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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

VIBRATION OF REINFORCED CONICAL SHELLS CONTACT WITH THE MEDIUM

Specialty: 3305.02 –Structural mechanics

Field of science: Engineering

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GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the topic.
Structures having the shape of a truncated cone can be used in engineering and civil engineering as an independent structure or as an element of a complex structure. A construction formed from the connection of cylindrical and spherical form elements can be shown as an example of a complex structure. When designing structures containing fluid in the form of a load, there arise complex problems related to their strength and stability. Here first of all the interaction force between the frequency and the form of free vibrations of liquid and also betüeen the wall of the shell and liquid must be determined.
Determination of interaction force between the wall of the shell and liquid plays an important role in recording the equation of motion of a mechanical system, in predicting dynamical behavior of the structure from the influence of external vibration, seismic nature different perturbation forces. Such issues usually arise at the stages of technical development of large-sized machines running on liquid fuel. In the general form, there are no methods for solving the problems of vibrations of liquid in a conical shell.
N.P.Abovski, I.Y.Amiro, M.B.Akhundov, J.A.Agalarov, H.A.Alumyaen, R.M.Bergman, E.H.Qrigolyuk, V.Z. Grisakin, A.N.Alizade, R.Y.Amanzade, M.F.Mekhtiyev, Kh.M.Mushtarin, I.N.Preobrajenski, I.T.Pirmammadov, F.S.Latifov, R.A.Iskenderov, A.I.Lourie, S.R.Timoshenko, V.Z.Vlasov, A.S.Volmir, A.A.Seyful-layev, V.A.Zarutski and others have studied dynamical rigidity characteristics of a liquid or solid medium-contacting conical shell.

Goals and objectives of the study. The goal of the work is to study free vibrations freguencies of the system consisting of a circular closed truncated cone stiffened with regularly located ribs and a mass associated to the shell with the same rigidity two springs at the diametrically opposite points to the shell.

Research methods. Second order Lagrange equation, Winkler and Pasternak models have been used in the implementation of the problem.

The main theses to be defended.

1. Physical and mathematical model of the problem was built to study free vibrations frequency of the system consisting of a circular closed truncated cone stiffned with ribs and interacting with inhomogeneous solid medium and a mass associated with the same rigidity two springs at diametrically opposite points to the shell was built;
2. Equations for finding free vibrations frequencies of the object under consideration were built by using the Lagrange equation;
3. The influence of physical and mechanical parameters characterizing the object to free vibrations frequencies of the object under consideration were studied.

Scientific novelty of the study. In this work , a model of the problem of studying free vibrations of a system consisting of a circular closed section of a conical shell reinforced by discretely arranged shafts on its surface, rings in contact with a homogeneous and inhomogeneous solid medium, and a mass combined by two springs of the same hardness at diametrically opposite points of the coating is constructed, and based on the Lagrange equation, equations are obtained for finding the frequency of free oscillations of the studied object. In the dissertation work, three variants of stiffening of the conic shell were considered: 1) with ribs stiffened in the direction of generatrix; 2) with ring-shaped ribs; 3) with ribs perpendicularly located on the surface. In the indicated cases, an equation for determining natural vibrations of the object under consideration was written and its roots were implemented by the numerical method.

The dependence of these frequencies on the quantities determining the object was studied.

Exatness of the results. Second kind Langrange equation, Winkler and Pasternak models have been used when implementing the problem.

Theoretical and practical importance of the study.

The results of the dissertation work can be used in calculating inhomogeneous medium-contacting, stiffened conic shell, in studying the stability of the objects used in industry and construction or their elements.

Author's personal contribution. In the dissertation work the stated scientific issues and the obtained main scientific results have been obtained by the author. Scientific results of the conducted research and the papers based on them and conference materials were discussed by the supervisor and coauthors. The main goals of the study and the problems for succeeding them were shown and discussed.

Approbation and application. The main scientific results of the work have been discussed in the seminars of the departments of "Mechanics", "Resistance of materials", "Construction structures", "Higher Mathematics" departments of Azerbaijan University of Architecture and Construction (2016-2019), of the departments of "Theoretical and continuum mechanics" (2019), in the departments of "Elasticity theory", "Wave Mechanics" of IMM of ANAS (2019), in the department of "Mathematics" ATU (2017-2019), in the International conference "Actual problems of applied mechanics and strength of constructions" organized by Ukranian National Academy and Zaparozhye National University (Zaporozhye city, 2017), in the International conference "Technical and physical problems in power engineering" (Turkey, 2017).

Publications. 11 scientific works related to the results of the topic of the dissertation work have been published in republican and foreign scientific-technical editions.

The name of the organization where the dissertation work was performed. The work was performed at the Azerbaijan University of Architecture and Construction

Total volume of the dissertation work in signs indicating separately the volume of each structural unit. The volume of the dissertation work consists of an introduction, four chapters, a conclusion and a list of used literature. The total volume of the work consists of 151 pages, which includes 13 figures, 45 graphs and a list of used literature. The dissertation work consists of 198754 characters, including the total number of characters for the introduction section 9260, chapter I 13509, chapter II 86307, chapter III 44914, chapter IV 41626, a conclusion and a list of used literature of 85 titles. The publication of the dissertation found its reflection in 11 scientific articles.

BRIEF CONTENT OF THE DISSERTATION WORK

The rationale of the topic of the dissertation work justified, information on the goal of the work, scientific noveeties, practical importance, the reliability of the obtained results, the theses to be defended, approbation of the work, its structure and volume was given in the introduction.

Chapter I consists of three sections and deals with brief summary of the work close to the topic of the dissertation work.

Chapter II was devoted to studying free vibrations of a system consisting of a circular closed truncated conical shell stiffened with ribs, rings and dynamically interacting with elastic medium and a mass associated with the same rigidity two springs at the diametrically opposite points to the shell. This chapter consists of four sestions. The statement of the problem of studying free vibrations of a system consisting of a circular close truncated conical shell stiffened with ribs, rings and interacting with elastic medium and a mass associated with the same rigidity two springs is given in section 1.

The work considers a system consisting of a medium-contacting circular, closed truncated conical shell stiffened with regularly located rods on the surface and a mass associated with the same rigidity two springs. It is considered that the ends of the springs were connected to the cone at the diametrically opposite points on the surface $\varphi = 0$. Three stiffenings of the conical shell were considered: 1) the ribs located along the generatrix (figure 1); 2) with ribgs to the surface perpendicular to the axis (figure 2); 3) the rods and rings forming an orthogonal network (figure 3). A coordinate system consisting of a varying r radius, two-sided φ angle between diametrical surfaces is used when solving the problem.

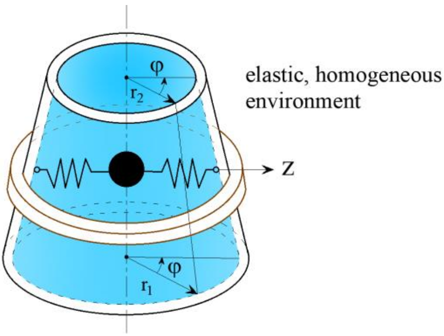

Figure1.

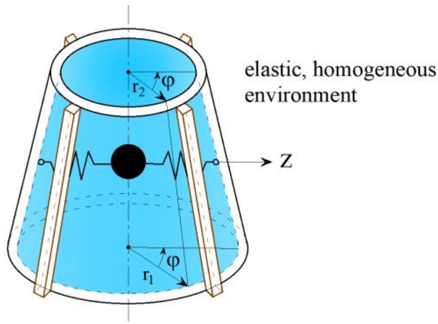

Figure 2.

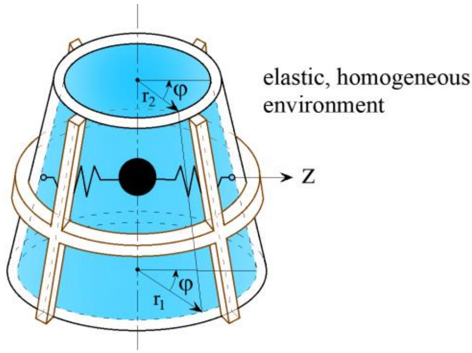

Figure 3.

Total energy of the system consisting of a circular, closed solid medium-contacting truncated cone stiffened with regularly distributed rods and a mass associated with the same rigidity two springs in the diametrically opposite points to the shell consists of the sum of the work done by the conical shell, potential and kinetic energies of longitudinal rods and rings, kinetic energy of the spring associated mass, of potential energies of the spring pressure force as viewed from medium at the displacements of the ponits of the conical shell:

$$L = \frac{Eh}{12\left(1-\nu^2\right)} \int_{r_2}^{r_1} \int_{0}^{2\pi} \left(\varepsilon_1^2 + \varepsilon_2^2 + 2\nu\varepsilon_1\varepsilon_2 + \frac{1-\nu}{2}\psi^2 \right) \frac{rdrd\varphi}{sin\gamma} + \qquad (1)$$

$$+\frac{D}{2}\int_{r_2}^{r_1}\int_{0}^{2\pi}\left(\chi_1^2+\chi_2^2+2\nu\chi_1\chi_2+2(1-\nu)\tau^2\right)\frac{rdrd\varphi}{sin\gamma}+$$

$$+\frac{1}{2}\sum_{i=1}^{k_1}\int_{r_2}^{r_1}\left[\tilde{E}_iF_1\left(\frac{\partial u}{\partial r}\sin\gamma\right)^2+\tilde{E}_iI_1\left(\frac{\partial^2 w}{\partial r^2}\sin^2\gamma\right)^2+\tilde{E}_i\tilde{I}_1\left(\frac{\partial^2\vartheta}{r^2\partial\varphi^2}\right)^2+\right.$$

$$\left.+\tilde{G}_iI_{1kp}\left(\frac{\partial^2 w}{r\partial r\partial\varphi}\sin\gamma\right)^2\right]_{\varphi=\varphi_i}\frac{dr}{\sin\gamma}+\left(z-w_0cos\gamma\right)^2+$$

$$+\frac{1}{2}\sum_{j=1}^{k_2}\int_{0}^{2\pi}\left[E_jF_2\left(\frac{\partial\vartheta}{r\partial\varphi}+\frac{w}{r}\right)+G_jI_{jkp}\left(\frac{\partial^2 w}{r\partial r\partial\varphi}sin\gamma\right)^2+E_jI_j\left(\frac{\partial^2 w}{r^2\partial\varphi^2}+\frac{w}{r^2}\right)^2\right]\Bigg|_{r=r_j}+$$

$$+A-\frac{\gamma_1 h}{2g}\int_{r_2}^{r_1}\int_{0}^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2\right]\frac{rdrd\varphi}{\sin\gamma}-\frac{1}{2}M\dot{z}^2-$$

$$-\frac{\gamma_1F_1}{2g}\sum_{i=1}^{k_1}\int_{r_2}^{r_1}\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2\right]_{\varphi=\varphi_i}\frac{dr}{\sin\gamma}-$$

$$-\frac{\gamma_1F_2}{2g}\sum_{j=1}^{k_2}\int_{0}^{2\pi}\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2\right]_{r=r_j}d\varphi$$

Here,

$$\varepsilon_1=\frac{\partial u}{\partial r}sin\gamma,\varepsilon_2=\frac{u}{r}sin\gamma+\frac{\partial\upsilon}{r\partial\varphi}+\frac{w}{r}cos\gamma,\psi=\frac{\partial u}{r\partial\varphi}+\frac{\partial\upsilon}{\partial r}sin\gamma-\frac{\upsilon}{r}sin\gamma,$$

$$\chi_1=-\frac{\partial^2 w}{\partial r^2}\sin^2\gamma;\chi_2=-\frac{u}{r^2}sin\gamma cos\gamma-\frac{w}{r^2}\cos^2\gamma-\frac{\partial^2 w}{r^2\partial\varphi^2}-\frac{\partial w}{r\partial r}\sin^2\gamma;$$

$$\tau=-\frac{cos\gamma}{r}\left(\frac{\partial u}{r\partial\varphi}-\frac{\partial\upsilon}{\partial r}sin\gamma+\frac{\upsilon}{r}sin\gamma\right)-\frac{2sin\gamma}{r}\left(\frac{\partial^2 w}{\partial r\partial\varphi}-\frac{\partial w}{r\partial\varphi}\right);\qquad (2$$

The influence of the solid medium to the conical shell is in the from of the work done by the external q_r radial force at the displacemenets of the shell:

$$A=-\int_{r_2}^{r_1}\int_0^{2\pi}\frac{q_r r}{\sin\gamma}drd\varphi \qquad (3)$$

In expressions (1)-(3) k_1 is the amount of longitudinal rods, k_2 is the amount of rings, E is a modulus of elasticity; h is the shell thickness; ν is a Poisson ratio; r_1, r_2 are the radii of the big and small seats of the shell; γ is an angle between the generatix and axis of the shell; u, v, w are the components of the displacement vector of the mean surface points of the shell in the direction of the generatrix, in the tangential direction and in the direction of the normal of the mean surface, $F_1, I_1,$ $\widetilde{I_1}$, $I_{1kpл}$ are the area of cross-section of the longitudinal rib and inertia moments of this section with respect to tangential and radial axes, respectively and also inertia moments in torsion; $F_2, I_2,, I_{2kp}$ is the area of the cross section of the ring and inertia moment coinciding with the generatrix of this area with respect to the axis; $\widetilde{E_\iota}, E_j$ are the module of elasticity of the cross-sections of the longitudinal rib and of the j −th rib; $\tilde{G}_\iota, G_j$ are module of elasticity in shear of the cross-sections of the i −th longitufinal rib and of the j −th ring in shear; k_1, k_2 are the amounts of longitudinal ribs and rings, respectively; φ_i, r_j are location coordinations of the longitudinal rib and ring, respectively; γ_1 is the gravity of the shell and rib materials, g is acceleration, q_r in the force of influence of the medium to the conical shell, M is the load mass

The following models were applied for taking into account the influence of the medium:

a) The Winkler model: $q=kw$, k – is a constant;

b) The Pasternak model: $q=\left(\tilde{q}+\tilde{q}_0\frac{d^2}{dx^2}\right)w$, $\tilde{q}$, $\tilde{q}_0$ and q, q_0 are elastic constants.

To the expression (1) we add the following contact and boundary conditions. It is considered that hard contact conditions berween the conic shell and rods are satisfied:

$$u_i(x) = u(x, \varphi_i), \vartheta_i(x) = \vartheta(x, \varphi_i), w_i(x) = w(x, \varphi_i), \tag{4}$$
$$u_j(x) = u\left(x_j, \varphi\right), \vartheta_j(x) = \vartheta\left(x_j, \varphi\right), w_j(x) = w\left(x_j, \varphi\right);$$

II kind Lagrange equation is used in solving the problem[1].

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \tag{5}$$

So, the study of vibrations of the system consisting of a circular, closed, solid-medium-contacting truncated shell stiffened with uniformly distributed rods in the surface and a mass associated with the same rigidity two springs at the diametrically opposite points to the shell is reduced to the integration of the Lagrange equation (5) taking into account the expression (1).

Section 2 studies free vibrations frequencies of solid-medium contacting circular closed truncated conical shell stiffened with ribs dynamically contacting with elastic medium and a mass associated with the same rigidity 2 springs at diametrically opposite points to the shell. In this case to obtain the expression for L in the expression (1) we should take $E_j = 0$ and $F_2 = 0$.

The displacements of the points of the shell contained in the expression (1) are sought in the form:

$$w = \frac{(r_2 + x sin\gamma)^2}{r_1^2} sin\frac{m\pi x}{l} \sum_{n=1}^{\infty} A_n(t) cosn\varphi\,; \tag{6}$$
$$\vartheta = \frac{(r_2 + x sin\gamma)^2}{r_1^2} sin\frac{m\pi x}{l} \sum_{n=1}^{\infty} B_n(t) \sin n\varphi$$
$$u = \frac{(r_2 + x sin\gamma)^2}{r_1^2} cos\frac{m\pi x}{l} \sum_{n=1}^{\infty} D_n(t) cosn\varphi$$

Here n are wave numbers in the circular direction, m are the wave numbers in the direction of the generatrix, $A_n(t)$, $B_n(t)$, $D_n(t)$ are unknown contants.

The total energy of the rows contained in (6) with respect to the n −th addend is as follows:

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[1] Bukhgols N.N. Basic course of theoretical mechanics, part two / - Publishing house "Nauka", Main Editorial Board of Physics and Mathematics Literature, Moscow: - 1969, - p. 332.

$$L=\varphi_{11}D_n^2(t)+\varphi_{22}A_n^2(t)+\varphi_{33}B_n^2(t)+\varphi_{44}A_n(t)D_n(t)+$$
$$+\varphi_{55}A_n(t)B_n(t)+(z-w_0cos\gamma)^2c++\varphi_{66}{A'_n}^2+\varphi_{77}{D'_n}^2+$$
$$+\varphi_{88}{B'_n}^2+\frac{1}{2}M\dot{z}^2 \qquad (7)$$

here

$$\varphi_{11}=\frac{\pi Eh}{2(1-\nu^2)r_1^4\sin\gamma}\left[\left(\frac{2+\alpha}{\alpha}R_4+\frac{\alpha^2}{48}R_6+\frac{5}{2\alpha}(r_1^2+r_2^2-12)R_2\right)+\frac{\sin^2\gamma}{\pi}R_4+\right.$$
$$\left.+\frac{3(r_1+r_2)sin\gamma r_1^4}{8\pi^2m^2}R_1+2\nu sin^2\gamma\frac{7}{20}R_4+\frac{4\alpha-5}{\alpha^2}R_2+\frac{60-24\alpha}{\alpha^4}\right]+$$
$$+\frac{D\pi}{4r_1^4}sin\gamma cos^2\gamma R_1+\frac{D(1-\nu)\pi}{4r_1^4}\frac{\cos^2\gamma}{\sin\gamma}R_2+\frac{\pi Eh}{4(1+\nu)r_1^4\sin\gamma}$$
$$\left(\frac{1}{8}R_4+\frac{3}{2\alpha^2}R_2\right)n^2;\ \varphi_{22}=\frac{\pi Eh}{2(1-\nu^2)r_1^4\sin\gamma}\times\frac{1}{8}cos^2\gamma R_4+$$
$$+\frac{D\pi}{2r_1^4}sin^3\gamma\left[2R_1+\left(\frac{3}{2\propto}+\frac{49}{8}\right)R_2-\left(\frac{\propto}{16}+\frac{5\propto^2}{16}\right)R_4+\frac{\propto^4}{32}R_6\right]+$$
$$+\left\{\frac{D\pi}{2r_1^4}\left[\frac{cos^4\gamma}{sin^2\gamma}T_1+4T_1-\frac{1}{2}T_2cos^2\gamma-T_2sin^2\gamma\right]+\left(\frac{\propto^2}{8}R_5+\frac{1}{2}R_3-\right.\right.$$
$$\left.3R_1\right)sin^2\gamma++\frac{\nu D\pi}{r_1^4}\left[2R_1sin^2\gamma+\frac{3(1+\alpha)}{8\alpha}R_2sin^2\gamma+R_2cos^2\gamma\right]$$
$$\frac{\alpha(8\alpha-1)}{64}R_4sin^2\gamma\ +\left\{\frac{D\pi}{2r_1^4}\left[\frac{1}{\alpha}R_1cos\gamma+\frac{cos^2\gamma}{2sin\gamma}R_1-2\left(R_1+\right.\right.\right.$$
$$\left.\left.\frac{1}{4}R_2\right)sin\gamma-\frac{1}{4}R_1cos^2\gamma\right]+\frac{\nu D\pi}{r_1^4}\times\left(-R_1-\frac{3+8\alpha}{8\alpha}R_2+\frac{\alpha}{64}R_4\right)+$$
$$\left.\frac{4(1-\nu)\pi D}{r_1^4}\mathrm{T}_4\sin\gamma\right\}n^2+\frac{D\pi n^4}{4r_1^4sin\gamma}R_1+$$
$$+\frac{1}{2}\sum_{i=1}^{k_1}\left[\frac{\tilde{E}_i\tilde{F}_i}{r_1^4}sin\gamma\left(\frac{8-6\alpha^2-3\alpha^3}{2\alpha^2}R_1-\frac{5}{6}R_3+\frac{\alpha^2}{8}R_5\right)\times\right.$$
$$\times\left(\sum_{n=1}^{\infty}D_n(t)cosn\varphi_i\right)^2+\tilde{E}_iI_isin^3\gamma\times$$

$$\times (3-\frac{3}{4}\alpha^2-\frac{3}{4}\alpha^3)R_1-\frac{5}{8}\alpha^2R_3+\frac{\alpha^4}{32}R_5\Big]cos^2n\varphi_i+$$

$$+\tilde{G}_iI_{ikp}sin\gamma\left(\frac{5}{4}R_1-\frac{1}{24}\alpha^2R_3\right)n^2sin^2n\varphi_i+\frac{k\pi Q(r_1,r_{2,}m)}{r_1^2};$$

$$\varphi_{33}=\frac{\pi Eh\sin\gamma}{4(1+\nu)r_1^4}\left[\left(\frac{\alpha^2}{8}+\frac{1}{2}\right)R_3-\frac{3}{2}R_4+\left(2-\frac{21}{2\alpha^2}\right)R_2+\right.$$

$$\left.\left(\frac{12}{\alpha^3}-3-\frac{4}{\alpha^2}\right)R_1\right]+\frac{(1-\nu)D}{2r_1^4}T_4sin^2\gamma cos^2\gamma+$$

$$+\frac{\pi Eh}{2(1-\nu^2)r_1^4\sin\gamma}\times\left(R_4-\frac{3}{8\alpha^2}R_2\right)n+\frac{\tilde{E}_iI_in^4}{2sin\gamma}R_1sin^2n\varphi_i;$$

$$\varphi_{44}=\frac{\pi Eh}{2(1-\nu^2)r_1^4}\Big[2\nu cos\gamma\Big(\frac{1}{2}R_4+\frac{3(1+\alpha)}{\alpha^2}R_2+\frac{6}{\alpha^2}R_1)+$$

$$\left.\frac{cos\gamma}{\alpha}\left(R_3-\frac{3R_1^2}{m\pi}\right)\right]+\frac{D\pi}{2r_1^4}\Big[-\frac{1}{2\alpha}\frac{cos^3\gamma}{sin\gamma}R_1+\frac{1}{2}T_3sin2\gamma+$$

$$\left.cos\gamma\left(-\frac{1}{2\alpha}R_1cos^2\gamma+T_3sin^2\gamma\right)\right]+\frac{\nu D\pi}{2r_1^4}sin2\gamma\left(\frac{7}{4\alpha}R_1+\frac{11}{24}\right);\ \varphi_{55}=$$

$$\frac{4D\pi(1-\nu)n}{2r_1^4}T_4sin2\gamma;$$

$$\varphi_{66}=\frac{1}{2gr_1^4sin\gamma}T_5\sum_{i=1}^{k_1}\tilde{\gamma}_i\tilde{F}_i\,cos^2n\varphi_i+\frac{\pi\gamma_1h}{2gr_1^4sin\gamma}T_7;$$

$$\varphi_{77}=\frac{1}{2gr_1^4sin\gamma}T_6\sum_{i=1}^{k_1}\tilde{\gamma}_i\tilde{F}_i\,cos^2n\varphi_i+\frac{\pi\gamma_1h}{2gr_1^4sin\gamma}T_8;$$

$$\varphi_{88}=\frac{1}{2gr_1^4sin\gamma}T_5\sum_{i=1}^{k_1}\tilde{\gamma}_i\tilde{F}_i\,sin^2n\varphi_i+\frac{\pi\gamma_1h}{2gr_1^4sin\gamma}T_7;$$

$$T_1=\frac{1}{6}R_3-\frac{1}{\alpha^2}R_1;\ T_2=R_3+\frac{6}{\alpha^2}R_1;\ T_3=-\frac{1}{\alpha}R_1+\frac{\alpha}{12}R_3;$$

$$T_4=\frac{1}{2}R_1+\frac{1}{8}R_2+\frac{1}{32}\alpha^2R_4;\ R_p=r_1^p-r_2^p,\ \text{p}=1,2,\ldots,6$$

$$T_5=\frac{1}{2}R_5-\frac{2}{\alpha^2}R_3-\frac{12}{\alpha^3}R_1;\ T_6=\frac{1}{2}R_5+\frac{2}{\alpha^2}R_3-\frac{12}{\alpha^3}R_1$$

$$T_7=\frac{1}{2}R_6-\frac{10}{\propto^2}R_4+\frac{60}{\propto^4}R_2; T_8=\frac{1}{2}R_6+\frac{10}{\propto^2}R_4-\frac{60}{\propto^4}R_2$$

Having written the expression (7) in the Lagrange equation (5) in the case of Winkler model we obtain a system consisting of the following ordinary differential equations:

$$M\ddot{z}+2c\big(z-\alpha_0 A_n(t)\big)=0$$
$$-2\alpha_0 cz+2\varphi_{66}A_n''(t)+(2\varphi_{22}+2\alpha_0^2 c)A_n(t)+$$
$$+\varphi_{44}D_n(t)+\varphi_{55}B_n(t)=0$$
$$2\varphi_{77}D_n''(t)+2\varphi_{11}D_n(t)+\varphi_{44}A_n(t)=0 \qquad (8)$$
$$2\varphi_{88}B_n''(t)+2\varphi_{33}B_n(t)+\varphi_{55}A_n(t)=0$$

Here, $\alpha_0=\frac{r_0^2}{r_1^2}sin\frac{m\pi(r_0-r_2)}{r_1-r_2}cos\gamma, n=1,3,5,\ldots$

In the case the Winkler model, for finding natural vibrations of an elastic médium-contacting conical shell stiffened with longitudinal ribs together with the spring associated mass, looking for the solution $z=z^*sin\omega t, A_n=A_n^*sin\omega t, B_n=B_n^*sin\omega t, D_n=$ $=D_n^*sin\omega t$ of the system (8) and writing it in the system, we obtain the system of algebraic equations with respect to the constants z^*, A_n^*, B_n^*, D_n^*:

$$\left(\frac{2c}{M}-\omega^2\right)z^*-\frac{2c\alpha_0}{M}A_n^*=0$$
$$-2\alpha_0 cz^*+(2\varphi_{22}-2\varphi_{66}\omega^2+2\alpha_0^2 c)A_n^*+\varphi_{44}D_n^*+\varphi_{55}B_n^*=0$$
$$\varphi_{44}A_n^*+(2\varphi_{11}-2\varphi_{77}\omega^2)D_n^*=0 \qquad (9)$$
$$\varphi_{55}A_n^*+(2\varphi_{33}-2\varphi_{88}\omega^2)B_n^*=0$$

Since the system (9) is a system of homogeneous algebraic equations, a necessary and sufficient condition for the existened of its non-trivial solution is that its main determinant is zero. As a result, for finding natural vibration frequencies, we obtain the following frequency equation $\lambda=\omega^2$:

$$8\varphi_{66}\varphi_{77}\varphi_{88}\lambda^4-\left(T_1+\frac{16c}{M}\varphi_{66}\varphi_{77}\varphi_{88}\right)\lambda^3+\left(T_2+\frac{2c}{M}T_1-\right.$$
$$\left.-16c^2\propto_0^2\varphi_{77}\varphi_{88}\right)\lambda^2-\left(T_3+\frac{2c}{M}T_2+16c^2\propto_0^2\varphi_{11}\varphi_{88}+\right.$$
$$\left.+16c^2\propto_0^2\varphi_{33}\varphi_{77}\right)\lambda-\frac{2c}{M}T_3-16c^2\propto_0^2\varphi_{33}\varphi_{77}=0 \qquad (10)$$

Having solved this equation by the Ferrari method, for the roots we obtain[2]:

$$\lambda_{1,2}=\frac{1}{2}\left[-\left(\frac{A}{2}+\eta_1\right)\pm\sqrt{\left(\frac{A}{2}+\eta_1\right)^2-4\left(\frac{y_0}{2}+\eta_2\right)}\right] \qquad (11)$$

$$\lambda_{3,4}=\frac{1}{2}\left[-\left(\frac{A}{2}-\eta_1\right)\pm\sqrt{\left(\frac{A}{2}-\eta_1\right)^2-4\left(\frac{y_0}{2}-\eta_2\right)}\right]$$

Here, $\eta_1^2=\frac{A^2}{4}-B+y_0;\;\; \eta_2^2=\frac{y_0^2}{4}-D$

$$A=(8\varphi_{66}\varphi_{77}\varphi_{88})^{-1}\left(T_1+\frac{16c}{M}\varphi_{66}\varphi_{77}\varphi_{88}\right);\, D=-\frac{2c}{M}T_3-16c^2\propto_0^2\varphi_{33}\varphi_{77}$$

$$B=(8\varphi_{66}\varphi_{77}\varphi_{88})^{-1}\left(T_2+\frac{2c}{M}T_1--16c^2\propto_0^2\varphi_{77}\varphi_{88}\right)$$

$$C=(8\varphi_{66}\varphi_{77}\varphi_{88})^{-1}\left(T_3+\frac{2c}{M}T_2+\right.$$
$$\left.+16c^2\propto_0^2\varphi_{11}\varphi_{88}+16c^2\propto_0^2\varphi_{33}\varphi_{77}\right)$$

The roots (11) of the obtained equation (10) were calculated numerical method. The following values were taken for the parameters: $r_1=160$ мм, $r_2=85$мм, the rod was taken as angled $5\times5\times1$ (in mm), $k_1=32,\, m=1,\, \gamma=\frac{13\pi}{180}$, the height of the shell was accepted 320 mm.

The results of calculations were given in figure 4 in the form of dependence of the frequency parameter $f^*=\frac{\omega}{2\pi}$ on n, in figures 5 and 6 in the form of dependence of the ratio of the minimum natural vibrations frequencies of the system to minimum natural vibrations

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[1] R.A. İskanderov, H. Shafiei Matanagh Free vibrations of a conical shell with spring associated mass and stiffened with a cross system of ribs in medium //. International Journal on Technical and Physical Problems of Engineering (IJTPE) - June 2020, Issue 43, - Volume 12, - Number 2, - pages 1-5.

frequencies of a shell stiffened with ribs on the load mass were given for various $\bar{k} = \frac{k}{D}$ values and for $\bar{c} = \frac{c}{D}$. In figure 7 the dependence of the ratio of minimum natural vibrations frequencies of the system to minimum natural vibrations frequencies of the shell stiffened with longitudinal ribs on the amount of longitudinal rods was shown. As can be seen from figure 4, as the number n increases, the minimum natural vibrations frequencies of the system at first decrease and then takes minimum value and increase.

It is shown in figures 5 and 6 that as the ratio M/M_p ($M_p$is the mass of the shell together with ribs) increases, minimum natural; vibrations frequencies of the system decrease and vice versa, as the ratio, $\bar{k} = \frac{k}{D}$ and the rigidity of the spring increases, the minimum natural vibrations frequencies of the system increase. This is explained by the fact that increase in the ratio $\bar{k} = \frac{k}{D}$causes increase in the rigidity of the medium.

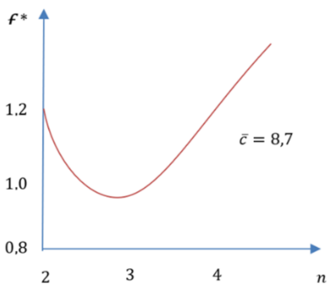

Figure 4. Dependence of natural vibrations of the shell on the wave numbers in the circular direction

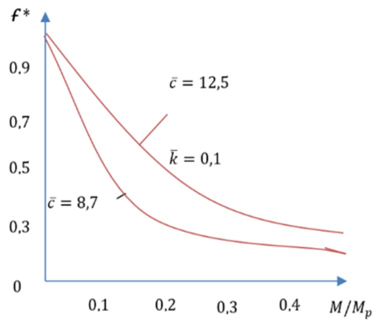

Figure 5. Dependence of the natural vibration frequencies of the shell on the load mass

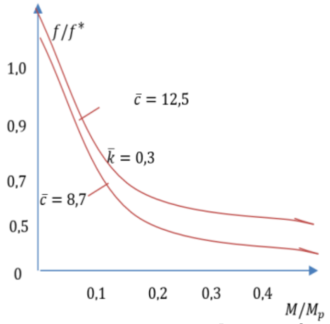

Figure 6. Dependence of natural vibrations frequencies of the shell on the load mass

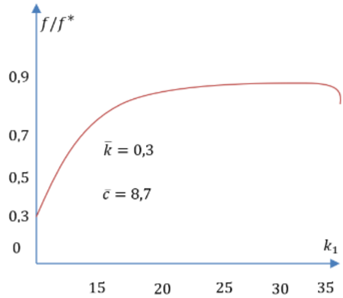

Figure 7. Dependence of natural vibration frequencies of the shell on the amount of longitudinal rods

As can be seen from figure 7, as the amount of the ribs increases natural vibrations frequencies of the system at first increases and then after some increase begins to decrease. The reason is that the further increase in the amoun of ribs increases the sum of their mass and as a final result causes the system to strength inertia action to the system vibrations.

The similar problem was performed for the Pasternak model as well.

Chapter III studies free vibrations frequencies of a system consisting of a circular closed truncated shell dynamically interacting with elastic medium and stiffened with ring-haped ribs and a mass associated with the same rigidity two springs at the points diametrically opposite to the shell. In this case, for obtaining the expression L in the expression (1) we should take $E_i = 0$ and $F_1 = 0$ The total energy of the rows included into the expressions (6) with respect to the $n -$ th addend is as follows:

$$L = \beta_{11} D_n^2(t) + \beta_{22} A_n^2(t) + \beta_{33} B_n^2(t) + \beta_{44} A_n(t) D_n(t) + \\ + \beta_{55} A_n(t) B_n(t) + (z - w_0 cos\gamma)^2 c + \\ + \beta_{66} {A'_n}^2 + \beta_{77} {D'_n}^2 + + \beta_{88} {B'_n}^2 + \frac{1}{2} M \dot{z}^2 \qquad (12)$$

Here

$$\beta_{11}=\frac{\pi Eh}{2(1-\nu^2)r_1^4\sin\gamma}\left[\left(\frac{2+\alpha}{\alpha}R_4+\frac{\alpha^2}{48}R_6+\frac{5}{2\alpha}(r_1^2+r_2^2-12)R_2\right)+\frac{\sin^2\gamma}{\pi}R_4+\right.$$

$$\left.+\frac{3(r_1+r_2)sin\gamma r_1^4}{8\pi^2m^2}R_1+2\nu sin^2\gamma\frac{7}{20}R_4+\frac{4\alpha-5}{\alpha^2}R_2+\frac{60-24\alpha}{\alpha^4}\right]+$$

$$+\frac{D\pi}{4r_1^4}sin\gamma cos^2\gamma R_1+\frac{D(1-\nu)\pi}{4r_1^4}\frac{\cos^2\gamma}{\sin\gamma}R_2+\frac{\pi Eh}{4(1+\nu)r_1^4\sin\gamma}\left(\frac{1}{8}R_4+\right.$$

$$\left.+\frac{3}{2\alpha^2}R_2\right)n^2$$

$$\beta_{22}=\frac{\pi Eh}{2(1-\nu^2)r_1^4\sin\gamma}\times\frac{1}{8}cos^2\gamma R_4+\frac{D\pi}{2r_1^4}sin^3\gamma\times$$

$$\times\left[2R_1+\left(\frac{3}{2\propto}+\frac{49}{8}\right)R_2-\left(\frac{\propto}{16}+\frac{5\propto^2}{16}\right)R_4+\frac{\propto^4}{32}R_6\right]+$$

$$+\left\{\frac{D\pi}{2r_1^4}\left[\frac{cos^4\gamma}{sin^2\gamma}T_1+4T_1-\frac{1}{2}T_2cos^2\gamma-T_2sin^2\gamma\right]+\left(\frac{\propto^2}{8}R_5+\frac{1}{2}R_3-\right.\right.$$

$$\left.3R_1\right)sin^2\gamma+$$

$$+\frac{\nu D\pi}{r_1^4}\left[2R_1sin^2\gamma+\frac{3(1+\alpha)}{8\alpha}R_2sin^2\gamma+R_2cos^2\gamma\right]+\frac{\alpha(8\alpha-1)}{64}R_4sin^2\gamma+$$

$$+\left\{\frac{D\pi}{2r_1^4}\left[\frac{1}{\alpha}R_1cos\gamma+\frac{cos^2\gamma}{2sin\gamma}R_1-2\left(R_1+\frac{1}{4}R_2\right)sin\gamma-\frac{1}{4}R_1cos^2\gamma\right]+\right.$$

$$\left.\frac{\nu D\pi}{r_1^4}\left(-R_1-\frac{3+8\alpha}{8\alpha}R_2+\frac{\alpha}{64}R_4\right)+\frac{4(1-\nu)\pi D}{r_1^4}\mathrm{T}_4\sin\gamma\right\}n^2+$$

$$\frac{D\pi n^4}{4r_1^4sin\gamma}R_1+\frac{k\pi Q(r_1,r_2,m)}{r_1^2}+\frac{\pi}{2r_1^4}\sum_j^{k_2}\left(E_jF_jr_j^2sin^2\frac{r_{j-}r_2}{2}\alpha+\right.$$

$$+(1-n^2)^2E_jF_j\,sin^2\frac{r_{j-}r_2}{2}\alpha\Big)+G_jI_{jkp}sin\gamma\left(4sin^2\frac{r_{j-}r_2}{2}\alpha\right.$$

$$\left.\left.+\alpha r_j\sin(r_{j-}r_2)\,\alpha+\frac{r_j^2\alpha^2}{4}cos^2\frac{r_{j-}r_2}{2}\alpha\right)n^2\right]$$

$$\beta_{33}=\frac{\pi Eh\sin\gamma}{4(1+\nu)r_1^4}\left[\left(\frac{\propto^2}{8}+\frac{1}{2}\right)R_3-\frac{3}{2}R_4+\left(2-\frac{21}{2\alpha^2}\right)R_2+\left(\frac{12}{\alpha^3}-3-\frac{4}{\alpha^2}\right)R_1\right]+\frac{(1-\nu)D}{2r_1^4}T_4sin^2\gamma cos^2\gamma+\frac{\pi Eh}{2(1-\nu^2)r_1^4\sin\gamma}\left(R_4-\frac{3}{8\alpha^2}R_2\right)n+\frac{\pi n^2}{2r_1^4}\sum_{j=1}^{k_2}E_jF_jr_j^2sin^2\frac{r_{j-}r_2}{2}\alpha;$$

$$\beta_{44}=\frac{\pi Eh}{2(1-\nu^2)r_1^4}\left[2\nu cos\gamma\left(\frac{1}{2}R_4+\frac{3(1+\alpha)}{\alpha^2}R_2+\frac{6}{\alpha^2}R_1\right)+\frac{cos\gamma}{\alpha}\left(R_3-\frac{3R_1^2}{m\pi}\right)\right]+$$
$$+\frac{D\pi}{2r_1^4}\left[-\frac{1}{2\alpha}\frac{cos^3\gamma}{sin\gamma}R_1+\frac{1}{2}T_3sin2\gamma+cos\gamma\left(-\frac{1}{2\alpha}R_1cos^2\gamma+T_3sin^2\gamma\right)\right]+\frac{\nu D\pi}{2r_1^4}sin2\gamma\left(\frac{7}{4\alpha}R_1+\frac{11}{24}\right);$$

$$\beta_{55}=\frac{4D\pi(1-\nu)n}{2r_1^4}T_4sin2\gamma+\frac{\pi}{2r_1^4}\sum_{j=1}^{k_2}E_jF_jr_j^2sin^2\frac{r_{j-}r_2}{2}\alpha$$

$$\beta_{66}=\frac{\pi}{2gr_1^4}\sum_{j=1}^{k_2}\gamma_jF_jr_j^4\,sin^2\frac{r_{j-}r_2}{2}\alpha+\frac{\pi\gamma_1h}{2gr_1^4sin\gamma}T_7$$

$$\beta_{77}=\frac{\pi}{2gr_1^4}\sum_{j=1}^{k_2}\gamma_jF_jr_j^4\,cos^2\frac{r_{j-}r_2}{2}\alpha+\frac{\pi\gamma_1h}{2gr_1^4sin\gamma}T_8$$

$$\beta_{88}=\frac{\pi}{2gr_1^4}\sum_{j=1}^{k_2}\gamma_jF_jr_j^4\,sin^2\frac{r_{j-}r_2}{2}\alpha+\frac{\pi\gamma_1h}{2gr_1^4sin\gamma}T_7$$

Substituting the expression (12) in the Lagrange equation (5) we obtain a system of ordinary differential equations in the case of Winkler model. Looking for the solution of the system obtained for finding natural vibration frequencies of an elastic-medium contacting conical shell stiffened with rings with a spring associated mass in the case of Winkler model in the form $z=z^*sin\omega t, A_n=A_n^*sin\omega t$, $B_n=B_n^*sin\omega t, D_n=D_n^*sin\omega t$ and substituting in the system we obtain a system of alegbraic equations with respect to the constants z^*, A_n^*, B_n^*, D_n^*. Since the obtained system is a system of homogeneous linear algebraic equations, its main determinant was equaled to zero, for finding natural vibratins frequencies a freguency equation was built, was solved by the Ferrari method and its roots were found.

Similar problem was performed for the case when the action of the medium is modeled as a Pasternak model.

In section 4 free vibrations of a conical shell dynamically contacting with elastic medium and stiffened with rods and rings in the form of a network together with spring associated mass, are studied.

The problem has found its solution both for Winkler and Pasternak models. Total energy of the rows contained in the expressions (6) was found with respect to the n -th addend of the row, the Lagrange equation was written and the system consisting of ordinary differential equations was built. In the same way, for finding natural vibrations frequencies, a system of algebraic equations was obtained and as a result, a freguency equation was built and was solved by the Ferrari method.

In chapter III joint vibrations of a stiffened conical shell and spring associated mass in homogeneous visco-elastic medium are studied. For taking into account the influence of the medium, the viscous-elastic Pasternak models were used. The chapter consists of four sections. Section 1 states the problem of studying of a stiffened conical shell and a spring associated mass in a homogeneous viscous-elastic medium.

For taking into account the influence of the medium, the dynamic Pasternak model for viscous-elastic-medium was used:

$$q=\left(\tilde{q}_0+\tilde{q}_0\frac{d^2}{dx^2}\right)w-\int_{-\infty}^{t}\Omega(t-\tau)w(\tau)d\tau \qquad (13)$$

Here, $\Omega(t-\tau)=A_* e^{-\Psi(t-\tau)}$, A_*, Ψ, $\tilde{q}_0 \tilde{q}_0$ – are the contants, w is the curvature of the shell.

Since the construction under consideration consists of a conical shell, viscous-elastic medium, stiffened ribs, spring associated load, we write the expressions of their energies as in chapter II, build a freguency equation by the Lagrange equation and find its roots.
Section 2 considers joint vibrations of a conical shell stiffened with longitudinal ribs and spring associated mass in a homogeneous viscous-elastic-medium. The following freguency equation was obtained:

$$8\varphi_{66}\varphi_{77}\varphi_{88}\lambda^5-\left(T_1+\left(\frac{16c}{M}-8\Psi^2\right)\ \varphi_{66}\varphi_{77}\varphi_{88}\right)\lambda^4$$
$$+\left(T_2+\frac{2c}{M}T_1-16c^2\propto_0^2\varphi_{77}\varphi_{88}-\right.$$
$$\left.-\left(T_1+\frac{16c}{M}\varphi_{66}\varphi_{77}\varphi_{88}\right)-16\tau\varphi_{77}\varphi_{88}\right)\lambda^3-\left(T_3+\frac{2c}{M}T_2+\right.$$
$$+16c^2\propto_0^2\varphi_{11}\varphi_{88}+16c^2\propto_0^2\varphi_{33}\varphi_{77}+T_2\Psi^2+16\tau\varphi_{33}\varphi_{77}$$
$$\left.+16\tau\varphi_{11}\varphi_{88}+\frac{2c}{M}(T_1-16c^2\propto_0^2\varphi_{77}\varphi_{88})\Psi^2\right)\lambda^2+$$
$$+\left(-\frac{2c}{M}T_3-16c^2\propto_0^2\varphi_{33}\varphi_{77}-T_3\Psi^2+16\tau\varphi_{11}\varphi_{33}-\right.$$
$$-\frac{2c}{M}(T_2\Psi^2+16\tau\varphi_{11}\varphi_{88})-16c^2\propto_0^2\Psi^2(\varphi_{11}\varphi_{88}+ \tag{14}$$
$$\left.+\varphi_{33}\varphi_{77})\right)\lambda-\frac{2c}{M}(T_3\Psi^2-16\tau\varphi_{11}\varphi_{33})-16c^2\propto_0^2\varphi_{11}\varphi_{33}\Psi^2=0$$

The roots of the equation (14) was found by the numerical method. The following values were taken for parameters: $r_1=160$ мм, $r_2=85$мм, the rod in the angled from $5\times5\times1$ (mm), $k_1=32$, $m=1$, $A_*=0{,}0997$, $\Psi=0{,}05$, $\gamma=\frac{13\pi}{180}$, $\frac{\tilde{q}_0}{\tilde{q}}=0{,}1$, the height of the shell was accepted 320 mm.

The results of calculations were given in figure 8 in the form of dependence of the frequency parameter $f^*=\frac{\omega}{2\pi}$ on n, in figure 9 and 10 in the form of dependence of the ratio of minimum natural vibrations frequencies of the shell stiffened with ribs to the minimum natural vibrations frequencies on the load mass were given for different values of $\bar{k}=\frac{k}{D}$ (solid lines), $\bar{q}=\frac{\tilde{q}}{D}$ (dashed lines) and for

$\bar{c} = \frac{c}{D}$. Figure 11 shows the dependence of the ratio of the minimum natural vibrations frequencies of the system to the minimum natural vibrations frequencies of the shell stiffened with rods on the amount of rods. The curves indicated by the prime denote the results obtained from the application of the Pasternak model. As can be seen from figure 8, increasing the number n, minimum vibrations frequencies of the system at first decrease, then attaining a minimum value they increase. Figures 9 and 10 show that as the ratio, M/M_p increases (M_p is the mass of the shell together with ribs) minimum natural vibrations freguencies of the system decrease and vice versa, in the case of the Pasternak model, only increasing the ratios $\bar{k} = \frac{k}{D}$, $\bar{q} = \frac{\tilde{q}}{D}$ of the elastic medium-contacting conic shell stiffened only with longitudinal ribs together with a spring associated mass and increasing the spring rigidity the minimum natural vibrations frequencies increase. This is explained by the fact that an increase in the ratio $\bar{k} = \frac{k}{D}$ and $\bar{q} = \frac{\tilde{q}}{D}$ causes an increase in the rigidity of the medium. As can be seen from figure 11 as the number of longitudinal ribs increases, the natural vibrations freguencies of the system at first increase and after some increase began to decrease.

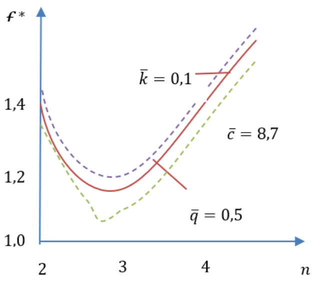

Figure 8. Dependence of natural vibration frequencies of the shell in a wave number in the circular direction

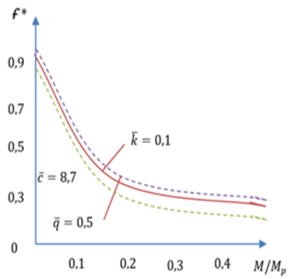

Figure 9. Dependence of natural vibration frequencies of the shell on the load mass.

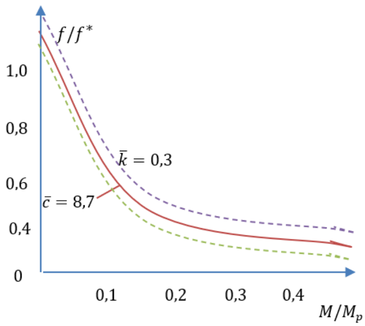

Figure 10. Dependence of natural vibration frequencies on the load mass

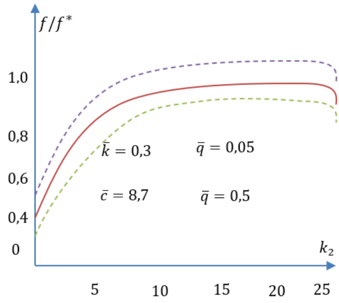

Figure 11. Dependence of natural vibration frequencies of the shell in the amount of ribs

The dotted curves in the graphs correspond to taking into account viscosity of the medium, the other two curves correspond to the elastic case of the medium. As can be seen, taking into account the medium viscosity leads to a decrease in the natural vibrations frequencies.

In the section 3 a similar problem, the joint vibrations of a conic shell stiffened with ring shuped ribs and a spring associated mass in a homogeneous viscous-elastic medium, in section 4 the joint vibrations of a conical shell stiffened with ribs forming a network and a spring associated mass were studied in a homogeneous viscous-elastic medium.

Chapter IV studies joint vibrations of a stiffened conical shell and spring associated mass in an inhomogeneous viscous-elastic medium. To take into account the influence of the medium, the Winkler method for elastic inhomogeneous media is used:

$$q_r = k_0\left(1 - \xi\frac{r - r_2}{r_1 - r_2}\right)w \tag{15}$$

Here k_0 is a constant, w is the curvature of the shell, l is the length of the generatrix of the cone, γ is an angle between the generatrix and axis of the cone, ξ is an inhomogeneity parameter and $\xi \epsilon [-1;1]$.

The chapter consists of four sections. Section 1 studies the problem of joint vibrations of a stiffened conical shell and spring associated mass in an inhomogeneous elastico-plastic medium. Section 2 considers joint vibrations of a conical shell stiffened with longitudinal ribs and spring associated mass in an inhomogeneous elastic medium. Section 3 studies joint vibrations of a conical shell stiffened with ring-shaped ribs and spring associated mass in an viscous-elastic medium, section 4 considers joint vibrations of a conical shell stiffened with ribs forming a network and a spring associated mass in an inhomogeneous elastic-medium.

The energetic method is used when solving the problem. Since the structure under consideration consists of a conical shell, viscous-elastic medium, ribs used in stiffening, spring associated mass to the conic shell, the expressions of their energies were given in (1). The difference is that the work A constained in (1) will be calculated as follows:

$$A = \frac{\pi k_0}{r_1^4 l}\left[\frac{1}{12}(r_2^6 - r_1^6) + \frac{5(r_2 - r_1)(r_1^2 - r_2^2)(r_1^2 + r_2^2 - 12)}{4\pi m} - \frac{\xi}{l}\Phi(r_1, r_2, m)\right](A_1^2(t) + A_3^2(t) + \cdots) =$$

$$= F(r_1, r_2, m)(A_1^2(t) + A_3^2(t) + \cdots); \Phi(r_1, r_2, m) = \frac{r_1^7 - r_2^7}{14}$$

$$-\frac{r_1^6(r_1 - r_2)}{4\pi m}\frac{sin 2\pi m(r_1 - r_2)}{r_1 - r_2} - \frac{3(r_1 - r_2)^2}{4\pi^2 m^2} \times \left(r_1^5 \frac{cos 2\pi m(r_1 - r_2)}{r_1 - r_2} - r_2^5\right) + \frac{45 r_1^4 (r_1 - r_2)^3}{4\pi^3 m^3} \times \times \frac{sin 2\pi m(r_1 - r_2)}{r_1 - r_2} + \frac{45(r_1 - r_2)^3}{\pi^3 m^3} \times \quad (16)$$

$$\times\left(r_1^3\frac{cos2\pi m(r_1-r_2)}{r_1-r_2}-r_2^3\right)-\frac{135(r_1-r_2)^5}{2\pi^5m^5}\left(r_1\frac{cos2\pi m(r_1-r_2)}{r_1-r_2}-r_2\right)-$$

$$-\frac{135r_1^2(r_1-r_2)^4}{2\pi^4m^4}\frac{sin2\pi m(r_1-r_2)}{r_1-r_2}-\frac{135(r_1-r_2)^6}{4\pi^5m^5}\times$$

$$\times\frac{sin2\pi m(r_1-r_2)}{r_1-r_2};F(r_1,r_2,m)=\frac{\pi k_0}{r_1^4 l}\left[\frac{1}{12}(r_2^6-r_1^6)+\right.$$

$$\left.+\frac{5(r_2-r_1)(r_1^2-r_2^2)(r_1^2+r_2^2-12)}{4\pi m}-\frac{\xi}{l}\Phi(r_1,r_2,m)\right]$$

Investigation of vibrations of the system consisting of a circular closed, inhomogeneous elastic medium-contacting truncated conical shell stiffened with regularly distributed rods on the surface and a mass associated with two springs of the same rigidity is reduced to integration of Lagrange equation (5) taking into account expressions (16). As a result, a frequency equation is obtained. It is shown that increasing the value of inhomogeneous parameters of the medium, since the rigidity of the medium increases, the natural vibrations frequencies of the system decrease.

CONCLUSIONS

The dissertation work studies free vibrations frequencies of a structure consisting of homogeneous and inhomogeneous visco-elastic medium contacting conical shell stiffened with ribs. Three variants of stiffening were considered: 1) with ribs stiffened in the direction of the generatrix; 2) with ring-shaped ribs; 3) with mutually perpendicular ribs.

In all three cases, an equation for finding vibration freguencies of the object under consideration was built and its roots were found. The second kind Lagrange equation, Winkler and Pasternak models were used for solbving the problem. The following results were obtained:

- With an increase of the ratio $\frac{M}{M_p}$ (M $-$is a load mass, M_p $-$is the mass of the shell together with ribs) minimum natural vibrations frequencies of the system decrease;

- With an increase in the amount of the ribs, natural vibrations frequensies of the system at first increases and then begins to decrease;

- Taking medium viscosity into account, natural vibrations frequencies of the system decrease;

- With an increase in the rigidity of the spring natural vibrations frequencies of the system increase;

- With an increase in the value of the inhomogeneous parameter, natural vibrations frequencies of the system increase.

The main results of the dissertation were published in the following works:

1. İskanderov, R.A., Shafiei Matanagh, H. M. Free vibrations of longitudinally reinforced conical shell with spring assoc. Problems of computational mechanics and strength of structures // - Dnepropetrovsk: National University named after Oles Honchar. -2017. vol.26, p 175-184.
2. Iskanderov, R.A., Shafiei Matanagh, H. M. Free oscillations of a conical roof reinforced with longitudinal bars with a spring-loaded load. Modern problems of construction and construction education. Conference, -Baku: -2017, p. 399-400.
3. İskanderov, R.A., Shafiei Matanagh, H. M. Free vibrations of lateral reinforced conical shell with spring associated mass in medium. IJTPE Journal International Journal on Technical and physical problems of engineering // -2017 vol. 32, p 48 – 52.
4. Shafiei Matanagh, H. M. Free dances of a conical cover reinforced with transverse rods and a mass fixed by a spring in a joint environment. Proceedings of the XXII Republican Scientific Conference of Doctoral Students and Young Researchers dedicated to the 100th anniversary of the Azerbaijan Democratic Republic. –Baku: -2018, pp. 398-400.
5. İskanderov, R.A., Shafiei Matanagh, H. M. Free vibrations of a longitudinally reinforced conical shell with a mass attached by springs, in contact with an inhomogeneous medium. Problems of computational mechanics and structural strength. Dnipro Lira //Dnepropetrovsk: -2019, No. 29, pp. 221–230.
6. İskanderov, R.A., Shafiei Matanagh, H. M. Free vibrations of a conical shell with spring associated mass and stiffened with a cross system of ribs in medium. International Journal on “Technical and Physical Problems of Engineering”// - 2020, vol. 43, No. 3, p 1 – 5.

7. Shafiei Matanagh, H. M. Free vibrations of a conical shell with spring associated mass and stiffened in medium. Engineering mechanics //-AzASU – 2020, No. 1, p 101 – 108.

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Personal contribution of the applicant to the published works

[1-11] – all scientific works, with the exception of the formulation of the problem, were carried out by the author.

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