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ABSTRACT

of the dissertation for the degree Doctor of Science

MAXIMAL REGULARITY PROPERTIES OF ABSTRACT CONVOLUTION DIFFERENTIAL OPERATOR EQUATIONS AND ITS APPLICATIONS

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GENERAL DESCRIPTION OF WORK

Relevance of the topic and the degree of development. The issue of maximum regularity has attracted attention in recent years, and significant progress has been made in this area. One reason for this is the importance of estimates for nonlinear problems. For example, using linearization methods, this allows you to find a simple and elegant approach to quasilinear problems. Some questions of maximum regularity with similar features were considered in the works of H.Amann, F.Clement, S.Li, J.Pruss, A.Lunardi, Shakhmurov V.B., etc.

The first general abstract result for the maximum-regularity of abstract equations in Hilbert spaces was obtained by L. Simon in 1964. He received a positive answer in this case without additional restrictions on the coefficient operator L_p. De Simon's proof uses the Plancherel theorem, which, as is known, is valid only in the case of Hilbert space.

One of the significant results on maximum L_p −regularity is given in the work of O. Ladyzhenskaya, V. Solonnikov and N. Uralceva. Their proof is based on potential theory and cannot be easily generalized.

At the end of the 20th century, N. Kalton showed that the result of L. Simon is not true in spaces $L_p(G)$ where $G \subset R^n$ there is a bounded region with a smooth boundary. Indeed, if the problem considered has the property of maximum L_p-regularity, then the Banach space must be isomorphic to the Hilbert space. Of particular interest is the example when the operator is a differential operator.

The first general result was obtained by L. Weis only in 2001. L. Weis established Mikhlin's theorem for operator functions (without restrictions), replacing boundedness with a stronger condition for R-boundedness. He obtained the characteristic of maximum L_p − regularity in the case when the space is a Banach space of class UMD in terms of R −boundedness. The theorem on operator-valued Fourier multipliers used is a theorem of Mikhlin type and was previously known only for Hilbert spaces.

By the Plancherel theorem, every Hilbert space is a UMD-space and, in addition, if E is a UMD-space, then closed subspaces are UMD-spaces, and $L_p(\mu, E)$ is a UMD-space for all σ finite-dimensional spaces with measure (Ω, μ) and $p \in (1, +\infty)$.

It was J. Bourgain who introduced the concept of R-boundedness. In the work of J. Bourgain, this is called the Riesz property and "R" was interpreted as "Randomized bounded".

In the work of R. Denk, M. Hieber, and J. Pruss, these results are developed, and the concepts of sectoriality and $R-$boundedness are combined to determine the class of $R-$ sectorial operators. Their properties and characteristics of maximum L_p- regularity are proved in terms of sectoriality introduced by L. Weis.

After the work of L. Weis, the theory of differential operator equations found wide application using the concepts of $R-$boundedness, $R-$sectorial operators, operator-valued Fourier multipliers. There are corresponding results for elliptic differential operator equations in Hilbert spaces. Some papers consider local boundary value problems for complete elliptic differential operator equations in Banach spaces.

The maximum regularity theory also applies to solving partial differential equations, such as the semi-linear heat equation, the incompressible Navier-Stokes equation, and so on.

The method for solving boundary value problems for elliptic differential operator equations in abstract spaces is based on abstract considerations in the famous book of S. G. Krein and from the idea given in the classic works of S. Agmon, A. Douglis and L. Nirenberg. Then this method was developed and presented in its final form in the book of S. Yakubov, Y. Yakubov in the framework of Hilbert spaces.

The concept of $R-$boundedness of families of operators and its connection with maximum regularity for linear Cauchy problems has undergone significant development in recent years. The purpose of this series of works is to generalize some important results in this direction and to demonstrate their strength in applications to linear and nonlinear partial differential equations of parabolic type. It is shown below how these results can be applied to linear and nonlinear parabolic partial differential equations. (e.g.; Navier-Stokes equation, Stefan type problem, etc.)

Maximum type regularity is also an important tool when working with quasilinear and non-autonomous parabolic type equations.L_p

For abstract elliptic boundary value problems in this direction, there are only a few works. Moreover, this method can be applied to very wide classes of partial differential equations.

One of our goals in this work is to expand the methods for solving abstract elliptic boundary value problems in the framework of weighted Banach spaces. For this, we used the concepts of $R-$boundedness and $R-$sectorial operators, the L. Weis theorem on operator-valued Fourier multipliers, and the areomorphism theorem of W. Arendt and M. Duelli. In addition, we have obtained the maximum L_p-regularity for elliptic convolutional differential-operator equations in Banach spaces UMD. Then we give the corresponding application of the obtained abstract results to boundary value problems for high-order elliptic equations (with or without a parameter).

This dissertation is devoted to the investigation of the maximum regularity of convolutional differential-operator equations with unbounded operator coefficients in Banach-valued weighted spaces.

Our main condition in the theorems is given in terms of the $R-$boundedness of a set, which is a direct generalization of the concept of normalized boundedness of a set in the framework of Banach spaces. In Hilbert spaces, these two concepts coincide. From the definition of $R-$boundedness it follows that each $R-$bounded family of operators is bounded. On the other hand, in a Hilbert space, each bounded set is $R-$bounded. Therefore, in a Hilbert space the concept of boundedness is equivalent to a $R-$bounded family of operators. The proof of the Fourier multiplicativity of the corresponding operator functions in weighted spaces is a decisive moment in the proof of the main theorems.

In this paper, we will often use an operator-valued version of the Mikhlin multiplier theorem in UMD-spaces, which in the version on the real line belongs to L. Weiss. We will also use versions of this theorem in space R^n.

In this paper, we also consider the problem, which consists of a degenerate integro-differential equation in a weighted Banach space.

Such equations appear in various applied problems. The monograph by A. Favini, A. Yagi is devoted to these problems and contains extensive applications to specific problems. The book of I. Melnikova and A. Filinkov also considers abstract degenerate equations. Evolutionary integro-differential equations usually arise in mathematical physics in the form of laws relating to materials for which memory effects are important, in combination with conventional conservation laws, such as energy balance or momentum balance.

In the case of Hilbert spaces, the solution of problems of this type is sufficient. This is due to the fact that Plancherel's theorem is available in Hilbert space. When the space is not a Hilbert space, to fulfill the Plancherel theorem for Banach-valued functions, it is required that the base space is isomorphic to the Hilbert space.

For non-degenerate integro-differential equations in both periodic and non-periodic cases, operator-valued Fourier multipliers were used by various authors to obtain correctness at various scales of function spaces. The results of correctness or maximum regularity are important in that they allow solving nonlinear problems.

Object and subject of research. Convolutional differential-operator equations and their maximal regularity in a weighted space.

The purpose and objectives of the study. The purpose of the thesis is to solve the following main problems:

- Study of convolutional differential operator equations (CDOE) in weighted Lp spaces;

- The study of uniform boundedness and R −boundedness of the operator function created when solving convolution equations;

- Study of the maximum regularity of degenerate convolutional elliptic equations;

- Research on the existence and uniqueness of a solution to a degenerate convolutional differential operator equation and obtaining coercive estimates;

- A study of the coercivity of degenerate convolutional differential-operator equations in Besov weighted spaces;

- Investigation of the Cauchy problem for parabolic differential-operator equations in Besov weighted spaces;

- The study of quasilinear convolutional elliptic equations and an infinite system of degenerate integro-differential equations;

- Research of a Wentzel-Robin boundary value problem for elliptic convolutional differential-operator equations and a boundary value problem for integro-differential equations;

- Study of the property of uniform separability of convolutional differential equations depending on the parameters.

Research Methods. The methods of the theory of linear differential operator equations in Banach spaces, the theory of functional spaces, the theory of integral representations of functions and embedding theorems, the theory of Fourier multipliers, the operator-valued version of the Mikhlin multiplier theorem, the theory of positive and sectorial operators, the theory of convolution in the sense of distributions (generalized functions) are used) and the theory of the semigroup of operators.

The main provisions to be defended.

Investigation of the separable properties of convolutional differential-operator equations with unbounded operator coefficients,

Studying maximally regular properties of convolutional differential-operator equations in Besov weighted spaces,

The proof of the coercivity of convolutional differential operator equations in weighted spaces and their application,

Studying some properties of convolutional differential-operator equations with parameters, including

- obtaining a solution of convolutional differential-operator equations in weighted Banach spaces and proof of the existence and uniqueness of the solution;

- evidence of uniform boundedness and R - boundedness of the operator function obtained in the study of the solvability of convolution equations;

- finding sufficient conditions guaranteeing maximum regularity; degenerate convolutional differential operator equations with a mixed derivative in the weighted space and a coercive estimate is obtained;

- prove the existence and uniqueness of a solution to the degenerate Cauchy problem for a convolutionally parabolic equation;

- prove the corresponding estimates of the resolvent of the operator generated by the considered problem;

- obtaining a coercive estimate for solving quasilinear elliptic convolutional differential-operator equations;

- evidence the fulfillment of coercive estimates and the fidelity of the corresponding resolvent estimates for infinite systems of degenerate integro-differential equations in the weight space;

- finding the conditions for the existence of a solution to the mixed problem with boundary conditions of the Wentzel-Robin type for degenerate integro-differential equations and obtaining a coercive estimate;

- investigate the maximum regularity of degenerate linear convolutional differential-operator equations with parameters and obtain a coercive estimate for solving the convolutional differential-operator equation depending on the parameter.

The scientific novelty of the study. In the dissertation, the following new results were obtained:

- representations of the solution of a convolutional differential-operator equation in a weighted Banach space are obtained;

- uniform boundedness and R −boundedness of the operator function obtained in the study of the solvability of convolutional equations are proved;

- for the first time, the existence and uniqueness of a solution to degenerate convolutional differential-operator equations with a mixed derivative in a weighted space is proved, and a coercive estimate is obtained;

- sufficient conditions have been found to guarantee the separability of linear problems in weighted Banach spaces;

- using coercive estimates, the existence and uniqueness of a solution to the degenerate Cauchy problem for a convolutionally parabolic equation is proved;

- the existence of the resolvent of the operator generated by the considered problem and the validity of the corresponding estimates are studied;

- an estimate is obtained for solving quasilinear elliptic convolutional differential-operator equations;

- the boundary value problem for convolutional differential equations of anisotropic type was studied and the most regular properties in weighted mixed norms were obtained;

- for infinite systems of degenerate integro-differential equations in the weighted space, the fulfillment of coercive estimates and the fidelity of the corresponding resolvent estimates are proved;

- conditions are found for the existence of a solution to the mixed problem with boundary conditions of the Wentzel-Robin type for degenerate integro-differential equations and an estimate is obtained;

- the maximum regularity of degenerate linear convolutional differential operator equations with parameters is described;

- the coercive estimate for solving the convolutional differential operator equation depending on the parameter is proved.

Coercive solvability theorems for degenerate CDOEs give a stimulating effect on the study of the corresponding concrete equations and boundary value problems. When solving specific problems, the methods obtained in the second and third chapters are used. Note that classical methods are not applicable here.

Theoretical and practical value of the study. Expanded methods for solving abstract elliptic boundary value problems in the framework of weighted Banach spaces. Different types of degenerate integro-differential and linear convolutional differential-operator equations are considered - in both cases the proved results are of undoubted interest in the theory of quasilinear elliptic convolutional differential-operator equations. First, sufficient conditions are formulated, implying the maximum regularity of the linear operator in the abstract structure.

The results of the second chapter can be applied to linear and nonlinear parabolic partial differential equations (for example, the Navier-Stokes equation, a Stefan type problem, etc.). In addition, the results obtained are applied to boundary value problems for high-order elliptic equations with or without a parameter.

The uniform R −boundedness of the operator function is well used during the proof of coercive estimates for solving convolutional differential operator equations.

It is well known that differential equations with parameters play an important role in modeling physical processes; therefore, convolutional differential-operator equations with small parameters also have significant applications in the development of the theory to problems of mathematical physics.

A large number of partial differential equations arising in physics and applied sciences, such as fluid flow through fractured rocks, thermodynamics, or in the theory of equations by dynamical systems, can be expressed by a model in the form of a degenerate convolutional differential-operator equation. The most typical example is when operator A is a Laplacian.

Maximum regularity is an important tool when working with quasilinear and non-autonomous parabolic equations. The problem, which consists of a degenerate integro-differential equation in a weighted Banach space, usually arises in mathematical physics in the form of laws relating to material for which memory effects are important, in combination with ordinary conservation laws, such as energy balance or momentum balance.

It is known that many classes of differential and pseudodifferential operators have the property of positivity and sectoriality (similar to R −positivity and R −sectoriality). Therefore, choosing specific spaces and specific operators acting in this space, we obtain the most regular properties of a different class of degenerate convolution equations in different spaces and the Cauchy problem for parabolic convolutional differential operator equations or their systems, respectively.

Since in the problems considered the Banach space E and the linear operator A are arbitrary, therefore, choosing the space E and the operators A, we can get different results on the regularity properties of convolution-elliptic operators and the properties of singular perturbations of the numerous classes of elliptic, quasielliptic equations and their systems that take place the widespread use of a variety of physical systems.

Approbation and application. The main provisions and results obtained in the dissertation were reported and discussed at various international and national conferences and seminars, including:

at the conference dedicated to the 80th anniversary of K.T.Akhmedov (Baku –1998), at the 61st scientific conference of young scientists (Baku-2000), at the scientific conference on the topic "Differential Equation and Their Application" (Baku-2002), at the scientific the conference dedicated to the 70th anniversary of professor G.K. Namazov (Baku-2002), “X International Conference on Mathematics and Mechanics devoted to the 45th anniversary of the Institute of Mathematics and Mechanics” (Baku-2004), “İnternational Conference on Mathematics and mechanics devoted to the 50th anniversary from birthday of member of the correspondent of NASA, professor İ.T. Mamedov ”(Baku-2005), at a scientific conference to the 100th anniversary of academician A. Huseynov (Baku-2007), at the Republican Congress a conference dedicated to the 85th anniversary of the National Leader of Azerbaijan, Heydar Aliyev (Baku-2008), “The 2nd International conference on control and optimization with Industrial Applications” (Baku-2008), “The 3d congress of the world mathematical society of Turkish countries” (Almaty-2009), at the international conference “Actual problems of mathematics and mechanics” (Baku -2010), “İnternational conference devoted to the 80th anniversary of academician F.G.Magsudov” (Baku-2010), “The 4d congress of the Turkish World Mathematical Society” (Baku-2011), “The international conference devoted to the 100- th anniversary of academician Z.İ.Khalilov ”(Baku-2011),“ The international conference devoted to the 100th anniversary of academician Z.İ.İbrahimov ”(Baku-2012), at the scientific conference“ Actual problems of mathematics and mechanics "(Baku-2012), at the Republican conference" Masters, doctorate of orants and young researchers ”(Baku-2012), at the Republican conference dedicated to the 90th anniversary of the National Leader Azerdbaizhan Heydar Aliyev, (Baku-2013), at the international conference “Actual problems of mathematics and mechanics” (Baku-2014), “The international conference devoted to the 55th anniversary of the İnstitute of Mathematics and Mechanics ”(Baku-2014), at the Republican Conference dedicated to the 91st anniversary of the National Leader Azerdbaizhan Heydar Aliyev (Baku-2014), at the scientific conference“ Actual Problems of Mathematics and Mechanics ”(Baku-2014), at the Republican Conference“ Undergraduates , doctoral students and young researchers ers "(Baku 2014), at the Republican conference devoted to 94 - anniversary Obshenatsionalnogo Azerdbayzhana Leader Heydar Aliyev (Baku 2017), at the Republican scientific conference dedicated to the 100 - anniversary of the corresponding member. ANAS prof. K.T. Akhmedova (Baku-2017), at the scientific conference “Actual Problems of Mathematics and Mechanics” (Baku-2018), “The 6th International Conference on“ Control and Optimization with industrial applications ”(Baku-2018),“ 3rd International Conference on “Operators in General Morry-type spaces and applications” (Kutahya-

2019), at the Republican conference dedicated to the 96th anniversary of the National Leader Azerdayjan Heydar Aliyev, (Baku-2019),“ The International conference devoted to the 60th anniversary of the Institute of Mathematics and Mechanics of Azerbaijan National Academy of Sciences (ANAS)”, 2019, at scientific seminars of the Research Institute of Applied Mathematics of BSU (headed by academician F.A. Aliyev), at general institute seminars (head by corresponding member of ANAS, prof. M.J. Mardanov) of the Institute of Mathematics and Mechanics of ANAS, at the scientific seminars of the department "Mathematical Analysis" (head-corresponding member of ANAS, prof. V.S. Guliyev) of the Institute of Mathematics and Mechanics of ANAS, at the seminars of the department "Differential and integral equations" BSU (head. -prof. Y.T.Megraliyev), as well as at seminars of the Department of Function Theory of ASPU, at seminars of the Department of Higher Mathematics of the University of Architecture and Construction, at scientific seminars of the Department of Natural Sciences of Stanbul Okan University.

The personal contribution of the author. All conclusions and obtained results belong to the author.

Author’s publications. Publications in the editions recommended by HAC under President of the Republic of Azerbaijan -30, abstracts of reports- 26.

The name of the institution where the dissertation was completed. The dissertation work was carried out in the "Inverse Problems and Nonlinear Equations" Department of the Research Institute of Applied Mathematics of Baku State University.

The total volume of the dissertation in signs, indicating the volume of each structural unit (individually). The title page consists of 250 symbols, a table of contents of 750 symbols, an introduction of 80000 symbols, conclusions of 2000 symbols, the main content of a dissertation of 368000 symbols (chapter I - 96000, chapter II - 44000, chapter III - 88000, chapter IV - 60000, chapter V - 80000). The total volume of the dissertation consists of 451000 symbols.

CONTENT OF THE DISSERTATION

The introduction substantiates the relevance of the topic, gives a brief overview of the works adjacent to the topic of the dissertation, and provides a brief summary of the dissertation.

In the dissertation, Chapter 1 is devoted to the study of convolutional differential-operator equations in weighted $L_p -$ spaces. Moreover, we consider the uniform boundedness of the operator function that arises in solving these problems.

Let a E $-$Banach space, and $\gamma = \gamma(x),\ x = (x_1, x_2, \dots, x_n)$ be a positively measurable weight function on a measurable subset $\Omega \in R^n$.

Let $\mathbb{C}$ the set of complex numbers and

$$S_\varphi = \{\lambda\,;\ \lambda \in \mathbb{C}\,, |arg\lambda| \leq \varphi\} \cup \{0\}, 0 \leq \varphi < \pi.$$

A closed linear operator function $A = A(x), x \in \mathbb{R}$ is called uniformly φ $-$positive in a Banach space E if $D\ (A\ (x))$ is dense in E and not dependent of x and there exists a positive constant M such that

$$\|(A(x) + \lambda I)^{-1}\|_{\mathcal{L}(E)} \leq M(1 + |\lambda|)^{-1}$$

for any $x \in \mathbb{R}\,, \lambda \in S_\varphi, \varphi \in [0,\pi)$, where I is the identity operator in E , and $\mathcal{L}(E)$ is the space of all bounded linear operators in E .

Suppose that $E_1$and E_2 are two Banach spaces. A function $\Psi \in L_\infty\left(R^n; \mathcal{L}(E_1, E_2)\right)$is called a multiplier from $L_{p,\gamma}(R^n; E_1)$ to $L_{p,\gamma}(R^n; E_2) p \in (1, \infty)$, if the map $u \to Tu = F^{-1}\Psi(\xi)Fu\,, u \in S(R^n; E_1)$is densely defined and extends to a bounded linear operator

$$T: L_{p,\gamma}(R^n; E_1) \to L_{p,\gamma}(R^n; E_2).$$

The space of Fourier multipliers from$L_{p,\gamma}(R^n; E_1)$ to $L_{p,\gamma}(R^n; E_2)$ on will be denoted by $M_{p,\gamma}^{p,\gamma}(E_1, E_2)$. For $E_1 = E_2$ we denote $M_{p,\gamma}^{p,\gamma}(E_1, E_2)$ by $M_{p,\gamma}^{p,\gamma}(E)$. Let $M\ (h)$ denote the set of some parameters.

Let $T_h = \{\Psi_h \in M_{p,\gamma}^{p,\gamma}(E_1, E_2), h \in M(h)\}$ be multipliers in $M_{p,\gamma}^{p,\gamma}(E_1, E_2)$. We say that T_h is a set of uniformly bounded multipliers (UBMs) if there exists a positive constant M independent of $h \in M(h)$, such that

$$\|F^{-1}\Psi_h F u\|_{L_{p,\gamma}(R^n;\, E_2)} \leq M\|u\|_{L_{p,\gamma}(R^n;\, E_1)}$$

for all $h \in M(h)$ and $u \in S(R^n, E_1)$.

We assume that the weight $\gamma\,(x)$ satisfies the condition A_p, i.e., $\gamma(x) \in A_p$, $1 < p < \infty$, if there exists a positive constant C such that

$$\sup_Q \left(\frac{1}{|Q|}\int_Q \gamma(x)dx\right)\left(\frac{1}{|Q|}\int_Q \gamma^{-\frac{1}{p-1}}(x)dx\right)^{p-1} \leq C$$

for all compacts $Q \subset R^n$.

A Banach space E is called a UMD space if the Hilbert operator

$$(Hf)(x) = \lim_{\varepsilon\to 0} \int\limits_{|x-y|>\varepsilon} \frac{f(y)}{x-y}dy$$

is bounded in $L_p(\mathbb{R}, E), p \in (1, \infty)$. UMD spaces include, for example, the spaces L_p, l_p and the Lorentz spaces $L_{pq}, p, q \in (1, \infty)$

A set $K \subset \mathcal{L}(E_1, E_2)$ is called R −bounded if there exists a constant $C >\ 0$ such that for all $T_1, T_2, \dots, T_m \in K$ and $u_1, u_2, \dots, u_m \in E_1 m \in \mathbb{N}$

$$\int\limits_0^1 \left\|\sum_{j=1}^m r_j\,(y)T_j u_j\right\|_{E_2} dy \leq C\int_0^1 \left\|\sum_{j=1}^m r_j(y)u_j\right\|_{E_1} dy,$$

where $\{r_j\}$ is a sequence of independent symmetric $\{-1,1\}$ -valued random variables on [0, 1], and $\mathbb{N}$ denotes the set of natural numbers. The smallest C for which the above estimate holds is called the R −boundary of the set K and is denoted by $R(K)$.

A Banach space E is called a space satisfying the condition of a weight multiplier with respect to p and a weight function γ if, for any $\Psi \in L_\infty(\mathbb{R}, \mathcal{L}(E))$, the R-boundedness of the set

$$\{|\xi|^k D^k \Psi(\xi): \xi \in \mathbb{R}\backslash\{0\}, k=0,1\}$$

implies that $.\Psi$ is the Fourier multiplier from $L_{p,\gamma}(\mathbb{R};E)$ to $L_{p,\gamma}(\mathbb{R};E)$, i.e., $\Psi \in M_{p,\gamma}^{p,\gamma}(E)$ for any $p \in (1,\infty)$.

Note that in Hilbert spaces every norm-bounded set is R −bounded. Therefore, in Hilbert spaces, all positive operators are R −positive.

In the first section of the first chapter, convolutional elliptic differential operator equations in weighted L_p −spaces are considered. The separability properties of convolutional differential-operator equations with unbounded operator coefficients in a Banach-valued weighted L_p −class are investigated. A coercive estimate of the resolvent of the corresponding operator is obtained.

In recent years, maximum regularity for differential-operator equations, especially for the parabolic and elliptic type, has been widely studied. Moreover, convolutional differential equations were considered, for example, in the works of A. Benedek, A. Calderon, V.B. Shakhmurov and others. The convolution equations in vector spaces were studied in the works of H. Amann, M. Girardi, L. Weis, C.Lizama, V. B. Shakhmurov, M. Warm, F. Zimmermann, etc. However, convolutional differential-operator equations have been studied relatively little.

The main goal of the first section is to establish the maximum regularity properties of the next convolutional differential-operator equation in $E-$ significant weight L_p- spaces

$$\sum_{k=0}^{l} a_k * \frac{d^k u}{dx^k} + A * u = f(x), \qquad (0.1)$$

where $A = A(x)$, generally speaking, a linear unbounded operator in a Banach space E, $a_k = a_k(x)$ are complex-valued functions on $\mathbb{R} = (-\infty;\ +\infty)$.

Applying the Fourier transform and the well-known Hausdorff-Young inequality, using the resolvent properties of positive operators, we prove the following lemma.

Lemma 0.1. Let $a_k \in L_1(\mathbb{R})$, $k = 0,1,2,\dots,l$, and $\hat{A}(\xi)$ be uniformly φ −positive in E, $\varphi \in [0,\pi)$. In addition, suppose that $L(\xi) = \sum_{k=0}^{l} \hat{a}_k(\xi)(i\xi)^k \in S_\varphi, \varphi_1 < \pi - \varphi$ and there exists a positive constant $C > 0$,such that

$$|L(\xi)| \geq C|\xi|^l \sum_{k=0}^{l} |\hat{a}_k(\xi)|,\ \ \xi \in \mathbb{R}\backslash\{0\}. \tag{0.2}$$

Then the operator functions

$$\lambda\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1},\ \hat{A}(\xi)\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1},$$

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \hat{a}_k(\xi)(i\xi)^k\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1}$$

obtained when solving the corresponding problems are uniformly bounded.

Lemma 0.2. Suppose that (0.2) holds and $\hat{A}(\xi)$ is uniformly R −positive E in. Then the following sets

$$\left\{\lambda\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1};\ \xi \in \mathbb{R}\backslash\{0\}\right\},$$

$$\left\{\hat{A}(\xi)\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1};\ \xi \in \mathbb{R}\backslash\{0\}\right\},$$

$$\left\{\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \hat{a}_k(\xi)(i\xi)^k\, \hat{A}(\xi)\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1};\ \xi \in \mathbb{R}\backslash\{0\}\right\}$$

evenly bounded.R −

Further, using the boundedness of the operator function, for each function f belonging to the weighted spaces L_p, we prove the uniform coercive estimate

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\| a_k * \frac{d^k u}{dx^k} \right\|_{L_{p,\gamma}(\mathbb{R};E)} + \|A * u\|_{L_{p,\gamma}(\mathbb{R};E)} + |\lambda|\|u\|_{L_{p,\gamma}(\mathbb{R};E)} \leq C\|f\|_{L_{p,\gamma}(\mathbb{R};E)}$$

for $\lambda \in S_\varphi$, $\varphi \in [0,\pi)$.

The second section of the first chapter is devoted to the study of the boundedness of operator-valued functions (operator functions).

Let $\hat{a}_k \in C^{(m)}(\mathbb{R})$, $\hat{A}^{(m)}(\xi)\hat{A}^{-1}(\xi) \in C^{(m)}\big(\mathbb{R}; \mathcal{L}(E)\big), \xi_0 \in \mathbb{R}$

$$\left|\hat{a}_k^{(m)}(\xi)\right| < M_1, \qquad |\xi^m \hat{a}_k(\xi)| \le M_2, \qquad \left\|\hat{A}^{(m)}(\xi)\hat{A}^{-1}(\xi)\right\|_{\mathcal{L}(E)} \le M_3, \ \left\|\xi^m \hat{A}^{(m)}(\xi)\hat{A}^{-1}(\xi)\right\|_{\mathcal{L}(E)} \le M_4, \quad (0.3)$$

where, $m = 1,2, \dots M_i, i = 1,2,3,4$ are the positive constants.

For simplicity, we denote, $H(\xi, \lambda) = \left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1}$ similarly, we have

$$G_1(\xi, \lambda) = \sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \hat{a}_k(\xi)(i\xi)^k H(\xi, \lambda), \ \ G_2(\xi, \lambda) = \hat{A}(\xi, \lambda) H(\xi, \lambda),$$

$$G_3(\xi, \lambda) = \lambda H(\xi, \lambda).$$

Under these conditions, it is proved that the operator functions $G_i{}'(\xi, \lambda) = \frac{d}{d\xi} G_i(\xi, \lambda)$. are uniformly bounded $i = 1,2,3,$

Summarizing the results obtained, we prove the following proposition:

Offer. If the above conditions are satisfied, then the operator functions $G_i^{(m)}(\xi, \lambda)$where $i = 1,2,3,\ m = 0,1,2,$ are uniformly bounded and the inequality

$$|\xi|^m \left\|G_i^{(m)}(\xi, \lambda)\right\|_{\mathcal{L}(E)} \le C.$$

In the third section of the first chapter, the R $-$boundedness of the operator function obtained in the study of solvability and the separable properties of convolutional differential operator equations in weighted L_p $-$spaces are proved.

Lemma 0.3. Let (0.2) and (0.3) be satisfied. Assume $\hat{A}(\xi)$ uniformly R $-$positive in E and there are positive constants C_1, C_2 and such that

$$R\left(\left\{\xi \, \frac{d}{d\xi} \hat{A}(\xi)\big(\hat{A}(\xi) + \xi\big)^{-1};\ \xi \in S_\varphi\right\}\right) \le C_1$$

$$\left|\xi \, \frac{d}{d\xi} \hat{a}_k(\xi)\right| \le C_2.$$

If Banach spaces E −satisfy the condition of weighted multipliers, then for $\lambda \in S_\varphi, \varphi \in [0,\pi)$ it is proved that the following sets

$$\left\{\xi \frac{d}{d\xi} G_i(\xi,\lambda);\ \xi \in \mathbb{R}\backslash\{0\}\right\}$$

uniformly R −bounded i.e.

$$\sup_\lambda R\left(\xi\{G_i{}'(\xi,\lambda)\}\right) \le C_i, \qquad i = 1,2,3.$$

It follows that the operator functions $G_i(\xi,\lambda)$ are a uniformly bounded collection of (family) of multipliers in $L_{p,\gamma}(\mathbb{R};E)$. Finally, we have that if $\hat{A}(\xi)$ is uniformly positive for $\varphi \in [0,\pi)$, then the operator L is a generator of the analytic semigroup in $L_{p,\gamma}(\mathbb{R};E)$.

The third section of Chapter 1 explores uniformly the R −positivity and R −sectoriality of the operators of the generated problem

$$D(L) = W^l_{p,\gamma}(R^n; E(A), E),$$

$$Lu = \sum_{k=0}^{l} a_k * \frac{d^k u}{dx^k} + A * u = f. \qquad (0.4)$$

In this section, it is also proved that for all $\lambda \in S_\varphi, |\lambda| \ge \lambda_0 > 0$ there exists a resolvent of the operator L and the following estimate holds

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\| a_k * \left[\frac{d^k}{dx^k}(L+\lambda)^{-1}\right]\right\|_{\mathcal{L}(X)} + \|A * (L+\lambda)^{-1}\|_{\mathcal{L}(X)}$$
$$+\|\lambda(L+\lambda)^{-1}\|_{\mathcal{L}(X)} \le C.$$

This means that the operator L is positive in $L_{p,\gamma}(\mathbb{R};E)$. In order to show the R-positivity of the operator L, we must prove the R −boundedness of the set $\{\lambda(L+\lambda)^{-1};\ \lambda \in S_\varphi\}$..

It is known that

$$\lambda(L+\lambda)^{-1} = F^{-1}\lambda\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1}\hat{f}, \qquad f \in L_{p,\gamma}(\mathbb{R};E).$$

It is similarly proved that the operator function $\lambda\left[\hat{A}(\xi)+\lambda+L(\xi)\right]^{-1}$ depends on the variable λ, the parameter ξ, and the uniformly bounded multiplier in $L_{p,\gamma}(\mathbb{R};E)$. Then, by the definition of $R-$boundedness, we have

$$\int_0^1 \left\|\sum_{j=1}^{m} r_j(y)\,\lambda_j\left(L+\lambda_j\right)^{-1} f_j\right\|_{L_{p,\gamma}(\mathbb{R};E)} dy \leq C\int_0^1 \left\|\sum_{j=1}^{m} r_j(y)\, f_j\right\|_{L_{p,\gamma}(\mathbb{R};E)} dy,$$

for all $\xi \in \mathbb{R}, \lambda_1, \lambda_2, \dots, \lambda_m \in S_\varphi,\ f_1, f_2, \dots, f_m \in L_{p,\gamma}(\mathbb{R};E)$.

In the same way, the $R-$sectoriality of the operators of the generated problem (0.4) is proved if the operator L is sectorial in $L_{p,\gamma}(\mathbb{R};E)$.

$R-$sectoriality and $R-$positivity of operators are widely used in the study of the most regular questions of linear and nonlinear convolutional differential-operator equations in weighted spaces and for UMD spaces. The research method for these issues is the theory of Fourier multipliers, representations of solutions, the theory of positive and sectorial operators, and convolution theory in the sense of distributions and so on.

Note that these results are used to obtain maximally regular properties of the Cauchy problem for the parabolic type of convolution-differential-operator equations.

The fourth section of the first chapter considers degenerate convolutional differential operator equations (CDOE)

$$\sum_{k=0}^{l} a_k * \frac{d^{[k]}u}{dx^{[k]}} + A*u+\lambda u = f(x), \tag{0.5}$$

where $\frac{d^{[k]}u}{dx^{[k]}} = \left(\gamma(x)\frac{d}{dx}\right)^k u(x), \gamma(x)-$is a positive measurable function in $\mathbb{R}$ A is a linear operator in a Banach space E. Convolution $a_k * \frac{d^{[k]}u}{dx^{[k]}}$ and $A*u$

are defined in the sense of distribution., We note that in connection with the study of boundary value problems for degenerate equations of elliptic type, a theory of weighted spaces of a function was created. By the way, the general theory of embedding of weight spaces was created by L.D. Kudryavtsev, the further development of which is devoted to the work of various authors.

Recently, degenerate equations have attracted the attention of many authors. Both first and second order equations were considered. Degenerate first order equation of the form

$$(Mu)'(t) = Au(t) + f(t), 0 \leq t \leq 2\pi$$

with a periodic boundary condition $Mu(0) = Mu(2\pi)$, was studied by C.Lizama and R. Ponce under suitable assumptions. They gave necessary and sufficient conditions for ensuring the correctness of this problem in Lebesgue-Bochner spaces $L_p(0,2\pi; X)$, in Besov spaces $B^s_{p,q}(0,2\pi; X)$ and in the Triebel-Lizorkin spaces $F^s_{p,q}(0,2\pi; X)$.

Recently, S. Bu studied the second-order degenerate equation with periodic boundary conditions and obtained the necessary and sufficient conditions to ensure the correctness of the problem in question in the corresponding spaces.

The maximum regularity of solutions for the following differential equation of degenerate (also called Sobolev) type

$$D^\alpha\big(Mu(t)\big) = Au(t) + f(t), \quad t \in \mathbb{R},$$

where A and M are two closed linear operators defined on a Banach space X with domains $D(A)$ and $D(M)$ respectively and $D(A) \cap D(M) \neq \{0\}$, the function $f: \mathbb{R} \to X$ belongs to some vector-valued space of functions $S(\mathbb{R}, X)$is studied in detail in the monographs of A. Favini and A. Yagi.

A large number of partial differential equations arising in physics and in applied sciences, such as fluid flow through fractured rocks, in thermodynamics, or in the theory of control of dynamical systems, can be expressed by the model in this form. The most typical example is the case when $A = \triangle$ is the Laplacian, and $M = m$ is the multiplier operator on the function $m(x)$. Then the degenerate differential equation (in the case $\alpha = 1$) describes the infiltration of water in unsaturated porous media in which saturation can occur.

In order to show the existence of a unique solution to equation (0.5), a uniform coercive estimate holds

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\| a_k * \frac{d^{[k]}u}{dx^{[k]}} \right\|_{L_p(\mathbb{R};E)} + \|A * u\|_{L_p(\mathbb{R};E)} + |\lambda| \|u\|_{L_p(\mathbb{R};E)} \leq C\|f\|_{L_p(\mathbb{R};E)},$$

use a substitution $y = \int_0^x \gamma^{-1}(z)dz$. Then, by virtue of a uniform coercive estimate, we obtain a statement for a non-degenerate problem.

Chapter 2 is devoted to the study of the separable properties of convolutional differential-operator equations with unbounded operator coefficients.

In the theory of convolutional differential equations, operator-valued Fourier multipliers are of particular interest. Differential, differential-operator and convolutional differential-operator equations are investigated in the works of various authors. In the work of J. Pruss, V.B. Shakhmurov, J. Goldstain, maximally regular properties of convolutional differential operator equations of elliptic type with linear operator coefficients were studied. The regularity properties of degenerate ordinary convolutional differential-operator equations are studied, for example, in the works of A. Desin, C. Lizama, and others.

The first section of the second chapter discusses the existence and uniqueness of the solution of convolutional differential-operator equations with a mixed derivative and the determination of sufficient conditions guaranteeing the separability of linear problems in weighted L_p spaces.

First, the questions of coercive solvability of convolutional differential operator equations (CDOE) of the following form are investigated

$$\sum_{|\alpha| \leq l} a_\alpha * D^\alpha u + (A + \lambda) * u = f(x), \qquad (0.6)$$

where $A = A(x)$ is a linear operator in E, $a_\alpha = a_\alpha(x)$ are complex functions, $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$, $\lambda -$ is a complex parameter.

The function $u \in W_{p,\gamma}^l(R^n; E(A), E)$ satisfying equation (0.6) almost everywhere on R^n are called solutions of equation (0.6). CDOE (0.6) is called uniformly separable in $L_{p,\gamma}(R^n; E)$ if equation (0.6) has a unique solution u for

$$f \in L_{p,\gamma}(R^n; E)$$

and the following coercive estimate is fair

$$\sum_{|\alpha| \le l} \|a_\alpha * D^\alpha u\|_{L_{p,\gamma}(R^n;E)} + \|A * u\|_{L_{p,\gamma}(R^n;E)} \le C\|f\|_{L_{p,\gamma}(R^n;E)},$$

where $C > 0$ the constant is independent of f.

To state this subheading, we introduce some notation and conditions. Let

$$L(\xi) = \sum_{|\alpha| \le l} \hat{a}_\alpha(\xi)(i\xi)^\alpha,$$

$$|L(\xi)| \ge C \sum_{k=1}^{n} |\hat{a}_{\alpha(l,k)}||\xi_k|^l, \alpha(l,k) = (0,0, \dots, l, 0,0, \dots, 0),$$

i. e. $\alpha_i = 0, i \ne k, \alpha_k = l$.

First, it is proved that under these conditions and for $\lambda \in S_{\varphi_2}, \varphi_2 \in [0, \pi)$, $\varphi + \varphi_1 + \varphi_2 < \pi$ operator functions

$$\sigma_0(\xi, \lambda) = \lambda D(\xi, \lambda),\ \sigma_1(\xi, \lambda) = \hat{A}(\xi) D(\xi, \lambda)$$

$$\sigma_2(\xi, \lambda) = \sum_{|\alpha| \le l} |\lambda|^{1 - \frac{|\alpha|}{l}} \hat{a}_\alpha(\xi)(i\xi)^\alpha D(\xi, \lambda),$$

where $D(\xi, \lambda) = \left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1}$

uniformly bounded, i.e.,

$$\|\sigma_i(\xi, \lambda)\|_{\mathcal{L}(E)} \le C, \qquad i = 0,1,2.$$

Let further

$$\hat{a}_\alpha \in C^{(n)}(R^n), \qquad \left[D^\beta \hat{A}(\xi)\right] \hat{A}^{-1}(\xi_0) \in C\left(R^n; \mathcal{L}(E)\right),$$

и $|\xi|^{|\beta|} \left|D^\beta \hat{a}_\alpha(\xi)\right| \le C_1$, $\beta_k \in \{0,1\}, \xi, \xi_0 \in R^n \backslash \{0\}, 0 \le |\beta| \le n,$

$|\xi|^{|\beta|}\left\|\left[D^{\beta}\hat{A}(\xi)\right]\hat{A}^{-1}(\xi_0)\right\|_{\mathcal{L}(E)} \leq C_2$. It is proved that the operator functions $|\xi|^{|\beta|}D_{\xi}^{\beta}\sigma_i(\xi,\lambda),\ i = 0,1,2$ are uniformly bounded.

Theorem 0.1. Let $E -$ be a Banach space satisfying the conditions of a uniform multiplier with respect to the weight function $\gamma \in A_p$ and $p \in (1,\infty)$, $\hat{A}$ be a uniformly $R-$sectorial operator in E, $\varphi \in [0,\pi)$, $\lambda \in S_{\varphi_2}, 0 \leq \varphi + \varphi_1 + \varphi_2 < \pi$. Then problem (0.6) has a unique solution u and a uniform coercive estimate holds

$$\sum_{|\alpha|\leq l} |\lambda|^{1-\frac{|\alpha|}{l}}\|a_\alpha * D^\alpha u\|_X + \|A * u\|_X + |\lambda|\|u\|_X \leq C\|f\|_X,$$

for all $f \in X = L_{p,\gamma}(R^n;E)$.

The main goal of the second section is to study the following degenerate elliptic convolutional differential-operator equations (CDOE)

$$\sum_{|\alpha|\leq l} a_\alpha * D^{[\alpha]}u + (A+\lambda) * u = f(x) \qquad (0.7)$$

in the $E-$valued weighted space $L_{p,\gamma}$, where the $E-$Banach space, $A = A\,(x)$ is a linear operator in E, $a_k = a_k(x)$ are the complex functions $\alpha = (\alpha_1,\alpha_2,\ldots,\alpha_n), a_k$- non-negative integers $\lambda -$ is a complex parameter, $\gamma = \gamma\,(x)$ is a positive measurable function on $\Omega \subset R^n$,

$$D^{[\alpha]} = D_{x_1}^{[\alpha_1]}D_{x_2}^{[\alpha_2]}\ldots D_{x_n}^{[\alpha_n]},\ D_{x_i}^{[\alpha_i]} = \left(\gamma(x)\frac{\partial}{\partial x_i}\right)^{\alpha_i}.$$

Convolution $a_\alpha * D^{[\alpha]}u$ and $A * u$ are defined in the sense of distribution. One of the main features of this paper is that convolution equations degenerate at some points $\mathbb{R} = (-\infty;+\infty)$.

Since such equations arise in applications, it is very important to prove the existence and uniqueness of a solution. Using the representation formula for the solution and operator-valued Fourier multipliers in $L_{p,\gamma}(R^n;E)$ spaces, it is proved that there is a unique solution to this problem. To solve the degenerate CDOE we use the substitutions

$$y_k = \int_0^{x_k} \gamma^{-1}(z)dz, \quad k = \overline{1,n}.$$

It is known that, using this replacement, the spaces $L_p(R^n; E)$ and $W_p^{[l]}(R^n; E(A), E)$ are displayed isomorphically in weighted spaces $L_{p,\tilde{\gamma}}(R^n; E)$ and $W_{p,\tilde{\gamma}}^{[l]}(R^n; E(A), E)$, respectively, where

$$\tilde{\gamma}(y) = \gamma(x(y)) = \gamma(x_1(y_1), x_2(y), \dots, x_n(y_n)).$$

Similarly, with this permutation, $D^{[\alpha]}u$ goes to $D^\alpha u$. In addition, with this change, the degenerate problem is transformed into the non-degenerate problem considered in the weight space $L_{p,\tilde{\gamma}}(R^n; E)$, where $a_\alpha = a_\alpha(\tilde{\gamma}(y)),\ u = u(\tilde{\gamma}(y)), A = A(\tilde{\gamma}(y)),\ f = f(\tilde{\gamma}(y))$.

Then, by virtue of Theorem 0.1, proved in the first subtitle of the second chapter, we obtain the statement.

The third section of Chapter 2 considers degenerate convolutional parabolic equations. The corresponding coercive estimate is proved similarly.

To this end, we study the degenerate Cauchy problem for CDOE. We first consider the non-degenerate Cauchy problem for a convolutionally parabolic equation

$$\frac{\partial u}{\partial t} + \sum_{|\alpha| \le l} a_\alpha * D^\alpha u + A * u + du = f(t,x), \qquad (0.8)$$

$$u(0,x) = 0, t \in \mathbb{R}_+, x \in R^n$$

where $d > 0$, a_α $-$complex-valued functions, $A -$ is a linear operator in the Banach space E. For $R_+^{n+1} = R^n \times \mathbb{R}_+$, $(p, p_1), Z = L_{p,\gamma}(R_+^{n+1}; E)$; E) will denote the space of all $p-$summable $E-$valued functions with mixed norm i.e., the space of all measurable $E-$valued functions f defined on R_+^{n+1} for which

$$\|f\|_Z = \left(\int_{R^n} \left(\int_{\mathbb{R}_+} \|f(x,t)\|_E^p \gamma(x)dx \right)^{\frac{p_1}{p}} dt \right)^{\frac{1}{p_1}} < \infty.$$

Let E_0 and E be two Banach spaces, E_0 continuously and densely embedded in E. $Z_0 = W_{\boldsymbol{p},\gamma}^{1,l}(R_+^{n+1}; E_0, E)$ denotes the space of all functions $u \in Z$ with the norm

$$\|u\|_{Z_0} = \|u\|_{Z(E_0)} + \|D_t u\|_Z + \sum_{k=1}^{n} \left\|D_k^l u\right\|_Z ,$$

where l is an integer, $D_t u, D_k^l u \in Z$ and $Z(E_0) = L_{\boldsymbol{p},\gamma}(R_+^{n+1}; E_0,)$

Theorem 0.2. Suppose that for $\varphi \in \left(\frac{\pi}{2}, \pi\right)$ all the conditions of Theorem 0.1 are satisfied. Then equation (0.8) has a unique solution, $u \in W_{\mathbf{p},\gamma}^{1,[l]}(R_+^{n+1}; E(A), E)$ and for sufficiently large d the following uniform coercive estimate

$$\left\|\frac{\partial u}{\partial t}\right\|_Z + \sum_{|\alpha|\leq l} \|a_\alpha * D^\alpha u\|_Z + \|A * u\|_Z \leq C\|f\|_Z.$$

Now consider the problem

$$\frac{\partial u}{\partial t} + \sum_{|\alpha|\leq l} a_\alpha * D^{[\alpha]} u + A * u = f(t,x),$$

$$u(0,x) = 0, \qquad t \in \mathbb{R}_+, x \in R^n \tag{0.9}$$

in E −valued mixed $L_{\boldsymbol{p}}$ spaces. It is known that using substitution

$$y_k = \int_0^{x_k} \gamma^{-1}(z)dz, \quad k = \overline{1,n}. \tag{0.10}$$

.

the degenerate problem (0.9) considered in $L_p(R^n; E)$ is transformed into the nondegenerate problem (0.8) in $L_{\boldsymbol{p},\gamma}(R^n; E)$. Then, by virtue of the previous results and from Theorem 0.1, we obtain the following result.

Theorem 0.3. Suppose that all the conditions of Theorem 0.2 are satisfied. and replacement (0.10), then problem (0.9) has a unique solution $u(t,x)$ for all $u \in L_{\boldsymbol{p}}(R_+^{n+1};E)$ and the uniform coercive estimate

$$\left\|\frac{\partial u}{\partial t}\right\|_{L_{\boldsymbol{p}}(R_+^{n+1};E)} + \sum_{|\alpha|\le l} \left\|a_\alpha * D^{[\alpha]}u\right\|_{L_{\boldsymbol{p}}(R_+^{n+1};E)} + \|A*u\|_{L_{\boldsymbol{p}}(R_+^{n+1};E)} \le C\|f\|_{L_{\boldsymbol{p}}(R_+^{n+1};E)}$$

Chapter 3 is devoted to the investigation of the most regular properties of convolutional differential-operator equations in Besov weighted spaces.

In a series of recent publications, operator-valued Fourier multiplier theorems on various vector-valued function spaces were studied. They are necessary for establishing existence and uniqueness, as well as regularity for differential equations in Banach spaces and, therefore, for partial differential equations.

Besov spaces form one class of functional spaces that are of particular interest. They can be defined as $B_{p,q}^s$ using the three indices $s \in \mathbb{R}, 1 \le p, q \le \infty$. The relatively complex definition is justified by very useful applications to differential equations. We also note that the space $B_{\infty,\infty}^s$ is nothing more than the familiar space of all Hölder continuous functions of index s if $s \in (0,1)$.

It was H. Amann who discovered yet another favorable property of vector-valued Besov spaces on the real line: a certain form (most efficiently) of the Mikhlin multiplier theorem holds for arbitrary Banach spaces.

In fact, H. Amann found that if satisfies the condition$m$

$$(m \in C^k(\mathbb{R}\backslash\{0\}; \mathcal{L}(X)), \sup_{t\in\mathbb{R}\backslash\{0\}} \left\|t^l m^{(l)}(t)\right\| < \infty, 0 \le l \le k$$

with $k = 2$, then m is a multiplier for Besov spaces and, in particular, for the space $C^\theta(\mathbb{R};X), 0 < \theta < 1$.

For Bochner spaces $L_p(R^n, X)$, additional hypotheses are necessary, in particular, extension is possible only when the basic

Banach space X has the property UMD, $1 < p < \infty$, and the set in $\{(1+|t|)^{|\alpha|}D^{\alpha}m(t); t \in \mathbb{R}^n, |\alpha| \le l)\}$ R −is bounded.

In sharp contrast with these results for $L_p(R^n, X)$, H. Amann and L. Weis independently discovered that for Besov spaces $B^s_{p,q}(R^n, X)$ additional restrictions on X and m are not needed and all indices $s \in \mathbb{R}$ and $p, q \in [1, \infty]$ hare allowed.

The first section of the third chapter discusses weighted vector-valued Besov spaces and B −separability of the considered equations. First, three equivalent definitions of the Besov space are introduced.

Vector-valued Besov spaces have been studied by many authors. In the classical case, Besov's space and its properties are broadly studied in the books of H. Tribella. Using interpolation technologies, Besov spaces are studied in articles by H. Amann, J. Borgh and J. Lofstrem.

It is known that maximally regular properties of differential operator equations have been studied in various papers. The main purpose of this subtitle is to establish the maximum regularity of a degenerate convolutional differential operator equation (CDOE)

$$Lu = \sum_{k=0}^{l} a_k * \frac{d^{[k]}u}{dx^{[k]}} + A * u + \lambda u = f(x) \qquad (0.11)$$

in Besov E −valued weight spaces. $A = A\,(x)$ is a linear operator in E, $a_k = a_k(x)$ are complex functions, λ is a complex parameter,

$$u^{[k]} = \left(\gamma(x)\frac{d}{dx}\right)^k u$$

where $\gamma(x)$ is a measurable positive function in $(-\infty; +\infty)$.

First, the Fourier multipliers from the Besov space $B^s_{p,q}(\mathbb{R}; X)$ to the Besov space $B^s_{p,q}(\mathbb{R}; Y)$ are determined.

Using the appropriate conditions, as in the previous chapters, we prove the uniform boundedness of the operator function obtained during the solution of this non-degenerate equation. Similarly, it turns out that the operator functions obtained are uniformly bounded multipliers in Besov.$-B_{p,q}^{s}(\mathbb{R};E)$ spaces.

It is easily proved that for a non-degenerate equation

$$\sum_{k=0}^{l} a_k * \frac{d^k u}{dx^k} + A * u + \lambda u = f(x) \quad (0.12)$$

has a unique solution $u \in B_{p,q,\gamma}^{l,s}(\mathbb{R};E(A),E)$ and the following uniform coercive estimate is true for $f \in B_{p,q,\gamma}^{l,s}(\mathbb{R};E),\ p,q \in [1;\infty)$,

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\| a_k * \frac{d^k u}{dx^k} \right\|_X + \|A * u\|_X + |\lambda| \|u\|_X \le C \|f\|_X. \quad (0.13)$$

If the operator Q of the generated task

$$D(Q) = B_{p,q,\gamma}^{l,s}(\mathbb{R};E(A),E),$$

$$Qu = \sum_{k=0}^{l} a_k * \frac{d^k u}{dx^k} + A * u = f \quad (0.14)$$

then for all $\lambda \in S_\varphi$ there exists a resolvent operator Q and the following estimate holds.

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\| a_k * \left[\frac{d^k}{dx^k} (Q+\lambda)^{-1} \right] \right\|_{\mathcal{L}(X)} + \|A * (Q+\lambda)^{-1}\|_{\mathcal{L}(X)}$$
$$+ \|\lambda (Q+\lambda)^{-1}\|_{\mathcal{L}(X)} \le C, \quad X = B_{p,q,\gamma}^{s}(\mathbb{R};E).$$

In particular, it is shown that the convolutional differential operator $Q + a, a > 0$, generated by equation (0.14) is a generator of the analytic semigroup.

To solve the degenerate equation (0.11), we consider the substitution (0.10).

It is clear that with this change of space, $B_{p,q}^{s}(\mathbb{R};E)$, $B_{p,q}^{[l],s}(\mathbb{R};E(A),E)$ is isomorphically mapped to the weighted spaces $B_{p,q,\tilde{\gamma}}^{s}(\mathbb{R};E)$ and $B_{p,q,\tilde{\gamma}}^{l,s}(\mathbb{R};E(A),E)$, respectively, where $\tilde{\gamma}(y) = \gamma(x(y))$. In addition, with this change, the degenerate problem (0.11) considered in $B_{p,q}^{s}(\mathbb{R};E)$ is transformed into the non-degenerate problem (0.12) considered in the weight space $B^{s}_{p,q,\tilde{\gamma}}(\mathbb{R}; E)$.

The corresponding coercive estimate for the degenerate equation (0.11) and the estimate for the resolvent of the operator of the degenerate case are proved similarly.

In this section, we also consider the Cauchy problem for the corresponding parabolic differential-operator convolution equation in an E −valued Besov space. Using the regularity properties of equation (0.11), we obtain the correctness of the Cauchy problem. This section also discusses specific examples, i.e., specific degenerate CDOE with weight functions of various types.In the second section of the third chapter, problems of a more general type are considered, i.e., CDOE with a mixed derivative in Besov weighted spaces.

In the works of V.B.Shakhmurov, the regularity of nondegenerate CDOEs in L_p spaces was investigated. In contrast, the main purpose of this section is to obtain the separable properties of the next elliptic degenerate CDOE

$$\sum_{|\alpha|\leq l} a_\alpha * D^{[\alpha]}u + A * u + \lambda u = f, \qquad (0.15)$$

and maximally regular properties of the Cauchy problem for the next parabolic CDOE

$$\frac{\partial u}{\partial t} + \sum_{|\alpha|\leq l} a_\alpha * D^{[\alpha]}u + A * u = f(t,x),$$

$$u(0,x) = 0, t \in \mathbb{R}_+, \qquad x \in R^n \qquad (0.16)$$

in the Besov E −weighted space.

First, consider a non-degenerate elliptic CDOE

$$\sum_{|\alpha|\leq l} a_\alpha * D^{a}u + A * u + \lambda u = f. \qquad (0.17)$$

Applying the Fourier transform to equation (0.17), we obtain

$$u(x) = F^{-1}\left[\hat{A}(\xi) + \lambda + L(\xi)\right]^{-1}\hat{f}\,, \quad L(\xi) = \sum_{|\alpha|\leq l} \hat{a}_\alpha\,(\xi)(i\xi)^{\alpha}.$$

In order to prove the main coercive estimate in the non-degenerate case, it suffices to show that the operator-functions obtained during the solution of these problems are uniformly bounded multipliers in the space $B^s_{p,q,\gamma}(R^n;E)$.

It is clear that with the substitution $z_k = \int_0^{x_k} \tilde{\gamma}_k{}^{-1}(y)dy$ of the space $B^s_{p,q}(R^n;E)$ and $B^{[l],s}_{p,q}(R^n;E(A),E)$ is isomorphically mapped onto the weighted spaces $B^s_{p,q,\tilde{\gamma}}(R^n;E)$ and $B^{l,s}_{p,q,\tilde{\gamma}}(R^n;E(A),E)$, respectively, where $\gamma = \prod_{k=1}^n \tilde{\gamma}_k\big(x_k(z_k)\big)$. In addition, with this substitution, problem (0.15) is transformed to problem (0.17).

Using the Fourier multiplier theorem in weighted Banach-valued Besov spaces, we obtain that for all $f \in B^s_{p,q,\gamma}(R^n;E)$ there exists a unique solution $u \in B^{[l],s}_{p,q}(R^n;E(A),E)$ of problem (0.15) and the uniform estimate holds

$$\sum_{|\alpha|\le l} |\lambda|^{1-\frac{|\alpha|}{l}} \left\| a_\alpha * D^{[\alpha]}u \right\|_X + \|A * u\|_X + |\lambda|\|u\|_X \le c\|f\|_X$$

for sufficiently large $\lambda \in S_\varphi$, where $X = B^s_{p,q,\gamma}(R^n;E)$.

Let the operator H generated by problem (0.15),

$$D(H) = B^{[l],s}_{p,q,}(R^n;E(A),E), \;\; Hu = \sum_{|\alpha|\le l} a_\alpha * D^{[\alpha]}u + A * u.$$

In this case, it is shown that for all $\lambda \in S_\varphi$ there exist operator resolvents H and has the estimate

$$\sum_{|\alpha|\le l} |\lambda|^{1-\frac{|\alpha|}{l}} \left\| a_\alpha * D^{[\alpha]}(H+\lambda)^{-1} \right\|_{\mathcal{L}(X)} + \|A * (H+\lambda)^{-1}\|_{\mathcal{L}(X)}$$
$$+ \|\lambda(H+\lambda)^{-1}\|_{\mathcal{L}(X)} \le C.$$

In the third section of the third chapter, the Cauchy problem of a degenerate parabolic CDOE (0.16) in the Besov weight space is considered.

Clearly, it means the space of all summable significant functions with a mixed norm.где $\boldsymbol{p} - E -$

It is clear that $B^{l,1,s}_{\boldsymbol{p},q,\gamma}(R^{n+1}_{+};E(A),E)=B^{1,s}_{p,q}(\mathbb{R}_{+};D(H),X)$ where $R^{n+1}_{+}=R^{n}\times\mathbb{R}_{+}$, $\boldsymbol{p}=(p,p_1), X=B^{s}_{p,q,\gamma}(R^{n};E)$ means the space of all p −summable E −valued functions with mixed norm.

It is easy to see that the problem of a non-degenerate case, i.e. the task

$$\frac{\partial u}{\partial t}+\sum_{|\alpha|\le l}a_{\alpha}*D^{\alpha}u+A*u=f(t,x)\,,\ \ u(0,x)=0 \qquad (0.18)$$

can be represented as

$$\frac{du(t)}{dt}+Hu(t)=f(t)\,,u(0)=0,t\in\mathbb{R}_{+}.$$

In view of the resolvent properties, the positivity of the operator H, and previous results, we obtain that the last equation has a unique solution $u\in B^{1,s}_{p,q}(R_{+};D(H),X)$ satisfying

$$\left\|\frac{du}{dt}\right\|_{X}+\|Hu\|_{X}\le \mathrm{c}\|f\|_{X}.$$

Hence we have, for all $f\in B^{s}_{p,q,\gamma}(\mathbb{R}_{+},X)$, there is a unique solution $u(t,x)$ to problem (0.15) satisfying the following coercive estimate

$$\left\|\frac{\partial u}{\partial t}\right\|_{Y}+\sum_{|\alpha|\le l}\left\|a_{\alpha}*D^{[\alpha]}u\right\|_{Y}+\|A*u\|_{Y}\le C\|f\|_{Y},$$

$$Y=B^{s}_{p,q,\gamma}(\mathbb{R}_{+},X).$$

In the fourth section of the third chapter, we study a system of degenerate integro-differential equations. The following system is considered.

$$\sum_{k=0}^{l}a_{k}*\frac{d^{[k]}u_{m}}{dx^{[k]}}+\sum_{j=1}^{\infty}\left(d_{j}+\lambda\right)*u_{j}(x)=f_{m}(x),$$

$$x\in\mathbb{R}, \qquad (0.19)$$

and we prove the uniform coercive estimate for (0.19). To this end, let $\left\{d_{j}(x)\right\}_{1}^{\infty}\in l_{q}$ be valid for all $x\in\mathbb{R}$ and some $x_{0}\in\mathbb{R}$, there exist positive constants C_1 and C_2 such that $C_1 \left|d_j(x_0)\right| \leq \left|d_j(x)\right| \leq C_2\left|d_j(x_0)\right|$.

Suppose that $\hat{a}_k, \hat{d}_j \in C^{(1)}(\mathbb{R})$ and there exist constants $M_i > 0, i = 1,2,$ such that

$$\left|\xi^j \frac{d^j}{d\xi^j}\hat{a}_k(\xi)\right| \leq M_1, \quad |\xi|^j\left|d_m^j(\xi)d_m^{-1}(\xi)\right| \leq M_2.$$

Denote $D(x) = \{d_m(x)\}$ and $D * u = \{d_m * u_m\}$

Then, under the above conditions and for $|L(\xi)| \geq C\max_{\mathrm{k}}|\hat{a}_k(\xi)|\,|\xi|^l$problem (0.19) has a unique solution $u(x) = \{u_m(x)\}_1^\infty$, which belongs to the space $B_{p,q,}^{[l],s}\left(\mathbb{R}; l_q(D), l_q\right)$, and the following coercive estimate holds:

$$\sum_{k=0}^{l}|\lambda|^{1-\frac{k}{l}}\left\|a_k * \frac{d^{[k]}u}{dx^{[k]}}\right\|_B + \|D * u\|_B + |\lambda|\|u\|_B \leq C\|f\|_B.$$

In addition, it is proved that for sufficiently large $|\lambda| > 0$, there exists a resolvent $(Q + \lambda)^{-1}$ and

$$\sum_{k=0}^{l}|\lambda|^{1-\frac{k}{l}}\left\|a_k * \frac{d^{[k]}}{dx^{[k]}}(Q + \lambda)^{-1}\right\|_{\mathcal{L}(B)} + \|D * (Q + \lambda)^{-1}\|_{\mathcal{L}(B)} + \|\lambda(Q + \lambda)^{-1}\|_{\mathcal{L}(B)} \leq C, \quad B = B_{p,q}^s\left(\mathbb{R}; l_q\right).$$

Further in the fourth section, we consider the Cauchy problem for these systems and prove the fulfillment of a similar coercive estimate.

Thus, in the third chapter, various types of degenerate convolutional differential-operator and degenerate integro-differential equations are considered - in both cases, the existence and uniqueness of the solution of these problems and the fulfillment of the corresponding coercive estimate are proved.

It is easily seen that from the coercive solvability and uniform positivity proved in Chapter 3, it follows that the operators generated by the considered problem are a generator of an analytic semigroup. In the proof of the theorems of the third chapter we use theories of operator semigroups, embedding theorems, theories of Fourier multipliers, methods of positive operators, and convolution theory.

The results proved in this chapter are of undoubted interest in the theory of quasilinear elliptic CDOEs.

It is known that there are many classes of differential and pseudo-differential operators with the property of positivity and sectoriality. Therefore, choosing specific spaces and specific operators acting in this space, we obtain the most regular properties of a different class of degenerate convolution equations in different spaces and the Cauchy problem for parabolic CDOEs or their systems, respectively.

Chapter 4 is devoted to the study of the coercivity of degenerate convolutional differential-operator equations in weighted spaces and their application.

The properties of separability or maximum regularity for degenerate CDOEs, especially with operator coefficients, have been studied relatively little.

The separable properties of non-degenerate CDOEs in weightless spaces have been studied quite extensively. Coercive solvability theorems for degenerate CDOEs give a stimulating effect to the study of the corresponding concrete equations and boundary value problems.

This chapter discusses specific tasks in which the methods used in the previous chapters are used to solve the problem. Note that classical methods are not applicable here.

In the first section of the fourth chapter, quasilinear elliptic CDOEs are considered.

$$\sum_{|\alpha|\leq l} a_\alpha * D^\alpha u + G(x, D^\sigma u)u = F(x, D^\sigma u) + f(x),$$

$$x \in R^n, \tag{0.20}$$

where G and F are nonlinear operators in the Banach space E, $a_\alpha = a_\alpha(x)$are complex-valued, $f -\ E$ −valued functions, D^σ −differential operators, such that $|\sigma| \leq l - 1$..

Using the embedding theorem and the trace theorem in Sobolev space, we have

$$\prod_{|\sigma|\leq l_0-1} \|D^\sigma u\|_{C(R^n;E_\sigma)} = \prod_{|\sigma|\leq l_0-1} \sup_{x\in R^n} \|D^\sigma u(x)\|_{E_\sigma} \leq \|u\|_Y,$$

where

$$X = L_{p,\gamma}(R^n;E),\ \ Y = W_{p,\gamma}^l(R^n;E(A),E), \gamma(x) = \prod_{k=1}^{n} |x_k|^\gamma,$$

$$0 \leq \gamma \leq \frac{p-1}{n}, E_\sigma = (E(A),E)_{x_{\sigma,p}},$$

$$x_\sigma = \frac{p|\sigma|+\gamma+n}{\rho l}, E_0 = \prod_{|\alpha|\leq l_0-1} E_\sigma$$

$l_0 = \left[l - \frac{\gamma+n}{p}\right]$ here stands for the whole part.$[s] - s > 0$

First, consider the corresponding linear equation. Using the theorems of Chapter 3, we obtain a coercive estimate in the space X.

Then, using the maximum regularity of the obtained linear problem, using the compression theorem, and using the Banach fixed point theorem, we obtain that there exists a unique solution $u \in Y$ of equation (0.20) and $\|u\|_Y \leq r, r > 0$.

In the second section of Chapter 4, we study the boundary-value problem for convolutional differential equations of anisotropic type and obtain maximally regular properties in mixed weighted norms.

Let $\widetilde{\Omega} = R^n \times \Omega$, where $\Omega \subset R^n$ is an open connected set with compact C^{2m} −boundary $\partial\Omega$, $\boldsymbol{p} = (p_1, p)$, weight function on Ω. $L_{p,\gamma}(\widetilde{\Omega})$ will denote the space of all p −summable scalar-valued functions with mixed norm i.e., the space of all measurable functions f defined on $\widetilde{\Omega}$ for which

$$\|f\|_{L_{\boldsymbol{p},\gamma}(\widetilde{\Omega})} = \left(\int_{R^n} \left(\int_{\Omega} |f(x,y)|^{p_1} \gamma(x) dx \right)^{\frac{p}{p_1}} dy \right)^{\frac{1}{p}} < \infty.$$

The rest of this section discusses the problem.

$$\sum_{|\alpha|\le l} a_\alpha * D^\alpha u + \sum_{|\alpha|\le 2m} \left(b_\alpha c_\alpha D_y^\alpha + \lambda\right) * u = f(x,y),$$
$$x \in R^n, y \in \Omega, \qquad (0.21)$$
$$B_j u = \sum_{|\beta|\le m_j} b_{j\beta}(y) D_y^\beta u(x,y) = 0,$$
$$y \in \partial\Omega, \qquad j = \overline{1,m}, \qquad (0.22)$$

where

$$D_j = -i\frac{\partial}{\partial y_j}, \;\; y = \left(y_1, y_2, \dots, y_\mu\right), a_\alpha = a_\alpha(x), \; b_\alpha = b_\alpha(x),$$
$$c_\alpha = c_\alpha(y),$$

and it is proved that under certain conditions for $f \in L_{\mathbf{p},\gamma}(\widetilde{\Omega})$ and $\lambda \in S_\varphi$, problem (0.21) - (0.22) has a unique solution $u \in W_{\mathbf{p},\gamma}^{l,2m}(\widetilde{\Omega})$ and the corresponding coercive estimate holds.

This problem is similarly solved in the Besov weight space. Let $B_{\boldsymbol{p},q,\gamma}^{s}(\widetilde{\Omega})$ denote the Besov space with the corresponding weighted mixed norm; then

$$B_{\boldsymbol{p},q,\gamma}^{s}(\widetilde{\Omega}) = B_{p,q,\gamma}\left(R^n; \; B_{p_1,q,\gamma}^{s}(\Omega)\right)$$
$$B_{\boldsymbol{p},q,\gamma}^{l,2m,s}(\widetilde{\Omega}) = B_{p,q,\gamma}^{l,s}\left(R^n; \; B_{p_1,q,\gamma}^{2m,s}(\Omega), B_{p_1,q,\gamma}^{s}(\Omega)\right)$$

By the way, we note that for $l \neq 2m$ equations (0.21) are anisotropic, for $l = 2m$ is isotropic.

Consider the operator A defined by the following equalities

$$D(A) = B_{p_1,q}^{2m,s}\left(\Omega, \;\; B_j u = 0\right),$$

$$A(x)u = \sum_{|\alpha|\le 2m} b_\alpha(x) C_\alpha(y) D^\alpha u(y).$$

A similar theorem is proved for problems (0.21) - (0.22) in the Besov space $B_{\boldsymbol{p},q,\gamma}^{s}(\widetilde{\Omega})$.

The third section of Chapter 4 explores an infinite system of degenerate integro-differential equations.

Consider the following infinite system of degenerate convolution equations in the space $L_{p,\gamma}(R^n; l_q)$,

$$\sum_{|\alpha|\le l} a_\alpha * D^{[\alpha]} u_m + \sum_{j=1}^{\infty} d_j * u_j = f_m, \qquad (0.23)$$

where

$$u_j = u_j(x), \qquad d_j = d_j(x), \qquad a_\alpha = a_\alpha(x),$$

$$\gamma(x) = \prod_{k=1}^{n} |x_k|^\gamma, -\frac{1}{n} < \gamma < \frac{p-1}{n}.$$

To put $1 < q < \infty$,

$$l_q = \left\{ \xi;\ \xi = \{\xi_i\}_{i=1}^{\infty};\ \|\xi\|_{l_q} = \left(\sum_{i=1}^{\infty} |\xi_i|^q \right)^{\frac{1}{q}} < \infty,\ \xi_i - complex\ numbers. \right\}$$

Then

$$L_{p,\gamma}(l_q) = L_{p,\gamma}(R^n; l_q) =$$

$$= \left\{ f; f = \{f_i(x)\}_{i=1}^{\infty},\ \|f\|_{L_{p,\gamma}(l_q)} = \left(\int_{R^n} \|\{f_i(x)\}\|_{l_q}^p \gamma(x) dx \right)^{\frac{1}{p}} \right\}.$$

Let

$$D(x) = \{d_m(x)\}, \qquad d_m > 0, \qquad u = \{u_m\}, D * u = \{d_m * u_m\},$$

$$l_q(D) = \left\{ u \in l_q, \qquad \|u\|_{l_q(D)} = \left(\sum_{m=1}^{\infty} |d_m(x) * u_m|^q \right)^{\frac{1}{q}} < \infty \right\},$$

$$1 < q < \infty.$$

Under specific conditions, it is proved that for all estimate$f(x) = \{f_m(x)\}_1^\infty \in L_{p,\gamma}\big(R^n; l_q(D)\big)$ and for $\lambda \in S_\varphi, \varphi \in [0,\pi)$ problem (0.23) has a unique solution $u = \{u_m(x)\}_1^\infty, Y$ belonging to Y and the following coercive estimate

$$\sum_{|\alpha|\le l} |\lambda|^{1-\frac{|\alpha|}{l}} \left\|a_\alpha * D^{[\alpha]}u\right\|_X + \|D * u\|_X \le C\|f\|_X,$$

where

$$X = L_p\big(R^n; l_q\big), Y = W_{p,\gamma}^{[l]}\big(R^n; l_q(D), l_q\big).$$

Denote by Q the differential operator in $L_p\big(R^n; l_q\big)$ generated by (0.23).

It can be proved similarly for $\lambda \in S_\varphi$ that there exists a resolvent $(Q+\lambda)^{-1}u$ and the corresponding resolvent estimate is true.

The fourth section of Chapter 4 considers a Wentzel-Robin boundary value problem for convolutional differential-operator equations. For a parabolic problem with Wentzel-Robin boundary conditions on some L_p −spaces $(1 < p < \infty)$ A.Favini, G.Goldstain and S.Romanelli did good research (generating the C_0 semigroup and holomorphy) and obtained some excellent results.

The second-order problem was investigated in the works of K. Engel and A. Favini, where the authors used the theory of cosine functions to prove correctness. In the articles of V. Keyantuo and M. Warma, the case of L_p −space is investigated. For proof. the correctness of the problem under consideration on suitable L_p −spaces on the interval [0,1] also uses the theory of cosine functions. Finally, at the end of the 20th century, A. Wentzel considered these issues in the multidimensional case, i.e. for regular bounded domains $\Omega \subset R^n$.

In this section, we take $E = L_2(0,1)$ and $A = A(x)$ as a differential operator with a generalized Wentzel-Robin type boundary condition defined by the formula

$$D(A) = \left\{u \in W_2^2(0,1), B_j u = Au(j) = 0, j = 0,1\right\},$$

$$A(x)u = a(x,y)u^{(2)} + b(x,y)u^{(1)} \;\; for\ all \;\; x \in R^n, y \in (0,1),$$

where $a(x,\cdot)$ and $b(x,\cdot)$ complex-valued functions on $(0,1)$ for all $x \in R^n$.

We consider mixed problems with boundary conditions of the Wentzel-Robin type for a degenerate integro-differential equation

$$\sum_{|\alpha|\leq l} a_\alpha * D^{[\alpha]}u + \left(a(x,y)\frac{\partial^2}{\partial y^2} + b(x,y)\frac{\partial}{\partial y} + \lambda\right) * u = f(x,y) \qquad (0,24)$$

$$B_j u = \left[a(x,j)\frac{\partial^2}{\partial y^2} + b(x,j)\frac{\partial}{\partial y}\right] u(x,j) = 0, \qquad j = 0,1, x \in R^n, y \in (0,1),$$

where $a_\alpha = a_\alpha(x)$ are complex-valued functions, λ is a complex parameter.

Let $\widetilde{\Omega} = R^n \times (0,1), \boldsymbol{p} = (2,p), L_p(\widetilde{\Omega})$ denote the space of all p −summable weighted scalar-valued functions with mixed norm, i.e. the space of all measurable functions f defined on $\widetilde{\Omega}$ for which

$$\|f\|_{L_{\boldsymbol{p}}(\widetilde{\Omega})} = \left(\int_{R^n}\left(\int_0^1 |f(x,y)|^2\, dy\right)^{\frac{p}{2}} dx\right)^{\frac{1}{p}} < \infty.$$

Similarly, $W_{\boldsymbol{p},\gamma}^{[l],2}(\widetilde{\Omega})$ denotes the Sobolev anisotropic space with the corresponding mixed norm, i.e. $W_p^{[l],2}(\widetilde{\Omega})$ denotes the space of all functions $u \in L_{\boldsymbol{p}}(\widetilde{\Omega})$ having the derivative $D_x^{[\alpha]}u \in L_{\boldsymbol{p}}(\widetilde{\Omega})$ with respect to x for $|\alpha| \leq m$ and the derivative $\frac{\partial^2 u}{\partial y^2} \in L_{\boldsymbol{p}}(\widetilde{\Omega})$ with the norm

$$\|u\|_{W_{\boldsymbol{p},\gamma}(\widetilde{\Omega})}^{[l],2} = \|u\|_{L_{\boldsymbol{p}}(\widetilde{\Omega})} + \sum_{|\alpha|=l}\left\|D_x^{[\alpha]}u\right\|_{L_{\boldsymbol{p}}(\widetilde{\Omega})} + \left\|\frac{\partial^2 u}{\partial y^2}\right\|_{L_{\boldsymbol{p}}(\widetilde{\Omega})} < \infty$$

This section presents the following result:

Theorem 0.4.

Suppose that all the conditions of Theorem 0.1 are satisfied and that $-a(x,\cdot) \in W_\infty^1(0,1), a(x,\cdot) \geq \delta > 0, b(x,\cdot) \in L_\infty(0,1)$ for all

$x \in R^n$. Then for $f \in L_{\boldsymbol{p}}(\tilde{\Omega})$ and $\lambda \in S_\varphi$ problem (0.24) has a unique solution $\lambda \in S_\varphi$ and the following coercive uniform estimate

$$\sum_{|\alpha| \leq l} |\lambda|^{1-\frac{|\alpha|}{l}} \left\| a_\alpha * D^{[\alpha]} u \right\|_{L_{\boldsymbol{p}}(\tilde{\Omega})} + |\lambda| \|u\|_{L_{\boldsymbol{p}}(\tilde{\Omega})} + \left\| \left(a \frac{\partial^2}{\partial y^2} + b \frac{\partial}{\partial y} \right) * u \right\|_{L_{\boldsymbol{p}}(\tilde{\Omega})} \leq C \|f\|_{L_{\boldsymbol{p}}(\tilde{\Omega})}.$$

For the resolvent $(Q + \lambda)^{-1}$ of the differential operator Q generated by problem (0.24), there exists a corresponding coercive estimate.

This subheading also considers some examples, for example, in the three-dimensional space R_+^3 and shows the maximum-regularity of the Cauchy problem for parabolic CDOEs.

Chapter 5 is devoted to the study of linear and nonlinear degenerate convolutional-elliptic operators and their applications. The regularity properties of degenerate or non-degenerate abstract convolution-elliptic equations with parameters are investigated. Sufficient conditions are found that guarantee the separability of linear problems in weighted $L_{\boldsymbol{p}}$ −spaces. It is proved that the corresponding convolution-elliptic operator is sectorial and is also a generator of an analytic semigroup.

Using these results, we obtain the existence and uniqueness of a maximally regular solution for the nonlinear convolution equation with parameters in weighted $L_{\boldsymbol{p}}$ spaces. In the appendix, the properties of maximally regularity of the Cauchy problem for a degenerate parabolic equation in mixed norms $L_{\boldsymbol{p}}$, a boundary value problem for an anisotropic convolutional elliptic equation, a boundary value problem for a degenerate integro-differential equation and their infinite systems are proved.

In the first section of Chapter 5, we study the maximum regularity properties of degenerate linear convolutional differential-operator equations with parameters. Consider the following degenerate CDOE

$$\sum_{|\alpha|\leq l} \varepsilon_\alpha a_\alpha * D^{[\alpha]}u + (A+\lambda) * u = f, \tag{0.25}$$

in E −valued spaces L_p, where l is a positive integer $a_\alpha = a_\alpha(x)$ are complex-valued functions $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, α_k are non-negative integers, $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)\varepsilon_\alpha = \prod_{k=1}^{n} \varepsilon_k^{\frac{\alpha_k}{l}}$, ε_k is a positive, λ −complex parameter, $A = A(x)$ is a linear operator in the Banach space E for $x \in R^n$.

Sufficient conditions are found that guarantee the separability of problem (0.25).

Assume that the following conditions are met:

Condition 0.1.

1) $L_\varepsilon(\xi) = \sum_{|\alpha|\leq l} \varepsilon_\alpha \hat{a}_\alpha(\xi)(i\xi)^\alpha \in S_{\varphi_1}, \varphi_1 \in [0,\pi)$

$$|L_\varepsilon(\xi)| \geq C \sum_{k=1}^{n} \varepsilon_k \left|\hat{a}_{\alpha(l,k)}\right| |\xi_k|^l, \qquad \alpha(l,k) = (0,0,\ldots,l,0,0,\ldots,0),$$

$$\alpha_i = 0, i \neq k, \alpha_k = l, \qquad for\ \xi \in R^n;$$

2) $\hat{a}_\alpha \in C^{(n)}(R^n)$, $|\xi|^{|\beta|}\left|D^\beta \hat{a}_\alpha(\xi)\right| \leq C_1$,

$\beta_k \in \{0,1\}, 0 \leq |\beta| \leq n;$

3) $\left[D^\beta \hat{A}(\xi)\right]\hat{A}^{-1}(\xi_0) \in C\big(R^n; \mathcal{L}(E)\big)$,

$|\xi|^{|\beta|}\left\|\left[D^\beta \hat{A}(\xi)\right]\hat{A}^{-1}(\xi_0)\right\|_{\mathcal{L}(E)} \leq C_2$,

$0 \leq |\beta| \leq n, \xi, \xi_0 \in R^n \backslash \{0\}$.

This section proves the following main result:

Theorem 0.5. Suppose that Condition 0.1 is fulfilled and an E − Banach space satisfying the multiplier condition with respect to the weight function $\in A_p$. Let $\hat{A}$ be uniformly R −sectorial operator in E with φ∈ [0, π), $\varphi \in [0,\pi), \lambda \in S_{\varphi_2} 0 \leq \varphi + \varphi_1 + \varphi_2 < \pi$. Then for all $f \in \tilde{X}$ there exists a unique solution to problem (0.25) and the following coercive uniform estimate holds

$$\sum_{|\alpha|\leq l}\varepsilon_\alpha\,|\lambda|^{1-\frac{|\alpha|}{l}}\left\|a_\alpha * D^{[\alpha]}\right\|_{\tilde{X}}+\|A*u\|_{\tilde{X}}+|\lambda|\|u\|_{\tilde{X}}\leq C\|f\|_{\tilde{X}},$$

$$\tilde{X}=L_{\mathrm{p}}(R^n;E),\tilde{Y}=W^{[l]}_{p,\gamma}(R^n;E(A),E)$$

Now we consider the Cauchy problem for a degenerate parabolic convolution equation with parameters

$$\frac{\partial u}{\partial t}+\sum_{|\alpha|\leq l}\varepsilon_\alpha a_\alpha * D^{[\alpha]}u+A*u+du=f(t,x)$$

$$u(0,x)=0, t\in\mathbb{R}_+,\qquad x\in R^n. \tag{0.26}$$

Using the previous remarks from Theorem 0.3, we obtain the following result.

Theorem 0.6. Let Condition 0.1 hold for $a_\alpha(y)$ and A with $\varphi\in\left(\frac{\pi}{2},\pi\right)$. Then problem (0.26) has a unique solution $u(t,x)$ for all $L_{\boldsymbol{p}}(R^{n+1}_+;E)$ in mixed norms and for sufficiently large d, the following coercive uniform estimate

$$\left\|\frac{\partial u}{\partial t}\right\|_{L_{\boldsymbol{p}}(R^{n+1}_+;E)}+\sum_{|\alpha|\leq l}\varepsilon_\alpha\left\|a_\alpha * D^{[\alpha]}u\right\|_{L_{\boldsymbol{p}}(R^{n+1}_+;E)}+\|A*u\|_{L_{\boldsymbol{p}}(R^{n+1}_+;E)}\leq C\|f\|_{L_{\boldsymbol{p}}(R^{n+1}_+;E)},\ R^{n+1}_+=R^n\times\mathbb{R}_+.$$

The results proved in the previous chapters are of undoubted interest and give a stimulating effect to the study of degenerate integro-differential equations with parameters. The results proved in the third and fourth chapters are widely used even in the case of $E=\mathbb{R}$ (i.e., in the numerical case) and $E=l_q$.

In the second section of the fifth chapter, we consider different types of boundary value problems for degenerate differential operator equations and prove the coercive solvability of these problems.

Our results on the maximum-regularity of problem (0.25) are well used for the existence and uniqueness of a maximum-regular solution for specific CDOEs.

$$\sum_{|\alpha|\le l} \varepsilon_\alpha a_\alpha * D^{[\alpha]} u_m + \sum_{j=1}^{\infty} d_j * u_j(x) = f_m(x),$$

$$x \in R^n, m = 1,2, \ldots, \qquad (0.27)$$

$$\text{где } \varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n), \varepsilon_\alpha = \prod_{k=1}^{n} \varepsilon_k^{\frac{\alpha_k}{l}},$$

$$\gamma(x) = \prod_{k=1}^{n} |x_k|^\gamma, -\frac{1}{n} < \gamma < \frac{p-1}{n}.$$

To prove the theorem on the existence and uniqueness of a solution to problem (0.27), we use the methods developed in the previous chapters. We similarly prove the existence of a resolvent of the operator of the generated problem (0.27) and show the corresponding estimate.

Further in this subheading we study the maximum regularity properties of a problem with boundary conditions of the Wentzel-Robin type in mixed norms for a degenerate integro-differential equation with parameters. Consider the operator $A = A(x)$ defined by the formula

$$D(A) = \{u \in W_2^2(0,1), \qquad Au(j) = 0\}, j = 0,1,$$

$$A(x)u = a(x,y)\frac{d^2u}{dy^2} + b(x,y)\frac{du}{dy}, \qquad x \in R^n, \qquad y \in (0,1).$$

Using the R $-$sectoriality of the operator A in the space $L_2(0,1)$, from Theorem 0.5 we obtain the following uniform coercive estimate

$$\sum_{|\alpha|\le l} \varepsilon_\alpha |\lambda|^{1-\frac{|\alpha|}{l}} \left\| a_\alpha * D^{[\alpha]} u \right\|_{L_p(\tilde{\Omega})} + |\lambda| \|u\|_{L_p(\tilde{\Omega})} \left\| \left(a\frac{\partial^2}{\partial y^2} + b\frac{\partial}{\partial y} \right) * u \right\|_{L_p(\tilde{\Omega})} \le C \|f\|_{L_p(\tilde{\Omega})}$$

The corresponding estimate for the resolvent of the operator generated by problem (0.25) with boundary conditions of the Wentzel-

Robin type is obtained from the resolution of the resolvent for problem (0.25) for specific values of the operator A in the space E.

The third section of Chapter 5 explores the separability properties of convolutional differential equations that depend on parameters. It is well known that differential equations with parameters play an important role in modeling physical processes. Convolutional differential-operator equations with small parameters also have significant applications in the development of the theory of the problem of mathematical physics.

In this section, we obtain a representation of a solution involving the semigroup of the operator A, which allows us to obtain the properties of maximum regularity of the DOE and exact coercive $L_p$estimates of the solution uniformly in the small and spectral parameters.

Consider the following boundary-value problem for a convolution-differential elliptic equation with small parameters

$$\begin{cases} Lu = -\varepsilon u''(t) + A_\lambda u(t) + \varepsilon^{\frac{1}{2}}(aA_1 * u')(t) + \\ \quad +(A_0 * u)(t) = f(t), \;\; t \in (0;\infty), \\ L_1 u = \varepsilon^{\frac{p+1}{2p}} \alpha u'(0) + \varepsilon^{\frac{1}{2p}} \beta u(0) = f_0 \end{cases} \tag{0.28}$$

where $u\,(t) = u\,(\varepsilon, t$ is the solution (0.28), $, A_1 = A_1\,(t), A_0 = A_0\,(t)$ are linear operators in the Banach space $A\,E$, $A_\lambda = A + \lambda I, a = a\,(t)$ the scalar function on (0; ∞), $f_0 \in E_p = (E\,(A), E)_{\theta,p}$, here $(E\,(A), E)_{\theta,p}$ denotes the real interpolation space between $E\,(A)$ and $E, p \in (1,\infty), \theta = \frac{1+p}{2p}$, $, \alpha, \beta$ are complex numbers, ε is a small positive, and λ is a complex parameter.

To study the main problem, we first consider the corresponding homogeneous problem of the main part of equation (0.28), i.e.,

$$\begin{cases} -\varepsilon u''(t) + A_\lambda u(t) = 0 \\ \quad L_1 u = f_0 \end{cases} \tag{0.29}$$

The following theorem is proved:

Theorem 0.7. Assume that the following conditions are met:

1) E is a Banach space satisfying the condition of a uniform multiplier for $p \in (1, \infty)$;

2) A is a uniformly R −positive operator in E for $0 \leq \varphi < \pi$, and $-\beta\alpha^{-1} \in S_{\varphi_1}, 0 \leq \varphi_1 + \varphi < \pi$. Then problem (0.29) for , $\forall f_0 \in E_p$ has a unique solution $u\,(t) \in W_p^2(\mathbb{R}_+;\ E\,(A), E)$ and the coercive estimate

$$\sum_{i=0}^{2} |\lambda|^{1-\frac{i}{2}} \varepsilon^{\frac{i}{2}} \left\| u^{(i)}(t) \right\|_{L_{p(\mathbb{R}_+;E)}} + \|Au(t)\|_{L_{p(\mathbb{R}_+;E)}} \leq C\left[|\lambda|^{1-\theta} \|f_0\|_E + \|f_0\|_{E_p} \right]$$

is performed uniformly in and for sufficiently large $\varepsilon, \lambda \in S_\varphi |\lambda|$.

Next, we consider the corresponding inhomogeneous problem i.e., the main part of equation (0.28). It is proved that there exists a unique solution and the following uniform coercive estimate holds

$$\sum_{i=0}^{2} |\lambda|^{1-\frac{i}{2}} \varepsilon^{\frac{i}{2}} \left\| u^{(i)}(t) \right\|_{L_{p(\mathbb{R}_+;E)}} + \|Au(t)\|_{L_{p(\mathbb{R}_+;E)}} \leq C\left[\|f\|_{L_{p(\mathbb{R}_+;E)}} + |\lambda|^{1-\theta} \|f_0\|_E + \|f_0\|_{E_p} \right].$$

Finally, we consider problem (0.28).

Theorem 0.8. Suppose that all the conditions of Theorem 0.7 are satisfied,

and $a\,(t) \in L_1\,(\mathbb{R}_+), A_1(t)A^{-\left(\frac{1}{2}-\mu_1\right)} \in L_\infty\left(\mathbb{R}_+; \mathcal{L}(E)\right)$,
$A_0(t)A^{-(1-\mu_2)} \in L_\infty\left(\mathbb{R}_+; \mathcal{L}(E)\right)$ for 0 <<1/2, 0 <<1. $\mu_1 \mu_2$

Then for all $f \in L_p(\mathbb{R}_+, E)$ and sufficiently large $|\lambda| > \lambda_0 > 0$ there exists a unique solution $u \in W_p^2\,(\mathbb{R}_+;\ E\,(A), E)$ of problem (0.28) and has place the following coercive uniform estimate

$$\sum_{i=0}^{2} \varepsilon^{\frac{i}{2}} |\lambda|^{1-\frac{i}{2}} \left\| u^{(i)} \right\|_{L_p(\mathbb{R}_+,E)} + \|Au\|_{L_p(\mathbb{R}_+,E)} \leq c\|f\|_{L_p(\mathbb{R}_+,E)}.$$

The results obtained can be used for a singular perturbation of the integro-differential parabolic equation. In addition, the properties of uniform separability of problem (0.28) make it possible to study the spectral properties of the elliptic problem, which depends on the parameter.

The main results of the thesis are published in 56 papers.

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THE RESULTS

The dissertation is devoted to the study of the property of maximal regularity of convolution differential operator equations in weighted Banach spaces. The following new science results have been obtained in the dissertation:

- representations of the solution of a convolution differential operator equations in a weighted Banach space are obtained;
- proved the uniform boundedness and R-boundedness of the operator-valued functions obtained in the study of the solvability of convolution equations;
- for the first time, the existence and uniqueness of the solution of degenerate convolution differential operator equations with a mixed derivative in the weighted space was proved and a coercive estimate was obtained;
- sufficient conditions were found to guarantee the separability of linear problems in weighted Banach spaces;
- using coercive estimates, the existence and uniqueness of the degenerate Cauchy problem for a convolution-parabolic equation was proved;
- the existence of the resolvent of the operator generated by the considered problem and the validity of the corresponding estimates are studied;

- an estimate was obtained for solving quasilinear elliptic convolution differential operator equations;

- the boundary value problem for convolution differential equations of anisotropic type was studied and the most regular properties are obtained in weighted mixed norm;

- for infinite systems of degenerate integro-differential equations in the weighted space, the satisfied of coercive estimates and the holds of the corresponding resolvent estimates are proved;

- found the conditions for the existence of a solution of a mixed problem with boundary conditions of the Wentzel-Robin type for degenerate integro-differential equations and obtained an estimate;

- maximal regularity of degenerate linear convolution differential operator equations with parameters are described;

- a coercive estimate is proved for solving a convolution differential operator equations depending on a parameter.

Coercive solvability theorems for degenerate CDOEs give a stimulating influence on the study of the corresponding specific equations and boundary value problems. When solving specific problems, the methods obtained in the second and third chapters are used. Note that the classic methods are not applicable here.

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1. Мусаев Г.К., Коэрцитивная разрешимость для дифференциально-операторных уравнений. *//Труды конференции, посвященной 80-летию К.Т.Ахмедова, Баку.- 1999. -с.52-55.*

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