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ABSTRACT

of the dissertation for the scientific degree of Doctor of Philosophy

DECISION MAKING IN PERSONNEL SELECTION UNDER Z-INFORMATION

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Applicant:	**Salman Yashar Salmanov**

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Scientific supervisor	Doctor of technical sciences, prof. **Latafat A. Gardashova**
Official opponents	1. Doctor of technical sciences, prof. **Mahammad A. Ahmadov** 2. Doctor of technical sciences, prof. **Kamala R. Aliyeva** 3. Doctor of philosophy on technical sciences **Babak G. Guirimov**

Dissertation council ED 2.48 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at Azerbaijan State Oil and Industry University.

Chairman of the Dissertation Council:	Doctor of technical sciences, prof.
____________	**Rafik A. Aliev**
Scientific secretariat of the Dissertation Council:	Doctor of philosophy on technical sciences, assoc. prof.
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GENERAL DESCRIPTION OF THE WORK

Relevance of the theme and degree of development. In the existing scientific literature, personnel selection problem is solved by using the methods based on the models that do not take uncertainty into account. Widely used AHP, TOPSIS, ELECTRE, VIKOR and other decision-making methods are created for a framework of precise information. Although their fuzzy versions are proposed, the yielded solution is reduced to the type of classical version. On the other hand, the aspect of reliability of information is not considered in these methods. In the presented dissertation, the approach relying on the theory of Z-numbers developed by Professor L. Zadeh which allows to describe uncertainty and reliability of information, is applied to the problem of personnel selection.

Research object and subject. The object of research is human resources, and the subject of research is multi-criteria decision-making in analysis of human resources.

Research goals and objectives. The aim of the dissertation work is to propose a systematic decision-making method for solving the problem of personnel selection under complex uncertainty in various fields relying on the procedure based on the theory of Z-numbers.

Research methods. The theory of Z-numbers including arithmetic of Z-numbers were used as research methods in the dissertation work.

Main highlights, brought forward for dissertation defense: The following **highlights** are submitted for defense in the dissertation work:

- Selection and analysis of criteria for the personnel selection problem under partially reliable information;
- Formulation of a 10-criteria-based decision-making problem for personnel selection under partially reliable information, determining the weight of each criterion based on eigen-solutions, deriving matrices that satisfy the consistency criterion;
- Investigation of the functionality and effectiveness of the proposed theoretical methods through computer simulation.

The main difference of the approach considered in personnel selection is that the addition of a new alternative to the set of alternatives or removing of alternatives from this set do not affect the ranking in the decision-making results. A series of computer experiments confirm the integrity and adequacy of the multi-criteria decision-making approach under Z-information.

Scientific novelty of the research. The novelty relies on solving the personnel selection problem based on the theory of Z-numbers.

1. For the first time, a 10-criteria-based decision making problem that comprehensively covers the aspects of personnel selection (quality of work, executive ability - quantity of work, degree of knowledge at work, effective decision-making in problem solving, honesty, punctuality, teamwork, development, adaptability to change, sustainability) is formulated and solved;
2. Solving the problem of multicriteria decision-making based on the theory of Z-numbers, under consideration of the reliability of relevant information and choice results;
3. Analysis of consistency of the preferences of the decision maker in the problem of personnel selection;
4. Solving the problem of selection of alternatives according to the Simple Average Weighting (SAW) method based on Z-numbers.

Theoretical and practical significance of research. The theoretical importance of the study is to propose theoretical basics for personnel selection relying on Z-numbers theory, and the practical importance is that it can be used in various real practical systems.

Approval and application. The theoretical and practical results of the dissertation were discussed at the following international conferences: ICSCCW-2021 – 11th International Conference on Theory and Application of Soft Computing, Prague, Czech Republic; ICAFS-2023 - 16th International Conference on Theory and Application of Fuzzy Systems, Budva, Montenegro.

The name of the institution where the dissertation was performed. Azerbaijan State Oil and Industry University, Research laboratory "Intelligent control and decision-making systems in industry and economics".

MAIN CONTENT OF THE WORK

The relevance of the subject area, the main objectives, research methods, theoretical and practical significance of the research are reflected in the introduction.

In the first chapter, the analysis of the state-of-the-art of the problem under consideration is conducted.

The problem of personnel selection plays a crucial role in human resources management. So far, this issue has been widely analyzed and studied in various literature sources. In essence, the personnel selection problem is a multicriteria decision making (MCDM) problem of choice of the best personnel from among many alternatives. If we consider real-world problems, we can witness the emergence of various methodologies and theories decision making under uncertainty. Fuzzy decision-making methods are becoming more and more relevant in personnel selection decisions. However, the traditional approach to the mentioned problem has been applied until now. Various types of decision-making approaches have been proposed for this purpose. For example, until now in this field of research AHP, ANP, TOPSIS, expert systems and their combination have been applied. Even the application of these methods has taken its place in existing literature in the form of various approaches under numeric, fuzzy, and Z-numbers-based information. The main purpose of the presented material is to review the research conducted in the field of personnel selection decisions, analyze the most applied approaches and reveal their limitations.

It can be briefly stated that in existing works, expert systems, linguistic variables, neural networks and various MCDM methods were applied to personnel selection problem. However, there is a lack for approaches that take into account reliability of information and all possible criteria. Mainly, lack of consideration of the incompleteness, subjectivity and uncertainty and reliability information, are the shortcomings of the studies conducted. To overcome these shortcomings, it is necessary to suggest a new and comprehensive approach to making decisions in the problem of personnel selection.

Information about fuzzy numbers and Z-numbers is given **in the second chapter**.

In the third chapter, the main criteria for personnel selection are determined and analyzed.

Determining how to evaluate employees can be difficult, especially if they perform different duties and functions. It is an important factor to consider when evaluating your employees[1].

Various evaluation criteria can be used to select alternatives during employee selection. In the dissertation, the following criteria are used:

Quality of work

The quality of work should be constantly improved, attracting people with new way of thinking and encouraging high commitment should be the most important goals of the organization. The old social contract forms should be replaced by new ones, people should be treated kindly and properly. Employees must be open to all innovations in a time of rapid change, receive training based on time, stress and conflict management, and think about long-term goals.

Performance

Business managers also want to know how well each employee is doing work in terms of quality and quantity. Both employees and managers can use different methods to determine job performance based on expectations. Employees should familiarize themselves with performance evaluation standards in advance.

Performance appraisal is done to check how well the employee is performing his/her job duties. The purpose of this assessment is to guide career development and identify the employee's strengths and capabilities. Performance appraisals help an employer identify

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[1] Kew J., Stredwick J.: Human resource management in a business context. CIPD - Kogan Page; 3rd edition, 608 p. (2023).

training and development opportunities as well as reward the best employees.

The degree of knowledge

Education and experience are important factors that determine success in any field. Employers are looking for candidates with both education and work experience. At workplaces, constant attention is paid to increasing the knowledge of employees in various fields.

Effective decision making in problem solving

Making the right decisions in effective problem solving requires a wide range of skills that allow individuals to achieve defined goals. To solve this problem on the part of the organization, it is important to be able to correctly define it, develop and implement approaches (tests) using creativity and analytical thinking. The tests assess candidates' ability to identify problems and analyze information to make sound decisions. This test helps identify candidates who use analytical skills to assess and respond to complex situations. A typical problem solution, such as making inferences based on information to correctly identify a qualified candidate, involves the following:

- To make a decision based on logic in accordance with certain information during a certain period of time;
- Determination and application of priorities based on identified situations
- Analyzing information to draw conclusions

Integrity

Integrity in business means adhering to ethical and established moral principles. If an organization has the right moral culture, it means that employees take their responsibilities seriously, are proactive in understanding their responsibilities, and are ultimately accountable for their actions. An honest employee behaves ethically and serves the following purposes:

- to value the right actions,
- to be responsible for his/her actions, respect himself and the people around, help those in need,

–to exhibit reliability,

– to show patience and flexibility when unexpected difficulties arise

Punctuality

Understanding the basics of punctuality and attendance helps you develop respectful, professional habits in the workplace to create a productive work environment. Being punctual and prompt, attending meetings on time and submitting assignments by the deadline are particularly required. Being punctual in a professional environment involves planning ahead and taking measures to ensure that your commitments are met on a strict schedule. Accurate behavior increases efficiency. Punctuality allows you to get more done, communicate consistently with people, improve your relationships with clients, and gain a reputation for quality, reliable work.

Teamwork

Teamwork is about working together towards a common goal. However, team success depends on encouraging team collaboration and supporting a shared vision. In healthy teams, team members value each other's contributions. They see their own skill gaps and support those around them as they work together to achieve a team goal. Instead of tackling difficult tasks alone, they take into account each other's strengths and organize work in a way that makes sense for everyone, making teamwork even more productive.

Development

Once you set goals, it's important to be intentional about accomplishing them. Development should be authentic, treated with honesty and kindness. Valuable and best skills should be developed, and quality should be at the forefront of every job done. In addition, each person should do his work with love. It is necessary to put serious effort into the career and try to become an expert in it.

Adaptability to change

Adaptability to change refers to the ability to quickly and successfully accept change under any circumstances and to adapt effectively in response. Accepting changes is important not only in the workplace, but in all areas of life. But adapting to change is not a natural skill for everyone. Coping with change, changing your mindset and adapting constructively is a learnable skill. The ability to adapt well to change is important for a long-term career, as it is one of the key skills employers look for in employees.

Sustainability

Sustainability is an important element in the human resource management system. Sustainability in an organization helps the organization achieve its goals and achieve high performance. Sustainability in an organization largely depends on employees' attitudes toward their jobs, which can usually be described by positive feelings about work as a result of job satisfaction assessments. The level of job satisfaction increases stability in the enterprise.

Sustainability refers to the value of human resources and the importance of employee engagement and the provision of a skilled workforce for the organization's existence and future operations. The process of continuous management of human resources includes the following:

– Employee development and evaluation;
- Health and safety of employees;
– External factors and partners;
– Employees and long-term strategy;
– Sustainability of the environment;
– Ethical behaviors, labor management relations;
– Welfare and benefits;
– Non-discrimination and equality.

In the **fourth chapter**, the issue of personnel selection is formulated as a scientific problem and a method for its solution is proposed.

This chapter considers a MCDM problem where the criteria values and the weighting coefficients of the criteria are Z-numbers as the relevant information is partially reliable[2]. The proposed method is based on a simple average weighting of all criteria values of an alternative. The final decision is made based on the preferences in terms of the fair price of the alternative.

Let us denote $A=\{A_1,A_2,...,A_n\}$ a set of considered alternatives, $M=\{M_1,M_2,...,M_m\}$ a set of decision criteria. Each M_j is characterized by a weight W_j determined on the basis of calculation of eigenvalue and eigenvector of matrix of criteria comparison. Evaluation of the alternative A_i ,i=1,...,n according to the criteria M_j $(j = \overline{1,m})$ under Z-information is expressed in the form:

$$A_{ij} = \{(Z(A_{i1}, B_{i1}), Z(A_{i2}, B_{i2}), \dots, Z(A_{ij}, B_{ij}), \dots, Z(A_{im}, B_{im})\},$$

where the evaluation of the alternative A_i according to the criterion $M_j$is Z-number $Z(A_{ij}, B_{ij})$. The values and weights of criteria are usually evaluated under uncertainty and partial reliability of information. Weights are described as

$$W_j = \{Z(A_j^w, B_j^w)\}\ ,\ j = 1, \dots, n$$

where A_j^w is a fuzzy evaluation of an importance weight of the j-th criterion, and B_j^wdenotes reliability of this value. Thus, the decision matrix D_{nm} (below denoted D) can be described as follows (Table 1)[3]:

Table 1.

Decision making matrix $D_{n\times m}$ with Z-numbers-based values of criteria

	M_1	$\cdots$	M_m
A_1	$Z(A_{11}, B_{11})$	$\cdots$	$Z(A_{1m}, B_{1m})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_n	$Z(A_{n1}, B_{n1})$	$\cdots$	$Z(A_{nm}, B_{nm})$

Constructing a matrix of criteria comparison for preferences on criteria

[2] Aliyev R.R.: A new comprehensive decision making method under bimodal information. Information Sciences, 657, 119989 (2024).
[3] Salmanov S., Gardashova L.A.: Using Z-Number-Based Information in Personnel Selection Problem. Lecture notes in network and systems, 362, 302-307 (2022).

Z-consistency analysis determines how consistent the constructed decision matrix is. The aim of this approach is to determine how compatible preference degrees of criteria are as provided by a decision maker (DM). In this case, the reciprocity (*Zij=1/Zji*) and transitivity pairwise comparison should be preserved. Since the consistency analysis involves different and conflicting criteria, we apply analysis of consistency of preference degrees of the criteria provided by a DM in MCDM problem of employee selection.

A matrix of criteria comparison contains information of objects or features as compared to each other. This matrix is used to show the degrees of similarity or dissimilarity between objects or features. There are many application areas such as multi-criteria decision making and ranking problems.

To create a criteria matrix of criteria comparison, follow these steps:

Make a comparison list: Make a list of the objects or features you want to compare.

Making Comparisons: Comparing each item in the list with the other items to determine the degree of similarity or dissimilarity for each pair. For example, in a list of features, we can express the degree of similarity between two features with a value between 0 and 1.

Matrix representation: A matrix is created using the degrees of similarity or dissimilarity obtained from the comparisons. The elements of this matrix represent the objects or properties in the list. Each element of the matrix indicates the degree of similarity of the two elements being compared.

The matrix of criteria comparison can be described as follows (see Table 2).

Table 2.

A matrix of pairwise comparison criteria

	M_1	...	M_n
M_1	Z_{11}		Z_{1n}
...	...	...	...
M_n	Z_{n1}		Z_{nn}

This matrix shows an example matrix of criteria comparison where criteria M_1, M_2, M_3 *and* M_n are compared to each other. Each element represents the degree of similarity of two criteria. Such matrices can be applied to multi-criteria decision making, ranking and comparison problems. To describe a fusion of fuzziness and probabilistic information on preference degrees provided by a DM (usually in linguistic form), Z-numbers are used.

MCDM is formalized in a form of criteria evaluations of alternatives and information on importance of criteria. The decision maker's choice can be described by a square matrix $[Z_{ij}]$ where Z_{ij} indicates a relative importance of the i- th criterion as compared to the j -th criterion.

The natural conditions used for $Z_{ij}$are $Z_{ii} = 1$ and $Z_{ji} = 1/Z_{ij}$ (reciprocity), $\forall\, i, j = 1, \ldots, n$. Conventionally, the consistency of (Z_{ij}) is based on the transitivity condition[4]:

$$Z_{ij}Z_{jk} = Z_{ik}, \forall i, j, k$$

This implies that the Z_{ij}-based preference degree is approximately equal to the product of the degrees of preference related to all the path variants from i to k over j – the requirement is usually violated as conditioned by restricted computational abilities of a human being. The criterion of inconsistency is used to measure deviations in terms of a multiplicative transitive condition. Considering this, various indices have been proposed to directly or indirectly measure this deviation. The inconsistency index is a mapping $I: C \to \mathcal{R}$ that transforms a matrix of pairwise comparison of criteria importance C into a set of real numbers $\mathcal{R}$.

Constructing a consistent matrix of pairwise comparison of criteria importance given an inconsistent one

Although these problems are difficult to solve, getting the right data and the right analysis is the best way to make the matrix of criteria

[4] Aliev R.A., Huseynov O.H., Aliyev R.R., Guirimov B.G.: A consistency-driven approach to construction of Z-number-valued pairwise comparison matrices. Iranian Journal of Fuzzy systems, 18(4), 37-49 (2021).

comparison consistent and reliable. We determine the eigenvalue and eigenvector of the matched matrix of criteria comparison.

Determination of the eigenvalue and the eigenvector of the matrix of Z-numbers (Z-matrix)

The goal is to measure the relationships between objects or features in a matrix of criteria comparison. These values are needed in many areas, such as ranking alternatives, setting priorities, or making optimal choices. Given a comparison matrix, you can follow these methods to determine an appropriate value for the objective:

- To rate only linguistically (in words): The objects or features we want to compare are evaluated with terms like "very good", "good", "average", "bad", "very bad", etc. we These terms describe the relationship of objects or features.
- Assigning numerical values: We can represent the elements in the comparison matrix by numbers. Basically, a scale of 1 (lowest) to 5 (highest) is used.
- Defining the linguistic scale: When defining the elements in the comparison matrix, we can evaluate the objects or features with a linguistic scale such as "I hesitate a lot", "How hesitant am? ", "I don't hesitate".

Analysis of Eigen solutions in the Z-pairwise comparison (dominance) matrix

A Z-number is represented a in form of tuple (A,B), where fuzzy number *A* is a fuzzy value of a random variable, B is a fuzzy numb r which expresses the reliability degree of this value. In this case, Z-pairwise comparison matrix is expressed as

$$(Z_{ij})=\begin{bmatrix} Z_{11} & \dots & Z_{1n} \\ . & \dots & . \\ Z_{n1} & \dots & Z_{nn} \end{bmatrix}$$

An eigenvalue of (Z_{ij}) matrix is expressed in the form[5]

$$\det(Z_{ij} - Z_\lambda I) = Z_0$$

Here, I is a numeric identity matrix, and $Z_0$is the zero Z-number. Thus, $Z_\lambda = (A_\lambda, B_\lambda)$, the Z-number-based eigenvalue, is a root of the equation:

$$Z_0 Z_\lambda{}^n + Z_1 Z_\lambda{}^{n-1} + Z_2 Z_\lambda{}^{n-2} ... + Z_{n-1} Z_\lambda + Z_n = Z(0),$$

Eigenvector of Z-numbers $(Z_Y) = (Z_{Y1}, \dots, Z_{Yn})$ is found as a solution of equation:

$$(Z_{ij})(Z_Y) = Z_\lambda (Z_Y)$$

Let us describe a problem of deriving of Z-number-valued eigenvalues $Z_{\lambda s} = (A_{\lambda s}, B_{\lambda s}), s = 1, \dots, n$ for (Z_{ij}). Note that B_{ij} is a fuzzy restriction for $P(A_{ij}) = \int_{\mathcal{R}} \mu_{A_{ij}}(x) p_{ij}(x) dx$. A set of probability density functions (pdfs) p_{ij} induce pdfs $p_{\lambda s}$. Given fuzzy numbers A_{ij} find A_λ such that:

$$\det(A_{ij} - A_\lambda I) = 0.$$

Given pdfs p_{ij} of random variables X_{ij} and the constraint $\det(X_{ij} - X_\lambda I) = 0$, determine pdf $p_{\lambda s}$ of $X_{\lambda s}$.

Next, one needs to determine Z-number-valued eigenvectors (Z_{Ys}) by solving linear system of equations of Z-number-valued components. (Z_{Ys}) can be found by determination of Z^+-numbers $(Z^+_{Ys} = (A_{Ys}, p_{Ys}))$.

Based on $A_{\lambda s}$, determine (A_{Ys}) that by solving the following system of linear equations:

$$(A_{ij})(A_{Y_s}) = A_{\lambda_s}(A_{Y_s})$$

Provided a random variable $X_{\lambda s}$ compute the random vector (Y_s) described by pdf p_{Ys} by solving the system of linear equations:

[5] Aliev R.A., Pedrycz W. Huseynov O.H., Aliyev R.R.: Eigensolutions of Partially Reliable Decision Preferences Described by Matrices of Z -Numbers. International Journal of Information technology & Decision Making, 19(06), 1429-1450 (2020).

$$\left(X_{ij}\right)(Y_s) = X_{\lambda_s}(Y_s)$$

Thus, given $Z^{+}_{\lambda s} = (A_{\lambda s}, p_{\lambda s})$, eigen vector $(Z^{+}_{Ys}) = \left((A_{Ys}, p_{Ys})\right)$, $s = 1, \dots, n$ is computed.

Example 1. Suppose that the preference matrix with elements expressed by Z-numbers is given:

pref_mat = ZMatrix([[Z11, Z12, Z13],
[Z21, Z22, Z23],
[Z31, Z32, Z33]])

where the Z-numbers are:

Z11 = zNum([[1,1,1,1],[0.6, 0.7, 0.7, 0.8]]);
Z12 = zNum([[0.22,0.25,0.25,0.285], [0.5, 0.6, 0.6, 0.7]]);
....
Z22 = zNum([[1,1,1,1],[0.6, 0.7, 0.7, 0.8]]);
....
Z32 = zNum([[0.18, 0.2, 0.2, 0.22], [0.7, 0.8, 0.8, 0.9]]);
Z33 = zNum([[1,1,1,1],[0.6, 0.7, 0.7, 0.8]]).

The calculated Z-number-valued eigenvalue (by using Z-lab software) is as follows:

eigen_value = [[2.727, 3.0247, 3.0247, 3.3774], [0.243, 0.2909, 0.2909, 0.3109]].

Computation of eigenvector of matrix of Z-numbers

The aim is the comparison and ranking of characteristics or objects. An eigenvector represents the priority and order of objects. One can use the following methods to convert a matrix of criteria comparison to an eigenvector:

Weighted average: Calculate the average value of each column in the matrix of criteria comparison and create an eigenvector by weighting these average values for each object. This will help ordering the objects.

Resolve inconsistency: The inconsistency ratio is used to measure the deviation of preference degrees in the comparison matrix. Calculate the inconsistency ratio and adjust preference degrees if the ratio exceeds a predefined threshold.

Rank all objects with the same rank: Using expert opinions and priorities to rank the objects in a matrix of criteria comparison with the same rank.

The eigenvalues and eigenvectors of the matrix of criteria comparison can be determined in different ways depending on the objectives and guidelines. An appropriate method should be chosen based on the requirements of the specific problem and the type of data.

Example 2. The eigenvector of the matrix in Example 1 computed (by using Z-lab software) is as follows:

eigen_vector =
ZColVector([
[[0.0323 0.1252 0.1252 0.1521] [0.2839 0.4021 0.4021 0.4618]],
[[0.1175 0.4282 0.4282 0.4924] [0.3134 0.5121 0.5121 0.6453]],
[[0.02 0.0732 0.0732 0.0884] [0.5299 0.6441 0.6441 0.6498]]]).

Evaluation of the consistency index of the Z -matrix.

A Z-matrix whose elements are Z-numbers is given by [3, 6]:

$$(Z) = \begin{bmatrix} Z_{11} & & & \cdots & Z_{1n} \\ Z_{21} & & & \cdots & Z_{21} \\ & & & \cdots & \\ \vdots & \vdots & \vdots & & \vdots \\ Z_{n1} & & & \cdots & Z_{nn} \end{bmatrix}$$

The consistency of this matrix is determined by two main conditions. The first is the reciprocity condition $Z_{12} = {}^{1}/_{Z_{21}}$ between elements of the matrix. The second is transitivity condition: $Z_{ij}Z_{jk} = Z_{ik}, \forall i, j, k$. Suppose that these conditions are not satisfied for the Z-matrix. Then it is necessary to determine such an updated matrix that these conditions are satisfied with a certain accuracy:

$$(Z') = \begin{bmatrix} Z'_{11} & \cdots & Z'_{1n} \\ & \cdots & \\ Z'_{n1} & \cdots & Z'_{nn} \end{bmatrix}$$

[6] Aliev, R. A.: Uncertain computation-based decision theory. Singapore: World Scientific, 521 p. (2017).

The difference between the (Z) and (Z') matrices can be measured based on distance between the Z-numbers (elementwise). An optimization technique is used to solve this problem.

The solution method consists of several stages. The Z-matrix of criteria comparison (Z'_{ij}) close to the given (Z_{ij}) can be constructed by the method proposed[5]. The- problem is formulated as follows. The elements of the Z-matrix of criteria comparison (Z'_{ij}) are treated as Z-valued decision variables. It is necessary to minimize the distance between the elements of the initially constructed inconsistent (Z_{ij}) matrix and the consistent (Z'_{ij}) matrix.

Inconsistency index for the Z-matrix of criteria comparison. The inconsistency index τ for the Z-matrix of criteria comparison (Z_{ij}) is defined as follows:

$$\tau((Z_{i\,j})) = \max_{i<j<k} \min \left\{ D\left((1,1), \left(\frac{Z_{i\,k}}{Z_{i\,j}\ z_{j\,k}}\right)\right) D\left((1,1), \left(\frac{Z_{i\,j}\ Z_{j\,k}}{Z_{i\,k}}\right)\right)\right\}$$

A normalization method for Z-valued decision matrix

The values of the different criteria of the alternatives are usually expressed in different units, so the decision matrix must be normalized. It is known that normalization of the decision matrix by different methods leads to different cases of ranking of alternatives.

When a new alternative is introduced, we apply a normalization approach that is free of rank reversal[7] [8].

For each criterion M_j, $j = 1, \dots, n$, the decision-maker determines the Z-value that he considers important for this criterion in the number r,

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[7] Alizadeh A.V., Aliyev R.R.: Rank Reversal Free Approach to Decision Making Under Z-information. Lecture notes in networks and systems, vol. 1, 335-346 (2024).

[8] Boza M., Zizovic M., Petojevic A., Damljanovic N.: New weighted sum model. Filomat, 31(10), 2991-2998 (2017).

$r \in N$: The normalized matrix $E = [Ze(i,j)]$ is defined as described below.
If $Zq(k+1,j)<Z(i,j)<Zq(k,j)$, then

$$Ze(i,j) = Zlq(k-1,j) + \frac{\left(Z(i,j)-Zq(k-1,j)\cdot\left(Zlq(k,j)-Zlq(k-1,j)\right)\right)}{Zq(k,j)-Zq(k-1,j)},$$

$i = 1,\dots,m, j = 1,\dots,n, k = 1,\dots,r.$
If $Z(i,j)>Zq(1,j)$, then $e(i,j) = \frac{Zq(1,j)}{Zq(1,j)}$.
If $Z(i,j)<Zq(r,j)$, then $Ze(i,j) = Zq(1,j) - Zq(1,j)$.

The fifth chapter analyzes the importance of criteria in MCDM problem. In this section, the problem of MCDM for employee selection under partially reliable information is considered. In this case, information on the criteria is important, and the criteria values for the alternatives are described by Z-numbers. In the solution approach, several methods of calculation with Z-numbers are used. First, the expert-provided Z-valued importance of the criteria is adjusted to achieve the necessary level of consistency of decision preference. Then, the weights of importance of the criteria are obtained by calculating eigenvectors of the Z-valued matrix. Finally, the best alternative is found by taking into account the given weights of the criteria and the values of criteria of the alternatives.
Let's assume that the MCDM problem in employee selection has 10 criteria - Quality of work (QW), Executive ability (EA), Degree of work knowledge (DW), Initiative and problem-solving skills (PS), Integrity(I), Persistence and punctuality(PP), Team work and communication (TWC), Development(D), Adaptation(A), Endurance(E) [3 9].
Consider the construction of a consistent matrix of criteria comparison expressed in Z-numbers that describes the relative importance of criteria in an employee selection problem under consideration (Table 3).

[9] Salmanov S.: Decision making on employee selection under uncertain environment. Lecture notes in network and systems (2024).

Table 3.

Z-matrix of criteria comparison expressed in Z-numbers

		QW	EA	DW	*PS*	*I*	*PP*	*TW*	*D*	*A*	*E*
	QW	*p*-$r_{1,1}$	*p*-$r_{1,2}$	*p*-$r_{1,3}$	*p*-$r_{1,4}$	*p*-$r_{1,5}$	*p*-$r_{1,6}$	*p*-$r_{1,7}$	*p*-$r_{1,8}$	*p*-$r_{1,9}$	*p*-$r_{1,10}$
	EA	*p*-$r_{2,1}$	*p*-$r_{2,2}$	*p*-$r_{2,3}$	*p*-$r_{2,4}$	*p*-$r_{2,5}$	*p*-$r_{2,6}$	*p*-$r_{2,7}$	*p*-$r_{2,8}$	*p*-$r_{2,9}$	*p*-$r_{2,10}$
	DW	*p*-$r_{3,1}$	*p*-$r_{3,2}$	*p*-$r_{3,3}$	*p*-$r_{3,4}$	*p*-$r_{3,5}$	*p*-$r_{3,6}$	*p*-$r_{3,7}$	*p*-$r_{3,8}$	*p*-$r_{3,9}$	*p*-$r_{3,10}$
	PS	*p*-$r_{4,1}$	*p*-$r_{4,2}$	*p*-$r_{4,3}$	*p*-$r_{4,4}$	*p*-$r_{4,5}$	*p*-$r_{4,6}$	*p*-$r_{4,7}$	*p*-$r_{4,8}$	*p*-$r_{4,9}$	*p*-$r_{4,10}$
D=	*I*	*p*-$r_{5,1}$	*p*-$r_{5,2}$	*p*-$r_{5,3}$	*p*-$r_{5,4}$	*p*-$r_{5,5}$	*p*-$r_{5,6}$	*p*-$r_{5,7}$	*p*-$r_{5,8}$	*p*-$r_{5,9}$	*p*-$r_{5,10}$
	PP	*p*-$r_{6,1}$	*p*-$r_{6,2}$	*p*-$r_{6,3}$	*p*-$r_{6,4}$	*p*-$r_{6,5}$	*p*-$r_{6,6}$	*p*-$r_{6,7}$	*p*-$r_{6,8}$	*p*-$r_{6,9}$	*p*-$r_{6,10}$
	TW	*p*-$r_{7,1}$	*p*-$r_{7,2}$	*p*-$r_{7,3}$	*p*-$r_{7,4}$	*p*-$r_{7,5}$	*p*-$r_{7,6}$	*p*-$r_{7,7}$	*p*-$r_{7,8}$	*p*-$r_{7,9}$	*p*-$r_{7,10}$
	D	*p*-$r_{8,1}$	*p*-$r_{8,2}$	*p*-$r_{8,3}$	*p*-$r_{8,4}$	*p*-$r_{8,5}$	*p*-$r_{8,6}$	*p*-$r_{8,7}$	*p*-$r_{8,8}$	*p*-$r_{8,9}$	*p*-$r_{8,10}$
	A	*p*-$r_{9,1}$	*p*-$r_{9,2}$	*p*-$r_{9,3}$	*p*-$r_{9,4}$	*p*-$r_{9,5}$	*p*-$r_{9,6}$	*p*-$r_{9,7}$	*p*-$r_{9,8}$	*p*-$r_{9,9}$	*p*-$r_{9,10}$
	E	*p*-$r_{10,1}$	*p*-$r_{10,2}$	*p*-$r_{10,3}$	*p*-$r_{10,4}$	*p*-$r_{10,5}$	*p*-$r_{10,6}$	*p*-$r_{10,7}$	*p*-$r_{10,8}$	*p*-$r_{10,9}$	*p*-$r_{10,10}$

The elements of this matrix are partially reliable criteria comparison degrees represented by Z-numbers p-$r_{i,j}$=$Z_{i,j}$ $i,j=1,...,10$ with trapezoidal components:

$Z_{1,1}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
$Z_{1,2}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{1,5}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{1,6}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
....
$Z_{1,9}$ = zNum([[5.0, 6.0, 6.0, 7.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{1,10}$ =zNum([[3.0, 4.0, 4.0, 5.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{2,1}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{2,2}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
....
$Z_{2,5}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])

$Z_{2,6}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{2,9}$ = zNum([[3.0, 4.0, 4.0, 5.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{2,10}$ =zNum([[4.0, 5.0, 5.0, 6.0], [0.8, 0.9, 0.9, 1.0]])

$Z_{3,1}$ = zNum([[1/5.0, 1/4.0, 1/4.0, 1/3.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{3,2}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{3,5}$ = zNum([[1/5.0, 1/4.0, 1/4.0, 1/3.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{3,6}$ = zNum([[1/7.0, 1/6.0, 1/6.0, 1/5.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{3,9}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{3,10}$ =zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])

$Z_{4,1}$ = zNum([[1/4.0, 1/3.0, 1/3.0, 1/2.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{4,2}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
....
$Z_{4,5}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
$Z_{4,6}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.7, 0.8, 0.8, 0.9]])
......
$Z_{4,9}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{4,10}$ =zNum([[2.0, 3.0, 3.0, 4.0], [0.7, 0.8, 0.8, 0.9]])

$Z_{5,1}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{5,2}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
....
$Z_{5,5}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
$Z_{5,6}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.8, 0.9, 0.9, 1.0]])
....
$Z_{5,9}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{5,10}$ =zNum([[2.0, 3.0, 3.0, 4.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{6,1}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
$Z_{6,2}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
....
$Z_{6,5}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.8, 0.9, 0.9, 1.0]])

$Z_{6,6}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
....
$Z_{6,9}$ = zNum([[2.0, 3.0, 3.0, 4.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{6,10}$ =zNum([[4.0, 5.0, 5.0, 6.0], [0.8, 0.9, 0.9, 1.0]])

Z_{71} = zNum([[1/6.0, 1/5.0, 1/5.0, 1/4.0], [0.7, 0.8, 0.8, 0.9]])
Z_{72} = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{7,5}$ = zNum([[1/4.0, 1/3.0, 1/3.0, 1/2.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{7,6}$ = zNum([[1/4.0, 1/3.0, 1/3.0, 1/2.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{7,9}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
$Z_{7,10}$ =zNum([[2.0, 3.0, 3.0, 4.0], [0.7, 0.8, 0.8, 0.9]])

$Z_{8,1}$ = zNum([[1/5.0, 1/4.0, 1/4.0, 1/3.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{8,2}$ = zNum([[3.0, 4.0, 4.0, 5.0], [0.8, 0.9, 0.9, 1.0]])
....
$Z_{8,5}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{8,6}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.8, 0.9, 0.9, 1.0]])
....
$Z_{8,9}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
$Z_{8,10}$ = zNum([[3.0, 4.0, 4.0, 5.0], [0.8, 0.9, 0.9, 1.0]])

$Z_{9,1}$ = zNum([[1/7.0, 1/6.0, 1/6.0, 1/5.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{9,2}$ = zNum([[1.0, 2.0, 2.0, 3.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{9,5}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{9,6}$ = zNum([[1/4.0, 1/3.0, 1/3.0, 1/2.0], [0.8, 0.9, 0.9, 1.0]])
....
$Z_{9,9}$ = zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])
$Z_{9,10}$ =zNum([[2.0, 3.0, 3.0, 4.0], [0.7, 0.8, 0.8, 0.9]])

$Z_{10,1}$ = zNum([[1/5.0, 1/4.0, 1/4.0, 1/3.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{10,2}$ = zNum([[1/3.0, 1/2.0, 1/2.0, 1/1.0], [0.8, 0.9, 0.9, 1.0]])
....

$Z_{10,5}$ = zNum([[1/4.0, 1/3.0, 1/3.0, 1/2.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{10,6}$ = zNum([[1/6.0, 1/5.0, 1/5.0, 1/4.0], [0.7, 0.8, 0.8, 0.9]])
....
$Z_{10,9}$ = zNum([[1/4.0, 1/3.0, 1/3.0, 1/2.0], [0.8, 0.9, 0.9, 1.0]])
$Z_{10,10}$ =zNum([[1,1,1,1],[0.8, 0.9, 0.9, 1.0]])

The inconsistency index of the given decision matrix *D* was calculated using the Zlab package.The program fragment is included:

```
consistency = Consistency(pref_mat)
new_consistent_matrix = consistency.consistent_matrix
inconsistency_index = consistency.inconsistency_index
printOut('inconsistency index : ', inconsistency_index)
printOut('consistent matrix : ',new_consistent_matrix

eig = Eigen(new_consistent_matrix)
printOut('Eigen vector')
printOut(eig.eigen_vector)
printOut('Weights vector')
printOut(eig.weights)
```

As a result, the following solution was obtained:
Inconsistency index : 0.7146114290473391
Consistent matrix obtained as a result of recalculation:

```
 [[[[0.95, 1.0, 1.0, 1.05], [0.97, 0.98, 0.98, 1.0]],
[[1.274, 1.341, 1.341, 1.408], [0.97, 0.98, 0.98, 1.0]],
[[4.113, 4.33, 4.33, 4.546], [0.97, 0.98, 0.98, 1.0]],
[[1.396, 1.47, 1.47, 1.543], [0.97, 0.98, 0.98, 1.0]],
[[[0.798, 0.84, 0.84, 0.882], [0.97, 0.98, 0.98, 1.0]],
[[2.254, 2.373, 2.373, 2.491], [0.97, 0.98, 0.98, 1.0]],
[[1.134, 1.194, 1.194, 1.254], [0.97, 0.98, 0.98, 1.0]],
[[2.204, 2.32, 2.32, 2.436], [0.97, 0.98, 0.98, 1.0]],
[[4.75, 5.0, 5.0, 5.25], [0.97, 0.98, 0.98, 1.0]],]
[[[0.71, 0.746, 0.746, 0.785], [0.97, 0.98, 0.98, 1.0]],
[[0.95, 1.0, 1.0, 1.05], [0.97, 0.98, 0.98, 1.0]],
[[2.033, 2.14, 2.14, 2.246], [0.97, 0.98, 0.98, 1.0]],
```

[[0.69, 0.726, 0.726, 0.763], [0.97, 0.98, 0.98, 1.0]],
....
[[1.311, 1.377, 1.377, 1.449], [0.97, 0.98, 0.98, 1.0]],
[[4.233, 4.444, 4.444, 4.678], [0.97, 0.98, 0.98, 1.0]],
[[0.95, 1.0, 1.0, 1.05], [0.97, 0.98, 0.98, 1.0]],
[[0.581, 0.611, 0.611, 0.642], [0.97, 0.98, 0.98, 1.0]],
[[0.36, 0.379, 0.379, 0.398], [0.97, 0.98, 0.98, 1.0]],
[[1.016, 1.07, 1.07, 1.123], [0.97, 0.98, 0.98, 1.0]],
[[0.511, 0.538, 0.538, 0.565], [0.97, 0.98, 0.98, 1.0]],
[[0.994, 1.046, 1.046, 1.098], [0.97, 0.98, 0.98, 1.0]],
[[2.142, 2.254, 2.254, 2.367], [0.97, 0.98, 0.98, 1.0]],]
[[[0.703, 0.738, 0.738, 0.777], [0.97, 0.98, 0.98, 1.0]],
[[1.422, 1.493, 1.493, 1.572], [0.97, 0.98, 0.98, 1.0]],
[[4.59, 4.82, 4.82, 5.074], [0.97, 0.98, 0.98, 1.0]],
[[1.558, 1.636, 1.636, 1.722], [0.97, 0.98, 0.98, 1.0]],
[[0.95, 1.0, 1.0, 1.05], [0.97, 0.98, 0.98, 1.0]],
[[0.39, 0.411, 0.411, 0.431], [0.97, 0.98, 0.98, 1.0]],
[[1.102, 1.16, 1.16, 1.218], [0.97, 0.98, 0.98, 1.0]],
[[0.555, 0.584, 0.584, 0.613], [0.97, 0.98, 0.98, 1.0]],
[[1.078, 1.135, 1.135, 1.191], [0.97, 0.98, 0.98, 1.0]],
[[2.323, 2.445, 2.445, 2.567], [0.97, 0.98, 0.98, 1.0]],]
....
[[1.631, 1.713, 1.713, 1.803], [0.97, 0.98, 0.98, 1.0]],
[[1.011, 1.061, 1.061, 1.117], [0.97, 0.98, 0.98, 1.0]],
[[2.856, 2.998, 2.998, 3.156], [0.97, 0.98, 0.98, 1.0]],
[[0.95, 1.0, 1.0, 1.05], [0.97, 0.98, 0.98, 1.0]],
[[1.224, 1.288, 1.288, 1.352], [0.97, 0.98, 0.98, 1.0]],
[1.288, 1.355, 1.355, 1.423], [0.97, 0.98, 0.98, 1.0]],

[[2.637, 2.775, 2.775, 2.914], [0.97, 0.98, 0.98, 1.0]],]
....
[[0.385, 0.405, 0.405, 0.426], [0.97, 0.98, 0.98, 1.0]],
[[1.244, 1.307, 1.307, 1.375], [0.97, 0.98, 0.98, 1.0]],
[[0.422, 0.444, 0.444, 0.467], [0.97, 0.98, 0.98, 1.0]],
[[0.39, 0.409, 0.409, 0.431], [0.97, 0.98, 0.98, 1.0]],

[[0.241, 0.253, 0.253, 0.267], [0.97, 0.98, 0.98, 1.0]],
[[0.682, 0.716, 0.716, 0.754], [0.97, 0.98, 0.98, 1.0]],
[[0.343, 0.36, 0.36, 0.379], [0.97, 0.98, 0.98, 1.0]],
[[0.667, 0.7, 0.7, 0.737], [0.97, 0.98, 0.98, 1.0]],
[[0.95, 1.0, 1.0, 1.05], [0.97, 0.98, 0.98, 1.0]],]],

The calculated eigenvector is as follows:

[[[0.134, 0.4368, 0.4368, 0.5889], [0.1812, 0.2974, 0.2974, 0.4092]],
[[0.0719, 0.2348, 0.2348, 0.3179], [0.3256, 0.8027, 0.8027, 0.8027]],
[[0.0241, 0.0648, 0.0648, 0.0809], [0.489, 0.8117, 0.8117, 0.9232]],
[[0.0774, 0.2496, 0.2496, 0.339], [0.0596, 0.2909, 0.2909, 0.385]],
[[0.0912, 0.2945, 0.2945, 0.4016], [0.1835, 0.7839, 0.7839, 0.879]],
[[0.1598, 0.5168, 0.5168, 0.7082], [0.1113, 0.4033, 0.4033, 0.4033]],
[[0.0614, 0.199, 0.199, 0.2311], [0.2558, 0.3423, 0.3423, 0.3904]],
[[0.1326, 0.43, 0.43, 0.5867], [0.1772, 0.3605, 0.3605, 0.4477]],
[[0.0741, 0.2406, 0.2406, 0.3299], [0.5078, 0.8989, 0.8989, 0.9315]],
[[0.0373, 0.1183, 0.1183, 0.1183], [0.3693, 0.6354, 0.6354, 0.7804]]]

The weights vector found by normalizing the calculated eigenvector is given below:
[[[0.1292, 0.1568, 0.1568, 0.2673], [0.1812, 0.2974, 0.2974, 0.4092]],
[[0.0693, 0.0843, 0.0843, 0.1443], [0.3256, 0.8027, 0.8027, 0.8027]],
[[0.0233, 0.0233, 0.0233, 0.0367], [0.489, 0.8117, 0.8117, 0.9232]],
[[0.0746, 0.0896, 0.0896, 0.1538], [0.0596, 0.2909, 0.2909, 0.385]],
[[0.0879, 0.1057, 0.1057, 0.1823], [0.1835, 0.7839, 0.7839, 0.879]],
[[0.154, 0.1856, 0.1856, 0.3214], [0.1113, 0.4033, 0.4033, 0.4033]],
[[0.0592, 0.0714, 0.0714, 0.1049], [0.2558, 0.3423, 0.3423, 0.3904]],
[[0.1278, 0.1544, 0.1544, 0.2663], [0.1772, 0.3605, 0.3605, 0.4477]],
[[0.0714, 0.0864, 0.0864, 0.1497], [0.5078, 0.8989, 0.8989, 0.9315]],

[[0.036, 0.0425, 0.0425, 0.0537], [0.3693, 0.6354, 0.6354, 0.7804]]].

Evaluation by Z-valued criteria

The considered criteria are used to evaluate three alternatives (employees): a_1, a_2, a_3. The information relevant to the criteria for evaluating the alternatives, and information on the preferences on the criteria is vague and partially reliabile. The decision matrix is given as follows:

D=
{{[6, 7, 8], [0.7, 0.8, 0.9]}, {[7, 8, 9], [0.8, 0.9, 1.0]},...
{[5, 6, 7], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[7, 8, 9], [0.8, 0.9, 1.0]}, {[7, 8, 9], [0.8, 0.9, 1.0]},...
{[6, 7, 8], [0.8, 0.9, 1.0]}, {[4, 5, 6], [0.8, 0.9, 1.0]},...
{[8, 9, 10],[0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]};

{[6, 7, 8], [0.7, 0.8, 0.9]}, {[4, 5, 6], [0.8, 0.9, 1.0]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[8, 9, 10],[0.8, 0.9, 1.0]}, {[7, 8, 9], [0.8, 0.9, 1.0]},...
{[3, 4, 5], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[4, 5, 6], [0.8, 0.9, 1.0]};

{[3, 4, 5], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[7, 8, 9], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[7, 8, 9], [0.8, 0.9, 1.0]}, {[3, 4, 5], [0.8, 0.9, 1.0]},...
{[6, 7, 8], [0.7, 0.8, 0.9]}, {[8, 9, 10],[0.8, 0.9, 1.0]}}.

For example, $Z_{i\,j} = \{[6, 7, 8], [0.7, 0.8, 0.9]\}$ is a Z-valued degree indicating the superiority of the i-th criterion over the j-th criterion.

In the first stage, we apply the Z-number-valued Simple Average Weighting (SAW) approach to evaluate each alternative, in the second stage, we use the fair price of each alternative Ai to compare the alternatives. The Z-number-based score (fair price) of alternatives is:

$$FP(Z(A,B)) = \int_0^1 K^-(\alpha)A^-(\alpha)d\alpha + \int_0^1 K^+(\alpha)A^+(\alpha)d\alpha + \\ + \int_0^1 L^-(\alpha)\ln(B^-(\alpha))d\alpha + \int_0^1 L^+(\alpha)\ln(B^+(\alpha))d\alpha,$$

where $K^{\pm}(\alpha)$ and $L^{\pm}(\alpha)$ are appropriate fuzzy functions.

For normalization, expert opinion-based Z-values matrices Q_j and L_j is obatined for criteria:

Q_j={
{ [8.5,9,9.5],[0.8,0.9,1.0]};
{[7.5,8,8.4],[0.8,0.9,1.0]};
{[5.6,6,6.5],[0.8,0.9,1.0]};
{[4.5,5,5.5],[0.8,0.9,1.0]};
{[1.5,2,2.5],[0.8,0.9,1.0]}
};
L_j={
{[0.8,0.9,1.0],[0.9,1.0,1.0]};
{[0.6,0.8,0.9],[0.9,1.0,1.0]};
{[0.4,0.6,0.8],[0.9,1.0,1.0]};
{[0.1,0.2,0.3],[0.9,1.0,1.0]};
{[0.0,0.1,0.2],[0.9,1.0,1.0]};
};

Part A of the largest eigenvalue Zlmax of the matrix is defined as (computed in Matlab) :

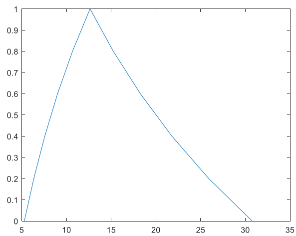

Figure 1. Diagram (Zlmax.As,Zlmax.A)

Part B of the largest eigenvalue Zlmax is defined as:

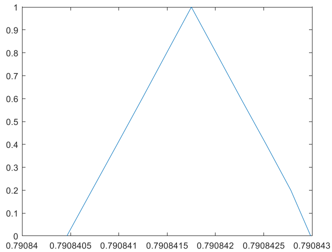

Figure 2. Diagram (Zlmax.Bs,Zlmax.B):

The crisp value of Zlmax obtained from the dezification transformation of the largest eigenvalue Zlmax is:

lmax=14.647468861420402

Consistency index corresponding to this value is calculated by the formula:

CI = (lmax-n)/(n-1);

For n=10:

CI=0.516385429046711

And the random consistency ratio for n=10:

RI=1.49

Since , the consistency ratio *CR=CI / RI* is obtained:

CR=0.346567402044773

The A and B parts of some components of the ZeigV eigenvector with Z-numbers corresponding to the largest eigenvalue Zlmax are defined as shown in Figs. 3-12 (computed in Matlab):

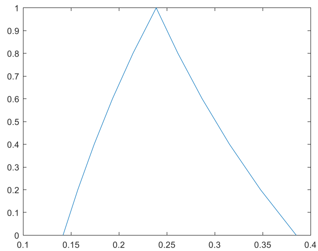

Figure 3. Diagram (ZeigV1.As,ZeigV1.A)

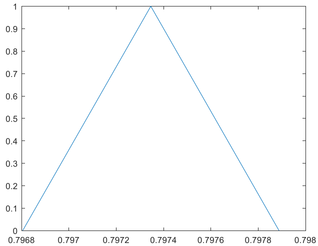

Figure 4. Diagram (ZeigV1.Bs,ZeigV1.B)

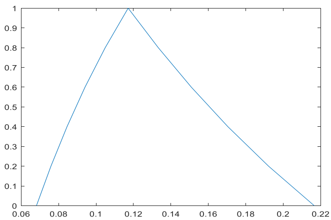

Figure 5. Diagram (ZeigV2.As,ZeigV2.A)

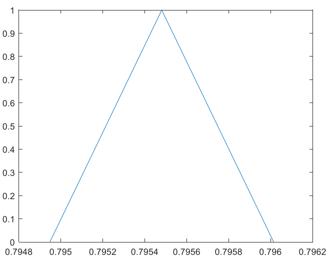

Figure 6. Diagram (ZeigV2.Bs,ZeigV2.B)

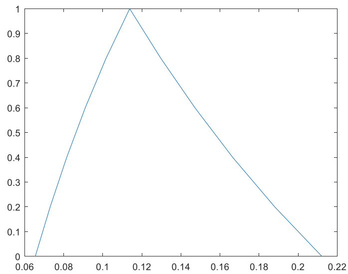

Figure 7. Diagram (ZeigV5.As,ZeigV5.A)

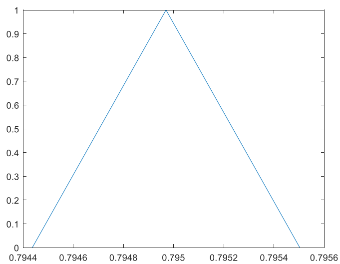

Figure 8. Diagram (ZeigV5.Bs,ZeigV5.B)

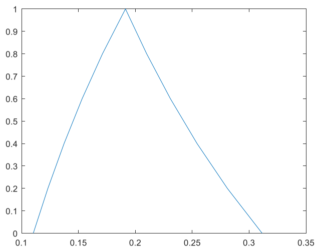

Figure 9. Diagram (ZeigV6.As,ZeigV6.A)

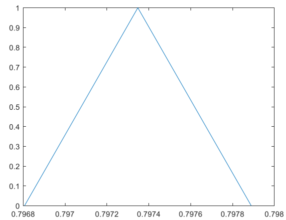

Figure 10. Diagram (ZeigV6.Bs,ZeigV6.B)

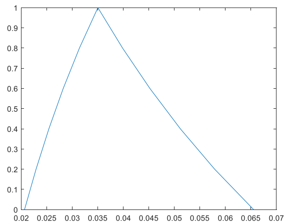

Figure 11. Diagram (ZeigV10.As,ZeigV10.A)

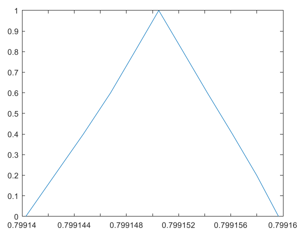

Figure 12. Diagram(ZeigV10.Bs,ZeigV10.B)

Some of the elements of the normalized decision matrix (Z-numbers) are defined as shown in Figs. 13-18 (computed in Matlab):

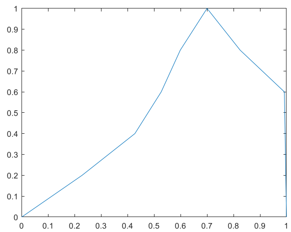

Figure 13. Diagram (e(1, 1).As,e(1, 1).A)

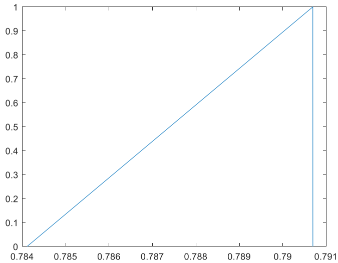

Figure 14. Diagram (e(1, 1).Bs,e(1, 1).B)

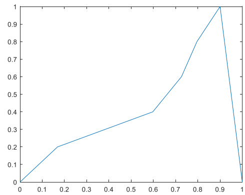

Figure 15. Diagram (e(2, 5).As,e(2, 5).A)

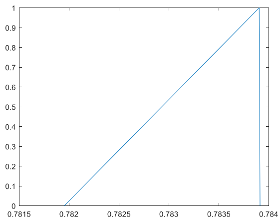

Figure 16. Diagram (e(2, 5).Bs,e(2, 5).B)

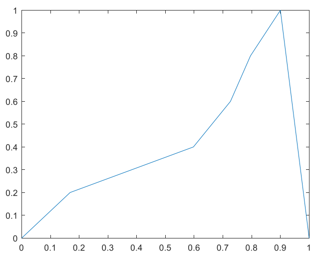

Figure 17. Diagram (e(3, 10).As,e(3, 10).A)

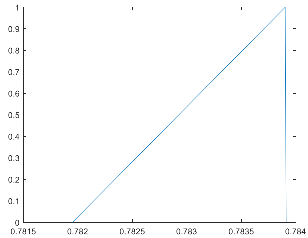

Figure 18. Diagram (e(3, 10).Bs,e(3, 10).B)

The weighted sum of criteria evaluations of each alternative is computed (in Matlab), Zwsi=ZmulRM(e,Wm). The weighted sum for alternative 1 is as follows, Zwsi(1) (table 4, support values for A and B parts are denoted ZwsiAs, ZwsiBs, and membership degrees by ZwsiA, ZwsiB):

Table 4.

The weighted sum for alternative 1

ZwsiAs	ZwsiA	ZwsiBs	ZwsiB
0	0	0.783108	0
0.069493	0.2	0.78312	0.2
0.137335	0.4	0.783132	0.4
0.296791	0.6	0.783144	0.6
0.463367	0.8	0.783156	0.8
0.684659	1	0.783168	1
0.997633	0.8	0.783168	0.8
1.364851	0.6	0.783168	0.6
1.738511	0.4	0.783168	0.4
2.226036	0.2	0.783168	0.2
3.058149	0	0.783168	0

The probability distribution Zwsi(1).p corresponding to Zwsi(1) are defined as:

Table 5.
Probability distribution Zwsi(1).p corresponding to Zwsi(1)

0.0000003	0.0000003	0.0000003	0.0000003	0.0000003	0.0000003
0.0000004	0.0000004	0.0000004	0.0000004	0.0000004	0.0000004
0.0000007	0.0000007	0.0000007	0.0000007	0.0000007	0.0000007
0.0000138	0.0000138	0.0000138	0.0000138	0.0000138	0.0000138
0.1956530	0.1956510	0.1956460	0.1956460	0.1956460	0.1956460
0.4100430	0.4100440	0.4100510	0.4100510	0.4100510	0.4100510
0.2657310	0.2657320	0.2657330	0.2657330	0.2657330	0.2657330
0.1285360	0.1285350	0.1285320	0.1285320	0.1285320	0.1285320
0.0000204	0.0000204	0.0000204	0.0000204	0.0000204	0.0000204
0.0000012	0.0000012	0.0000012	0.0000012	0.0000012	0.0000012
0.0000008	0.0000008	0.0000008	0.0000008	0.0000008	0.0000008

Part A of the weighted sum of the 1st alternative is determined as follows (figure 19,computed in Matlab):

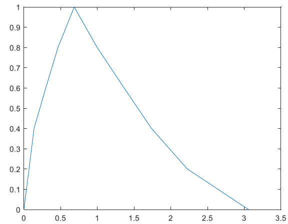

Figure 19. Diagram (Zwsi1.As,Zwsi1.A)

Part B of the weighted sum for alternative 1 is determined as follows:

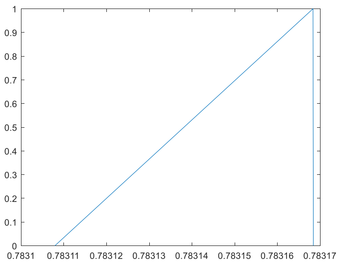

Figure 20. Diagram (Zwsi1.Bs,Zwsi1.B)

The weighted sum for alternative 2 is as follows (Table 6):

Table 6.

The weighted sum for alternative 2

ZwsiAs	ZwsiA	ZwsiBs	ZwsiB
0	0	0.757205	0
0.05647	0.2	0.757208	0.2
0.125074	0.4	0.757212	0.4
0.242388	0.6	0.757215	0.6
0.376744	0.8	0.757218	0.8
0.576612	1	0.757222	1
0.867996	0.8	0.757222	0.8
1.210546	0.6	0.757222	0.6
1.586637	0.4	0.757222	0.4
2.103864	0.2	0.757222	0.2
2.951975	0	0.757222	0

The probability distribution Zwsi(2).p corresponding to Zwsi(2) are defined as follows (Table 7):

Table 7.

Probability distribution Zwsi(2).p corresponding to Zwsi(2)

0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
0.0000002	0.0000002	0.0000002	0.0000002	0.0000002	0.0000002	0.0000002
0.0000030	0.0000030	0.0000030	0.0000030	0.0000030	0.0000030	0.0000030
0.1889950	0.1889950	0.1889940	0.1889930	0.1889930	0.1889930	0.1889950
0.4012160	0.4012160	0.4012170	0.4012180	0.4012180	0.4012180	0.4012160
0.2641370	0.2641370	0.2641370	0.2641370	0.2641370	0.2641370	0.2641370
0.1348840	0.1348840	0.1348840	0.1348840	0.1348840	0.1348840	0.1348840
0.0108000	0.0108000	0.0108000	0.0108000	0.0108000	0.0108000	0.0108000
0.0000004	0.0000004	0.0000004	0.0000004	0.0000004	0.0000004	0.0000004
0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001

Part A of the weighted sum for alternative 2 is determined as follows (Figure 21):

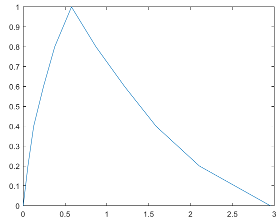

Figure 21. Diagram (Zwsi2.As,Zwsi2.A)

Part B of the weighted sum for alternative 2 is determined as follows (Figure 22):

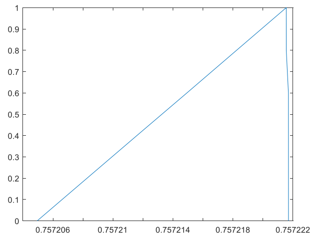

Figure 22. Diagram(Zwsi2.Bs,Zwsi2.B)

The weighted sum Zwsi(3) of the 3rd alternative is as follows:

Table 8.

The weighted sum Zwsi(3) of the 3rd alternative

Zwsi(3).As	Zwsi(3).A	Zwsi(3).Bs	Zwsi(3).B
0	0	0.764281	0
0.047483	0.2	0.764285	0.2
0.083384	0.4	0.764288	0.4
0.182084	0.6	0.764292	0.6
0.305501	0.8	0.764296	0.8
0.499526	1	0.7643	1
0.82125	0.8	0.7643	0.8
1.126889	0.6	0.7643	0.6
1.491204	0.4	0.7643	0.4
1.992121	0.2	0.7643	0.2
2.81664	0	0.7643	0

The probability distribution Zwsi(3).p corresponding to Zwsi(3) are defined as:

Table 9.

Probability distribution Zwsi(3).p corresponding to Zwsi(3)

0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
0.0000002	0.0000002	0.0000002	0.0000002	0.0000002	0.0000002	0.0000002
0.0000024	0.0000024	0.0000024	0.0000024	0.0000024	0.0000024	0.0000024
0.1858640	0.1858640	0.1858630	0.1858620	0.1858620	0.1858620	0.1858640
0.4001340	0.4001340	0.4001350	0.4001360	0.4001360	0.4001360	0.4001340
0.2687870	0.2687870	0.2687870	0.2687880	0.2687880	0.2687880	0.2687870
0.1348540	0.1348540	0.1348540	0.1348540	0.1348540	0.1348540	0.1348540
0.0104000	0.0104000	0.0104000	0.0104000	0.0104000	0.0104000	0.0104000
0.0000004	0.0000004	0.0000004	0.0000004	0.0000004	0.0000004	0.0000004
0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001

Part A of the weighted sum of the 3rd alternative is determined as follows (Figure 23):

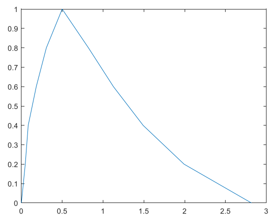

Figure 23. Diagram (Zwsi3.As,Zwsi3.A)

Part B of the weighted sum of the 3rd alternative is determined as follows (Figure 24):

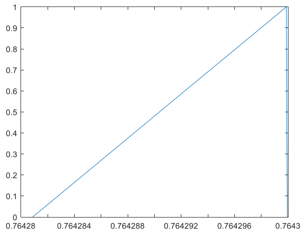

Figure 24. Diagram(Zwsi3.Bs,Zwsi3.B)

By calculating the simple weighted average values of the alternatives taking into account the weight coefficients of the criteria, their ranking was checked by 3 methods: 1) DeZifying the respective weighted average values of the alternatives; 2) using the respective fair prices of the alternatives and 3) using the proximity to the positive ideal solution. The obtained results are shown below accordingly.
DeZifying the respective weighted average values:
ZwsiC=[0.972474948733158; 0.887033256377744; 0.819605508730566];
It is clear that the order of the alternatives is as follows:

$Alternative1 \succ Alternative2 \succ Alternative3.$
If we apply the Fair Price-based evaluation method:

FairPrice= [1.871709079128489; 1.696285496568227; 1.567863001910343], the order is $Alternative1 \succ Alternative2 \succ Alternative3.$

The indexes of the alternatives according to the degree of closeness to the ideal solutions are given in a vector form:
r= [0.999547452656621; 0.120521362430677; 0.0373944862139085].
That is, again one has $Alternative1 \succ Alternative2 \succ Alternative3.$
After adding the new (fourth) alternative, the decision matrix becomes:

D={
{[6, 7, 8], [0.7, 0.8, 0.9]}, {[7, 8, 9], [0.8, 0.9, 1.0]},...
{[5, 6, 7], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[7, 8, 9], [0.8, 0.9, 1.0]}, {[7, 8, 9], [0.8, 0.9, 1.0]},...
{[6, 7, 8], [0.8, 0.9, 1.0]}, {[4, 5, 6], [0.8, 0.9, 1.0]},...
{[8, 9, 10],[0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]};

{[6, 7, 8], [0.7, 0.8, 0.9]}, {[4, 5, 6], [0.8, 0.9, 1.0]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[8, 9, 10],[0.8, 0.9, 1.0]}, {[7, 8, 9], [0.8, 0.9, 1.0]},...
{[3, 4, 5], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[4, 5, 6], [0.8, 0.9, 1.0]};

{[3, 4, 5], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[7, 8, 9], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[7, 8, 9], [0.8, 0.9, 1.0]}, {[3, 4, 5], [0.8, 0.9, 1.0]},...
{[6, 7, 8], [0.7, 0.8, 0.9]}, {[8, 9, 10],[0.8, 0.9, 1.0]};

{[6, 7, 8], [0.7, 0.8, 0.9]}, {[4, 5, 6], [0.8, 0.9, 1.0]},...
{[4, 5, 6], [0.8, 0.9, 1.0]}, {[6, 7, 8], [0.7, 0.8, 0.9]},...
{[8, 9, 10],[0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[6, 7, 8], [0.8, 0.9, 1.0]}, {[5, 6, 7], [0.8, 0.9, 1.0]},...
{[6, 7, 8], [0.7, 0.8, 0.9]}, {[5, 6, 7], [0.8, 0.9, 1.0]};
};

As a result of normalization and simple weighted averaging, the weighted sum Zwsi(4) of alternative 4 is:

Table 10.

The weighted sum Zwsi(4) of the 4th alternative

Zwsi(4).As	Zwsi(4).A	Zwsi(4).Bs	Zwsi(4).B
0	0	0.779006	0
0.067325	0.2	0.779018	0.2
0.150661	0.4	0.77903	0.4
0.248068	0.6	0.779042	0.6

0.383238	0.8	0.779054	0.8
0.617266	1	0.779066	1
0.998371	0.8	0.779066	0.8
1.38559	0.6	0.779066	0.6
1.770664	0.4	0.779066	0.4
2.276702	0.2	0.779066	0.2
3.047841	0	0.779066	0

The probability distribution of Zwsi(4).p corresponding to Zwsi(4) are defined as:

Table 11.
Probability distribution Zwsi(4).p corresponding to Zwsi(4)

0.00000001	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001
0.00000002	0.00000002	0.00000002	0.00000002	0.00000002	0.00000002	0.00000002
0.00000004	0.00000004	0.00000004	0.00000004	0.00000004	0.00000004	0.00000004
0.00000042	0.00000042	0.00000042	0.00000042	0.00000042	0.00000042	0.00000042
0.18505700	0.18505500	0.18505300	0.18504800	0.18504800	0.18504800	0.18505700
0.40285000	0.40285200	0.40285300	0.40285800	0.40285800	0.40285800	0.40285000
0.26973500	0.26973600	0.26973700	0.26973900	0.26973900	0.26973900	0.26973500
0.13746200	0.13746200	0.13746200	0.13746300	0.13746300	0.13746300	0.13746200
0.00490000	0.00489000	0.00489000	0.00489000	0.00489000	0.00489000	0.00490000
0.00000007	0.00000007	0.00000007	0.00000007	0.00000007	0.00000007	0.00000007
0.00000001	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001

The ranking of the four alternatives is shown below.
ZwsiC=[0.972474948585615; 0.887033231158182; 0.819605492305809; 0.962337134274546].
It means:
$Alternative1 \succ Alternative4 \succ Alternative2 \succ Alternative3.$

FairPrice= [1.87170907910337; 1.69628549365423; 1.56786300217767; 1.85694253454992].
It means:
$Alternative1 \succ Alternative4 \succ Alternative2 \succ Alternative3$.
Ideal solution method results:
r= [0.938701764998138; 0.198583368499463; 0.0622415322007566; 0.482506849293076].
It means: $Alternative1 \succ Alternative4 \succ Alternative2 \succ Alternative3$.

Thus, the order of the previous alternatives is not disturbed.

CONCLUSION

1. A comprehensive review of the existing literature proved that not all the necessary criteria are considered in systematic approach to decision-making of personnel selection . For the first time, the decision-making problem of personnel selection based on ten criteria (work quality, execution ability-work quantity, degree of knowledge at work, decision-making in effective problem solving, honesty, punctuality, teamwork, development, ability to adapt to change, stability) comprehensively have been stated and solved solution.
2. The problem of multicriteria decision-making based on the theory of Z-numbers has been solved considering reliability of available information and selection results.
3. In this research work, the consistency of the decision-maker's preference in the personnel selection problem was studied, and a matrix of pairwise comparison of criteria importance satisfying consistency criterion was constructed.Based on the consistent matrix, the weighting coefficients of the criteria were computed.
4. Based on Z-numbers, the problem of ranking alternatives according to the Simple Average Weighting method was solved.
5. In contrast to all existing studies, the proposed decision-making procedure for personnel selection satisfies the rank-reversal condition.

The main content of the dissertation is published in the following works:

1. Salmanov S. Çoxmeyarlı yanaşma əsasında kadr seçimi // Azərbaycan Mühəndislik Akademiyasının xəbərləri, -2017, - Cild 9, №4, - s. 113-118
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4. Salmanov S. The development history of decision making and creation of this system // İnternational conference: process management and scientific development (UK), -2021, - ISBN 978-5-905695-64-4 UDC 330, - pp. 130-133
5. Salmanov S. Kadr Seçimində Qərar Qəbuletmə üsullarina baxış // Azərbaycan Ali Texniki Məktəblərinin Xəbərləri, -2023, - Volume 34(04) Issue 11, - pp.294-301
6. Salmanov S. İnsan Resurslarının İdarə Edilməsi // Azərbaycan Ali Texniki Məktəblərinin Xəbərləri, -2023, - 35(04) 12, - s. 429-437
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8. Salmanov S., L. A. Gardashova. Using Z-Number-Based Information in Personnel Selection Problem // Lecture notes in network and systems, - 2022, - Volume 362, - pp.302-307
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Author's individual participation in the published works

[8] - Author of the idea, calculations and analysis of findings.

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