}W = O\left(\varepsilon^{n+1}\right). \qquad (38)$$

It was determined that for the functions $V_j;\ j = 0,1,...,n+1$ to vanish at *y=0* and satisfy also the condition

$$\left(W+V\right)\Big|_{y=0} = 0 \qquad (39)$$

of the sum $W+V$, when the function $F(x,y,u)$ is linearly dependent on the variable u satisfies the condition

$$\frac{\partial^i f(1,0)}{\partial x^{i_1}\partial y^{i_2}} = 0;\ i = i_1 + i_2;\ i = 0,1,...,2n+3, \qquad (40)$$

when is linearly independent satisfies the condition

$$\frac{\partial^i F(1,0,0)}{\partial x^{i_1}\partial y^{i_2}\partial u^{i_3}}=0;\ i=i_1+i_2+i_3;\ i=0,1,...,2n+3. \qquad (41)$$

Near *y=1*, the boundary layer type function is structured in the form $\eta=\sum_{j=0}^{n+1}\varepsilon^j\eta_j$ as the solution of the equation

$$L_{\varepsilon,2}(W+V+\eta)-L_{\varepsilon,2}(W+V)=O\left(\varepsilon^{n+1}\right). \qquad (42)$$

To determine the functions η_j from the equality (42) we get the following differential equations:

$$\frac{\partial^2\eta_s}{\partial t^2}+\frac{\partial\eta_s}{\partial t}=P_s;\ s=0,1,...,n+1. \qquad (43)$$

Here, the functions $P_0\equiv 0,\ P_k$ are known functions dependent on the functions $W_0,W_1,...,W_{k-1};\ V_0,V_1,...,V_{k-1},\ \eta_0,\eta_1,...,\eta_{k-1}\ \ k=1,2,...,n+1.$

For differential equations (43) the boundary conditions are found from the equality

$$(W+V+\eta)\big|_{y=1}=0. \qquad (44)$$

Using the fact that all the functions $W_i(x,y);\ i=0,1,...,n$ vanish for $x=y$, we get that the functions η_j satisfy the conditions

$$\eta_j\big|_{x=1}=0,\ \frac{\partial\eta_j}{\partial x}\big|_{x=1}=0;\ \ j=0,1,...,n+1.$$

Hence we get that the sum $W+V+\eta$ satisfies the boundary conditions

$$\left(W+V+\eta\right)\big|_{x=1}=0,\ \frac{\partial}{\partial x}(W+V+\eta)\big|_{x=1}=0 \qquad (45)$$

as well.

When the function $F(x,y,u)$ is linearly dependent on the variable u satisfies the condition

$$\frac{\partial^i f(0,1)}{\partial x^{i_1}\partial y^{i_2}}=0;\ i=i_1+i_2;\ i=0,1,...,2n+3, \qquad (46)$$

when is lienarly indepedent satisfies the condition

$$\frac{\partial^i F(0,1,0)}{\partial x^{i_1} \partial y^{i_2} \partial u^{i_3}} = 0;\ i = i_1 + i_2 + i_3;\ i = 0,1,...,2n+3, \tag{47}$$

the structured sum $W + V + \eta$ satisfies the condition

$$(W + V + \eta)\big|_{x=0} = 0$$

as well.

Denoting by z, the difference between exact solution u of boundary value problem (27)-(29) and the structured $\tilde{u} = W + V + \eta$ in this section we get the following asymptotic expansion for solving the problem under consideration in this section:

$$u = \sum_{i=0}^{n} \varepsilon^i W_i + \sum_{j=0}^{n+1} \varepsilon^j V_j + \sum_{j=0}^{n+1} \varepsilon^j \eta_j + z. \tag{48}$$

Theorem 4. Assume that when the function $F(x, y, u) \in C^{2n+3}(D \times (-\infty, +\infty))$ is linearly dependent on the variable u, satisfies the conditions (34), (40), (46), when is linearly indepedent of the variable u satisfies the conditions (35), (41), (47) and the condition

$$\frac{\partial F(x, y, u)}{\partial u} \geq \alpha^2 > 0; (x, y, u) \in (D \setminus \{(x, y) \in D | x = y\}) \times$$
$$\times (-\infty, +\infty). \tag{49}$$

Then for solving the boundary value problem (27)-(29) the asymptotic expnasion (48) is valid. Here the functions W_i are determined in the first iterative process, the functions V_j near $x = 1$, the functions η_j near $y = 1$ are boundary layer type functions and are determined in approximate iterative processes, z is a residual term and the estimation

$$\varepsilon^2 \left\| \frac{\partial z}{\partial x}\bigg|_{x=0} \right\|^2_{L_2(0,1)} + \varepsilon \left[\left\| \frac{\partial z}{\partial x} \right\|^2_{L_2(D)} + \left\| \frac{\partial z}{\partial y} \right\|^2_{L_2(D)} \right] + C_1 \|z\|^2_{L_2(D)} \leq C_2 \varepsilon^{2(n+1)}$$

is valid for it the constants $C_1 > 0, C_2 > 0$ are independent of ε.

The results obtained in section 2.1 were published in the author's papers [7], [8], [9], [13], [16], [17].

In section 2.2 we consider boundary value problem (27)-(29). In the value $\varepsilon = 0$ of the small parameter this problem degenerates into the boundary value problem

$$\frac{\partial W}{\partial x} + \frac{\partial W}{\partial y} + F(x, y, W) = 0 \qquad (50)$$

$$W\Big|_{x=0} = 0;\ (0 \le y \le 1),\ W\Big|_{y=0} = 0;\ (x \le 0 \le 1). \qquad (51)$$

The derivatives of the function $W(x, y)$ have a singularity in the domain D on the line $y = x$. At first we find such an inner boundary layer type function $\eta = \varepsilon\eta_0$ in D near the line $y = x$ and add to the solution of the problem (50), (51) that the function $W + \eta$ and its first derivatives be continuous. The equation for the function η_0 is obtained from the equality

$$L_{\varepsilon,1}(W + \eta) - L_{\varepsilon,1}W = O(\varepsilon)$$

and is in the form:

$$\frac{\partial^3 \eta_0}{\partial \xi^3} + \frac{\partial^2 \eta_0}{\partial \xi^2} = 0. \qquad (52)$$

For $\xi > 0$, we take the solution of differential equation (52) in the form $\eta_0 = \varphi(x_1)(e^{-\xi} - 1)$, for $\xi < 0$ the trivial solution $\eta_0 \equiv 0$. The unknown function $\varphi(x_1)$ is found from the requirement that the function $\frac{\partial}{\partial x}(W + \eta)$ is a continuous function on the line $y = x$. It is shown that when the function $\varphi(x_1)$ is found from this requirement, the function $\frac{\partial}{\partial y}(W + \eta)$ is also continuous on the line $y = x$.

After at first we construct a boundary layer type function $V = V_0 + \varepsilon V_1$ near $x = 1$, and then a boundary layer type function $\psi = \psi_0 + \varepsilon\psi_1$ near $y = 1$.

For solving the bounday value problem (27)-(29) in the first approximation we get the following asymptotic expansion:

$$u = W + \eta + V + \psi + z. \qquad (53)$$

Theorem 5. Assume that the function $F(x, y, u)$ enters the space $C^3\big(D \times (-\infty, +\infty)\big)$, satisfies the conditions $F(1,0,u) = 0$;

$F(0,1,u)=0$ and

$$\frac{\partial F(x,y,u)}{\partial u} \geq \alpha^2 > 0; \ (x,y,u) \in D \times (-\infty,+\infty).$$

Then for the solution of the boundary value problem (27)-(29) in the first approximation the asymptotic expansion (53) is valid. Here W is the solution of the degenerated problem, η is a boundary layer type function in the domain D near the line $y = x$, the functions V, ψ are boundary layer type functions near $x=1$ and $y=1$, respectively, for z the estimation

$$\|z\|_{L_2(D)} \leq C\varepsilon, \ \left\|\frac{\partial z}{\partial x}\right\|_{L_2(D)} + \left\|\frac{\partial z}{\partial y}\right\|_{L_2(D)} \leq C$$

is valid. The constant $C > 0$ is independent of ε.

The results obtained in section 2.2 were published in the author's paper [14].

In section 2.3 in the infinite semi-strip $P_+ = \{(x,y) | 0 \leq x \leq 1, 0 \leq y < +\infty\}$ we consider the following boundary value problem:

$$L_\varepsilon u \equiv \varepsilon^2 \frac{\partial}{\partial x}(\Delta u) - \varepsilon \Delta u + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = f(x,y), \tag{54}$$

$$u\big|_{x=0} = u\big|_{x=1} = 0, \ \frac{\partial u}{\partial x}\bigg|_{x=1} = 0, \ (0 \leq y < +\infty), \tag{55}$$

$$u\big|_{y=0} = 0, \ \lim_{y \to +\infty} u = 0, \quad (0 \leq x \leq 1). \tag{56}$$

In the first iterative process, looking for the approximate solution (54) of differential equation in the form $W = \sum_{i=0}^{n} \varepsilon^i W_i$. For determining the functions W_i the following statement is proved.

Lemma 5. Assume that the function $f(x,y)$ is contained in the space $C^{2n+3}(P_+)$, satisfies the condition (34) and on the domain P_+ the condition

$$\left|\frac{\partial^i f(x,y)}{\partial x^{i_1}\partial y^{i_2}}\right| \le c^{(1)}_{i_1,i_2}\exp(-\lambda y);\; i=i_1+i_2;\;\; i=0,1,...,2n+3;\; c^{(1)}_{i_1,i_2}>0, \lambda>0. \quad (57)$$

Then boundary value problem has a unique solution , this solution is contained in the space $C^{2n+3}(P_+)$, satisfies the condition (36) and condition

$$\left|\frac{\partial^i W_0(x,y)}{\partial x^{i_1}\partial y^{i_2}}\right| \le c^{(2)}_{i_1,i_2}\exp(-\lambda y);\; i=i_1+i_2;\;\; i=0,1,...,2n+3;\; c^{(2)}_{i_1,i_2}>0 . \quad (58)$$

As a result, the function $W=\sum_{i=0}^{n}\varepsilon^i W_i$ constructed in the first iterative process satisfies the following boundary conditions:

$$W\big|_{x=0}=0,\; W\big|_{y=0}=0,\;\; \lim_{y\to+\infty} W=0. \quad (59)$$

Near $x=1$ we construct a boundary layer type function $V=\sum_{j=0}^{n+1}\varepsilon^j V_j$.

Theorem 6. Assume that $f(x,y)\in\; C^{2n+3}(P_+)$ and for the function $f(x,y)$ the conditions (34), (57) and

$$\frac{\partial^k f(1,0)}{\partial x^{k_1}\partial y^{k_2}}=0; k=k_1+k_2;\;\; k=0,1,...,n$$

are satisfied. Then the asymptotic expansion of the solution of the boundary value problem (54)-(56) with respect to the small parameter in the form

$$u=\sum_{i=0}^{n}\varepsilon^i W_i+\sum_{j=0}^{n+1}\varepsilon^j V_j+\varepsilon^{n+1}z\;.$$

Here the functions W_i are determined in the first iterative process, the functions V_j are determined in the second iterative process, $\varepsilon^{n+1}z$ is a residual term and for the function z the following estimation is valid:

$$\varepsilon^2\left\|\frac{\partial z}{\partial x}\Big|_{x=0}\right\|^2_{L_2(0,+\infty)}+\varepsilon\left[\left\|\frac{\partial z}{\partial x}\right\|^2_{L_2(P_+)}+\left\|\frac{\partial z}{\partial y}\right\|^2_{L_2(P_+)}\right]+c_1\|z\|^2_{L_2(P_+)}\le c_2\;,$$

the constants $c_1 > 0, c_2 > 0$ are independent of ε.

The results obtained in section 2.3 were published in the author's papers [11], [12], [15], [18], [19].

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The list of scientific works on the topic of the dissertation:

1. Сабзалиев, М.М., Керимова, М.Е. Асимптотика решения краевой задачи в прямоугольнике для однохарактеристического дифференциального уравнения, вырождающегося в параболическое уравнение // Материалы Международной научной конференции, посвящ. 85-летнему юбилею академика А.Х.Мирзаджанзаде, - Баку: - 21-22 ноября, -2013, - с.217-219.

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