REPUBLIC OF AZERBAIJAN

On the right of the manuscript

SPECIFICITY OF FUZZY RULE BASES

Specialty: 3338.01 – "Systems analysis"

Field of science: Technics

Applicant: **Nigar Adilova**

A B S T R A C T

of the dissertation for the scientific degree of Doctor of Philosophy

Baku - 2021

Dissertation was accomplished at the research laboratory "Intelligent Control and Decision Making Systems in Industry and Economics" of Azerbaijan State Oil and Industry University.

Scientific supervisor	Doctor of technical sciences, prof. **Rafik A. Aliev**
Official opponents	1. Doctor of technical sciences, prof. **Latafat A. Gardashova** 2. Doctor of technical sciences, prof. **Sayyaddin M. Jafarov** 3. Doctor of technical sciences, prof. **Javanshir F. Mammadov**

Dissertation council ED 2.02 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at Azerbaijan State Oil and Industry University.

Chairman of the Dissertation Council: ______________	Doctor of technical sciences, prof. **Mustafa B. Babanli**
Scientific secretariat of the Dissertation Council: ______________	Doctor of philosophy on technical sciences, assoc. prof. **Tahir G. Jabbarov**
Chairman of the Scientific Seminar: ______________	Doctor of technical sciences, prof. **Tarlan S. Abdullayev**

GENERAL DESCRIPTION OF THE DISSERTATION

The actuality of the topic. In contemporary times, fuzzy If-Then model of control systems, which is the main research object of a series of fields, such as systems analysis, automation, is a model based on knowledge, decision support system, and it has a great importance in monitoring the state of processes without human intervention. At the same time, the assessment of specificity parameter, which reflects the semantics of information granulation in fuzzy If-Then models of control systems, provides a great opportunity for the decision maker to describe both the current situation and the outcome. In addition, considering such parameters as accuracy, interpretability, inconsistency, which are the quality criteria of fuzzy If-Then model, has a positive effect on improving the efficiency of the rule base. Unfortunately, there is very little work in the scientific literature on the specificity of fuzzy rules and to take into account other quality criteria, and there is a serious theoretical and practical need for a comprehensive approach to this problem. For this purpose, the dissertation is devoted to the study of theoretical aspects and practical application of the main quality parameters for the fuzzy If-Then model of the a control system.

Goal of the dissertation. The goal of the research consists of a multi-criterial analysis of the quality of fuzzy and Z-based If-Then model of control systems and decision-making.

Main highlights, brought forward for dissertation defense. In this dissertation the following problems are considered:

- The assessment of specificity of fuzzy and Z-based rules and suggest a more efficient, alternative solution;
- Identification and analysis of quality criteria (accuracy, coverage, complexity, partition, inconsistency) for fuzzy If-Then model of control systems and decision-making;
- Synthesis of a model close to the optimal for fuzzy If-Then model of control systems with the application of a multi-criteria optimization method based on the principle of ideal solution and Pareto-optimality;
- Investigation of the activity and efficiency of the proposed theoretical methods by using computer simulation.

Research methods. Research methods and methodologies used in this dissertation are fuzzy and Z-based sets, fuzzy and Z-rule bases, Pareto-optimality method, approximate reasoning, etc.

Scientific value of the thesis. Complex approach to the specificity of fuzzy rules, as well as other quality criteria (accuracy, complexity, coverage, partition, inconsistency) and creation of a balance close to the optimal by using multi-criteria optimization method.

Theoretical and practical value of the thesis. The theoretical value of the thesis is to offer theoretical provisions for establishing a balance among the quality criteria of fuzzy and Z-based If-Then model for decision-making and control systems. The practical value of the dissertation is that it can be used to create various real practical fuzzy systems.

Approbation of dissertation. Main results of dissertation were presented at the local and international conferences:

- ***ICSCCW-2017*** – 9th International Conference on Theory and Application of Soft Computing, Computing with Words and Perceptions, Budapest, Hungary;
- ***ICAFS-2018*** – 13th International Conference on Theory and Application of Fuzzy Systems and Soft Computing, Warsaw, Poland;
- ***WCIS-2018*** – 10th World Conference on Intelligent Systems for Industrial Automation, Tashkent, Uzbekistan;
- ***ICSCCW-2019*** – 10th International Conference on Theory and Application of Soft Computing, Computing with Words and Perceptions, Prague, Czech Republic;
- ***ICAFS-2020*** – 14th International Conference on Theory and Application of Fuzzy Systems and Soft Computing, Budva, Montenegro;
- V International Scientific Conference of Young Researchers dedicated to the 98th anniversary of national leader Heydar Aliyev, held by Baku Engineering University, April 29-30, 2021, Baku, Azerbaijan.

Organization where dissertation was realized: Azerbaijan State Oil and Industry University, Research laboratory “Intelligent Control and Decision Making Systems in Industry and Economics”.

Structure of dissertation. Manuscript of the dissertation includes introduction, 5 chapters, conclusion, and references.

Publications. Although author has 11 published works including 8 works in SCOPUS database, Web of Science Core Collection, Conference Proceedings, the obtained results in dissertation were published in 9 works.

MAIN CONTENT OF THE WORK

In the Introduction part reflects the actuality of the topic, the objectives, research methods, theoretical and practical significance of the research.

In the first chapter an overview of investigations on the specificity and other quality criteria of fuzzy rules, the actuality and the statement of the problem are given. Studies have shown that there are theoretical provisions and practical applications on quality criteria of fuzzy rules in an individual basis, howevere there hasn’t been a complex approach to determine values of criteria close to optimal. In the investigations of linguistic fuzzy modeling, the relationship between interpretability and accuracy parameters, as well as other quality parameters, has been considered as ad hoc, ie without mathematical generalization or a comprehensive approach to all objectives. There are also shortcomings in selecting an alternative solution from the methods used to determine the specificity for fuzzy and Z-based rules.

Based on all this, the next chapters of the dissertation are devoted to the solution of these problems.

The second chapter of the dissertation is dedicated to the study of specificity and other quality criteria of fuzzy, as well as Z-based rules.

Fuzzy IF-THEN rules

The fuzzy IF-THEN rules proposed by Lotfi Zadeh characterize fuzzy conditions to support decision-making and analysis of complex systems. Fuzzy IF-THEN rules are described as follows:

$$\text{If x is } A \text{ then y is } B \tag{1}$$

where X – antecedent, Y – consequent of fuzzy rules, A and B are linguistic terms. In other words, A and B define the relationship between the antecedent and consequent of fuzzy rules.

Fuzzy rules can consist of more than one antecedent and consequent. In this case, a connection between antecedents and consequents are provided by "AND", “OR” operators. Such cases are considered for complex systems. Multi-input and multi-output fuzzy rules are shown as follows:

$$\begin{gathered}\text{If } X_1 \text{ is } A_1 \text{ and } X_2 \text{ is } A_2 \text{ and ..... and } X_n \text{ is } A_n \\ \text{then } Y_1 \text{ is } B_1 \text{ and } Y_2 \text{ is } B_2 \text{ and ..... and } Y_m \text{ is } B_m\end{gathered} \tag{2}$$

where A_i and B_i are information granules.

Specificity. Specificity of fuzzy sets and rules

For the sake of the concept of specificity is closely related to the information granulation, this section is dedicated to the granularity of data, and fuzzy sets.

Fuzzy granulation is carried out according to the rules. The fuzzy sets used in the antecedent and consequent parts of fuzzy rules are derived from the decomposition of input and output domains. As the carrier describing the antecedent increases, granulation also increases correspondingly. As shown in Figure 1, the carriers of fuzzy sets in the decomposition of the domain are indifferent sizes, accordingly, the rule base can consist of rules with different granulations.

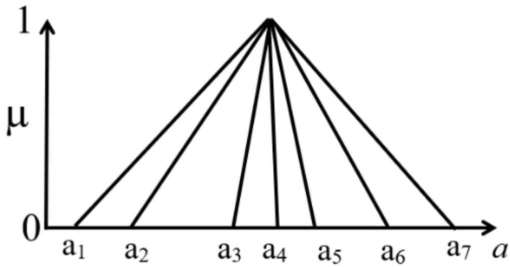


Figure 1. Domain decompozition

Fuzzy granulation necessitated the usage of two variables (coverage and specificity), which are the main characteristics of fuzzy sets. The description of these two variables also uses the cardinality variable, which reflects the number of elements. For instance, the cardinality on the given fuzzy set *A*, is calculated as follows for the set X = {x_1, x_2, …, x_n}:

$$card(A) = \sum_{i=1}^{n} A(x_i) \qquad (3)$$

If we describe (3) as an integral, the result will be as follows:

$$card(A) = \int_{x} A(x)dx \qquad (4)$$

where $A(x)$ describes information granulation. The cardinality can be specified as the number of elements for fuzzy sets, but it should be noted that each element has a certain degree of membership.

The specificity of a normalized fuzzy set *A* on a finite domain *S*, can be computed as:

$$Sp(A) = \sum_{i=1}^{n} \frac{\mu_A(s_i) - \mu_A(s_{i+1})}{i} \qquad (5)$$

where the n elements of S are ordered according to the decreasing values of μ_A and $\mu_A(s_{n+1})=0$, and *Sp(A)* is formalized for the probability of the element which has the greatest membership degree[1].

According to Yager, the measure of specificity of a fuzzy number is described as follows.[2]

$$SpA(\pi) = \int_0^{\alpha_{max}} \frac{1}{Card(\pi_\alpha)} d\alpha \tag{6}$$

$\alpha_{max} = \underset{x}{Max}\,\pi(x), \pi_\alpha$- is the subset of α-cuts.$Card(\pi_\alpha)$ characterizes the cardinality for α-cut. (6) may be expressed in another way.

$$Sp(A) = \int_0^{hgt(A)} \frac{1}{\left|A^\mu\right|} d\mu \tag{7}$$

where $\left|A^\mu\right|$ is a cardinality of μ -cut of A.

Specificity for fuzzy rules may be defined in 3 different ways[3]:

1. using the mean-value-type measure:

$$Sp_1 = \frac{1}{card\ X'} \sum_{x_i \in X'} Q(x_i) \tag{8}$$

where $X' = \{x_i \in X : \exists R_j \in Q : CR(x_i, R_j) = 1\}$. $Q(x_i)$ defines specificity for each fuzzy rule.

2. using the product-type measure:

---

[1] Dubois, D., Prade H.: A note on measures of specificity for fuzzy sets // International journal of general systems, - 1985, 10(4), - p. 279–283.

[2] Yager, R. R.: On the Specificity of a Possibility Distribution // Fuzzy sets and systems, - 1992, vol. 50, - p. 279-292.

[3] Kacprzyk, J.: On measuring the specificity of IF-THEN rules // International journal of approximate reasoning, - 1994, 11, - p. 29-53.

$$Sp_2 = \prod_{x_i \in X'} Q(x_i) \quad (9)$$

3. using the min-type measure:

$$Sp_3 = \min_{x_i \in X'} Q(x_i) \quad (10)$$

Z-based rules

Z-based rules can be defined as follows:

$$\text{IF } (X, A_X, B_X) \text{ THEN } (Y, A_Y, B_Y).$$

Specificity of Z-information

The specificity of a Z-number is determined by the specificity of its fuzzy *A* and *B* parts. Specificity measures of Z – numbers indicate the degree to which a fuzzy vector with components *A* and *B* designates a unique *Z*-number.

Assume that, Z_i and Z_j are Z-numbers with normal fuzzy set components and $Z_i \leq Z_j$. Then the specificity of Z_i is greater than Z_j.

For Z-numbers Z_1 and Z_2 with triangular fuzzy numbers Z_1 defined by peak points p_{1A}, p_{1B} and bases r_{1A}, r_{1B} is more specific than Z_2 defined by peak points $p_{1A} = p_{2A}$, $p_{1B} = p_{2B}$ and bases r_{1A}, r_{1B} if and only if[4]:

$$\alpha r_{1A} + (1-\alpha) r_{1B} \leq \alpha r_{2A} + (1-\alpha) r_{2B} \quad (11)$$

where α is preference index between *A* and *B* components of Z-numbers Z_1 and Z_2.

For simplicity we have considered specificity of Z-numbers with fuzzy set components to be defined by the cardinality of its support. For example, the specificity for Z-number (with TFN) is determined as

---

[4] Aliev, R.A.: Operations on Z-numbers with acceptable degree of specificity // Procedia Computer Science, - 2017, 120, - p. 9-15.

$$2\alpha r_{1A} + 2(1-\alpha) r_{1B} \tag{12}$$

$$Sp(Z) = \gamma Sp(A) + (1-\gamma) Sp(B) \tag{13}$$

where, γ, $(1-\gamma) \in [0,1]$ are importance weights that describe relative importance of the components. The specificity of fuzzy numbers *A* and *B*, which are carriers of Z-numbers, is calculated in this way.

$$Sp(A) = 1 - \frac{\int_0^1 Length(A^{\mu}) d\mu}{\text{supp}(A)}, \quad Sp(B) = 1 - \frac{\int_0^1 Length(B^{\nu}) d\nu}{\text{supp}(B)} \tag{14}$$

Distance-related measures of specificity of fuzzy components *A* and *B* is found as below:

$$Sp(A) = 1 - Dist(A, A_{crisp}), \quad Sp(B) = 1 - Dist(B, B_{crisp}) \tag{15}$$

Quality criteria of fuzzy IF-THEN rules

The definition of quality criteria of fuzzy rules is linked with 5 different criteria in addition to specificity. These 5 criteria include complexity, coverage, partition, inconsistency, and accuracy.

In general, interpretability can be distinguished as an index that determines how easily people understand the rules. The most commonly used expression to define interpretability index is Nauck index. Nauck index is calculated as follows[5]:

$$Nauck\ index = comp \times \overline{cov} \times \overline{part} \tag{16}$$

where, $comp$ - complexity, cov – coverage, $part$ – partition.

The complexity index indicates how complex the rules base is for the user and is determined for the scale of interval [1-0]. Complexity

[5] Nauck, D.D.: Measuring interpretability in rule-based classification systems // USA: FUZZ-IEEE, - 2003, vol. 1, - p. 196–201.

is measured by the total number of rules, the total number of premises, and the total number of labels defined by input

$$comp = m/\sum_{i=1}^{r} n_i \tag{17}$$

where m is the number of classes (outputs), r is the number of rules and n_i is the number of variables used in the i-th rule.

The next parameter to calculate the Nauck index is $\overline{cov}$ – coverage index, which determines the given values are in the range [0-1]:

$$\overline{cov} = \sum_{i=1}^{r} cov_i/n_i \tag{18}$$

Individually, the coverage criterion is one of the most important criteria for characterizing the quality of a fuzzy rule base. Thus, the coverage criterion describes the statement of the consequent part of fuzzy rules with degrees. The calculation of this criterion also allows a clearer implementation of research results. If X_i is the domain of ith input variable partitioned by p_i MFs $\left\{\mu_i^{(1)}, \ldots, \mu_i^{p_i}\right\}$, then cov_i is computed as:

$$cov_i = \frac{\int x_i \widehat{h_i}(x)dx}{N_i}$$

$$\widehat{h_i}(x) = \begin{cases} h_i(x) & if \quad 0 < h_i(x) < 1 \\ \dfrac{p_i - h_i(x)}{p_i - 1} & otherwise \end{cases} \tag{19}$$

$$h_i(x) = \sum_{k=1}^{p_i} \mu_i^{(k)}(x)$$

where $h_i(x)$ is the total MFs of ith input variable with $N_i = \int_{X_i} dx$ for continuous domains. The integral is replaced by a sum for discrete finite domains with $N_i = |X|$. Then, $\overline{cov}$ is computed as normalized coverage cov_i for all input variables.

The last parameter for Nauck index is partition index, which is computed as the inverse of the number of MFs minus one for each input variable:

$$\overline{part} = \sum_{i=1}^{r} part_i / n_i \qquad (20)$$

$$part_i = \frac{1}{p_i - 1} \qquad (21)$$

where p_i - the number of MFs in the ith input variable. $part_i$ –is normalized partition index.

The next method to define interprtability is the fuzzy index[6].

Accuracy index

Accuracy is measured by the number of correctly classified training patterns[7]:

$$J = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - y_i)^2} \qquad (22)$$

where *n* - is number of samples (current input vectors), $y_i = FS_{all}(x_i)$ is response of fuzzy system with all rules, $y_i = FS(x_i)$, x_i is the *i*-th sample (current input vector).

Inconsistency index

Let's have a look at the definition of inconsistency criterion, which is one of the most important quality criteria in fuzzy rules for decision making. Assume that the fuzzy If-Then model of a control system consists of 7 fuzzy rules.

---

[6] Razak, T.R., Garibaldi, J.M., Wagner, C.: Pourabdollah, A. Soria, D.: Interpretability indices for hierarchical fuzzy systems // IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), - 2017, - p. 1-6.

[7] Gacto, M. J., Alcala, R., Herrera, F.: Interpretability of linguistic fuzzy rule-based systems: An overview of interpretability measure // Information Science, - 2011, 181, - p. 4340-4360.

Let us first consider the semantic and functional definition of the inconsistency criterion. Two fuzzy rules are inconsistent if they have the same if-part, but different then-parts as noticed above[8]:

$$IF\ X_1\ is\ A_1\ and\ X_2\ is\ A_2,\ THEN\ Y\ is\ C.$$
$$IF\ X_1\ is\ A_1\ and\ X_2\ is\ A_2,\ THEN\ Y\ is\ D. \tag{23}$$

Inconsistency index will be identified based on consistency index. To calculate the consistency of two fuzzy rules, Similarity of Rule Premise (SRP) and Similarity of Rule Consequent (SRC) are obtained. Two fuzzy rules R_1 and R_k are given as below:

$$R_1 : If\ x_1\ is\ A_{i1}(x_1)\ and\ x_2\ is\ A_{i2}(x_2)$$
$$and \ldots x_n\ is\ A_{in}(x_n),\ then\ y\ is\ B_i(y) \tag{24}$$
$$R_k : If\ x_1\ is\ A_{k1}(x_1)\ and\ x_2\ is\ A_{k2}(x_2)$$
$$and \ldots x_n\ is\ A_{kn}(x_n),\ then\ y\ is\ B_k(y)$$

Then SRP and SRC between rule *i* and rule *k* are defined by using fuzzy similarity measure as following:

$$SRP(i,k) = \wedge_{j=1}^{n} S(A_{ij}, A_{kj}) \tag{25}$$

$$SRC(i,k) = S(B_i, B_k) \tag{26}$$

where n is the total number of the input variables. The consistency between rule *R(i)* and *R(k)* can be defined as[8]:

$$Cons(R(i),R(k)) = \exp\left\{ -\frac{\left(\frac{SRP(i,k)}{SRC(i,k)} - 1.0\right)^2}{\left(\frac{1}{SRP(i,k)}\right)^2} \right. \tag{27}$$

---

[8] Yaochu, J., von Seelen, W., Sendhoff, B.: An approach to rule-based knowledge extraction. IEEE International Conference on Fuzzy Systems Proceedings, -1998, -p. 1188-1193.

Thus, an inconsistency index can be defined by this formula:

$$f_{incons} = \sum_{i=1}^{N} \sum_{\substack{1 \le k \le n \\ k \ne 1}} \left[1.0 - Cons(R(i), R(k))\right] \quad (28)$$

Quality criteria of Z-based rules

Defining the quality criteria for Z-based rules is the same as defining the quality criteria for fuzzy rules. However, because of Z-based rules have a more complex structure, the calculation of quality criteria of Z-based rules is somewhat different. For example, let's have a look at the calculation of interpretability (complexity, coverage, partition) of the given Z-based rules.

1. IF x is $Z_{1A} = [(-10, -10, -7), sure]$ THEN y is $Z_{1B} = [(-1, -0.9, -0.7), sure]$
2. IF x is$Z_{2A} = [(-10, -7, -3), very\ sure]$ THEN y is $Z_{2B} = [(-0.7, -0.6, -0.4), sure]$
3.IF x-$Z_{3A} = [(-7, -4, -1), very\ sure]$ THEN y- $Z_{3B} = [(-0.4, -0.25, -0.1), very\ sure]$
4. IF x is $Z_{4A} = [(-3,0,3), az\ əmin]$ THEN y is $Z_{4B} = [(0,0.1,0.3), less\ sure]$
5. IF x is $Z_{5A} = [(0,3.5,7), sure]$ THEN y is $Z_{5B} = [(0.25,0.4,0.55), less\ sure]$
6. IF x is $Z_{6A} = [(3,5,8), very\ sure]$ THEN y is $Z_{6B} = [(0.6,0.8,0.85), sure]$
7. IF x is $Z_{7A} = [(7,10,10), less\ sure]$ THEN y is $Z_{7B} = [(0.85,0.9,1), less\ sure]$

where, *less sure*, *sure*, *very sure* are the *B* part of Z-number, defines the degree of belief. The values for *B* part are as follows:

$$less\ sure = [0.4; 05; 0.6]\ ,\ sure = [0.7; 0.8; 0.9]\ ,\ very\ sure = [0.8; 0.9; 1]$$

Firstly, let's calculate the complexity index.

$$comp = m/\sum_{i=1}^{r} n_i \quad comp = \frac{7+3}{2+2+2+2+2+2+2} = 0.714.$$

The coverage will have the same value if the fuzzy or Z-based rules consist of the same input variables.

$$\overline{cov} = 0.811144986.$$

The partition index for the given control system will be as below:

$$part_i = \frac{1}{p_i - 1} = \frac{1}{10-1} = \frac{1}{9} \qquad \overline{part} = \sum_{i=1}^{r} \frac{part_i}{n_i} \approx 0.11.$$

Thus, interpretability index for Z-based model of control system will be

$$Nauck\ index = comp \times \overline{cov} \times \overline{part} = 0.714 \times 0.811 \times 0.11 = 0.06.$$

Thus, we investigated the calculation of the interpretability index in the Z-based model of control system.

The third chapter proposes a solution to the optimization problem based on the interval and information granulation of fuzzy rules.

Statement of the problem for multi-criterial optimization of information granules in fuzzy If-Then rules

Assume that the cardinality $C([a,b])$ of interval $[x_{\min}, x_{\max}] \subset R$, $[a,b] \subset [x_{\min}, x_{\max}]$ is defined by the characteristic functon $I_{[a,b]}$ of interval:

$$C([a,b]) = \int_{x_{\min}}^{x_{\max}} I_{[a,b]}(x)dx = (b-a) \tag{29}$$

The specificity $Sp([a,b])$ of interval $[a,b] \subset [x_{\min}, x_{\max}]$ is as follows:

$$Sp([a,b]) = \frac{b-a}{x_{\max} - x_{\min}} \tag{30}$$

We will consider two types of granules: interval granule and fuzzy granule. Graphical representation of an interval granule is shown in Figure 2.

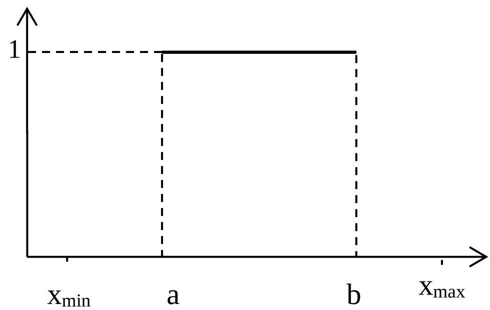


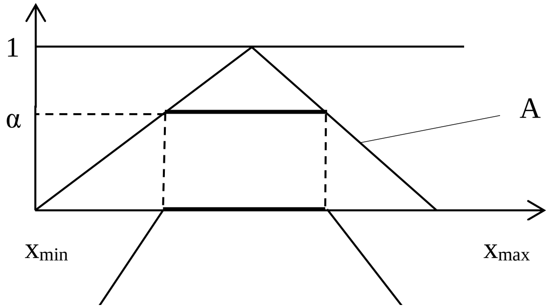
Figure 2. An interval granule

The problem of an interval granule $[a,b] \subset R$ is formulated as follows:

$$\max(C([a,b]),Sp([a,b])) \tag{31}$$

$$\begin{gathered} x_{\min} < a < x_{\max}, \quad x_{\min} < b < x_{\max}, \\ a < b, \quad b - a > l, l > 0 \end{gathered} \tag{32}$$

Graphical representation of a fuzzy granule is shown in Figure 3.



$$\left(\max\left\{x \middle| A^{-1}(y) = \alpha\right\}\right) \quad \left(\min\left\{x \middle| A^{-1}(y) = \alpha\right\}\right)$$

Figure 3. A fuzzy granule

The problem of a fuzzy granule is formulated as follows[9]:

$$\max(C(A) = \int_{x_{\min}}^{x_{\max}} A(x)dx), \tag{33}$$

$$max(Sp(A) = \int_0^1 \left(1 - \frac{h(\alpha)}{x_{\max} - x_{\min}}\right) d\alpha)). \tag{34}$$

$$\begin{gathered} A:[x_{\min}, x_{\max}] \to [0,1] \\ \underset{\alpha>0}{h(\alpha)} = \left(\max\left\{x \middle| A^{-1}(y) = \alpha\right\}\right) - \left(\min\left\{x \middle| A^{-1}(y) = \alpha\right\}\right) \\ \operatorname{supp} A > l, l > 0 \end{gathered} \tag{35}$$

9 Aliev, R. A., Huseynov O. H., Adilova N. E.: Multi-criterial optimization of information granules in fuzzy IF-THEN rules // B-quadrat verlags,- 2018,- p. 52-55.

In (35) the constraint rules out reduction of a fuzzy granule interval to a minimum point. Considering (31)-(35), we propose goal programming approach:

$$(C(A), Sp(A)) \rightarrow (C_g, Sp_g) \quad (36)$$

$$A:[x_{\min}, x_{\max}] \rightarrow [0,1] \quad (37)$$

where C_g, Sp_g describe the cardinality and specificity to create a trade-off between conflicting criteria.

Let's look at the solution of optimization problem based on interval and information granulation (Table 1).

Table 1.

C and *Sp* results for [5, 10] interval granulation

A	*b*	*C*	*Sp*
7	8	1	0.8
5	9	4	0.2
5.5	8	2.5	0.5
7	7.5	0.5	0.9
8	10	2	0.6
5.4	7	1.6	0.68
6.3	9.8	3.5	0.3
5.9	6.2	0.3	0.94
7.7	8.3	0.6	0.88
6	8.7	2.7	0.46
5	6.1	1.1	0.78
7.1	9.2	2.1	0.58
8	8.08	0.08	0.984

Presume that, we should find the range in which the cardinality and specificity variables get the optimal values after interval granulation. Optimal values for cardinality and specificity are found by considering the expressions in (31) - (32). $C_g = 1.2,\ Sp_g = 0,95$. By applying goal programming approach (36) -(37), we have obtained the values close to the optimal ones. The graphical representation of cardinality and specificity criteria is shown in Figure 4.

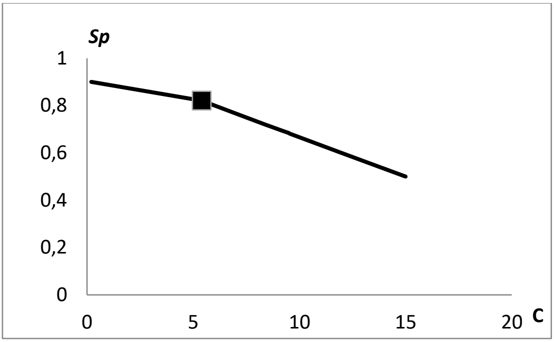


Figure 4. The front of the *C* and *Sp* criteria for interval granules

As seen from the Table 1, the obtained interval granule $[a,b]=[5,6.1]$ is sufficiently acceptable considering the closeness of its specificity and cardinality values to the predefined goal values (C=1.1 and Sp=0.78). Thus, the interval $[a,b]=[5,6.1]$ is an optimal interval for the trade-off between cardinality and specificity. Consider problem of a fuzzy granulation (33)-(35) within the interval $[x_{\min}, x_{\max}]=[0,30]$. The graphical representation of fuzzy granulation between cardinality and specificity criteria is shown in Figure 5.

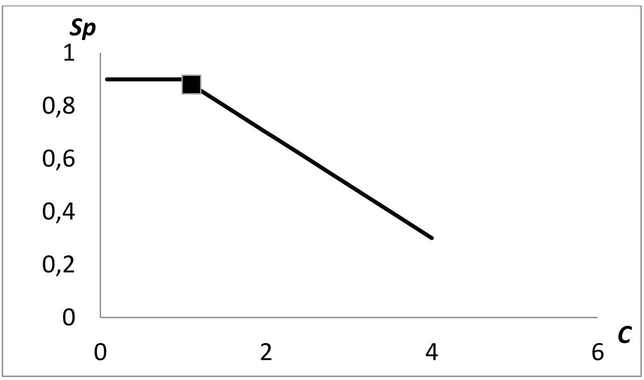


Figure 5. Dependence between *C* and *Sp* criteria for fuzzy granules

By applying goal programming approach (34)-(35), we have found the solution (Table 2):

Table 2.

C və *Sp* results in [0, 30] interval for fuzzy granulation

b_1	b_2	b_3	C	Sp
0	15	30	15	0.5
2	25	28	13	0.566667
5.5	18	29.3	11.9	0.603333
6	15	23	8.5	0.716667
11	13	30	9.5	0.683333
23	27	29	3	0.9
10.5	17	27.4	8.45	0.718333
9.9	11	29	9.55	0.681667
14	20.3	28	7	0.766667
6	13	21.7	7.85	0.738333
5	17.7	19	7	0.766667
7.1	9.2	25.5	9.2	0.693333
18	23	28.8	5.4	0.82
5.03	16.9	29.4	12.185	0.593833

Based on expressions (36) - (37), the cardinality and specificity variables are closest to the given goal functions, and the fuzzy granulation will be as follows:

$$C = 5.4,\ \ Sp = 0.82\,,$$

$$A = (18, 23, 28.8).$$

It seems that the specificity and cardinality of the obtained fuzzy granule $A = (18, 23, 28.8)$ are close to the predefined goal values.

Thus, we achieved the solution of the problem of multi-criteria optimization in information granules estimation (antecedent and consequent parts of fuzzy rules)[9].

In the ***fourth chapter***, the solution of multi-criterial optimization problem in 2 different ways according to the quality criteria of the fuzzy rule bases is considered.

Multi-crierial optimization problem for fuzzy If-Then rules by Ideal solution method

It is known that the quality criteria of fuzzy If-Then model of a control system include the calculation of indices of complexity (com), coverage (cov), partition $(\overline{part})$, inconsistency $Incons(i)$ and accuracy (RMSE - J), in addition to specificity (*Sp*). Thus, these 5 criteria play an important role in estimation and optimization of the model.

Let's presume that these following 27 rules are involved in fuzzy rule-based system (Table 3).

Each fuzzy rule is represented by its 3 antecedents (X_1, X_2, X_3) and a consequent (Y). In accordance with three inputs (X_1, X_2, X_3), the membership functions are:

$$\text{for } X_1 \quad Less = (0;6;10) \quad Medium = (7;11;25) \quad High = (20;27;50)$$

$$\text{for } X_2 \quad Less = (0;7;10) \quad Medium = (7;19;25) \quad High = (20;26;50)$$

$$\text{for } X_3 \quad Less = (1000;1200;1500) \quad Medium = (400;900;1200) \quad High = (50;400;500)$$

$$\text{for } Y \quad Less = (0;7;10) \quad Medium = (8;24;30) \quad High = (25;30;60)$$

---

[9] Aliev, R. A., Huseynov O. H., Adilova N. E.: Multi-criterial optimization of information granules in fuzzy IF-THEN rules // B-quadrat verlags, - 2018, - p. 52-55.

Table 3.

Fuzzy IF-THEN rule base[10]

Rules	Antecedent			Consequent
№	X_1	X_2	X_3	Y
1	H	L	L	S
2	H	L	M	M
3	H	L	H	H
4	H	M	L	M
5	H	M	M	M
6	H	M	H	H
7	H	H	L	H
8	H	H	M	H
9	H	H	H	H
10	M	L	L	S
11	M	L	M	M
12	M	L	H	M
13	M	M	L	M
14	M	M	M	M
15	M	M	H	M
16	M	H	L	H
17	M	H	M	H
18	M	H	H	M
19	L	L	L	S
20	L	L	M	M
21	L	L	H	M
22	L	M	L	M
23	L	M	M	H
24	L	M	H	M
25	L	H	L	S
26	L	H	M	M
27	L	H	H	H

Graphical representations of membership functions for 3 inputs and an output for original case are described in Figure 6 (respectively (a,b,c,d)[11] .

---

[10] Mohanaselvi, S., Shanpriya, B.: Application of fuzzy logic to control traffic signals // AIP Conference Proceedings, - 2019, vol. 2112, - p. 1-9.

[11] Adilova, N. E.: Quality criteria of fuzzy IF-THEN rules and their calculations. Advances in Intelligent Systems and Computing // Springer International Publishing, - 2021, vol. 1306, - p. 55-62.

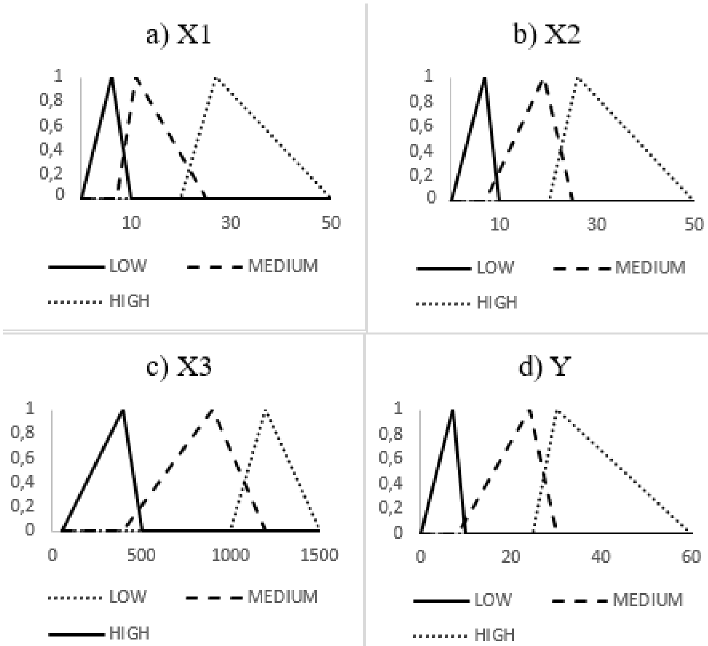


Figure 6. Membership functions for original case

The problem is to compute the measures for Fuzzy Rule based System (FRBS) based on 5 criteria. Corresponding to (17) - (28) formulas quality criteria is obtained for original case (Table 4).

Table 4.

Computatioal results for quality criteria

Quality criteria	Original case
The number of rules	27
Complexity measure	0,037
Coverage degree	0,588
Partition measure	0,5
Inconsistency measure	0,0172
Accuracy measure (RMSE)	0,089

The second case demonstrates the results based on randomly decreased 5 rules. The result will be changed as below (Table 5):

Table 5.
Comparison of computatioal results of quality criteria for 2 cases

Quality criteria	Original case	Second case
The number of rules	27	22
Complexity measure	0,037	0,045
Coverage degree	0,588	0,592
Partition measure	0,5	0,5
Inconsistency measure	0,0172	0,0184
Accuracy measure (RMSE)	0,089	8,824

The computational results show that, the number of rules affects the change of the values for quality criteria except from partition measure.

In order to improve the quality criteria for the fuzzy If-Then model of control systems, let us consider the trade-off between accuracy and interpretability index. For that purpose, an approach to solve the multi-criteria optimization problem is proposed by ideal solution.

Presume that FRBS consists of 27 rules given in Table 3. Fuzzy terms of input variables described by TFNs:

$$A_i^j = (a_{il}^j ,\ a_{im}^j ,\ a_{ir}^j) \qquad (38)$$

where, j – represents the number of antecedents $j = 1 \div 3$. For example, for X_1 antecedent $j = 1$, for X_2 antecedent $j = 2$, for X_3 antecedent $j = 3$. i – describes membership functions of linguistic terms $i = 1 \div 3$, for example, for low case – L, $i = 1$, for medium case – M, $i = 2$, for High case – H, $i = 3$. l, m and r are left, medium, right sides of TFN, respectively.

We need to obtain the trade-off between the following criteria of FRBS:

1. Complexity measure
2. Coverage degree
3. Partition measure
4. Inconsistency
5. Accuracy

The following objective functions are formulated to solve the multi-criteria optimization problem based on quality criteria.

$$comp \to \max\,,\ \text{cov} \to \max\,,\ part_i \to \max$$

$$f_{incons} = \sum_{i=1}^{N} Incons(i) \to \min\,,\ J = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2} \to \min \tag{39}$$

Thus, 3 quality criteria (*comp*, *cov* and $part_i$) must get the maximum value within FRBS. Other 2 criteria (f_{incons} and *J*) must approach to minimum value.

Moreover, the following restrictions must consider achieving the good trade-off given in (39):

Constraints of fuzzy terms

1) Sequence low, *i=1,…,M*

$$a_{il}^{j} \le a_{im}^{j} \le a_{ir}^{j}$$
$$a_{i,\,l}^{j} \le a_{(i+1),\,l}^{j} \le a_{(i+2),\,l}^{j}\,;\ a_{i,\,m}^{j} \le a_{(i+1),\,m}^{j} \le a_{(i+2),\,m}^{j}\,; \tag{40}$$
$$a_{i,\,r}^{j} \le a_{(i+1),\,r}^{j} \le a_{(i+2),\,r}^{j}$$

2) Overlapping low

$$a_{i+1\,,\,l}^{j} \le a_{i,\,r}^{j} \tag{41}$$

For example,

$$a_{2\,,\,l}^{1} \le a_{1,\,r}^{1}\ ;\ a_{3\,,\,l}^{1} \le a_{2,\,r}^{1}\,;$$
$$a_{2\,,\,l}^{2} \le a_{1,\,r}^{2}\ ;\ a_{3\,,\,l}^{2} \le a_{2,\,r}^{2}\,; \tag{42}$$
$$a_{2\,,\,l}^{3} \le a_{1,\,r}^{3}\ ;\ a_{3\,,\,l}^{3} \le a_{2,\,r}^{3}\,.$$

3) Length of fuzzy number

$$a_{i\,m}^{j} - a_{i\,l}^{j} \ge d_{i\,l}^{j} \quad a_{i\,r}^{j} - a_{i\,m}^{j} \ge d_{i\,r}^{j} \tag{43}$$

This rule indicates that in each input variable, the difference between the medium and left sides of the term must be greater than or eqal to the length of its left side, and the difference between the right and medium sides must be greater than or equal to the length of the right side.

Constraints on collection of the rules

This constraint defines that the collection of the rules must be greater than or equal to *K*. In this case, the value of *K* is taken as 2/3 of all rules (minimum 2/3 of all rules may be used K=2/3R):

$$Rule\ 1 + Rule\ 2 + \ldots + Rule\ 27 \geq K \tag{44}$$

For a special case, if we take into account that there are 27 rules in the FRBS, then the value of *K* will be equal to 18:

$$Rule\ 1 + Rule\ 2 + \ldots + Rule\ 27 \geq 18$$

Constraints on collection of the terms:

The collection of the terms (Term 1, Term 2,…,Term Q) must be higher than or equal to N*S (minimum S terms are used for each input):

$$\text{Term } 1 + \text{Term } 2 + \ldots + \text{Term } Q \geq N*S \tag{45}$$

In this FRBS the collection of all terms will be

$$Q1 + Q2 + \ldots + Q9 \geq 6.$$

This means that at least 2 terms were used for each input variable.

Differential evolution method is proposed for the solution of multi-criterial optimization problem.

If 27 rules are included in the FRBS, the change of 5 quality parameters will be as follows based on 22 rules after 5 rules (Table 6). For simplicity and clarity, let's look at the following 4 cases.

Table 6.

Measure of 5 criteria for 4 cases

	comp	*cov*	*part*	f_{incons}	RMSE
1st case	0,045	0,588	0,5	0,0153	8,704
2nd case	0,045	0,584	0,5	0,0178	6,578
3rd case	0,045	0,592	0,5	0,0184	8,824
4th case	0,045	0,617	0,5	0,017	34,92

Before solving the multi-criterial optimization problem by ideal solution method, a normalized decision matrix consisting of *m* alternatives (cases) and *n* attributes (criteria) will be created by the following expression:

$$D = \begin{Bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{Bmatrix} \qquad (46)$$

Then (46) matrix is formalized:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad (47)$$

Thus, the elements of the matrix *D* are normalized. From the normalized elements, the values closest to the ideal solution are determined.

If we write the values given in Table 6, the following matrix is obtained according to 22 rules consisting of 4 cases (alternatives) 5 criteria (attributes).

$$D = \begin{Bmatrix} 0.045 & 0,588 & 0.5 & 0,0153 & 8.704 \\ 0.045 & 0,584 & 0.5 & 0,0178 & 6.578 \\ 0.045 & 0,592 & 0.5 & 0,0184 & 8.824 \\ 0.045 & 0,617 & 0.5 & 0.017 & 34.92 \end{Bmatrix}$$

Thus, the normalized values among parameters are as below (Table 7):

Table 7.

Normalized values for 5 criteria

	comp	*cov*	*part*	f_{incons}	RMSE
1st case	0,5	0,493795	0,5	0,44568547	0,231281767
2nd case	0,5	0,490436	0,5	0,5185099	0,17478992
3rd case	0,5	0,497154	0,5	0,53598776	0,234470394
4th case	0,5	0,518149	0,5	0,49520608	0,927890544

Then based on a normalized matrix, positive and negative ideal solutions can be identified.

$$A^{+} = \left\{ \left(\max_i v_{ij} \mid j \in J \right), \left(\min_i v_{ij} \mid j \in J' \right) \mid i = 1, 2, ..., m \right\} = \left\{ v_1^{+}, v_2^{+}, ..., v_j^{+}, ..., v_n^{+} \right\} \quad (46)$$

$$A^{-} = \left\{ \left(\min_i v_{ij} \mid j \in J \right), \left(\max_i v_{ij} \mid j \in J' \right) \mid i = 1, 2, ..., m \right\} = \left\{ v_1^{-}, v_2^{-}, ..., v_j^{-}, ..., v_n^{-} \right\} \quad (47)$$

where $J = \{ j = 1, 2, ..., n \}$, $J' = \{ j = 1, 2, ..., n \}$ and v_{ij} characterize parameters of normalized decision matrix - D_{norm}.

Thus, the positive and negative ideal solutions are as follows (Table 8 a, b):

Table 8 a.

Positive ideal solution

	Comp	*cov*	*part*	f_{incons}	RMSE
Ideal solution	0,5	0,518149	0,5	0,44568547	0,17478992

Table 8 b.

Negative ideal solution

	comp	*cov*	*part*	f_{incons}	RMSE
Ideal solution	0.5	0.490436	0.5	0.535988	0.927891

For the next step, the values closest to the ideal solution are calculated by using the Euclidean distance (S_i^{+}, S_i^{-}). In the final step, the values farthest from the negative ideal solution and closest to the positive ideal solution are determined.

$$c_i^* = \frac{S_i^-}{(S_i^+ + S_i^-)}, \quad 0 < c_i^+ < 1, \ \text{i=1,2,…,m} \tag{48}$$

c_i^* is computed in Table 9.

Table 9.

Determining proximity to ideal solutions

S_i^+	S_i^-	c_i^*
0.061518	0.702445	0.919475
0.077919	0.753303	0.90626
0.110259	0.693453	0.862813
0.754727	0.049307	0.061324

According to Table 9, the case closest to the positive ideal solution (shortest distance) and farthest from the negative ideal solution (longest distance) is Case 1. In comparison, we can state the result as follows:

$$Case-1 \succsim Case-2 \succsim Case-3 \succsim Case-4. \tag{49}$$

Multi-crierial optimization problem for fuzzy If-Then rules by fuzzy Pareto-Optimality

Assume that 2 solutions $A_1, A_2 \in \mathrm{A}$ are given. $nb(A_1, A_2)$, $ne(A_1, A_2)$, $nw(A_1, A_2)$ describes A_1 is better than A_2, A_1 is equal to A_2, A_1 is worse than A_2 respectively[12]. For the two solutions *nbF, neF, nwF* will be as:

$$nbF(A_1, A_2) = \sum_{i=1}^{M} \mu_b^i (f_i(A_1) - f_i(A_2)) \tag{50}$$

$$neF(A_1, A_2) = \sum_{i=1}^{M} \mu_e^i (f_i(A_1) - f_i(A_2)) \tag{51}$$

[12] Aliev, R. A.: Uncertain computation-based decision theory / Singapure: World Scientific, - 2017. 521p.

$$nwF(A_1,A_2)=\sum_{i=1}^{M}\mu_w^i(f_i(A_1)-f_i(A_2)) \quad (52)$$

A new matrix d(A_1, A_2) is created based on the variables *nbF, neF, nwF*.

$$d(A_1,A_2)=\begin{cases}0, & if \quad nbF \le \dfrac{M-neF}{2}\\ \dfrac{2\cdot nbF+neF-M}{nbF}, & otherwise\end{cases} \quad (53)$$

Briefly, if $d(A_1,A_2)=1$ then A_1 is pareto-dominant solution for A_2, if $d(A_1,A_2)=0$ then A_1 is not pareto-dominant solution for A_2.

In the last step, the degree of optimality $do:\mathcal{A}\to[0,1]$ must be calculated for the general case $A\in\mathcal{A}$.

$$do(A^*)=1-\max_{A\in\mathcal{A}} d(A,A^*) \quad (54)$$

Thus, according to Table 7, the values are normalized for the expressions (55) - (56).

$$x_{ij}^{norm}=\frac{x_{ij}-x_j^{min}}{x_j^{max}-x_j^{min}} \quad (55)$$

$$x_{ij}^{norm}=\frac{x_j^{max}-x_{ij}}{x_j^{max}-x_j^{min}} \quad (56)$$

As a result, the normalized decision matrix will be as follows (Table 10):

Table 10.

The values of normalized decision matrix

Cases	*comp*	*cov*	*part*	f_{incons}	RMSE
1st case	0	0.121212121	0	1	0.924988
2nd case	0	0	0	0.193548387	1
3rd case	0	0.242424242	0	0	0.920754
4th case	0	1	0	0.451612903	0

The values in Table 10 will be applied to Matlab software by using the following program code:

```
A(:,:,1)=[0  0.1212  0  1  0.92498; 0 0 0 0.1935  1;  0  0.2424  0  0  0.9207; 0  1  0
0.4516  0];
A(:,:,2)=[0  0.1212  0  1  0.92498; 0 0 0 0.1935  1;  0  0.2424  0  0  0.9207; 0  1  0
0.4516  0];
A(:,:,3)=[0  0.1212  0  1  0.92498; 0 0 0 0.1935  1;  0  0.2424  0  0  0.9207; 0  1  0
0.4516  0];
W=[0.2 0.2 0.2; 0.2 0.2 0.2; 0.2 0.2 0.2; 0.2 0.2 0.2; 0.2 0.2 0.2];  m = 4; n = 5;
```

Finally, the degree of optimality(*do*) for 4 cases will be as in Table 11.

Table 11.

Degree of optimality for 4 cases

Cases	*do*
1^{st} case	1
2^{nd} case	0.0809
3^{rd} case	0.1207
4^{th} case	0.5965

Thus, summarized Pareto-optimality result will be as follows:

$$Case-1 \succsim Case-4 \succsim Case-3 \succsim Case-2 .$$

In the fifth chapter, the application of specificity and other quality criteria of fuzzy rules are shown. Assume that fuzzy control system includes the following 7 rules[13].

Membership functions for error-*e* and control action-*u* are specified as below:

[13] Aliev, R. A., Aliev, F., Babaev, M.: Fuzzy Process Control and Knowledge Engineering in Petrochemical and Robotic Manufacturing // Köln: Verlag, - 1991, – 146 p.

Rule 1

$$\mu_{E1}(e) = 1.00/-10 + 0.73/-7 + 0.34/-3 + 0.20/0 + 0.13/3 + 0.08/7 + 0.06/10$$

$$\mu_{U1}(u) = 1.00/-1 + 0.92/-0.7 + 0.67/-0.3 + 0.50/0 + 0.37/0.3 + 0.26/0.7 + 0.20/1$$

Rule 2

$$\mu_{E2}(e) = 0.73/-10 + 1/-7 + 0.61/-3 + 0.34/0 + 0.20/3 + 0.11/7 + 0.08/10$$

$$\mu_{U2}(u) = 0.91/-1 + 1.00/-0.7 + 0.86/-0.3 + 0.67/0 + 0.50/0.3 + 0.34/0.7 + 0.26/1$$

Rule 3

$$\mu_{E3}(e) = 0.34/-10 + 0.61/-7 + 1.00/-3 + 0.73/0 + 0.41/3 + 0.20/7 + 0.13/10$$

$$\mu_{U3}(u) = 0.61/-1 + 0.86/-0.7 + 1.00/-0.3 + 0.92/0 + 0.74/0.3 + 0.50/0.7 + 0.37/1$$

Rule 4

$$\mu_{E4}(e) = 0.20/-10 + 0.34/-7 + 0.73/-3 + 1.00/0 + 0.73/3 + 0.34/7 + 0.20/10$$

$$\mu_{U4}(u) = 0.50/-1 + 0.67/-0.7 + 0.92/-0.3 + 1.00/0 + 0.92/0.3 + 0.6726/0.7 + 0.50/1$$

Rule 5

$$\mu_{E5}(e) = 0.13/-10 + 0.20/-7 + 0.41/-3 + 0.73/0 + 1.00/3 + 0.61/7 + 0.34/10$$

$$\mu_{U5}(u) = 0.37/-1 + 0.50/-0.7 + 0.74/-0.3 + 0.92/0 + 1.00/0.3 + 0.86/0.7 + 0.67/1$$

Rule 6

$$\mu_{E6}(e) = 0.08/-10 + 0.11/-7 + 0.20/-3 + 0.34/0 + 0.61/3 + 1.00/7 + 0.73/10$$

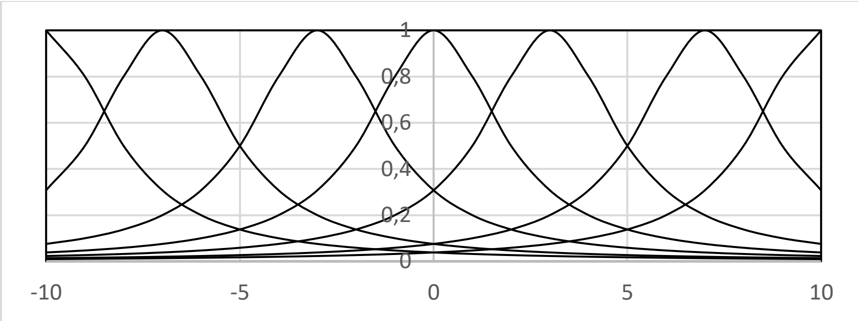
$$\mu_{U6}(u) = 0.26/-1 + 0.34/-0.7 + 0.50/-0.3 + 0.67/0 + 0.86/0.3 + 1.00/0.7 + 0.91/1$$

Rule 7

$$\mu_{E7}(e) = 0.06/-10 + 0.08/-7 + 0.13/-3 + 0.20/0 + 0.34/3 + 0.73/7 + 1.00/10$$

$$\mu_{U7}(u) = 0.20/-1 + 0.26/-0.7 + 0.37/-0.3 + 0.50/0 + 0.67/0.3 + 0.92/0.7 + 1.00/1$$

The graphical representation of membership functions is given in Figure 7:



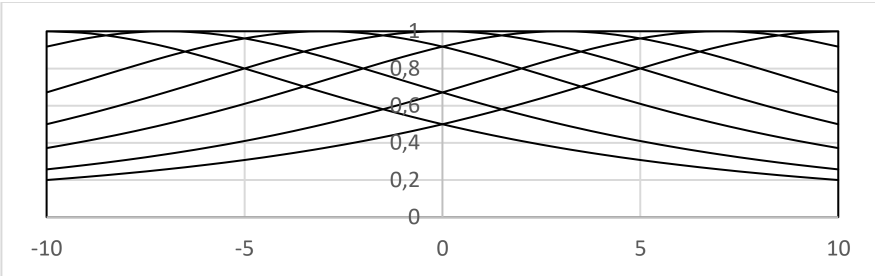


Figure 7. Membership functions of error and control action

The composed fuzzy relation matrix is as follows:

1	0.92	0.67	0.5	0.37	0.26	0.2
0.73	0.73	0.67	0.5	0.37	0.26	0.2
0.34	0.34	0.34	0.34	0.34	0.26	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.08	0.08	0.08	0.08	0.08	0.08	0.08
0.06	0.06	0.06	0.06	0.06	0.06	0.06

The problem is to investigate and increase specificity of the control system[14] .

0.5	1	0.92	0.67	0.5	0.37	0.26	0.2
1	0.73	0.73	0.67	0.5	0.37	0.26	0.2
0.65	0.34	0.34	0.34	0.34	0.34	0.26	0.2
0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.3	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.15	0.08	0.08	0.08	0.08	0.08	0.08	0.08
0.1	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Consequents are formalixed based on the folloüing formula. Then obtained relation matrix will be:

$$\mu_U = \max_e \ \min \ [\mu_E(e^c_{SC}), \mu_R(u,e)] \qquad (57)$$

0.5	1	0.92	0.67	0.5	0.37	0.26	0.2
1	0.73	0.73	0.67	0.5	0.37	0.26	0.2
0.65	0.34	0.34	0.34	0.34	0.34	0.26	0.2
0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.3	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.15	0.08	0.08	0.08	0.08	0.08	0.08	0.08
0.1	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	0.73	0.73	0.67	0.5	0.37	0.26	0.2

The degree of specificity for antecedent and consequent variables of the relation matrix will be calculated by using (7).

Sp(x)=0.527, *Sp*(u)=0.181.

---

[14] Adilova, N. E.: Construction of Fuzzy Control System Rule-Base with Predefined Specificity // Advances in Intelligent Systems and Computing, Springer, - 2018, - p. 901 - 904.

where $Sp(x)$-specificity for antecedent, $Sp(u)$- specificity for consequent part

Assume that different antecedent values were given to the control system. Then the consequent values and specificity will be as:

1	1	0.92	0.67	0.5	0.37	0.26	0.2
0.5	0.73	0.73	0.67	0.5	0.37	0.26	0.2
0.1	0.34	0.34	0.34	0.34	0.34	0.26	0.2
0	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.65	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.3	0.08	0.08	0.08	0.08	0.08	0.08	0.08
0	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	1	0.92	0.67	0.5	0.37	0.26	0.2

$Sp(\text{x})=0.561667$; $Sp(\text{u})=0.354738$.

In conclusion, the degree of specificity of the relation matrix was modified with a new relationship matrix. The degree of specificity has been increased for both antecedent and consequent part of the matrix. The obtained results imply that IF-THEN model for considered control system is not perfect. We can update it by altering (increasing) predefined specificity.

As another application area, the estimation of specificity and other quality criteria in terms of impact factors to the customer's purchase intention in supermarkets was considered.

Primary data for the investigation were collected through a questionnaire-based survey from 300 customers. The main purpose is to calculate quality criteria of rule base and define how much impact the environmental factors have on the purchase intention of customers.

A customer’s purchase intention may depend on a complex set of factors, such as ambient factors, design factors, social factors, and customer experience. Figure 8 shows the conceptual model of the research.

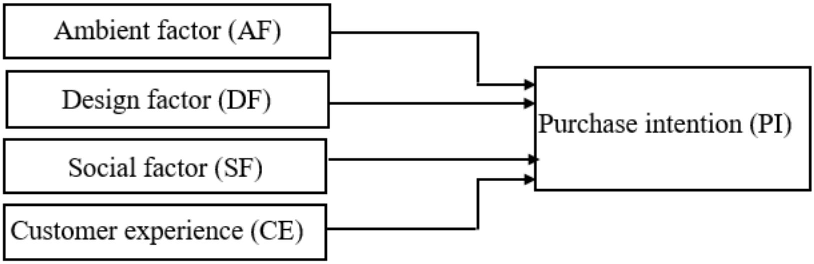


Figure 8. The conceptual framework of the relationship between in-store environmental attributes

The conceptual model consists of 4 factors (Ambient, Design, Social factor, and Customer experience) that play an influential role in the process and 1 output factor (Purchase intention). The result of questionnairy is structured in FRBS regarding to the opinion of the experts (Table 12).

Table 12.

IF-THEN rules

Rule	**IF**				**THEN**
№	**AF**	**DF**	**SF**	**CE**	**PI**
1	VH	VH	VH	H	VH
2	L	H	L	N	N
3	H	H	H	N	H
4	VH	VH	VH	N	VH
5	N	N	N	N	N
6	H	H	H	H	H
7	H	N	H	N	H
8	N	VH	N	N	H
9	H	VH	VH	H	VH
10	L	N	L	N	L
11	L	L	L	L	L
12	VL	VL	VL	VL	VL
13	N	L	N	L	L

VL, *L*, *N*, *H*, *VH* are linguistic terms for *Very low*, *Low*, *Neutral*, *High*, *Very high* respectively. The codebook for linguistic terms is described in Table 13.

Table 13.

Values of linguistic terms

Very low	(0, 0.2, 0.35)
Low	(0.2, 0.35, 0.5)
Neutral	(0.35, 0.5, 0.75)
High	(0.5, 0.75, 1)
Very high	(0.75, 1, 1)

The graphical description of rule base corresponding to 5 linguistic terms shown in Table 13 is described by the Matlab software (Figures 9 - 10).

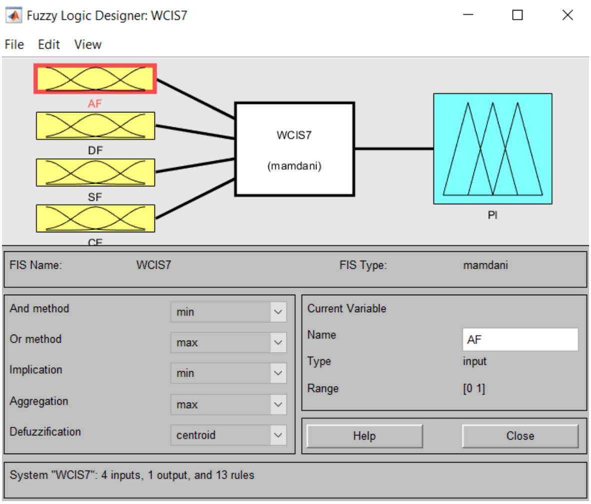


Figure 9. Antecedents and consequent for fuzzy rule base

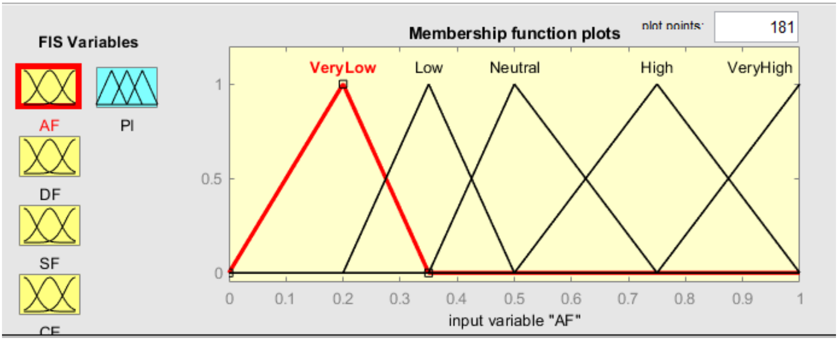


Figure 10. Membership functions for Ambient factor

The rules formed on Table 12 are shown in the software (Figure 11).

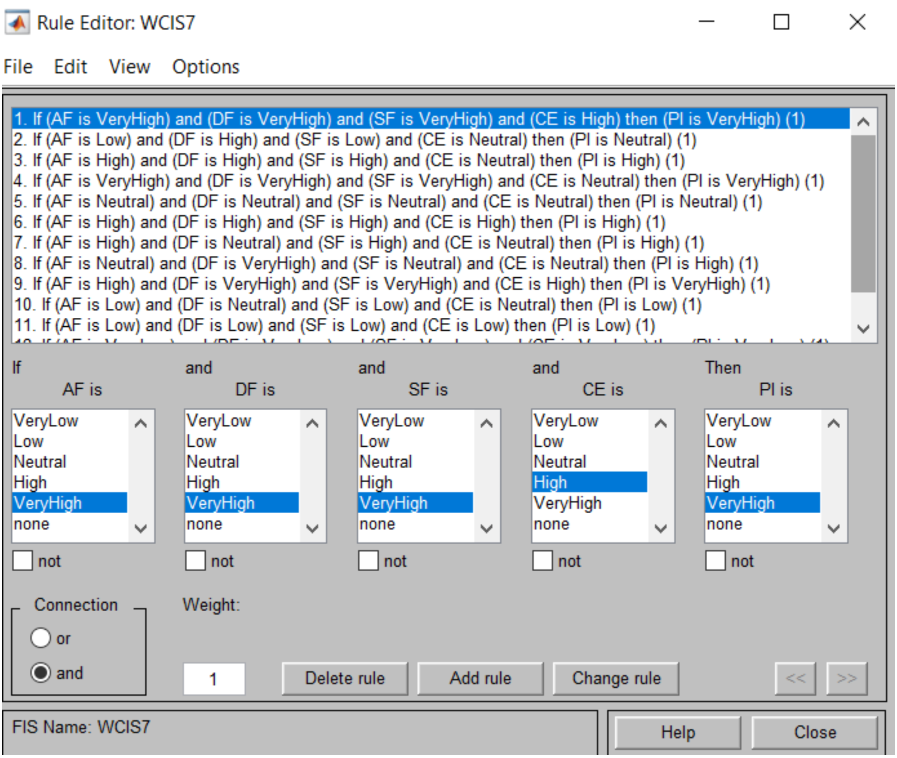


Figure 11. Fuzzy rules

Then the specificity is calculated based on (8). As a result, specificity will be

$$Sp = 0.82.$$

The coverage index will be computed considering 20 granulation points of each input variable (Table 14):

Table 14.

Calculation of coverage index

	X	μ_{A-1}	μ_{A-2}	μ_{A-3}	μ_{A-4}	μ_{A-5}	$h_i(x)$	$h_i(x)$
x1	0	0	0	0	0	0	0	0
x2	0.1	0.5	0	0	0	0	0.5	0.5
x3	0.15	0.75	0	0	0	0	0.75	0.75
x4	0.2	1	0	0	0	0	1	1
x5	0.25	0.666667	0.333333	0	0	0	1	1
x6	0.3	0.333333	0.666667	0	0	0	1	1
x7	0.35	0	1	0	0	0	1	1
x8	0.4	0	0.666667	0.333333	0	0	1	1
x9	0.45	0	0.333333	0.666667	0	0	1	1
x10	0.5	0	0	1	0	0	1	1
x11	0.55	0	0	0.8	0.2	0	1	1
x12	0.6	0	0	0.6	0.4	0	1	1
x13	0.65	0	0	0.4	0.6	0	1	1
x14	0.7	0	0	0.2	0.8	0	1	1
x15	0.75	0	0	0	1	0	1	1
x16	0.8	0	0	0	0.8	0.2	1	1
x17	0.85	0	0	0	0.6	0.4	1	1
x18	0.9	0	0	0	0.4	0.6	1	1
x19	0.95	0	0	0	0.2	0.8	1	1
x20	1	0	0	0	0	1	1	1

According to Table 14, the coverage index (16) will be:

$$cov = 0.9125.$$

The complexity index (15) will be determined for the number of linguistic terms of the each input variables in the rule base

$$comp = 0.217.$$

Then partition index is obtained for (17) formula:

$$part = 0.045.$$

So interpretability index based on 3 measures (coverage, complexity, partition) will be obtained

$$I = comp * cov * part = 0.9125 * 0.217 * 0.045 = 0.09.$$

This result is explained by the fact that the rules consist of 5 linguistic terms, 4 different input variables, and as a result, the interpretability index decreases.

As the next application, let us investigate improving the quality of fuzzy If-Then model for a control system, including the specificity of the.

For simplicity, suppose that IF-THEN model of a control system is constructed based on the journey schedule as below:

1. IF *Distance* is close, *vehicle's speed* is fast THEN *duration* of the journey is short.
2. IF *Distance* is far, *vehicle's speed* is slow THEN *duration* of the journey is long.
3. IF *Distance* is too far, *vehicle's speed* is very slow THEN *duration* of the journey is too long.

The model is organized with 2 input variables (Distance and Vehicle's speed) and 1 Output variable (Duration of the journey) generally including the following triangular fuzzy values for the linguistic terms:

For *Distance*: *too far* = (0; 0.13; 0.25); *far* = (0.3; 0.35; 0.45); *close* = (0.75; 0.8; 0.88);

For *Vehicle's speed*: *very slow* = (0.05; 0.15; 0.3); *slow* = (0.25; 0.4; 0.45); *fast* = (0.65; 0.72; 0.8);

For *Duration*: *too long* = (-0.99; -0.85; -0.7); *long* = (-0.6; -0.55; -0.45); *short*=(0.5; 0.65; 0.75);

Figure 12 characterizes the description of membership functions. $\mu(A_1)$, $\mu(A_2)$, $\mu(A_3)$ reflects the first, second and third linguistic terms, respectively.

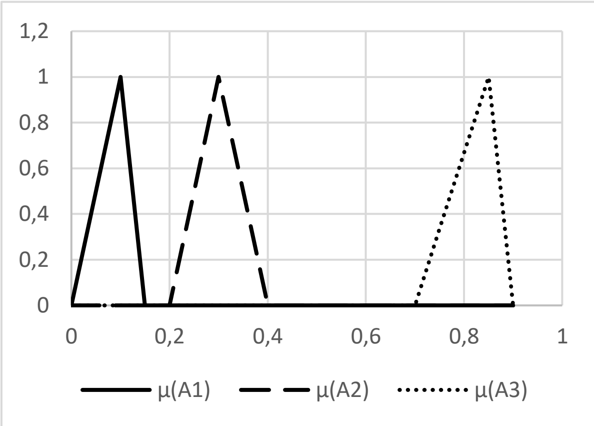


Figure 12. Description of membership functions

Proportionately, the computational result for the mentioned quality criteria and specificity is stratified in Table 15.

Table 15.

Measures of quality criteria for the initial IF-THEN model

Criteria of IF-THEN model	Values
Complexity	0.5
Coverage	0.298
Partition	0.2
Inconsistency	0.124
Specificity	0.934

The reason why the accuracy criterion, which is considered one of the quality criteria for a fuzzy rule base, is not considered for this If-Then model, is that the model consists of 3 rules. İf at least one rule is removed from the rule base, then informativeness of the model can be lost. Thus, 5 quality criteria, including specificity, were calculated.

Consequently, interpretability index based on the product of first 3 criteria will be I=0.0298.

For the next step, investigation will be expanded by changing the degrees of membership functions and gaining a better result.

For Distance: *too far* = (0; 0.2; 0.35); *far* = (0.2; 0.35; 0.5);
close = (0.5; 0.75; 1);

For *Vehicle's speed*: *very slow*=(0; 0.1; 0.15); *slow*=(0.2; 0.3; 0.4);
fast=(0.7; 0.85; 0.9);

For *Duration*: *too long* = (-1; -0.7; -0.55); *long* = (-0.55; -0.4; -0.15);
short = (0.2; 0.35; 0.55);

Thus, the values of the quality criteria for the updated fuzzy If-Then model of the journey schedule are summarized as in Table 16.

Table 16. Measures of quality criteria for the second IF-THEN model

Criteria of IF-THEN model	Values
Complexity	0.5
Coverage	0.446
Partition	0.2
Inconsistency	0.087
Specificity	0.870

Clearly seems from the experimental outcome, which is reported in Table 16, interpretability measure is I=0.0446.

Final step of the experimental analysis is devoted to create a balance among criteria with the assistance of the following goal functions:

$$comp \to \max,\ \mathrm{cov} \to \max,\ part_j \to \max,\ 1 - f_{incons} \to \max,$$
$$Sp(A) \to \max$$

At the end, optimality degrees between the initial and the second case will be summarized as below (Table 17):

Table 17. Degrees of optimality for 2 cases

Alternatives	*do*
Case-1	0.1811
Case-2	1

The obtained results show that the fuzzy IF-THEN model for the journey schedule control system is not perfect. The author updated rule-base by investigation of the quality criteria. Thereupon Case-2 is the case that represents a better model for the control system.

RESULTS

The main **scientific results** obtained in the dissertation are as follows:

1. Different methods of the assessment of specificity of fuzzy and Z-based rules have been explored, and suggested a more efficient and alternative solution based on comparative analysis.
2. Quality criteria (accuracy, coverage, complexity, partition, inconsistency) for fuzzy If-Then model of control systems were identified, and a systematic approach to achieving a balance among them was proposed.
3. The problem of multi-criterial optimization of information and fuzzy granulation in fuzzy If-Then rules has been solved.
4. A method based on an ideal solution for multi-criteria optimization problem has been proposed.
5. A method based on the principle of pareto-optimality for optimizing the quality of fuzzy If-Then models of decision-making and control systems has been investigated and synthesis of a model close to the has been obtained.
6. Theoretical findings suggested in this dissertation were applied for the purpose of investigation of the activity and efficiency of the methods by using computer simulation. Experimental results show the validity and usefulness of the proposed approaches.

The main content of the dissertation is published in the following works:

1. Valiev, A. A., Abdullayev, T.S., Alizadeh, A.V., Adilova, N.E. Comparison of measures of specificity of Z-numbers // Procedia Computer Science, - 2017, - vol. 120, - p. 466-472.
2. Adilova, N. E. Construction of Fuzzy Control System Rule-Base with Predefined Specificity // Advances in Intelligent Systems and Computing, Springer, - 2018, - p. 901-904.
3. Aliev, R. A. Huseynov O. H., Adilova N. E. Multi-criterial optimization of information granules in fuzzy IF-THEN rules // B-quadrat verlags, - 2018, - p. 52-55.
4. Adilova, N. E. Consistency of Fuzzy IF-THEN rules for control system // Advances in Intelligent Systems and Computing, Springer, - 2019, - p. 137-143.
5. Adilova N.E. Qeyri-səlis idarəetmə sistemlərində qaydalar bazasının spesifikliyinin yoxlanılması. Azərbaycan Neft Təsərrüfatı, 12.2019, İSSN 0365-8554, səh. 49-51
6. Adilova, N.E. Qeyri-səlis idarəetmə sistemlərində qaydalar bazasının anlaşıqlıq meyarının ölçülməsi // Azərbaycan Neft Təsərrüfatı jurnalı, 01.2021, №256, səh. 35-36
7. Adilova, N. E. Quality criteria of fuzzy IF-THEN rules and their calculations. Advances in Intelligent Systems and Computing // Springer International Publishing, - 2021, vol. 1306, - p. 55-62.
8. Huseynov, O.H., Adilova, N. E. Multi-criterial optimization problem for fuzzy IF-THEN rules // Advances in Intelligent Systems and Computing, Springer International Publishing, - 2021, 1306, - p. 80-88.
9. Nigar E. Adilova, Qeyri-səlis proqnozlaşdirma əsasinda sərnişin daşinmaları fəaliyyətinin analizi // Bakı Mühəndislik Universitetinin keçirdiyi ümummilli lider Heydər Əliyevin anadan olmasının 98-ci ildönümünə həsr olunmuş gənc tədqiqatçıların V beynəlxalq elmi konfransı, 29-30 aprel 2021. – səh. 887-891.

Author's individual participation in the published works

[1]-Problem statement, analysis of findings;

[3]-Author of the idea, calculations and analysis of findings;

[8]-Author of the idea, problem statement and computer simulation.

The defense of the dissertation will be held on January 11, 2022 at 1 PM at the meeting of the Dissertation Council ED 2.02 operating under the Azerbaijan State Oil and Industry University.

Address: AZ1010, Baku, Azadlig Avenue 34

The dissertation is available in the library of the Azerbaijan State Oil and Industry University.

Electronic versions of the dissertation and abstract are posted on the official website of the Azerbaijan State Oil and Industry University.

The abstract was sent to the necessary addresses on November 29, 2021.