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ABSTRACT

of the dissertation for the degree of Doctor of Science

EXISTENCE AND ASYMPTOTICS OF SOLUTION TO ELLIPTIC AND PARABOLIC TYPE EQUATION IN DIFFERENT DOMAINS

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Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THIS WORK

Rationale of the work. The dissertation work has been devoted to one direction of pure mathematics, actual in a theoretical aspect along with applied aspects, more precisely, to the existence, nonexistence of global solutions and asymptotic of solutions near infinity of semilinear elliptic, parabolic equations and systems in unbounded domains. Mathematical modeling of various phenomena and processes of mechanics, physics, hydrodynamics, and biology is reduced to studying some issues for partial differential equations. Up to the fifties of the XX century, the main object for studying partial differential equations was linear equations. It should be noted that the present theory on elliptic and parabolic-type partial differential equations has been well studied, and a sufficient number of books, monographs, and papers are devoted to such equations.

Around the middle of the last century, the development of functional analysis gave a strong impetus to the development of the theory of nonlinear equations. The rapid development of theory on nonlinear equations is due to their close connection with problems of applied character. For example, a quasilinear heat conductivity equation in certain conditions describes the processes of electronic and ionic thermal conductivity in plasma, adiabatic filtration of gas and liquids in porous media, diffusion of neutrons and alpha particles; it appears during mathematical modeling of the processes of chemical kinetics, various kinds of biochemical reactions, the methods of growth and migration of populations, etc.

In the early XX century, in connection with the astrophysical research of Emden and Fowler, the equation

$$y'' \pm x^{\alpha} |y|^{q-1} y = 0$$

emerged. It can also be encountered in nuclear physics when studying the behavior of electrons in a heavy atom in the works of Thomas and Fermi.

Mathematical modeling of some physical processes, such as forecasting of some heat transfer processes, self-ignition of coal collected in mines, filtration of air in a certain medium, gas separation process in a closed medium, etc., is reduced to studying the existence, uniqueness, and stability of solutions to nonlinear differential equations. The fact that the state of mechanical systems described by nonlinear equations can be predicted at any moment in time makes the existence of global solutions (that can be determined at any time) of these equations relevant.

At present, the problems related to studying the existence and nonexistence of nontrivial global solutions of nonlinear differential equations and inequalities are of great interest, and there are a lot of works devoted to these problems. For example, we can recite the works of the following authors: V.A.Kondratyev, A.A.Konkov, S.D.Eidelman, S.I.Pokhozhaev, V.A.Galaktianov, G.G.Laptev, V.V.Kurta, Sh.M.Nasibov, F.I.Mamedov, A.B.Aliev, B.Sirakov, C.Bandle, M.Essen, H.Brezis, H.Berestycki, L.Nirenberg, M.-F.Bidaut-Veron, P.Lions, L.Vazquez, H.Chen, M.Fall, B.Gidas, J.Spruck, T.Kato, I.Kombe, V.Liskevich, Z.Sobol, V.Moroz, İ.I.Skrypnik, Y.Pinchover, R.G.Pinsky, A.Tesei, J.Serrin, H.Zou, M.Surnachev, S.Terracini, Y.Uda, C.P.Wang, S.Zheng, X.Xu, H.Xiong, Y.T.Shen., Y.Yao, Qi.S.Zhang.

Historically, the Cauchy problem for a second-order semilinear parabolic equation in the half-space $R^n \times (0,+\infty)$ was studied first. In 1966, Fujita[1], in his famous work, has considered the following problem

$$\frac{\partial u}{\partial t} = \Delta u + u^q, \quad (q > 1), \quad (x,t) \in R^n \times (0,+\infty) \tag{1}$$

$$u\big|_{t=0} = u_0(x) \geq 0, \quad u_0(x) \neq 0. \tag{2}$$

It is known that for arbitrary $q > 1$ problem (1), (2) has a local classic solution. A question arises: When does the global solution exist? Fujita proved that there exists such a solution $q_{cr} = 1 + \frac{2}{n}$ that,

[1]Fujita, H. On the blowing-up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$ // J. Fac. Sci.Unix, Tokyo, Sect. I, 13, 1966, p.109-124.

if $1 < q < q_{cr}$, then no matter how little $u_0 \geq 0$ there is no global solution and in the bounded domain with respect to t, the solution grows unlimitedly. At the same time, he showed that such a solution exists in the case $q > q_{cr}$, depending on the initial function. Later, it was proven that, in the case $q = 1 + \frac{2}{n}$ there are no global solutions either. Thus, there appears the so-called critical exponent of nonlinearity separating range of values of degree q in which the problem has or does not have any global solution. The absence of a global solution means that no matter how little initial excitation of the system, at some final value of time, the solution goes to infinity, i.e., the process under consideration is not stable.

After this famous work, Fujita began to widely study the problem of how the dimension of nonlinearity affects the presence and absence of global solutions. Fujita's work expanded in differential directions. For example, instead of R^n different bounded and unbounded domains, instead of the Laplace operator, other differential operators, and instead of nonlinearity in equation (1), another type of nonlinearities was considered. One of the possible directions for the extension of Fujita's results is to study the existence of global solutions for Fujita's reaction-diffusion type system of equations. The review of these is given in the paper by K.Deng and H.A.Levine[1], M.Fila, A.Levine, Y.A.Uda[2], in the monograph of S.I.Pohozhaev, E.Mitideri[3] and the books of A.A.Samarski,

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[1] Deng, K., Levine, H.A. The role of criticial exponents in blow-up theorems: the sequel. J. Math. Anal. Appl. 243 (2000), no. 1, 85-126.

[2] Fila, M., Levine, A., Uda, Y.A. Fujita-type global existence-global nonexistence theorem for a system of reaction diffusion equations with differing diffusivities, Math. Methods Appl. Sci. 17 (1994), 807-835.

[3] Похожаев, С.И., Митидиери Э. Априорные оценки и отсутствие решений нелинейных уравнений и неравенств в частных производных. // Труды МИАН, 234, Наука, М.,2001, 1-383.

V.A.Galaktionov, S.P.Kurdyumov, A.P.Mikhailov[1], P.Quittner and P.Souplet[2]. Note that positive solutions are of particular interest due to the variety of applications that involve unknown positive quantities.

One of the main criteria for the undying interest in the existence of global solutions of elliptic, parabolic, and hyperbolic type nonlinear equations is that such problems have broad applications in some internal problems of mechanics, physics, and mathematics at the same time; for example, in the theory of chemical reactors, quantum mechanics, fluid mechanics, biology, ecology, etc.

The existence and nonexistence of global solutions of nonlinear elliptic and hyperbolic equations and inequalities were actively studied alongside parabolic equations. For example, in 1980, Kato[3] proved that the critical exponent for the absence of a global solution for a hyperbolic problem is $q_{cr} = 1 + \frac{2}{n-1}$.

Questions about the existence of global solutions to stationary semilinear equations and inequalities defined in unbounded domains also receive significant attention in the mathematical literature. Stationary equations describing stationary states for reaction-diffusion equations that can be found everywhere in applications are also widely used in some internal mathematics problems. For example, the problem of finding exact estimators in embedding theorems in Sobolev spaces is reduced to the problem of the existence of global solutions of stationary nonlinear equations. Stationary nonlinear equations in the whole R^n have important applications in geometry as well (see the works of W.Y.Ding, and

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[1] А.А. Самарский, В.А. Галактионов, С.П. Курдюмов, А.П. Михайлов Режимы с обострением в задачах для квазилинейных параболических уравнений. М.: Наука, 1987.
[2] P. Quittner and P. Souplet Superlinear Parabolic Problems, Berlin, 2007.
[3] Като, T. Blow-up of solutions of some nonlinear hyperbolic equations // Commun. Pure and Appl. Math. 1980. V. 33. P. 501–505.

W.-M. Ni[1], Qi.S. Zhang[2]), and the results on the nonexistence have a direct relation to geometrical applications (see the works of C.E.Kenig and W.-M.Ni[3]). For stationary equations in unbounded domains, there are two historically established statements of the problem: find the conditions guaranteeing that any solution from the given class is trivial (the so-called Liouville type theorems) and a problem of finding exact estimators as $r \to \infty$ of the maximum value of the solution on the sphere of radius r centered at some fixed point (Fragmen-Lindeloff type theorems). It is well known that any non-negative harmonic function in R^n is a constant. Interest in the whole solutions of stationary nonlinear equations arose after the appearance of the works of J.B. Keller and R.Osserman. Since then, for more than half a century, interest in this issue has not waned, and a massive number of publications have been accumulated. We mention only the work of the authors: Kh.Berestuki, I.Kapuzzo-Doltset, L.Nirenberg, F.Hammel, L.Rossi, M.-F.Bido-Veron, Kh.Brezis, V.A.Strauss, K.Kabre, J.L.Vazkez, L.Veron, B.Guidas, J.Spruk, V.A.Kondratyev, E.M.Landis, V.V.Kurt, E. Mitidieri and S.I.Pohozhaev. There is also a series of great monographs and reviews containing a comprehensive bibliography. The absence of positive solutions of an elliptic equation means that the existing solutions hesitate, which is also important information in applications.

The present dissertation is devoted to obtaining these kinds of results. First, we study the existence of global solutions to semilinear, second-order parabolic equations with periodic time coefficients and the system of these equations in an unbounded domain. Then, we

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[1] Ding, W. Y. and Ni, W. -M. On the elliptic equation Δu + Ku(n+2)/(n−2) = 0 and related topics, Duke Math. J. 52 (1985), 485–506

[2] Zhang, Qi S. An optimal parabolic estimate and its applications in prescribing scalar curvature on some open manifolds with Ricci > 0, Math. Ann. 316 (2000), 703–731.

[3] Kenig, C. E. and Ni, W. -M. An exterior Dirichlet problem with applications to some nonlinear equations arising in geometry, Amer. J. Math. 106 (1984), no. 3, 689–702.

study the absence of non-negative global solutions to semilinear elliptic, parabolic equations and the system of second and fourth-order equations with a singular potential in external domains. In the conclusion, we study the asymptotic of solutions to second-order semilinear elliptic equations around the infinity in a cylindrical domain.

Object and subject of the study. The main objects of the work are semilinear elliptic and parabolic equations, as well as the systems of these equations. The subject of the study is the absence of global solutions of semilinear elliptic, parabolic equations and a system of equations in an infinite domain and the study of the asymptotic behavior of solutions to elliptic equations in a semi-cylinder.

Goals and objectives of the study. The main goal of the work is finding sufficient conditions that ensure the absence of positive global solutions to semilinear second-order parabolic equations with periodic time coefficients and systems of these equations in an infinite domain to obtain Fujita-type results for second and fourth-order parabolic equations and a system of such equations with a singular potential. The objectives also include studying the near-infinity asymptotics of the solutions of semilinear elliptic equations on a semi-cylinder satisfying the Neumann homogeneous boundary condition in later surface.

Investigation methods. When studying the considered problems, some methods of functional analysis, the theory of ordinary differential and partial differential equations, the theory of function spaces, and the method of trial functions were used.

The main points of the study.

1. In the cylinder where the base is the exterior of some compact, studying the existence of positive global solutions of semilinear parabolic equations with time-periodic coefficients with a power and logarithmic nonlinearity, finding exact and sufficient conditions for the absence of such solutions and refining the accuracy of the obtained conditions.
2. In the cylinder where the base is the exterior of some compact, studying the existence of positive global solutions of a semilinear

parabolic equation with time-periodic coefficients and with lowest terms, finding exact sufficient conditions for the absence of such solutions and refining the accuracy of the obtained conditions.

3. In the cylinder where the base is the exterior of some compact, studying the existence of positive global solutions of a semilinear parabolic equation with weight and time-periodic coefficients, finding exact sufficient conditions for the absence of such solutions, and refining the accuracy of the obtained conditions.
4. In the cylinder where the base is the exterior of some compact, studying the existence of positive global solutions of a weakly coupled system of parabolic equation with a weight and time-periodic coefficient, find exact sufficient conditions for the absence of such solutions and refining the accuracy of the obtained conditions.
5. Depending on the nonlinearity degree, singularity coefficient, and dimension of the space, obtaining a Fujita-type result for a system of semilinear parabolic equations with a singular potential.
6. Depending on the degree of nonlinearity, the singularity coefficient, and the dimension of space, obtaining a Fujita-type result for a system of semilinear parabolic equations with a singular potential.
7. Studying the absence of global solutions to a semilinear biharmonic equation with singular potential and, depending on the nonlinearity degree, singularity coefficient, and dimension of space, finding the exact critical exponent.
8. Studying the absence of global solutions of a semilinear Bauend-Grushin type with a biharmonic equation with singular potential and, depending on the nonlinearity degree, singularity coefficient, and dimension of space, finding the exact critical exponent.
9. Studying the absence of global solutions of a weakly coupled system of semilinear biharmonic equations with singular potential and, depending on the nonlinearity degree, singularity coefficient, and dimension of the space, finding the exact critical exponent.
10. Studying the semi-cylinder asymptotic behavior of solutions near the infinity of a second-order semilinear elliptic equation with a Laplace operator in the principle part and with Neumann

homogeneous boundary condition on the lateral part of the semi-cylinder.

11. Studying the cylinder asymptotic behavior of solutions near the infinity of a semilinear second-order elliptic equation with a divergent principal part and with Neumann homogeneous boundary condition on the lateral surface of the cylinder.

The scientific novelty of the study. In the dissertation work, the following new results were obtained:

1. A sufficient condition providing the absence of a positive global solution in an unlimited domain of semilinear parabolic equations with time-periodic coefficients, with power and logarithmic nonlinearity was found; in all the cases, the accuracy of the obtained sufficient conditions have been proven;
2. In the cylinder where the base is the exterior of some compact, the critical exponent of the absence of positive global solutions of a semilinear parabolic equation with lowest terms and time-periodic coefficients has been found;
3. In the cylinder where the base is the exterior of some compact, a sufficient condition providing the absence of positive global solutions of a semilinear parabolic equation with a weight in the principal part and time-periodic coefficients has been found;
4. In the cylinder where the base is the exterior of some compact, a sufficient condition providing the absence of positive global solutions of a weakly coupled system of semilinear parabolic equations with periodic functions has been found;
5. Depending on weight, nonlinearity degree, singularity coefficient, and dimension of the space, a Fujita-type result has been obtained for a semilinear parabolic equation with a weight and singular potential;
6. Depending on the nonlinearity degree, singularity coefficient, and dimension of the space, a Fujita-type result has been obtained for a system of semilinear second-order equations with a singular potential;
7. Depending on the nonlinearity degree, singularity coefficient, and dimension of the space, an exact critical exponent of the absence

of global solutions of a semilinear biharmonic equation with a singular potential has been found;

8. Depending on the nonlinearity degree, singularity coefficient, and dimension of the space, the exact critical exponent of the absence of global solutions of a Bauendi-Grushin type semilinear biharmonic equation with a singular potential has been found;
9. A sufficient condition providing the absence of global solutions of a weakly coupled system of semilinear biharmonic equations with a singular potential has been found;
10. Asymptotic behavior at infinity of positive and sign-changing solutions of second-order semilinear elliptic equations in a cylinder with Neumann homogeneous boundary condition on the lateral surface has been obtained.

Theoretical and practical value of the study. The results obtained in the dissertation have theoretical character along with applied aspects. These results can be used in the theory of differential equations, embedding theory, and some problems in mechanics, physics, biology, and many other fields.

Approbation and application. The main results of the work have been reported in the following seminars: the seminar of the Mechanics and Mathematics faculty of Baku State University (prof. Z.S.Aliyev), Mechanics and Mathematics faculty of BSU (prof. N.Sh. Iskenderov), in the Institute seminar of the Institute Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan (corr.-member of ANAS, prof. M.J.Mardanov), in the seminars of the Department of "Differential and integral equations" of BSU (prof. Y.T.Mehraliyev), in the seminar of the Department of "Equations of mathematical physics" of the IMM (corr.-member of ANAS, prof. R.V.Huseynov), in the seminar of the Department of "Equations of mathematical physics" of the IMM (A.F.Guliyev), in the seminar of the Department of "Differential equations" of the IMM (prof. A.B.Aliyev).

Furthermore, the reports have been discussed at the following scientific conferences: at the International Conference in Mathematics and Mechanics, devoted to the 50 years of corr.-

member of ANAS, prof. I.T.Mamedov (Baku - 2005), at the International Conference in Mathematics and Mechanics devoted to the 70th anniversary of corr.-member of ANAS, prof. B.A.Iskenderova (Baku, 2006), at the conference devoted to 100-years of corr.-member of ANAS, prof. G.Akhmedov (Baku-2007), at the International conference "Actual Problems of Mathematics and Mechanics," devoted to the 80th anniversary of the corr.-member of ANAS, prof. Yahya Mamedov (Baku, 2010), at the International Conference "Theory of functions and problems of harmonic analysis," devoted to the 100th anniversary of acad. I.I.Ibrahimov (Baku, 2012), at the VI International conference on "Distortion Damping and Control" (Chicago, USA 2012), at European Mathematical congress (TCM- Krakow, 2012), at the international conference "Actual Problems of Mathematics and Computer Science" devoted to the 90 years of National Leader Heydar Aliyev (Baku, 2013), at the scientific conference "Actual Problems of Mathematics and Mechanics", devoted to the 95th anniversary of Baku State University (2014), at the International conference "Actual Problems of Mathematics and Mechanics", devoted to the 55th anniversary of the Institute of Mathematics and Mechanics of ANAS (Baku 2014), at the VII International conference on "Mathematical analysis, differential equations and their applications" MADEA-7" (Baku 2015), at the International seminar on "Nonharmonic analysis and differential operators" (Baku 2016), at the International conference "Modern problems of mathematics and mechanics" devoted to the 80th anniversary of academician Akif Hajiyev" (Baku 2017), at the International conference "Modern problems of mathematics and mechanics" devoted to the 60-th anniversary of the Institute of Mathematics and Mechanics of ANAS (Baku 2019), at the International conference "Operators, functions and systems in mathematical physics", devoted to the 70th anniversary of prof. G.A.Isakhanly (Baku, 2018), at the International seminar "Spectral theory and its applications," devoted to the 80th anniversary of the outstanding mathematician, acad. Mirabbas Gasimov (Baku 2019), at the Scientific conference dedicated to the 99th anniversary of the birth of the National Leader of the Azerbaijani people, Heydar

Aliyev (2022), at the International Conference "Modern Problems of Mathematics and Mechanics" devoted to the 110-th anniversary of acad. Ibrahim Ibragimov (Baku, 2022), at the International Conference "Modern Problems of Mathematics and Mechanics," devoted to the 100th anniversary of National Leader Heydar Aliyev (Baku,2023).

Author's personal contribution. All the results obtained in the work belong to the author.

Published scientific works. The content of the work has been reflected in 19 articles and 23 abstracts, 9 of which are included in international generalizing and indexing periodicals.

The name of the institution where the work was done. The dissertation work was performed at the Department of "Differential and integral equations" of the Mechanics and Mathematics faculty of Baku State University.

Structure and volume of the dissertation (in signs, indicating the volume of each structural subsection separately). The general volume of the work is 379,055 signs (title page - 455, contents - 3,472, introduction 69,010, I chapter - 136,000, II chapter - 28,000, III chapter - 95401, IV chapter - 44000, conclusion - 2717). A list of references with 244 names.

MAIN CONTENT OF THE DISSERTATION WORK

The dissertation consists of an introduction, four chapters with 12 sections, a conclusion, and a list of references.

Chapter I, consisting of four sections, is devoted to studying the existence of positive global solutions of semilinear parabolic equations with periodic coefficients in a cylinder, where the base is the exterior of a compact containing the origin of coordinates. The existence of positive time periodic global solutions of parabolic equations was the subject of much scientific research (see the works of M.J. Esteban, Y. Giga and N. Mizoguchi, N.Hirano and N. Mizoguchi, J.Huska). Among these problems, we can note the work of I. Seidman[1], where the existence of a nontrivial periodic solution of the problem

$$\frac{\partial u}{\partial t} = \Delta u + a(x,t)u^q, \quad (x,t) \in \Omega \times (0,+\infty),\ u\big|_{\partial\Omega} = 0,$$

was studied, here $q > 1$, $a(x,t)$ is a function, periodic in t, $\Omega \subset R^n$ is a bounded domain. Then, the existence of a periodic solution to this problem for $q > 0$ was studied by many authors. In the work of A. Beltramo, P. Hess[2], it was shown that for $q = 1$ under specially selected $a(x,t)$, this problem can have periodic solutions. In the work of M.J. Esteban[3], it was established that when $n \leq 2$ for any $q > 1$, and when $n > 2$ for any $q \in \left(1, \frac{n}{n-2}\right)$, this problem has positive periodic solutions for any positive function $a(x,t)$. Furthermore, it is proven that if $n > 2$ and $q \geq \frac{n+2}{n-2}$, then there are no positive periodic solutions. Under some constraints on $a(x,t)$, in the work of P. Quittner[4], it was established that for any $q \in \left(1, \frac{n+2}{n-2}\right)$, the problem has positive periodic solutions.

First, we introduce the following denotations:

$$x = (x_1, ..., x_n) \in R^n, \quad n \geq 1, \quad r = |x| = \sqrt{x_1^2 + ... + x_n^2}\ , B_R = \{x : |x| < R\},$$

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[1] Seidman, T. I. Periodic solutions of a non-linear parabolic equation, Journal of Differential Equations, vol. 19, no. 2, pp. 242-257, 1975.

[2] A. Beltramo, and P. Hess On the principal eigenvalue of a periodic-parabolic operator, Communications in Partial Differential Equations, vol. 9, no. 9, pp. 919-941, 1984.

[3] Esteban, M. J. A remark on the existence of positive periodic solutions of superlinear parabolic problems, Proceedings of the American Mathematical Society, vol. 102, no. 1, pp. 131-136, 1988.

[4] P. Quittner Multiple equilibria, periodic solutions and a priori bounds for solutions in superlinear parabolic problems, Nonlinear Differential Equations and Applications, vol. 11, no. 2, pp. 237-258, 2004.

$B_{R_1,R_2} = \{x : R_1 < |x| < R_2\}$, $B'_R = \{x : |x| > R\}$, $Q_T^R = B_R \times (0,T)$,
$Q_T^{R_1,R_2} = B_{R_1,R_2} \times (0,T)$, $Q_T^{R,\infty} = B'_R \times (0,T)$, $Q'_R = B'_R \times (-\infty,+\infty)$,
$S_R = \{x : |x| = R\} \times (-\infty,+\infty)$, $S_T^R = \{x : |x| = R\} \times (0,T)$, Ω - domain in R^n, $Q_T = \Omega \times (0,T)$, $Q = \Omega \times (-\infty,+\infty)$.

By $W_2^{1,1/2}(Q_T)$ we mean a space of functions $u(x,t)$ for which $u(x,t+T) = u(x,t)$, $u(x,t) \in W_2^{1,0}(Q_T)$ and

$$\sum_{k=-\infty}^{\infty} |k| \int_\Omega |u_k(x)|^2 dx < \infty,$$

where

$$u_k(x) = \frac{1}{T}\int_0^T u(x,t)\exp\left\{-ik\frac{2\pi}{T}t\right\}dt\,.$$

The norm in this space is determined by the equality

$$\|u\|^2_{W_2^{1,1/2}(Q_T)} = \|u\|^2_{L_2(Q_T)} + \|\nabla u\|^2_{L_2(Q_T)} + \sum_{k=-\infty}^{\infty} |k| \int_\Omega |u_k(x)|^2 dx\,.$$

By $\overset{\circ}{W}{}_2^{1,1/2}(Q_T)$ we denote completion $C^{0,\infty}(Q_T)$ in the norm $\|\cdot\|_{W_2^{1,,/2}(Q_T)}$, where $C^{0,\infty}(Q_T)$ is a set of infinitely smooth, T - periodic functions with respect to t that is equal to zero in the vicinity of $\partial\Omega$.

Section 1.1 begins with the existence of positive global solutions of the equation

$$\frac{\partial u}{\partial t} = div(A(x,t)\nabla u) + a_0(x,t)|u|^{p-1}u \qquad (3)$$

in the cylinder $Q = \Omega \times (-\infty;+\infty)$, where Ω is the exterior of the compact in R^n containing the origin of coordinate, $p > 1$, $A(x,t) = \left(a_{ij}(x,t)\right)_{i,j=1}^n$ is a symmetric matrix, $a_{ij}(x,t)$, $a_0(x,t)$ - are such bounded measurable functions that $a_{ij}(x,t+T) = a_{ij}(x,t)$, $a_0(x,t+T) = a_0(x,t)$ and

$$\lambda_1|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \le \lambda_2|\xi|^2,\ \lambda_1,\ \lambda_2 = const > 0, \qquad (4)$$

for any $(x,t) \in Q, \xi = (\xi_1,...,\xi_n) \in R^n$.

Here

$$A\nabla u = \left(\sum_{j=1}^{n} a_{ij} \frac{\partial u}{\partial x_j} \right)_{i=1}^{n},\ (A\xi,\eta) = \sum_{i,j=1}^{n} a_{ij}\xi_i\eta_j,\ \xi = (\xi_1,...,\xi_n),$$

$$\eta = (\eta_1....,\eta_n),\ div(A\nabla u) = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}(a_{ij}(x,t)\frac{\partial u}{\partial x_j}).$$

The solution of the equation (3) is the function $u(x,t) \in W_{2,\mathrm{loc}}^{1,1/2}(Q_T) \cap L_{\infty,\mathrm{loc}}(Q_T)$, that satisfies the integral identity

$$2\pi \sum_{k=-\infty}^{\infty} (ik) \int_{\Omega} u_k(x)\varphi_{-k}(x)dx + \int_{Q_T} \sum_{i,j=1}^{n} a_{ij}(x,t) \frac{\partial u}{\partial x_j} \frac{\partial \varphi}{\partial x_i} dxdt =$$

$$= \int_{Q_T} a_0(x,t)|u|^{p-1} u\varphi\, dxdt$$

for any function $\varphi(x,t) \in \overset{\circ}{W}{}_2^{1,1/2}(Q_T)$.

Denote

$$L_0 u := div(A(x,t)\nabla u) - \frac{\partial u}{\partial t}.$$

When proving the main theorems of this section, we use the following two important lemmas.

Lemma 1.1. *Let* $n \ge 3$ *and* $u(x,t) \in W_{2,loc}^{1,\frac{1}{2}}(Q_T^{R,\infty})$ *be such a continuous, non-negative function in* $\overline{Q}_T^{R,\infty} = Q_T^{R,\infty} \cup S_T^R$, *that* $L_0 u \le 0$ *in* $Q_T^{R,\infty}$ *and* $u(x,t) > 0$ *on* S_T^R. *Then there exists such a* $\alpha_0 = const > 0$ *that* $u(x,t) \ge \alpha_0 |x|^{2-n}$, $(x,t) \in Q_T^{R,\infty}$.

Let us consider the following linear equation

$$\frac{\partial u}{\partial t} = div(A(x,t)\nabla u) + Q(x,t)u \quad \text{в} \quad Q_T^{R,\infty}, \qquad (5)$$

where the coefficients $a_{ij}(x,t)$ satisfy the condition (4),

$$Q(x,t+T)=Q(x,t), Q(x,t)\in L_{\infty,loc}(Q_T^{R,\infty}).$$

Lemma 1.2. *There exists such a constant $C_0>0$, depending on n,λ_1 and λ_2 that if $Q(x,t)\geq \frac{C_0}{|x|^2}$, the equation (5) has no positive supersolutions in $Q_T^{R,\infty}$.*

We have the following theorem.

Theorem 1.1. *Let $n\geq 3$, $a_0(x,t)\geq C|x|^\sigma$, $C=const>0$, $\sigma>-2$ and $2+\sigma+(2-n)(p-1)\geq 0$. Then equation (3) has no positive solutions in Q_T.*

At first, it is shown that the estimation obtained in the theorem is exact, i.e. in the case $2+\sigma+(2-n)(p-1)<0$ the equation of the form (3) can have positive solutions.
For that, in $Q_T^{R,\infty}$ we consider the equation

$$Lu\equiv -\frac{\partial u}{\partial t}+\Delta u+a_0|x|^\sigma |u|^{p-1}u=0, \tag{6}$$

where $a_0=const>0,\ p>1,\ \sigma>-2,\ n\geq 3$. Then by the direct calculation we can show that the function

$$u(x,t)=a_0^{-\frac{1}{p-1}}\left[\frac{(n-2)(\sigma+2)}{p-1}-\frac{(\sigma+2)^2}{(p-1)^2}\right]^{\frac{1}{p-1}}|x|^{-\frac{\sigma+2}{p-1}}>0$$

is a positive solution of equation (6) provided that $2+\sigma+(2-n)(p-1)<0.$

Theorem 1.2. *Let $n\geq 3$, $a_0(x,t)\geq C|x|^\sigma \ln^s|x|$, $C=const>0$. If $2+\sigma+(2-n)(p-1)>0$, $s\in(-\infty,+\infty)$ or $2+\sigma+(2-n)(p-1)=0$, $s\geq -1$, then equation (3) has no positive solutions in $Q_T^{R,\infty}$, $R>1$.*

It is shown that the conditions of theorem 1.2 are exact, i.e. for $2+\sigma+(2-n)(p-1)<0$, $s\in(-\infty,+\infty)$ or $2+\sigma+(2-n)(p-1)=0$,

$s<-1$ equation (3) can have positive solutions. For that, in $Q_T^{R,\infty}$ we consider the equation

$$-\frac{\partial u}{\partial t}+\Delta u+a_0|x|^{\sigma}(\ln^s|x|)|u|^{p-1}u=0, \qquad (7)$$

where $a_0=const>0, p>1, \sigma>-2, n\geq 3$, and it is shown that under the indicated conditions, the equation (7) has a positive solution. This solution is sought in the form of $u(x)=u(r)$. Then after some substitutions, equation (7) is reduced to the ordinary differential equation

$$\omega_{rr}+a_0(n-2)^{-(s+2)}\tau^{-2+(\sigma+2(2-n)(p-1))/(n-2)}(\ln^s\tau)|\omega|^{p-1}\omega=0. \qquad (8)$$

It is known that the equation

$$\omega_{\tau\tau}+q(\tau)|\omega|^{p-1}\omega=0,\ p>1,\ q(\tau)>0$$

has a constant sign solution if

$$\int_a^{\infty}\tau|q(\tau)|d\tau<\infty. \qquad (9)$$

Obviously, if $2+\sigma+(2-n)(p-1)<0$, $s\in(-\infty,+\infty)$ or $2+\sigma+(2-n)(p-1)=0$, $s<-1$, then for equation (8) the condition (9) is fulfilled. So, equation (8) has a positive solution. Then coming back, we obtain that equation (7) also has a positive solution.

At the end of section 1.1 in the cylinder $Q=\Omega\times(-\infty;+\infty)$ we consider the equation

$$\frac{\partial u}{\partial t}=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j}\right)+a_0(x,t)\ln^{p-1}(1+|u|)u, \qquad (10)$$

where $p>1$, the coefficients $a_{ij}(x,t), a_0(x,t)$ are bounded, T periodic functions with respect to t and satisfy condition (4).

For this equation we prove the following theorem.

Theorem 1.3. *Let* $n\geq 3,\ \ a_0(x,t)\geq C|x|^{\sigma}$, $C=const>0$ *and* $2+\sigma+(2-n)(p-1)\geq 0.$ *Then equation (10) has no positive solutions in* Q.

It is shown that the estimation obtained in the theorem

$2+\sigma+(2-n)(p-1)<0$ is exact, i.e.in the case the equation (10) has positive solutions. For that, in $Q_T^{R,\infty}$ we consider the equation

$$Lu \equiv -\frac{\partial u}{\partial t}+\Delta u+a_0|x|^\sigma \ln^{p-1}(1+|u|)u=0, \qquad (11)$$

where $a_0=const>0,\ p>1,\ \sigma>-2,\ n\geq 3$.

By the method of upper and lower functions we prove that for $2+\sigma+(2-n)(p-1)<0$ the equation (11) has a positive solution.

Theorem 1.4. *Let* $n=1$ *or* $n=2,\ a_0(x,t)\geq C|x|^\sigma,\ \sigma>-2$, $C=const>0$. *Then for any* $p>1$ *the equation (10) has no positive solutions in* $Q_T^{R,\infty}$.

In section 1.2 in the cylinder $Q=\Omega\times(-\infty;+\infty)$ we study the existence of positive solutions in

$$\frac{\partial u}{\partial t}=div(A(x,t)\nabla u)+h(x,t,u), \qquad (12)$$

where Ω is the exterior of the compact in R^n containing the origin of coordinate, $n\geq 3,\ \ A(x,t)=\left(a_{ij}(x,t)\right)_{i,j=1}^n$, $a_{ij}(x,t)$ are also bounded, measurable, T periodic functions with respect to t and satisfy condition (4), $h(x,t,u):Q\times[0,+\infty)\to R$ is a measurable function and $h(x,t,u(x,t))\in L_{\infty,loc}(Q_T)$.

By the solution of equation (12) we mean the function $u(x,t)\in W_{2,\mathrm{loc}}^{1,1/2}(Q_T)\cap L_{\infty,\mathrm{loc}}(Q_T)$, that satisfies the integral identity

$$2\pi\sum_{k=-\infty}^{k=+\infty} ik\int_\Omega u_k(x)\varphi_{-k}(x)dx+\int_{Q_T}(A(x,t)\nabla u,\nabla\varphi)dxdt=\int_{Q_T}h(x,t,u)\varphi dxdt$$

for any $\varphi(x,t)\in \overset{0}{W}{}_2^{1,\frac{1}{2}}(Q_T)$.

As it can be seen, here the nonlinearity has a more general form. More exactly, it is assumed that for all $(x,t)\in Q$ $h(x,t,u)\geq \tilde{h}(x,u)\geq 0$ and $\tilde{h}:\Omega\times[0,+\infty)\to[0,+\infty)$ is a function satisfying the following conditions (H):

a) for any $x \in B'_e$

$$\frac{\tilde{h}(x,s_1)}{s_1} \geq \frac{\tilde{h}(x,s_2)}{s_2} \quad \text{if} \quad s_1 \geq s_2 > 0,$$

b) for any $\tau > 0$,

$$\liminf_{|x|\to+\infty} \tilde{h}(x,\tau|x|^{2-n})|x|^n > C_0,$$

If b) is not satisfied, then it is assumed that

b1) there exists such $\sigma_1 \in (0,1)$ that for any $\tau > 0$

$$\liminf_{|x|\to+\infty} \tilde{h}(x,\tau|x|^{2-n})|x|^n (\ln|x|)^{\sigma_1} > 0,$$

or

b2) there exists such $\gamma > 1$ that for any $\tau > 0, \beta \geq 0$

$$\liminf_{|x|\to+\infty} \tilde{h}(x,\tau|x|^{2-n}(\ln|x|)^{\beta})|x|^n (\ln|x|)^{-\beta\gamma+1} > 0,$$

and

b3) there exists such $\sigma_2 > 0$ that for any $\tau > 0$,

$$\liminf_{|x|\to+\infty} \frac{\tilde{h}(x,\tau|x|^{2-n}(\ln|x|)^{\sigma_2})}{\tau|x|^{-n}(\ln|x|)^{\sigma_2}} > C_0.$$

We have the following theorem.

Theorem 1.5. *Let $n \geq 3$, $A(x,t)$ satisfy the condition (4). İf the condition (H) is fulfilled, then the equation (12) has no positive solutions in Q.*

In section 1.3 in the domain Q'_R we consider the equation

$$\frac{\partial u}{\partial t} = div\left(|x|^{\alpha} A\nabla u\right) + a_0(x,t)|u|^{q-1}u, \tag{13}$$

where $q > 1$, $\alpha < 2$, $A = A(x,t) = \left(a_{ij}(x,t)\right)_{i,j=1}^{n}$, $a_{ij}(x,t)$, $a_0(x,t)$ are bounded, measurable, T periodic functions with respect to t and the condition (4) is fulfilled.

The function $u(x,t)$ is called the solution of equation (13) in $Q_T^{R,\infty}$, if $u(x,t) \in W_2^{1,\frac{1}{2}}(Q_T^{R,\infty}) \cap L_{\infty,loc}(Q_T^{R,\infty})$ and for any

$\varphi(x,t)\in \overset{0}{W}{}_2^{1,\frac{1}{2}}(Q_T^{R,\infty})$ the following integral identity is fulfilled:

$$2\pi\sum_{k=-\infty}^{+\infty}(ik)\int_{\Omega}u_k(x)\varphi_{-k}(x)dx+\iint_{Q_T^{R,\infty}}|x|^{\alpha}(A\nabla u,\nabla\varphi)dxdt=$$

$$=\iint_{Q_T^{R,\infty}}a_0(x,t)|u|^{q-1}u\varphi dxdt.$$

We prove the following auxiliary facts.

We introduce the denotation:

$$Lu\equiv div\left(|x|^{\alpha}A\nabla u\right)-\frac{\partial u}{\partial t}.$$

Lemma 1.3. *Let* $W(x,t)\in L_{\infty,loc}(Q_T^{R,\infty})$, $W(x,t+T)=W(x,t)$. *If* $0<u(x,t)$ *is the solution of the inequality* $Lu+Wu\le 0$, *then for any* $f(x)\in C_0^{\infty}(B'_R)$ *the following inequality is fulfilled:*

$$\iint_{Q_T^{R,\infty}}W(x,t)f^2(x)dxdt\le C\int_{B'_R}|x|^{\alpha}(\nabla f)^2dx.$$

Lemma 1.4. *Let* $n\ge 3,\ 2-n\le\alpha<2$, *a non-negative, continuous on* $\overline{Q_T^{R,\infty}}$ *function* $u(x,t)\in W_{2,loc}^{1,\frac{1}{2}}(Q_T^{R,\infty})$ *satisfy the inequality* $Lu\le 0$ *in* $Q_T^{R,\infty}$ *and* $u(x,t)>0$ *on* S_R. *Then there exists such* $\beta_0=const>0$, *that* $u(x,t)\ge\beta_0|x|^{2-n-\alpha}$ *for* $(x,t)\in Q_T^{R,\infty}$.

Lemma 1.5. *Let* $\alpha<2$ *and* $\upsilon(x,t)$ *be a non-negative solution of the equation* $L\upsilon+\beta^2|x|^{\alpha-2}\upsilon=0$ *in* $Q_T^{R,\infty}$. *Then for* $\rho>2R$ *the following inequality is valid*

$$\iint_{Q_T^{\rho,2\rho}}|x|^{\alpha}|\nabla\upsilon|^2dxdt\le C\int_{Q_T^{\rho/2,5\rho/2}}|x|^{\alpha-2}\upsilon^2dxdt.$$

Lemma 1.6. *Let* $\alpha<2,\ 0\le W(x,t)\in L_{\infty,loc}(Q_T^{R,\infty})$, $W(x,t+T)=W(x,t)$ *and* $|x|^{2-\alpha}W(x,t)\to\infty$ *as* $x\to\infty$. *Then the inequality* $Lu+W(x,t)u\le 0$ *has no positive solution in* $Q_T^{R,\infty}$.

Lemma 1.7. Let $\alpha = 2-n,\ 0 \le W(x,t) \in L_{\infty,loc}(Q_T^{R,\infty})$, $W(x,t+T) = W(x,t)$ *and* $|x|^n \ln|x| W(x,t) \to \infty$ *as* $x \to \infty$. *Then the inequality* $Lu + W(x,t)u \le 0$ *has no positive solution in* $Q_T^{R,\infty}$.

Using these lemmas we prove the following theorems.

Theorem 1.6. *Let* $n \ge 3,\ q > 1,\quad 2-n \le \alpha < 2$, $a_0(x,t) \ge C|x|^\sigma,\ \sigma \in R,\ C = const > 0.$ *If* $\sigma + 2 - \alpha + (2-n-\alpha)(q-1) \ge 0$, *then equation (13) has no positive solutions in* $Q_T^{R,\infty}$.

Theorem 1.7. *Let* $n \ge 3,\ q > 1,\ \alpha < 2-n,\quad a_0(x,t) \ge C|x|^\sigma$, $\sigma \in R,\quad C = const > 0$. *Then for each* $\sigma \in (-\infty,+\infty),\ q > 1$ *the equation (13) has no positive solutions in* $Q_T^{R,\infty}$.

It is shown that the obtained estimation on the nonexistence of positive solutions is exact. For that, we consider the following equation

$$\frac{\partial u}{\partial t} = div\left(|x|^\alpha \nabla u\right) + |x|^\sigma |u|^{q-1} u \tag{14}$$

and the solutions are sought in the form

$$u(x,t) = A|x|^{-\mu}.$$

Substituting this into the equation, we show that for $\sigma + 2 - \alpha + (2-n-\alpha)(q-1) < 0$ the equation (14) has the positive solution

$$u(x,t) = \left[-\frac{\sigma+1-\alpha}{q-1}\,\frac{\sigma+1-\alpha+(2-n-\alpha)(q-1)}{q-1}\right]^{\frac{1}{q-1}} |x|^{-\frac{\sigma+2-\alpha}{q-1}}.$$

In section 1.4 of chapter I in the cylinder $Q = \Omega \times (-\infty,+\infty)$ we consider the following equation

$$\frac{\partial u}{\partial t} = Lu + C|x|^\sigma |u|^{p-1} u, \tag{15}$$

where

$$L \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_j} \right) + \sum_{i=1}^{n} b_i(x,t)\frac{\partial u}{\partial x_i} \equiv div(A\nabla u)(x,t) + B(x,t)\nabla u(x,t),$$

$$A = A(x,t) = (a_{ij}(x,t))_{i,j=1}^{n}, \quad B(x,t) = (b_1(x,t),...,b_n(x,t)).$$

It is assumed that $n \geq 3$, $p > 1$, $\sigma > -2$, the coefficients $a_{ij}(x,t)$, $b_i(x,t)$ are measurable, T periodic functions with respect to t in $R^n \times (-\infty,+\infty)$ and $a_{ij}(x,t)$ satisfy the following conditions:

I) $A(x,t)$ is a symmetric function, $a_{ij}(x,t)$ are Holder continuous functions in $R^n \times (-\infty,+\infty)$ and these exists such a constant $\lambda > 1$ that $\lambda^{-1}I \leq A(x,t) \leq \lambda I$ for all $(x,t) \in R^n \times [0,T]$.

Semilinear equations with lowest terms were considered in the works of the authors: C.Budd, V.Galaktionov, V.Kondratiev, V.Liskevich, Z.Sobol, V. Moroz, D.Smets, A.Tesei, Qi S.Zhang.

The solution of the equation is understood in the generalized sense.

The function $u(x,t)$ is called a generalized solution of equation (15) if $u(x,t) \in W_{2,loc}^{1,\frac{1}{2}}(Q_T) \cap L_{\infty,loc}(Q_T)$, $B\nabla u \in L_{1,loc}(Q_T)$ and for any $\varphi(x,t) \in \overset{0}{W}{}_2^{1,\frac{1}{2}}(Q_T)$ the following integral identity is fulfilled:

$$2\pi \sum_{k=-\infty}^{+\infty}(ik)\int_{\Omega} u_k \varphi_{-k}(x)dx + \int_{Q_T}\sum_{i,j=1}^{n} a_{ij}(x,t)\frac{\partial u}{\partial x_j}\frac{\partial \varphi}{\partial x_i}dxdt -$$

$$-\int_{Q_T}\sum_{i=1}^{n} b_i(x,t)\frac{\partial u}{\partial x_i}\varphi\, dxdt = C\int_{Q_T}|x|^{\sigma}|u|^{p-1}u\varphi\, dxdt.$$

Definition. *Let $H(x) \in L_{1,loc}(R^n)$. We say that $H(x)$ belongs to the class $\hat{K}_{n+1,\infty}$, if*

$$M_{n+1}(H) \equiv \sup_{x\in R^n}\int_{R^n}\frac{|H(y)|}{|x-y|^{n-1}}dy < \infty .$$

It is assumed that $B(x,t)$ satisfies the following conditions:

II) $|B(x,t)| \le C_3 |V(x)|$, where $V(x) \in \hat{K}_{n+1,\infty}$ and there exists such $\varepsilon > 0$, that $M_{n+1}(V) < \varepsilon$.

III) There exist such constants that $C_4 > 0,\ \beta \in (0,1)$ for any $\varphi(x,t) \in \overset{0}{W}{}_2^{1,\frac{1}{2}}(Q_T^{R,\infty})$

$$\int_{Q_T^{R,\infty}} |\overline{B}|^2 \varphi^2 dxdt \le C_4 \int_{Q_T^{R,\infty}} |\nabla \varphi|^2 dxdt,$$

$$(1-\beta) \int_{Q_T^{R,\infty}} \sum_{i,j=1}^{n} a_{ij}(x,t) \frac{\partial \varphi}{\partial x_j} \frac{\partial \varphi}{\partial x_i} dxdt - \int_{Q_T^{R,\infty}} \sum_{i,j=1}^{n} \overline{b}_i(x,t) \frac{\partial \varphi}{\partial x_i} \varphi dxdt \ge 0,$$

where $\overline{B} := BX_{Q_T^{R,\infty}}$, $X_{Q_T^{R,\infty}}$ is a characteristic function of the domain $Q_T^{R,\infty}$.

We prove the following auxiliary lemmas.

Lemma 1.8. *Let $A(x,t), B(x,t)$ satisfy the conditions I), II), III) and $u(x,t)$ be such a non-negative solution of the inequality $Lu - \frac{\partial u}{\partial t} \le 0$ in $\overline{Q}_T^{R_0,\infty}$ that $u\big|_{|x|=R_0} > 0$. Then there exist such constants $C_0, R > R_0$, that $u(x,t) \ge C_0 |x|^{2-n}$ in $Q_T^{R,\infty}$.*

Lemma 1.9. *Let conditions of lemma 1.8 be fulfilled, $0 \le W(x,t) \in L_{loc}^{\infty}(Q_T^{R_0,\infty})$, $W(x,t+T) = W(x,t)$ and $|x|^2 W(x,t) \to \infty$ as $x \to \infty$. Then in the cylinder $Q_T^{R_0,\infty}$ the inequality*

$$Lu + W(x,t)u - \frac{\partial u}{\partial t} \le 0.$$

has no positive solutions.

In the cylinder $Q_T^{R_0,\infty}$ we consider the equation

$$-\frac{\partial v}{\partial t} + L^* v = 0, \qquad (16)$$

where

$$L^* v \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial v}{\partial x_j}\right) - \sum_{i=1}^{n}\frac{\partial}{\partial x_i}(b_i(x,t)v).$$

Lemma 1.10. *Let conditions I), II), III) be fulfilled. There exists* $\varepsilon > 0$ *such that the condition II) imply the existence of a solution* $v(x,t)$ *to the equation (16) in* $Q_T^{R_0,\infty}$ *and constants* $0 < C_9 < C_{10} < \infty$, *such that* $C_9 \le v(x,t) \le C_{10}$.

In $Q_T^{R,\infty}$ we consider the linear equation

$$-\frac{\partial v}{\partial t} + Lv + \alpha_1 |x|^{-2} v = 0, \tag{17}$$

where α_1 is a rather small positive number.

Lemma 1.11. *Let the conditions of lemma 1.10 be fulfilled and* $v(x,t)$ *be a such non-negative solution of equation (17) in* $Q_T^{R,\infty}$, *that* $v\big|_{|x|=R_0} > 0$. *Then there exist such constants* $C_{11} > 0,\ R_1 > R$, *that* $v(x,t) \ge C_{11}|x|^{2-n}\log|x|$ *for* $|x| \ge R_1$.

Using these lemmas we prove the following theorem.

Theorem 1.8. *Let conditions I), II), III) be fulfilled and* $n \ge 3,\ p > 1,\ \sigma > -2$. *Then there exists such* $\varepsilon > 0$ *that when the condition II) is fulfilled with such* ε, *equation (15) for* $2 + \sigma + (2-n)(p-1) \ge 0$ *has no positive solutions in* Q_T.

In the last section of chapter I, in the domain Q_R' we study the following weakly coupled system of equations with respect to the unknown functions u and v:

$$\frac{\partial u}{\partial t} = div(A\nabla u) + |x|^{\sigma_1}|v|^{q_1},\ \frac{\partial v}{\partial t} = div(A\nabla v) + |x|^{\sigma_2}|u|^{q_2}, \tag{18}$$

where $q_1, q_2 > 1$ are constants, $A = (a_{ij}(x,t))_{i,j}^{n}$ is a symmetric matrix $(a_{ij} = a_{ji})$, the elements $a_{ij}(x,t)$ are bounded, measurable, T periodic functions and satisfy condition (4).

The solution of the system (18) is such pair of functions $(u(x,t), v(x,t))$, that $u, v \in W_{2,loc}^{1,1/2}\left(Q_T^{R,\infty}\right) \cap L_{\infty,loc}(Q_T^{R,\infty})$ and for any

$\varphi(x,t) \in \overset{\circ}{W}{}_2^{1,1/2}\left(Q_T^{R,\infty}\right)$ the following integral identities hold :

$$2\pi \sum_{k=-\infty}^{+\infty} (ik) \int_{B'_R} u_k(x)\varphi_{-k}(x)dx + \iint_{Q_T^{R,\infty}} (A\nabla u, \nabla\varphi)dxdt = \iint_{Q_T^{R,\infty}} |x|^{\sigma_1}|v|^{q_1}\varphi dxdt\,,$$

$$2\pi \sum_{k=-\infty}^{+\infty} (ik) \int_{B'_R} v_k(x)\varphi_{-k}(x)dx + \iint_{Q_T^{R,\infty}} (A\nabla v, \nabla\varphi)dxdt = \iint_{Q_T^{R,\infty}} |x|^{\sigma_2}|u|^{q_2}\varphi dxdt\,.$$

We have the following theorem.

Theorem 1.9. *Let* $n \ge 3,\ q_1 > 1,\ q_2 > 1,\ \sigma_1 \in R,\ \sigma_2 \in R$. *If the following inequality*

$$\max\left\{\frac{q_1(\sigma_2+2)+\sigma_1+2}{q_1q_2-1}, \frac{q_2(\sigma_1+2)+\sigma_2+2}{q_1q_2-1}\right\} \ge n-2$$

is fulfilled, then the system (15) has no positive solutions in the domain Q'_R *.*

It is shown that for

$$\theta_1 = \frac{q_1(\sigma_2+2)+\sigma_1+2}{q_1q_2-1} < n-2 \text{ and } \theta_2 = \frac{q_2(\sigma_1+2)+\sigma_2+2}{q_1q_2-1} < n-2\,,$$

the system

$$\frac{\partial u}{\partial t} = \Delta u + |x|^{\sigma_1}|v|^{q_1}\,, \quad \frac{\partial v}{\partial t} = \Delta v + |x|^{\sigma_2}|u|^{q_2}$$

has a global positive solution (u,v) in the form of $u = A|x|^{-\theta_1}$, $v = B|x|^{-\theta_2}$, where the constants A and B satisfy the inequalities

$$A\theta_1(\theta_1+2-n) + B^{q_1} = 0\,, \quad B\theta_2(\theta_2+2-n) + A^{q_2} = 0.$$

In chapter II, we study Fujita type problems for a semilinear parabolic equation with a singular potential and the systems of such equations. The existence of solutions of linear, nonlinear elliptic, parabolic equations and systems of such equations with lowest derivatives or singular potentials were earlier studied by many authors.

In section 1, in a cylinder where the base is the exterior of some compact, containing the origin of coordinates, we consider a semilinear parabolic equation with a weight, with a singular potential; then in section 2, in the same cylinder we consider the system of second order semilinear parabolic equations with a singular potential. Depending on the weight, nonlinearity degree, singularity coefficient, dimension of space, we find a sufficient condition providing the absence of global solutions. Fujita type results for the system of nonlinear elliptic, parabolic equations earlier were studied in the papers of Gabriella Caristi; H. Chen, R. Peng and F. Zhou; M. Escobedo, M.A. Herrero; M. Escobedo and H. A. Levine; Wei Guo, Wenjie Gao, Bin Guo; K. Mochizuki and Q. Huang; A. Quaas, B. Sirakov; J. Serrin, H. Zou; Faten Toumi; Y. Uda.

We use the following denotations:
$Q_R = B_R \times (0,+\infty)$, $Q'_R = B'_R \times (0,+\infty)$, $C^{2,1}_{x,t}(Q'_R)$ is the set of functions twice continuously differentiable with respect to x and continuously differentiable with respect to t in Q'_R.

In section 2.1, in the domain Q'_R we consider the following problem:

$$\frac{\partial u}{\partial t} = div\left(|x|^{\alpha}\nabla u\right) + C_0|x|^{\alpha-2}u + |x|^{\sigma}|u|^{q}, \tag{19}$$

$$u\big|_{t=0} = u_0(x) \geq 0, \tag{20}$$

where

$$q > 1,\ \alpha < 2,\ \sigma + 2 - \alpha > 0,\ 0 \leq C_0 \leq \left(\frac{\alpha + n - 2}{2}\right)^2,\ u_0(x) \in C(B'_R).$$

We study the existence of non-negative global solutions of this problem. The solution is understood in the classic sense

We introduce the denotations:

$$C_0^* = \left(\frac{\alpha + n - 2}{2}\right)^2,\ D = \sqrt{C_0^* - C_0},\ \lambda_{\pm} = -\frac{\alpha + n - 2}{2} \pm D.$$

The following theorem is proved.

Theorem 2.1. *Let* $n \ge 3,\ q > 1,\ \alpha < 2,\ \sigma + 2 - \alpha > 0$ *and* $q \le 1 + \frac{\sigma + 2 - \alpha}{\lambda_+ + n}$ *for* $0 \le C_0 < C_0^*$, $q < 1 + \frac{\sigma + 2 - \alpha}{\lambda_+ + n}$ *for* $C_0 = C_0^*$. *Then problem (19), (20) has no global non-negative solutions.*

In section 2.2, in the domain Q_R' we consider the system of equations

$$\begin{cases} \dfrac{\partial u}{\partial t} = \alpha_1 \Delta u + \dfrac{C_1}{|x|^2} u + |x|^{\sigma_1} v^{q_1} \\ \dfrac{\partial v}{\partial t} = \alpha_2 \Delta v + \dfrac{C_2}{|x|^2} v + |x|^{\sigma_2} u^{q_2} \end{cases} \tag{21}$$

with the initial condition

$$u\big|_{t=0} = u_0(x), v\big|_{t=0} = v_0(x), \tag{22}$$

where $u_0(x), v_0(x) \in C(B_R')$, $u_0(x) \ge 0,\ v_0(x) \ge 0$, $\alpha_i > 0,\ \sigma_i > -2,$ $0 \le C_i < \alpha_i \left(\frac{n-2}{2} \right)^2$, $q_i > 1,\ i = 1,2$.

By the global solution of problem (21), (22) we mean such pairs of functions (u, v) that, $u, v \in C_{x,t}^{2,1}(Q_R') \cap C(B_R' \times [0, +\infty))$ and u, v satisfy the system (21) at each point of Q_R' and the initial condition (22) for $t = 0$.

We introduce the denotations:

$$D_1 = \sqrt{\left(\frac{n-2}{2} \right)^2 - \frac{C_1}{\alpha_1}},\quad D_2 = \sqrt{\left(\frac{n-2}{2} \right)^2 - \frac{C_2}{\alpha_2}},$$

$$\lambda_1^+ = -\frac{n-2}{2} + D_1,\qquad \lambda_1^- = -\frac{n-2}{2} - D_1,$$

$$\lambda_2^+ = -\frac{n-2}{2} + D_2,\qquad \lambda_2^- = -\frac{n-2}{2} - D_2,$$

$$\theta_1 = \frac{\sigma_1 + 2 + q_1(\sigma_2 + 2)}{q_1 q_2 - 1} - \lambda_1^+ - n,$$

$$\theta_2 = \frac{\sigma_2 + 2 + q_2(\sigma_1 + 2)}{q_1 q_2 - 1} - \lambda_2^+ - n.$$

Theorem 2.2. *Let* $n > 2,\ \alpha_i > 0,\ \sigma_i > -2,\quad 0 \le C_i < \alpha_i\left(\frac{n-2}{2}\right)^2$ *and* $\max(\theta_1, \theta_2) \ge 0,\ \ i = 1,2$. *If* $u(x,t) \ge 0,\ v(x,t) \ge 0$ *is the solution of the problem (21), (22), then* $u(x,t) \equiv 0,\ v(x,t) \equiv 0$.

Chapter III consisting of three sections is devoted to the existence of non-negative solutions of semilinear elliptic, parabolic equations with a biharmonic operator in the principal part and a system of such equations.

In the first two sections of this chapter, we study the absence of global solutions of fourth order semilinear elliptic, parabolic equations with singular potential and depending on the nonlinearity degree, singularity coefficient and dimension of the space, we find the exact critical exponent. In section 3, we find the exact critical exponent of the absence of global solutions of the system of fourth order semilinear parabolic equations with a singular potential.

Similar issues for weakly nonlinear equations with a biharmonic operator were considered by various authors as Q.Q.laptev, Yu.V.Volodin, X.Xu, V.Ghergu, S.D.Taliaferro, H.Xiong, Y.T.She, P.C.Carriao, R.Demarque, O.H.Miyagaki, Y.Yao, R.Wang, Y.Shen.

In section 3.1 at first in B_R' we study the equation

$$\Delta^2 u - \frac{C}{|x|^4} u - |x|^\sigma |u|^q = 0, \tag{23}$$

where $q > 1,\ 0 \le C \le (n(n-4)/4)^2,\ \ \sigma > -4,\ \ \Delta^2 u = \Delta(\Delta u)$.

We study the nonexistence of the global solution of the equation (23), satisfying the conditions

$$\int_{\partial B_R} u dx \ge 0, \quad \int_{\partial B_R} \Delta u dx \le 0. \tag{24}$$

The global solution to problem (23), (24) is understood a function $u(x) \in C^4(\overline{B'_R})$ that satisfies condition (24) on the boundary and equation (23) at each point B'_R.

For brevity of the notation we introduce the denotations:

$$\sqrt{(n-2)^2 + C} = D, \quad \sqrt{\left(\frac{n-2}{2}\right)^2 + 1 \pm D} = \alpha_{\pm}.$$

The following theorem is proved.

Theorem 3.1. *Let* $n > 4$, $\sigma > -4$, $0 \le C < (n(n-4)/4)^2$ *and* $1 < q \le 1 + \dfrac{\sigma + 4}{(n-4)/2 + \alpha_-}$. *If* $u(x)$ *is the solution of the equation (23) in* B'_R, *satisfying the condition (24), then* $u \equiv 0$.

In section 3.1 we consider the problem

$$\begin{cases} |x|^{\lambda} \dfrac{\partial u}{\partial t} = -\Delta^2 u^p + \dfrac{C_0}{|x|^4} u^p + |x|^{\sigma} |u|^q & (25) \\ u\big|_{t=0} = u_0(x) \ge 0, & (26) \\ \int_0^T \int_{\partial B_R} u\,dx\,dt \ge 0, \int_0^T \int_{\partial B_{yR}} \Delta u^p\,dx\,dt \le 0, \ \forall T > 0 & (27) \end{cases}$$

in the domain Q'_R, where $n > 4$, $q > 1$, $0 \le C_0 < \left(\dfrac{n(n-4)}{4}\right)^2$, $\sigma > -4$, $u_0(x) \in C(\overline{B'_R})$.

The function $u(x,t) \in C^{4,1}_{x,t}(Q'_R) \cap C^{2,0}_{x,t}(\overline{B'_R} \times [0,+\infty))$ is said to be the solution of problem (25)-(27), if $u(x,t)$ satisfies the equation (25) at each point of Q'_R, condition (26) for $t = 0$ and the condition (27) for $|x| = R$. Here $C^{4,1}_{x,t}(Q'_R)$ is the set of functions four times continuouly differentiable with respect to x and continuously differentiable with respect to t in Q'_R.

We have the following theorem.

Theorem 3.2. *Let* $n>4,\ \sigma>-4,\ 1\le p<q$,

$0\le C_0<\left(\frac{n(n-4)}{4}\right)^2$ *and* $q\le p+\dfrac{\sigma+4}{\frac{n+4}{2}+\lambda+\alpha_-}$. *If* $u(x,t)$ *is the solution of problem (25)-(27), then* $u(x,t)\equiv 0$.

In section 3.2, we study the existence of non-negative solutions of Bauendi-Grushin type equations with a biharmonic operator in the principal part. Bauendi-Grushin type equations have been considered in the works of M.S.Baouendi; Lorenzo D'Ambrosioa and Sandra Lucente; V.A.Grushin; I.Kombe; Farman Mamedov etc..

We introduce the following denotations:

$x=(x_1,...,x_n)\in R^n,\ y=(y_1,...,y_m)\in R^m,\ |x|=\sqrt{\sum_{i=1}^{n}x_i^2},\ |y|=\sqrt{\sum_{i=1}^{m}y_i^2}$,

$B_x(r)=\{x\in R^n:|x|<r\},\ B_y(r)=\{y\in R^m:|y|<r\}$,

$S_x(r)=\{x\in R^n:|x|=r\},\ S_y(r)=\{y\in R^m:|y|=r\}$,

$B(r)=B_x(r)\times B_y(r),\ B_x(r_1,r_2)=\{x\in R^n:r_1<|x|<r_2\}$,

$B_y(r_1,r_2)=\{x\in R^m:r_1<|y|<r_2\},\ B(r_1,r_2)=B_x(r_1,r_2)\times B_y(r_1,r_2)$,

$B'_x(R)=R^n\setminus B_x(R), B'_y(R)=R^m\setminus B_y(R),\ B'(R)=B'_x(R)\times B'_y(R)$,

$\partial B'(R)=S_x(R)\times B'_y(R)\cup B'_x(R)\times S_y(R),\ B'_x(1)=B'_x, B'_y(1)=B'$.

In the domain $B'(R)$ we consider the equation

$$\Delta_x\left(|x|^{\alpha}\Delta_x u\right)+\Delta_y^2 u-\frac{C_1}{|x|^{4-\alpha}}u-\frac{C_2}{|y|^4}u-|x|^{\sigma_1}|y|^{\sigma_2}|u|^q=0, \qquad (28)$$

where

$$\alpha < 4,\ \sigma_1, \sigma_2 \in R,\ \ q > 1,\ \ 0 \le C_1 < \left(\frac{(n-\alpha)(n+\alpha-4)}{4} \right)^2,$$

$$0 \le C_2 < \left(\frac{m(m-4)}{4} \right)^2,\ \Delta_x = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2},\ \Delta_y^2 = \Delta_y(\Delta_y),\ \Delta_y = \sum_{i=1}^{m} \frac{\partial^2}{\partial y_i^2}.$$

We study the absence of the solution of the equation satisfying the following conditions:

$$u \ge 0,\ \ \Delta u \le 0 \ \text{ on } \partial B'(R). \qquad (29)$$

The solution of the problem (28), (29) is understood in the classic sense, more exactly, the function $u(x, y) \in C^4(B'_R) \cap C^3(\overline{B'_R})$ satisfying the equation (28) at each point and the condition (29) on $\partial B'_R$, is said to be a global solution to problem (28), (29).

We introduce the denotations:

$$D_1 = \sqrt{\left(\frac{(n-2)(\alpha-2)}{2} \right)^2 + C_1},\quad D_2 = \sqrt{(m-2)^2 + C_2},$$

$$\alpha_{\pm} = \sqrt{\left(\frac{n-2}{2} \right)^2 + \left(\frac{\alpha-2}{2} \right)^2 \mp D_1},\qquad \text{for } \ 2 \le \alpha < 4,$$

$$\alpha_{\pm} = \sqrt{\left(\frac{n-2}{2} \right)^2 + \left(\frac{\alpha-2}{2} \right)^2 \pm D_1},\qquad \text{for } \ \ \alpha < 2,$$

$$\beta_{\pm} = \sqrt{\left(\frac{m-2}{2} \right)^2 + 1 \pm \sqrt{D_2}},\qquad \gamma = \frac{\dfrac{2\sqrt{D_1}}{|\alpha-2|} - \alpha_+}{\alpha_-},$$

$$a = \frac{m+4}{2} + \beta_- - \sigma_2 \frac{1}{q-1} = \frac{m+4}{2} + \beta_- - \sigma_2(q'-1),$$

$$b = \frac{n+4-\alpha}{2} + \alpha_- - \sigma_1(q'-1),$$

where $\dfrac{1}{q} + \dfrac{1}{q'} = 1$.

The following theorem is the main result of this section:

Theorem 3.3. *Let* $\alpha < 4,\ n > 4-\alpha,\ m > 4,$

$$0 \le C_1 < \left(\frac{(n-\alpha)(n+\alpha-4)}{4}\right)^2,\ 0 \le C_2 < \left(\frac{m(m-4)}{4}\right)^2 \ u$$

$$a)\ \sigma_1 > 0,\ \sigma_2 > 0,\ 1 < q \le \min\left\{1+\frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}, 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\},$$

$$b)\ \sigma_1 > 0,\ \sigma_2 > -4,\ q > \max\left\{1, 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\} \ and$$

$$q < 1+\frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}, q \le 1+\frac{\sigma_2+4}{\frac{m+4}{2}+\beta_- - 4} \ or$$

$$q = 1+\frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}, q < 1+\frac{\sigma_2+4}{\frac{m+4}{2}+\beta_- - 4},$$

$$c)\ \sigma_1 > \alpha-4,\ \sigma_2 > 0,\ q > \max\left\{1, 1+\frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-}\right\} \quad and$$

$$q < 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-},\ q \le 1+\frac{\sigma_2+4-\alpha}{\frac{n+\alpha-4}{2}+\alpha_- - 4} \ or \ q = 1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-},$$

$$q < 1+\frac{\sigma_1+4-\alpha}{\frac{n+\alpha-4}{2}+\alpha_-},$$

$$d)\ 4-\alpha+\sigma_1+\frac{4-\alpha}{4}\sigma_2 > 0,$$

$$\max\left\{1,1+\frac{\sigma_1}{\frac{n-\alpha+4}{2}+\alpha_-},1+\frac{\sigma_2}{\frac{m+4}{2}+\beta_-}\right\}<$$

$$< q \le 1+\frac{4-\alpha+\sigma_1+\frac{4-\alpha}{\alpha}\sigma_2}{\frac{n-\alpha+4}{2}+\alpha_- +\frac{4-\alpha}{4}\left(\frac{m-4}{2}+\beta_-\right)}.$$

If $u(x,y)$ is the solution of problem (28), (29), then $u\equiv 0$.

In section 3.3 in the domain Q'_R we consider the system of equations

$$\begin{aligned}\frac{\partial u_1}{\partial t}+\Delta^2 u_1-\frac{C_1}{|x|^4}u_1&=|x|^{\sigma_1}|u_2|^{q_1}\\ \frac{\partial u_2}{\partial t}+\Delta^2 u_2-\frac{C_2}{|x|^4}u_2&=|x|^{\sigma_2}|u_1|^{q_2}\end{aligned} \qquad (30)$$

with the initial condition

$$|u_i|_{t=0}=u_{i0}(x)\ge 0 \qquad (31)$$

and the conditions

$$\int_0^T\int_{\partial B_R} u_i\,dxdt\ge 0,\ \int_0^T\int_{\partial B_R}\Delta u_i\,dxdt\le 0, \forall T>0, \qquad (32)$$

where

$$n>4,\ q_i>1,\ \sigma_i\in R,\ 0\le C_i<\left(\frac{n(n-4)}{4}\right)^2,\ u_{i0}(x)\in C(B'_R), \Delta^2 u=\Delta(\Delta u).$$

By the global solution of problem (30)-(32) we mean a pair of such functions (u_1,u_2), that

$$u_1(x,t),u_2(x,t)\in C^{4,1}_{x,t}(Q'_R)\cap C^{3,0}_{x,t}(\overline{B}'_R\times(0,+\infty))\cap C(\overline{B}'_1\times[0,+\infty))$$

and satisfy the system (30) at each point, initial condition (31) and conditions (32).

We use the following denotations:

$$D_i = \sqrt{(n-2)^2 + C_i}, \quad \lambda_i^{\pm} = \sqrt{\left(\frac{n-2}{2}\right)^2 + 1 \pm D_i},$$

$$\mu_i = \frac{1}{2}\left(1 + \frac{D_i - \lambda_i^+}{\lambda_i^-}\right), \quad \overline{\mu}_i = \frac{1}{2}\left(1 - \frac{D_i - \lambda_i^+}{\lambda_i^-}\right),$$

$$\alpha_1 = \frac{\lambda_1^- + \sigma_1 + \frac{n+4}{2}}{\lambda_2^- + \frac{n+4}{2}}, \ \alpha_2 = \frac{\lambda_2^- + \sigma_2 + \frac{n+4}{2}}{\lambda_1^- + \frac{n+4}{2}},$$

$$\beta_1 = \frac{\lambda_1^- + \sigma_1 + 4 + \frac{n+4}{2}}{\lambda_2^- + \frac{n-4}{2}}, \ \beta_2 = \frac{\lambda_2^- + \sigma_2 + 4 + \frac{n+4}{2}}{\lambda_1^- + \frac{n-4}{2}},$$

$$\theta_1 = \frac{\sigma_1 + 4 + q_1(\sigma_2 + 4)}{q_1 q_2 - 1} - \lambda_1^- - \frac{n+4}{2},$$

$$\theta_2 = \frac{\sigma_2 + 4 + q_2(\sigma_1 + 4)}{q_1 q_2 - 1} - \lambda_2^- - \frac{n+4}{2}, \quad i = 1,2.$$

Theorem 3.4. *Assume that* $n > 4$, $\beta_i > 1$, $0 \le C_i < \left(\frac{n(n-4)}{4}\right)^2$ *and* $1 < q_i \le \beta_i$, $\max(\theta_1, \theta_2) \ge 0$, $(q_1, q_2) \ne (\alpha_1, \beta_2)$ *in the case* $\alpha_1 > 1$, $(q_1, q_2) \ne (\beta_1, \alpha_2)$ *in the case* $\alpha_2 > 1$, $i = 1,2$. *Then problem (30)-(32) has no nontrivial global solutions.*

Chapter IV has been devoted to the study of asymptotic behavior near the infinity of the solutions of second order semilinear elliptic equations on a half-cylinder satisfying the homogeneous Neumann condition on the lateral surface.

In section 4.1, we study the asymptotic behavior of solutions near the infinity of a second order semilinear elliptic equation with a Laplace operator in the principal part. Similar problems with nonlinearity of the form $|u|^{\sigma-1}u$ have been studied in the works of Wei Guo, Xinyue Wang and Mingjun Zhou; T.S. Khachlayev; V.A.

Kondratiev, Yu.V Egorov, O.A Oleinik; V.A.Kondratiev, L.Veron; A. Pazy; Y. Pinchover.

Let G be a bounded in R^n with a Lipchitz boundary.

We introduce the denotations:

$\Pi_{a,b} = G\times(a,b), \Pi_{a,\infty} = \Pi_a, \Gamma_{a,b} = \partial G\times(a,b)$, $\Gamma_{a,\infty} = \Gamma_a$.

In section 4.1, we study the behavior at infinity of the solution of the equation

$$u_{tt} + \Delta u - |u|^{\sigma} = 0 \quad \text{in } \Pi_0, \tag{33}$$

satisfying the condition

$$\frac{\partial u}{\partial n} = 0 \qquad \text{on} \qquad \Gamma_0, \tag{34}$$

where $\sigma > 1$, n is a unit vector of the external normal to ∂G.

As the solution of the problem (33), (34) we understand a generalized solution. The function $u(x,t)$ is said to be a generalized solution of equation (33), satisfying the condition (34), if $u(x,t) \in W_2^1(\Pi_{a,b}) \cap L_\infty(\Pi_{a,b})$ for any $0 < a,b < \infty$ we have the equality:

$$\int\limits_{\Pi_{a,b}} u_t\varphi_t dxdt + \sum_{i=1}^{n} \int\limits_{\Pi_{a,b}} u_{x_i}\varphi_{x_i} dxdt + \int\limits_{\Pi_{a,b}} |u|^{\sigma}\varphi dxdt = 0$$

for any function $\varphi(x,t) \in W_2^1(\Pi_{a,b})$ such that $\varphi(x,a) = \varphi(x,b) = 0$.

The following theorem is the main result of this section.

Theorem 4.1. *a) If* $u(x,t) > 0$ *is the solution of the equation (33), satisfying the condition (34), then* $u(x,t) = O\left(t^{-\frac{2}{\sigma-1}}\right)$;

б) If $u(x,t)$ *is the solution of the equation (33), satisfying the condition (34), that changes sign in each domain* Π_a, $a > 0$, *then*

$$u(x,t) = O(e^{-ht}),$$

where h *is independent of* $u(x,t)$.

In section 4.2 we study the behavior at infinity of the solution of the equation:

$$u_{tt} + Lu - (t+1)^{\mu}|u|^{\sigma} = 0 \quad in \quad \Pi_0 \tag{35}$$

satisfying the condition:

$$\frac{\partial u}{\partial \nu} = \sum_{i,j=1}^{n} a_{ij} \frac{\partial u}{\partial x_j} \cos(x_i, n) \quad on \quad \Gamma_0, \tag{36}$$

where $\sigma > 1$, $\mu > -2$, n is a unit vector of the external normal to ∂G and L is an operator of the form

$$L \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x) \frac{\partial}{\partial x_j} \right) + \sum_{i=1}^{n} a_i(x) \frac{\partial}{\partial x_i},$$

the coefficients $a_{ij}(x), a_i(x)$ are bounded, measurable functions, $a_{ij} = a_{ji}$ and the condition (4) is fulfilled.

The following theorem is the main result of this section.

Theorem 4.2. *a) If $u(x,t) > 0$ is the solution of equation (35), satisfying condition (36), then $u(x,t) = O(t^{-\frac{\mu+2}{\sigma-1}})$.*

б) If $u(x,t)$ is the solution of the equation (35), satisfying the condition (36), that changes sign in each domain Π_a, $a > 0$, then

$$u(x,t) = O(e^{-ht}).$$

where h is independent of $u(x,t)$.

CONCLUSIONS

The dissertation work is devoted to the existence of global solutions and the asymptotic of solutions near the infinity of semilinear elliptic, parabolic equations of second and fourth orders, the systems of such equations in unbounded domains.

At first, we solve a problem on the absence of global solutions to semilinear parabolic equations with time periodic coefficients and the systems of such equations and then of semilinear elliptic and parabolic equations of second and fourth order with a singular potential and the systems of such equations. We study asymptotic behavior of the solutions of semilinear elliptic equations near the infinity in a cylindrical domain.

In the dissertation work the following new results were obtained:

1. A sufficient condition providing the absence of a positive global solution in an unlimited domain of semilinear parabolic equations with time periodic coefficients, with a power and logarithmic nonlinearity is found; in all the cases the exactness of the obtained sufficient conditions is proved;
2. In the cylinder where the base is the exterior of some compact, the critical exponent of the absence of positive global solutions of a semilinear parabolic equation with lowest terms and time periodic coefficients is found;
3. In the cylinder where the base is the exterior of some compact, a sufficient condition providing the absence of positive global solutions of a weakly-connected system of semilinear parabolic equation with time periodic coefficients is found;
4. In the cylinder where the base is the exterior of some compact, a sufficient condition providing the absence of positive global solutions of a weakly coupled system of semilinear parabolic equations with periodic functions is found;
5. Depending on weight, nonlinearity degree, singularity coefficient and dimension of the space, a Fujita-type result has been obtained for a semilinear parabolic equation with a weight and singular potential;
6. Depending on nonlinearity degree, singularity coefficient and dimension of the space, a Fujita type result has been obtained for a system of semilinear second order equations with a singular potential;
7. Depending on the nonlinearity degree, singularity coefficient and dimension of the space, an exact critical exponent of the absence of global solutions of a semilinear biharmonic equation with a singular potential has been found;
8. Depending on the nonlinearity degree, singularity coefficient and dimension of the space, the exact critical exponent of the absence

of global solutions of a Bauendi-Grushin type semilinear biharmonic equation with a singular potential has been found;
9. A sufficient condition providing the absence of global solutions of a weakly coupled system of semilinear biharmonic equations with a singular potential and depending on nonlinearity degree singularity coefficient and dimension of the space was found;
10. The complete asymptotic behavior at the infinity of the positive solutions of second order semilinear elliptic equations in a semi-cylinder with Neumann homogeneous boundary condition on its lateral surface was obtained.

The main results of the dissertation work have been published in the following works:

1. Багыров, Ш.Г., О существовании положительного решения нелинейного параболического уравнения второго порядка с периодическими коэффициентами по времени // Тезисы Международной конференции по математике и механике, посвященной 50-летию со дня рождения чл.-корр. НАНА, профессора И.Т. Мамедова, Баку – 2005, с.48.
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