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ABSTRACT<br>of the dissertation for the degree of Doctor of Philosophy

# DEVELOPMENT AND APPLICATION OF METHODS AND ALGORITHMS FOR CONTROLLING FLYING OBJECTS ON THE BASIS OF AUTONOMOUS NAVIGATION DATA 

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## GENERAL DESCRIPTION OF THE DISSERTATION

The relevance and maturity of the topic. Since the beginning of the last century, armies of industrially developed countries have considered the use of aircrafts in military operations to be very promising. The invention of the flight data recorder system, which became known in scientific and technical literature as "black box", opened a very progressive page in avionics in 1939. The invention of the black box began to play an important role in solving problems such as the perfect design of aircrafts operating on different principles, their correct operation, the identification of technical defects during the flight, the evaluation of pilots' skills. The black box data processing algorithm and the range of problems solved with their involvement depend on the number of different sensors (transmitters) involved in data acquisition, the physical principle on which their operation is based, their design and other additional information.

The plotting of the flight path is one of the most important of these issues. Both the investigation of the post-flight path and the determination of the aircraft's current location during flight operations require accurate processing of black box data. Undoubtedly, there are many algorithms and appropriate software that process the data collected by companies that design and manufacture avionics for aircrafts. An example of such software systems is the Monster-2012 software and hardware system. However, design and production enterprises do not provide such systems to anyone, and the information about them found in the scientific and technical literature is very simple. On the other hand, information on missions and training flights performed by military aircrafts is classified and it is not reasonable to send the flight data collected in the black box to companies located in foreign countries for processing. From this point of view, the
development of black box data processing algorithms is a relevant scientific and technical issue.

Determining the current location of the aircraft on the basis of online processing of recorded sensor data allows one to control its movement along the intended trajectory (route). This makes it possible to create an automatic control system for long-range military unmanned aerial vehicles. The recently revealed possibility of involvement of unmanned aerial vehicles, including groups of them, in military operations necessitates addressing a number of issues related to their control. From the point of view of ensuring the failsafety and reliability of UAVs during operations, it is not advisable to purchase their software from foreign countries. Thus, the solution of problems related to the design, set up and control of unmanned aerial vehicles, including their special type, quadcopters, which are widely developed for military purposes in Azerbaijan, is a relevant issue investigated in the dissertation.

The object and subject of the research. The problems investigated in the dissertation cover three research objects. The first is the data obtained from the sensors placed on board a military aircraft; research methods of functional analysis have been applied depending on the task set during its processing and the hypotheses put forward.

The second object is a flying object modeled as a solid body. Empirical approaches, differential equations and methods of the theory of boundary value problems put forward in solid state mechanics have been used during the modeling.

The third research object is a group of unmanned aerial vehicles. The determination of the flight strategy of each unmanned aerial vehicle in the group performing a join flight and the development of a current movement decision-making mechanism are investigated.

The subject of the research is the development of approaches and methods for solving the problems of control of an unmanned aerial vehicle and a joint flight of a group of unmanned aerial vehicles, which arise in the processing of sensor data.

The aim and objectives of the research. The aim of the dissertation is to calculate the trajectory of a military aircraft on the basis of sensor data after the flight, to determine its current location during the flight, to develop a system of automatic control of unmanned aerial vehicles. To achieve this aim, the following main objectives are set in the dissertation:

- Elimination of load distortions in the flight data of a military aircraft equipped with a TECTEP-У3-type device;
- Development of an algorithm for calculating the data fusion coefficients based on the loads, orientation angles and speed data recorded in the black box for the initial flight period of the military aircraft;
- Development of a neural network method for the recovery of flight data lost during recording in the black box and a numerical computation algorithm to calculate the coefficients of the network;
- Development of an algorithm for filtering the angle of the aircraft, adjusting the flight speed according to the data of the Doppler device and adjusting the angle of inclination;
- Development of an algorithm based on the Newton-Raphson iteration method to fuse the data of the flight path equation as a linear combination and to find the coefficients of the equations;
- Construction of a mathematical model of the problem of flight data fusion in the form of a problem of minimization of the corresponding functional;
- Reducing the minimization problem to the boundary value problem for fourth-order ordinary differential equations with constant coefficients using the variation method;
- Development of an analogue of the classical sweep method to solve boundary value problems;
- Substantiation of replacement of regular metrics with integral metrics during the adjustment of data received from transmitters on board unmanned aerial vehicles;
- Construction of a mathematical model expressing the relationship between the trajectory and spatial orientation of a
quadcopter-type unmanned aerial vehicle and the factors and control parameters affecting it;
- Development of an algorithm for calculating and refining a number of characteristic physical and technical parameters of a quadcopter that change due to the addition of various devices and loads during operation;
- Development of the calculation algorithm of the control to eliminate the deviations that occur during the movement of a quadcopter-type unmanned aerial vehicle on the intended trajectory;
- Development of a method of constructing a bijective mapping between the geographical coordinates and the images on the camera monitor to solve the problem of determining the location of the unmanned aerial vehicle from the video and photo images taken by it;
- Construction of a mathematical model of the problem of determining the optimal trajectory of movement along the intended route on the basis of a geometric map of obstacles and irregularities of the terrain in the area given as spherical elements;
- Development of a heuristic numerical computation algorithm to find one of the possible efficient trajectories;
- Describing a group of unmanned aerial vehicles as a system of systems to create an effective control mechanism for the joint flight of unmanned aerial vehicles by one operator;
- Development of a universal decision-making algorithm to control the movement of each unmanned aerial vehicle participating in a joint flight along the intended trajectory by regulating its speed;
- Obtaining formulas for calculating the speed mode that allows an unmanned aerial vehicle to participate in a safe joint flight without colliding with other UAVs in different parts of the trajectory under consideration;
- Development of the principles for determining the debugging parameters for the model of joint flight control of unmanned aerial vehicles.

Research methods. The methods used in the research are based on mathematical modeling and approximate calculation methods based on systems analysis.

Research methods of functional analysis, such as approximation of experimental data, the least square method, the finite difference method, the iteration method, etc., were used for determining the state of flight objects based on the processing of sensor data.

Empirical approaches, differential equations and methods of the boundary value theory, as well as approximate calculation methods were used in modeling the motion of quadcopter-type aircraft, calculating the parameters in the dynamic model and solving the problems of the control of the aircraft on the planned trajectory.

Methods of abstract algebra, analytical geometry and computational mathematics, such as the bijective mapping principle, methods of successive approximations, etc., were used in the study of the problem of determining the location of unmanned aerial vehicles from video and photo images.

The principles and methods of the System-of-Systems ideology were used in the control of joint flights in order for each unmanned aerial vehicle to make independent decisions. Empirical formulas and methods of vector calculus and analytical geometry, as well as methods of the algorithm theory were used to determine the position in space relative to others for each vehicle regarded as an independent object and to calculate the current velocity required.

## Main theses put forward for defense:

1. Formulas for solving the problem of eliminating load distortions in the flight data of a military aircraft equipped with a strapdown navigation system
2. An algorithm for calculating data fusion coefficients based on loads, orientation angles and speed data recorded in the black box for the initial flight period of a military aircraft with a platform-type navigation system.
3. A neural network method for recovering flight data lost during recording in the black box and numerical computation algorithm for calculating network coefficients.
4. Algorithms for filtering the yaw angle of the aircraft, correcting the flight velocity according to the data of the Doppler velocity sensor and adjusting the pitch angle.
5. A method for fusing the data of the equation of the flight path of a military aircraft as a linear combination and algorithm based on the Newton-Raphson iteration method for finding the coefficients of the equations.
6. Constructing the mathematical model of the problem of the fusion of aircraft flight data as a problem of minimization of a functional. Reducing the minimization problem to the boundary value problem for fourth-order ordinary differential equations with constant coefficients using the variation method and an analogue of the classical sweep method to solve the boundary value problem.
7. Substantiation of the possibility of applying the approximation criterion in the $L_{2}$ metric instead of the regular approximation criterion when adjusting the data received from the transmitters on board unmanned aerial vehicles with the help of additional flight information.
8. An algorithm for calculating and refining a number of characteristic physical and technical parameters of a quadcopter that have changed due to the addition of various devices and loads during operation on the basis of data related to the initial flight period based on the mathematical model that expresses the relationship between the trajectory and spatial orientation of a quadcopter-type unmanned aerial vehicle and the factors and control parameters that affect it.
9. An algorithm for solving the problem of the control of a quadcopter-type unmanned aerial vehicle by eliminating the deviations that occur during its movement along the intended trajectory.
10. An algorithm for calculating the geographical coordinates of the current location of the unmanned aerial vehicle from the video and photo images taken by it.
11. A mathematical model of the problem of determining the optimal trajectory of movement along the intended route on the basis of a geometric map of obstacles and irregularities of the terrain in the
area given as spherical elements. A heuristic numerical computation algorithm to find one of the possible efficient trajectories.
12. Basic universal principles of describing a group of unmanned aerial vehicles as a system of systems and the control system in order to create an effective control mechanism for the joint flight of unmanned aerial vehicles by one operator.
13. Universal decision-making algorithm to control the movement of each unmanned aerial vehicle participating in a joint flight along the intended trajectory by regulating its speed.

Scientific novelty of the research. The scientific novelty of the research consists in the following:

1 The problem of eliminating load distortions in the flight data of a military aircraft equipped with a TECTEP-У3-type device has been studied, and a formula for calculating the loads relative to the normal coordinate system has been proposed. The refinement coefficients calculated during the processing of flight data of a military aircraft equipped with a TECTEP-У3-type device can be used to determine the loads of that aircraft relative to the earth during the next flight.

2 An algorithm has been given for calculating the data fusion coefficients based on the loads, orientation angles and speed data recorded in the black box for the initial flight period of a military aircraft with a platform-type navigation system has been given. These coefficients make it possible to obtain a satisfactory trajectory in real time by applying the fusion algorithm in the post-flight period. The results of numerical experiments give reason to believe that this algorithm can be successfully applied.

3 A neural network method has been developed for recovering flight data lost during recording in the black box. A numerical computation algorithm has been given to calculate the coefficients of the neural network for large amounts of data. This algorithm allows filling in the gaps in the black box data and analyze the flight in more detail.

4 Pre-processing problems related to filtering the yaw angle of the aircraft, determining the moment of its takeoff, correcting the flight velocity according to the data of the Doppler velocity sensor and adjusting the pitch angle have been studied, and algorithms have been proposed to solve them. With the application of these algorithms, data can be processed automatically and used to create an automated decision-making system.

5 A method has been developed for fusing the data of the equation of the flight path of a military aircraft as a linear combination. An algorithm based on the Newton-Raphson iteration method has been proposed for finding the coefficients of the equations.

6 The mathematical model of the problem of the fusion of aircraft flight data has been constructed as a problem of minimization of a functional. The minimization problem has been reduced to the boundary value problem for fourth-order ordinary differential equations with constant coefficients using the variation method. An analogue of the classical sweep method has been developed for solving the boundary value problem.

7 The possibility of applying the approximation criterion in the $L_{2}$ metric instead of the regular approximation criterion when adjusting the data received from the transmitters on board unmanned aerial vehicles with the help of additional flight information has been substantiated.

8 An algorithm has been developed for calculating and refining a number of characteristic physical and technical parameters of a quadcopter that have changed due to the addition of various devices and loads during operation on the basis of data related to the initial flight period based on the mathematical model that expresses the relationship between the trajectory and spatial orientation of a quadcopter-type unmanned aerial vehicle and the factors and control parameters.

9 The problem of the control of a quadcopter-type unmanned aerial vehicle by eliminating the deviations that occur during its movement along the intended trajectory has been investigated, an
algorithm has been developed for calculating the corresponding control.

10 Constructing a bijective mapping between the geographical coordinates and the images on the camera monitor has been proposed to solve the problem of determining the location of the unmanned aerial vehicle from the video and photo images taken by it; an iterative computation algorithm has been developed for calculating the geographical coordinates of the current flight location.

11 A mathematical model of the problem of determining the optimal trajectory of movement along the intended route has been constructed on the basis of a geometric map of obstacles and irregularities of the terrain in the area given as spherical elements. The concept of efficient trajectory has been given; a heuristic numerical computation algorithm has been proposed for finding one of the possible efficient trajectories.

12 A group of unmanned aerial vehicles has been described as a system of systems to create an effective control mechanism for the joint flight of unmanned aerial vehicles by one operator; basic universal principles of the control system have been proposed.

13 A universal decision-making algorithm has been developed for controlling the movement of each unmanned aerial vehicle participating in a joint flight along the intended trajectory by regulating its speed. Formulas have been given for calculating the speed mode that allows an unmanned aerial vehicle to participate in a safe joint flight without colliding with other UAVs in different parts of the trajectory under consideration. The principles for determining the debugging parameters for the model of joint flight control of unmanned aerial vehicles have been developed.

Theoretical and practical significance of the research. The results obtained during the research are both of theoretical and of practical significance.

The theoretical significance of the results obtained during the research is that the proposed ideas, methods and algorithms can be used in the investigation and solution of the problems of processing of
sensor data of aerial vehicles, a stable flight of a controlled drone or group of drones along the trajectory and other similar problems.

The practical significance of the results obtained during the processing of aircraft flight data is that it is possible to create automated software systems that allow determining the current location and orientation of military aircrafts on the basis of the indicators recorded in the black box, both in the post-flight period and in online mode.

The practical significance of the results obtained in the field of unmanned aerial vehicle research is that these results can be used in the design, calculation and automation of drones of various purposes.

The practical significance of the results obtained in the study of joint flight of unmanned aerial vehicles is that they can be used to create an independent decision-making system for each aircraft in the group on the basis of information on the trajectory obtained in advance.

The proposed approaches, developed mathematical models and algorithms can be used by research organizations and planning and design offices engaged in the tracking of the movement of aerial vehicles and the development of automated control systems based on their sensor data, as well as the development of joint flight systems.

Validation and implementation of the results. The main results of the dissertation were presented at the 3rd and 4th International scientific conferences on System Identification and Control Problems ("Идентификация систем и задачи управления", Moscow, 2004-2005), the 2nd International Scientific-Practical Conference on Topical Problems of Modern Science and Education ("Актуальные вопросы современной науки и образования", Penza, 2020), 10th International Conference on Modern Directions of Development of the Information and Communication Technologies and Control Systems (Kharkiv, 2020), the scientific-practical conference on Military Science and Security Problems (Baku, 2019), the 7th International Conference on Control and Optimization with Industrial Applications (2020, Baku), the 4th International Scientific
and Practical Conference on Modeling, Control and Information Technologies (2020, Rivne), as well as discussed at the seminars of the Institute of Control Systems of the Azerbaijan National Academy of Sciences and the Research Institute of Applied Mathematics of Baku State University.

The proposed approaches and the developed mathematical models have been applied to the study of the solution of the problems under consideration and to the development of appropriate solution algorithms.

The author's personal contribution is the formalization of the correct statement of the problems and the choice of research direction to achieve the set aim. All the results obtained and research methods also belong to the author personally.

The author's published works. The author has 21 research works published as part of his work on the dissertation, 14 articles, 7 theses in publications recommended by the Supreme Attestation Commission under the President of the Republic of Azerbaijan.

Name of the organization where the dissertation work was performed. The dissertation work was performed at the Institute of Control Systems of the Azerbaijan National Academy of Sciences.

Length and structure of the dissertation. The dissertation consists of an introduction, 5 chapters, main results of the work, and a list of references of 223 entries. The total length of the dissertation is 234 pages, the main body of the manuscript is 210 pages ( 400,000 symbols), including 1 table and 16 figures. The length of Chapter 1 is 70,000 symbols, Chapter 2 - 98,000 symbols, Chapter 3 - 64,000 symbols, Chapter 4 - 96,000 symbols, and Chapter 5 - 56,000 symbols.

## THE CONTENT OF THE DISSERTATION

Chapter 1 consists of three sections and is devoted to a review of the existing literature in three areas related to the problems under investigation.

The first area, which is mainly devoted to the study of materials published in the open-access scientific and technical literature and Internet resources, covers the algorithms for processing aircraft black box data. Thus, in modern aircrafts, a portion of the flight data measured by various sensors is pre-processed and displayed on the dashboard to be used by the pilot to control the flight. On the other hand, it is digitized with the application of conversion procedures and recorded into the black box, making it possible to study the progress of the flight afterwards. However, due to errors in operations such as measurement, encoding, recording, it becomes difficult to determine how the aircraft flew on the basis of data obtained directly from the black box. A brief overview of the articles devoted to the construction of the flight path by filtering the relevant data and fusing the data of measuring devices operating on various physical principles is given; it is shown that known filtering algorithms are ineffective when the probabilistic characteristics of the technical characteristics of sensor devices are not known, and other new approaches need to be developed.

It is also pointed out that the approaches employed in the software packages developed in different countries for the processing of flight data are not published in the open-access literature, being part of military systems on the one hand, and commercial-oriented scientific and technical products on the other. Therefore, a flight data processing system for military aircrafts used in Azerbaijan should be created by the country's own specialists.

The second area includes the approaches proposed for planning the flight path, controlling the aircraft's flight along the flight path, elimination of deviations, autonomous control and a number of other
theoretical and practical problems related to the control of unmanned aerial vehicles and performance of various tasks by them. It is noted that the successful application of the proposed solutions depends on the type, design, specifications, operating conditions and functional requirements of a particular aircraft. Therefore, when designing new devices aimed at solving said problems, it is necessary to look at the different options proposed in the literature and, if necessary, to develop new approaches.

The third area of the review is devoted to the discussion of the problems and approaches studied by various authors on the control of a group flight of unmanned aerial vehicles. The main focus here is on the architecture of decision-making systems in the field of control, their comparative analysis is given. It is shown that from the point of view of multi-agent system classification, a purposeful hybrid system based on a model operating on the principle of utility can be considered the most suitable control system. In this regard, it would be quite effective for each unmanned aerial vehicle performing a joint flight to have a priori information on the area of operation and the environment, to pursue a common goal and to make independent based on the exchange of limited information about nearby drones.

The studies described in Chapter 1 were published in [16, 17, 19].

Chapter 2, consisting of four sections, is devoted to the issues arising during the recovery of the flight path of a military aircraft on the basis of black box data and the problem of their elimination.

Section 1 of Chapter 2 explores the problem of determining the trajectory of a military aircraft equipped with a TESTER-UZ platformtype navigation device that has returned to the airfield after completing a flight. It is assumed that the coordinates $s_{x}(0), s_{y}(0), s_{z}(0)$ of the aircraft in the normal coordinate system and the components $v_{x}(0), v_{y}(0), v_{z}(0)$ of the velocity vector at the instant $t=0$ of the loss of contact with the runway during takeoff, as well as the coordinates $s_{x}(T), s_{y}(T), s_{z}(T)$ and the components $v_{x}(T), v_{y}(T), v_{z}(T)$ of the
velocity vector at the instant $t=T$ of contact with the runway during landing are known.

As the load distortions in the flight data do not allow the trajectory to be constructed satisfactorily with the integration of the loads, the causes of the deviations are suggested and a method is proposed to eliminate them. It is hypothesized that, using the known conversion functions $k_{x}\left(p_{x}\right), k_{y}\left(p_{y}\right), k_{z}\left(p_{z}\right)$, the encoded values $p_{x}, p_{y}$ of the loads ${ }^{1}$ relative to the local coordinate system of the aircraft measured and recorded into the black box give certain intermediate values $\left\{\hat{n}_{x}=k_{x}\left(p_{x}\right), \hat{n}_{y}=k_{y}\left(p_{y}\right), \hat{n}_{z}=k_{z}\left(p_{z}\right)\right\}$ such that the relationship between them and the actual loads is described with linear dependence formulas:

$$
\left\{\begin{array}{l}
n_{x}=\alpha_{x} \hat{n}_{x}+\beta_{x}, \\
n_{y}=\alpha_{y} \hat{n}_{y}+\beta_{y}, \\
n_{z}=\alpha_{z} \hat{n}_{z}+\beta_{z} .
\end{array}\right.
$$

It is required to find such unknown coefficients $\alpha_{x}, \alpha_{y}, \alpha_{z}, \beta_{x}, \beta_{y}, \beta_{z}$ that the trajectory recovered on the basis of the calculated loads ensures that the aircraft has the coordinates $s_{x}(T), s_{y}(T), s_{z}(T)$ and the velocity vector $v_{x}(T), v_{y}(T), v_{z}(T)$ at the instant $t=T$. Finding these coefficients from real flight data and appropriate calculations show that the obtained trajectories are quite close to the real trajectory.

It should be noted that the data from strapdown navigation devices are given relative to the $O x y z$ coordinate system fixed directly to the aircraft. The transformation matrix of the coordinates generated on the basis of orientation angles measured by gyroscopes when calculating this data relative to the $O X Y Z$ earth-fixed coordinate

[^0]system ( $\psi-$ the yaw angle, the angle between the $O X$ axis of the $O X Y Z$ coordinate system and the projection of the longitudinal axis of the aircraft on the $O X Y$ plane, $\vartheta$ - the pitch angle, the angle between the longitudinal $O X$ axis of the aircraft and the $O X Y$ plane, $\gamma$ - the roll angle, the angle between the lateral $O Y$ axis of the aircraft and the $O Y$ axis of the $O X Y Z$ coordinate system) should be multiplied.

The studies described in this section were published in [7].
Section 2 of Chapter 2 investigates the problem of processing flight data of an aircraft with an ИКВ ${ }^{2}$ platform-type navigation system by real-time fusion method. The main difference of the ИКВtype devices from other known navigation systems is that its accelerometers are mounted on a gyroscopic platform that maintains its orientation in space.

The essence of the issue is that the flight path can be determined on the basis of two data channels, one by one-time integration of the measured speed of the aircraft, and the other by double integration of accelerations determined based on the loads. However, due to the distortions and errors in the parameters recorded in the black box, neither of these channels allows determining the flight path with sufficient accuracy during the direct processing of flight parameters.

Because the loads $n_{x}, n_{y}$ and $n_{z}$ are written to the list of flight parameters recorded in the black box, for each instant $t$ considered in the initial period of the flight, on the one hand, the accelerations $a_{x}, a_{y}, a_{z}$ from the load channel and the velocity vector $\mathbf{V}(t)=\left(V_{x}(t), V_{y}(t), V_{z}(t)\right)$ found as their integral are calculated. On the other hand, the vector $\mathbf{U}(t)=\left(U_{x}(t), U_{y}(t), U_{z}(t)\right)$ coming from the velocities channel is taken. It is proposed to find matrices of the constants $\mathbf{E}_{v}$ and $\mathbf{E}_{u}$ such that the part of the trajectory

[^1]$\int_{0}^{t}\left(\mathbf{E}_{v} \mathbf{V}(\tau)+\mathbf{E}_{u} \mathbf{U}(\tau)\right) d \tau \quad$ calculated with their help on the basis of the
fusion of the velocities $\mathbf{V}(t)$ and $\mathbf{U}(t)$ should be as close as possible in the quadratic metric to the trajectory obtained on the basis of GPS data. This method allows calculating data fusion coefficients based on the loads, orientation angles and speed data recorded in the black box for the initial period of flight of a military aircraft. These coefficients allow obtaining a satisfactory trajectory in real time by applying the fusion algorithm in the post-flight period.

The results of numerical experiments give grounds to believe that this algorithm can be successfully applied (Fig. 1).


Fig. 1. Comparison of the flight path with the trajectories plotted on the basis of various data: (A) - based on the loads; (B) - based on the velocity data; (C) - based on the results of data fusion; (D) - based on the GPS data.

The studies described in this section were published in [10, 15].
Section 3 of Chapter 2 is devoted to the recovery of flight data lost in the process of recording in the black box.

It is known that the flight data recorded in the black box of an aircraft consists of a series of frames, each of which is received from the sensors and converted into binary code and consists of $N=256$
code. In terms of information technology, a "frame" is a "record" of binary components. Each frame covers a 1 -second period of flight time. Practice shows that in some cases, when flight data is written into the black box or transferred from the black box to other magnetic media, some of it is lost. This is manifested in the fact that in some cases, instead of 256 binary codes, a certain, smaller, number of $M(M<N)$ codes is written in a frame during data recording. It is not known in advance what code or codes are missing in the frame, therefore, during the processing of flight data, such frames are considered useless. It is proposed to apply the neural network method as follows to find the locations of lost quantities in the frame and to recover their values.

Suppose that $\mathbf{x}\left(t_{i}\right)=\left(x_{1}\left(t_{i}\right), x_{2}\left(t_{i}\right), \ldots, x_{N}\left(t_{i}\right)\right), \quad i=1,2, \ldots$, $t_{i}=i \cdot \Delta t, \Delta t=$ const is a vector notation of the sequence of quantities recorded in a frame. Normally, three groups of quantities that have a strong correlation with each other are distinguished: longitudinal channel, lateral channel and altitude channel. Let us denote by the vector function $\mathbf{y}\left(t_{i}\right)=\left(y_{1}\left(t_{i}\right), y_{2}\left(t_{i}\right), \ldots, y_{m}\left(t_{i}\right)\right)$ such subset requiring the recovery of the components of the vector $\mathbf{x}\left(t_{i}\right)$ that they have a strong correlation with one of the mentioned channels. As input data, let us consider the time series $\left\{\mathbf{y}\left(t_{i}\right), i=1,2, \ldots, p\right\}$. It is proposed to construct a linear autoregression model (neural network) with respect to $\mathbf{y}\left(t_{i}\right)$ for a certain number $k<p$, depending on the unknown coefficients $w_{1}, w_{2}, \ldots, w_{k}$, as follows:

$$
\mathbf{y}\left(t_{s}\right)=\sum_{j=1}^{k} w_{j} \mathbf{y}\left(t_{s-k-i+j}\right), \quad s=k+1, k+2, \ldots, p
$$

Given the large amount of real black box data, the Cholesky scheme was applied during the development of the algorithm for calculating the weight coefficients in their processing ${ }^{3}$.

The studies described in this section were published in [1, 4, 5].
Section 4 of Chapter 4 is devoted to the pre-processing of some flight parameters collected in the black boxes of fighter aircrafts. It is well known that during the processing of black box data, there is a need to solve another specific problem. This problem is related to the measurement system gauging various indicators, the setting of sensors and the conditions for the participation of these quantities in the computation algorithms. Therefore, sensor data should be preprocessed to prepare it for use. For this purpose, Section 4 describes the algorithms for such problems as filtering the yaw angle, determining the take-ff moment of the aircraft, correcting the flight velocity according to the data of the Doppler velocity sensor ${ }^{4}$ and adjusting the pitch angle.

The studies described in this section were published in [21].
Chapter 3 consists of three sections and is devoted to other theoretical approaches to the fusion of black box data of a military aircraft and the substantiation of the applied integral-quadratic criterion.

In Section 1 of Chapter 3, it is proposed to search for the righthand side of the differential equation constructed for calculating of the real flight trajectory of an aircraft in the form of a combination of measured quantities. The essence of the problem can be explained as follows. Ideally, the first and second derivatives of the actual flight path $\mathbf{s}(t)=(x(t), y(t), z(t))$, the equation of which is written with respect to the inertial system, should coincide with the velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$, respectively, obtained from the primary

[^2]processing of black box data. However, in reality we observe $\frac{d}{d t} \mathbf{s}(t) \neq \mathbf{v}(t)$ and $\frac{d^{2}}{d t^{2}} \mathbf{s}(t) \neq \mathbf{a}(t)$, and the readings $h(t)$ of the barometric altimeter differ from the real function $y(t)$. In view of the above, a flight data fusion hypothesis is put forward, suggesting that there is a linear combination of indicators recovered from flight data such that its value coincides with the corresponding combination recorded for the actual flight path.

Based on this, the vector $\mathbf{s}=(x(t), y(t), z(t))$, which determines the position of the aircraft, can be sought as a solution to the following linear system of differential equations:

$$
\left\{\begin{array}{c}
\frac{d^{2}}{d t^{2}} x(t)-k_{1} \frac{d}{d t} x(t)=a_{x}(t)-k_{1} \sqrt{v^{2}(t)-\left(\frac{d}{d t} y(t)\right)^{2}} \cos \psi(t) \\
\frac{d^{2}}{d t^{2}} y(t)-k_{2} y(t)=a_{y}(t)-k_{2} h(t) \\
\frac{d^{2}}{d t^{2}} z(t)-k_{3} \frac{d}{d t} z(t)=a_{z}(t)-k_{3} \sqrt{v^{2}(t)-\left(\frac{d}{d t} y(t)\right)^{2}} \sin \psi(t) \\
t \in(0, T)
\end{array}\right.
$$

Here, $a_{x}=a_{x}(t), a_{y}=a_{y}(t), a_{z}=a_{z}(t)$ are the accelerations calculated on the basis of the loads on the $O x, O y$ and $O z$ axes of the earth-fixed coordinate system, respectively, $h=h(t)$ is the flight altitude calculated by the barometric altimeter, $\psi=\psi(t)$ is the yaw angle, $v=v(t)$ is the actual velocity, $k_{1}, k_{2}, k_{3}$ are the fusion coefficients, and $T$ is the duration of the flight.

To show the relationship between the function $y(t), x(t), z(t)$ and the fusion parameters $k_{1}, k_{2}, k_{3}$, we can write them as follows:

$$
x=x\left(k_{1}, t\right), \quad y=y\left(k_{2}, t\right), \quad z=z\left(k_{3}, t\right)
$$

According to the introduced hypotheses, it is required to determine the coefficients $k_{1}, k_{2}, k_{3}$ so that the differences $\left|x\left(k_{1}, T\right)-x_{T}\right|, \quad\left|y\left(k_{2}, T\right)-y_{T}\right|, \quad\left|z\left(k_{3}, T\right)-z_{T}\right|$ are minimal; here $\left(x_{T}, y_{T}, z_{T}\right)$ is the coordinates of the landing place at the airfield to which the aircraft (airplane) returns.

This formulation of the fusion problem requires the calculation of the coefficients of the differential operators. From this point of view, the problem under investigation can be classified as an inverse mathematical problem.

The structure of the problem is such that it can be divided into independent problems and, in turn, solved relative to $k_{2}, k_{1}$ and $k_{3}$. First, the approximate value of the coefficient $k_{2}$ is determined with a certain accuracy. Assuming $k_{2}>0, \alpha=\sqrt{k_{2}}$ is written. The following nonlinear equation with respect to $\alpha$ is obtained by applying the Lagrange method of variation of constants to the general solution constructed using the fundamental solutions $\left\{e^{\alpha t}, e^{-\alpha t}\right\}$ of the equation written with respect to $y\left(k_{2}, t\right)$ :

$$
\begin{gathered}
\frac{y_{0}\left(e^{\alpha T}-e^{-\alpha T}\right)}{2 \alpha}-y_{T}+ \\
+\frac{1}{2 \alpha} \int_{0}^{T}\left(e^{\alpha(T-\tau)}-e^{-\alpha(T-\tau)}\right)\left(a_{y}(\tau)-\alpha^{2} h(\tau)\right) d \tau=0 .
\end{gathered}
$$

To solve this equation, the Newton-Raphson iteration method ${ }^{5}$ is applied and the coefficient $k_{2}$ is found. The coefficients $k_{1}$ and $k_{3}$ can also be calculated with a similar computation scheme.

[^3]This method is of theoretical nature. As we know, flight data is measured at discrete instants of time and recorded in the black box. Therefore, for the practical implementation of the method, its discrete analogue should be developed and applied to process large amounts of flight data of every aerial vehicle (airplane). It is assumed that the combined coefficients may be different for each specific aerial vehicle (airplane), and the mean values of the coefficients calculated for different flights of that aerial vehicle (airplane) can be used for data fusion.

The studies described in this section were published in [2, 3, 11].
In Section 2 of Chapter 3, an algorithm is given for reducing the problem of the fusion of data received from different channels to the boundary value problem for a simple differential equation to calculate the actual flight path of an aircraft and solving it. This problem can be formulated as follows:

- It is necessary to find a trajectory that has a parametric dependence on time, such that the velocity it creates is close to the actual velocity obtained from the processing of black box data of an aircraft (airplane), and the acceleration is close to the acceleration determined by the loads.

It should be noted that, given that the measurement of the flight altitude with barometric and/or radio altimeter gives satisfactory results, this section focuses on the horizontal indicators of movement.

The mathematical formalization of the problem can be written as follows.

Suppose that as a result of the pre-processing of black box data from different channels, the components of the velocity and acceleration of the aircraft (airplane) for the entire flight duration have been determined: $\mathbf{v}(t)=\left(v_{x}(t), v_{y}(t)\right), \mathbf{a}(t)=\left(a_{x}(t), a_{y}(t)\right)$.

It is required to find the trajectory of the aircraft (airplane) given as a vector function $\mathbf{S}(t)=(x(t), y(t))$ such that its first-order
derivative $\frac{d}{d t} \mathbf{S}(t)$ is "close" to $\mathbf{v}(t)$, and its second-order derivative $\frac{d^{2}}{d t^{2}} \mathbf{S}(t)$ is "close" to the vector $\mathbf{a}(t)$.

The "closeness" of the two functions means that the sum of the squares of their differences is minimal. Since the components $x=x(t)$ and $y=y(t)$ behave as mutually independent functions, the problem of finding the required trajectory is reduced to the problem of finding the minimum of the following two mutually independent functionals $\mathfrak{I}_{x}(x(t))$ and $\mathfrak{J}_{y}(y(t))$ :

$$
\begin{aligned}
& \mathfrak{J}_{x}(x(t)) \equiv \int_{0}^{T}\left(\left(\frac{d^{2}}{d t^{2}} x(t)-a_{x}(t)\right)^{2}+\left(\frac{d}{d t} x(t)-v_{x}(t)\right)^{2}\right) \rightarrow \mathrm{min} \\
& \mathfrak{J}_{y}(y(t)) \equiv \int_{0}^{T}\left(\left(\frac{d^{2}}{d t^{2}} y(t)-a_{y}(t)\right)^{2}+\left(\frac{d}{d t} y(t)-v_{y}(t)\right)^{2}\right) \rightarrow \mathrm{min} .
\end{aligned}
$$

These requirements are supplemented by boundary conditions that express known values of the coordinates and the velocity of the aircraft at the beginning and at the end of the trajectory:

$$
\begin{aligned}
& \begin{cases}x(0)=x_{00}, & \frac{d}{d t} x(0)=x_{01} \\
x(T)=x_{T 0}, & \frac{d}{d t} x(T)=x_{T 1}\end{cases} \\
& \begin{cases}y(0)=y_{00}, & \frac{d}{d t} y(0)=y_{01} \\
y(T)=y_{T 0}, & \frac{d}{d t} y(T)=y_{T 1}\end{cases}
\end{aligned}
$$

Here the values $x_{00}, x_{01}, x_{T 1}, x_{T 1}, y_{00}, y_{01}, y_{T 1}, y_{T 1}$ are considered known quantities. In order to solve the problem, the Euler equations of the following form with respect to the functions $x(t)$ and $\mathrm{y}(\mathrm{t})$ are obtained from the zero equality condition of the variation of the functions $x(t)$ and $y(t)$ :

$$
\begin{aligned}
& \frac{d^{4}}{d t^{4}} x(t)-\frac{d^{2}}{d t^{2}} x(t)=\frac{d^{2}}{d t^{2}} a_{x}(t)-\frac{d}{d t} v_{x}(t), \quad t \in(0, T), \\
& \frac{d^{4}}{d t^{4}} y(t)-\frac{d^{2}}{d t^{2}} y(t)=\frac{d^{2}}{d t^{2}} a_{y}(t)-\frac{d}{d t} v_{y}(t), \quad t \in(0, T) .
\end{aligned}
$$

As mentioned above, navigation devices measure flight parameters at different discrete instants of time. With this in mind, their discrete analogues have been written to solve the boundary value problems written for the Euler equations. Since these are of fourthorder equations, an analog of the sweep method has been developed for their numerical solution.

The studies described in this section were published in $[6,8]$.
Section 3 of Chapter 3 is entitled "Substantiation of the application of the integral criterion in fusion problems". Although the parameters of an aircraft have a quite smooth time dependence, approaches to eliminate distortions in fact use the least square method instead of regular metrics. In such cases, it should be clarified to what extent the replacement of regular metrics with integral-quadratic metrics is justified.

To explain the essence of the problem, let us consider the example of load filtering algorithm. Basically, it is necessary to find the coefficients $\alpha_{1}, \alpha_{2}, \ldots, \beta_{3}$ such that the functional

$$
\mathfrak{I}_{C}\left(\alpha_{1}, \alpha_{2}, \ldots, \beta_{3}\right) \equiv \max _{t \in[0, T]}\left|\mathbf{V}(t)-\int_{0}^{t} \mathbf{a}(\tau) d \tau\right|
$$

takes on the minimum value, but instead of that functional, the functional

$$
\mathfrak{I}_{L_{2}}\left(\alpha_{1}, \alpha_{2}, \ldots, \beta_{3}\right) \equiv\left\|\mathbf{V}(t)-\int_{0}^{t} \mathbf{a}(\tau) d \tau\right\|
$$

is applied. From a mathematical point of view, such an approach considers a solution to the problem in the space $W_{2}^{16,7}$, instead of in the space of continuous functions.

On the other hand, it should be noted that the functions employed in problems of filtering are subject to certain constraints, as they reflect the velocity of the aircraft. Thus, the acceleration that the device can receive depends on the power of its motors and is always limited to a certain constant. Within the condition of this limitation, it can be shown that the series that minimizes the functional under consideration in the $L_{2}$ metric is also fundamental for the regular metric. In fact, the value of the regular metric has the upper limit of $2 / 3$ of the value of the $L_{2}$ metric. The mathematical formalization of the above for the elements of the space $W_{2}^{1}[0,1]$ can be given as follows.

Theorem. Let us denote by $\Omega$ the set of functions that satisfy the conditions $u \in W_{2}^{1}[0,1]$ and $\left|u^{\prime}(t)\right| \leq K$. Then, for the sought $u \in \Omega$,

$$
\|u\|_{C[0,1]}^{\frac{3}{2}} \leq \sqrt{3 K}\|u\|_{L_{2}[0,1]}
$$

[^4]here, $\|u\|_{C[0,1]}=\max _{t \in[0,1]}|u(t)|,\|u\|_{L_{2}[0,1]}=\sqrt{\int_{0}^{1}|u(t)|^{2} d t}$.

This theorem has been proved in the dissertation. The studies described in this section were published in [9].

Chapter 4 is devoted to the problems of control of the flight of quadcopter-type unmanned aerial vehicles. The reduction of a number of risks in the use of unmanned aerial vehicles compared to other technical means has given an impetus to their widespread use. For this reason, many companies and specialists in different countries are engaged in the development of unmanned aerial vehicles and the solution of related problems. In recent years, enterprises engaged in the development and production of unmanned aerial vehicles have begun to emerge in Azerbaijan as well. At one time, the High Technology Park of ANAS, as well as in a number of special enterprises on the commission of the Ministry of Defense and the State Border Services began to work on the production of unmanned aerial vehicles. While such enterprises often produce standard models of unmanned aerial vehicles in the early stages of their activities, there is a need to modernize them, add new features, and solve problems and new issues that arise during operation. Thus, a number of problems related to the operation of unmanned aerial vehicles arises. These problems include calculating the characteristics of the unmanned aerial vehicle, controlling the flight along the planned trajectory, determining the current location of the unmanned aerial vehicle on the basis of various feedback data. To solve the above-mentioned problems, first of all, an adequate mathematical model of the unmanned aerial vehicle under consideration must be constructed.

For this reason, Section 1 of Chapter 4 is devoted to the development of a mathematical model of a quadcopter-type unmanned aerial vehicle.

During the construction of the mathematical model, the following stipulations were reached based on the design of the
quadcopter, the technical conditions for its operation and its functional requirements:

- unmanned aerial vehicles can be considered as solid bodies;
- the quadcopter arms are sufficiently symmetric;
- the quadcopter has 4 symmetrically fixed identical motors that rotate its blades;
- if we indicate the quadcopter motors as shown in Fig. 2, blades 1 and 3 create a clockwise torque, and blades 2 and 4 create a counterclockwise torque;
- the force of resistance of air is directed in the opposite direction to the speed of movement and proportional to its square;
- as the angular rotational velocities are small, the effect of the gyroscopic forces produced by the movement of the quadcopter can be neglected;
- gravity and aerodynamic drag affect the center of mass of the unmanned aerial vehicle, so these forces do not create a torque;
- the mass of the quadcopter is distributed only along its arms, in other words, the rotary inertia matrix of the unmanned aerial vehicle is diagonally shaped.

Let us denote the current coordinates of the quadcopter by $x(t), y(t), y(t)$.


Fig. 2. A schematic representation of a quadcopter-type unmanned aerial vehicle: 1 st and 3rd blades create a clockwise torque, and 2nd and 4th blades create a counterclockwise

Based on the laws of solid-state mechanics, we can describe the mathematical model of quadcopter motion with the following system of equations:

According to the balance of forces:

$$
\left\{\begin{array}{c}
\frac{d}{d t} v_{x}(t)+\frac{C_{a}}{m} \sqrt{v_{x}^{2}(t)+v_{y}^{2}(t)+v_{z}^{2}(t)} v_{x}(t)- \\
-w_{z}(t) v_{y}(t)+w_{y}(t) v_{z}(t)=\frac{g}{m} \sin \vartheta(t) \\
\frac{d}{d t} v_{y}(t)+w_{z}(t) v_{x}(t)+\frac{C_{a}}{m} \sqrt{v_{x}^{2}(t)+v_{y}^{2}(t)+v_{z}^{2}(t)} v_{y}(t)- \\
-w_{x}(t) v_{z}(t)=\frac{g}{m} \sin \gamma(t) \cos \vartheta(t), \\
\frac{d}{d t} v_{z}(t)-w_{y}(t) v_{x}(t)+w_{x}(t) v_{y}(t)+ \\
+\frac{C_{a}}{m} \sqrt{v_{x}^{2}(t)+v_{y}^{2}(t)+v_{z}^{2}(t)} v_{z}(t)=\frac{g}{m} \cos \gamma(t) \cos \vartheta(t)+\frac{k}{m} \sum_{j=1}^{4} \omega_{j}^{2}
\end{array}\right.
$$

and according to the balance of torques,

$$
\left\{\begin{array}{c}
\sin \psi(t) \frac{d^{2}}{d t^{2}} \vartheta(t)+\cos \psi(t) \sin \vartheta(t) \frac{d^{2}}{d t^{2}} \gamma(t)+ \\
+\frac{J_{x x}-J_{y y}}{J_{x x}} w_{y}(t) w_{z}(t)=\frac{k l}{J_{x x}}\left(\omega_{2}^{2}-\omega_{4}^{2}\right) \\
-\cos \psi(t) \frac{d^{2}}{d t^{2}} \vartheta(t)+\sin \psi(t) \sin \vartheta(t) \frac{d^{2}}{d t^{2}} \gamma(t)+ \\
\quad+\frac{J_{x x}-J_{z z}}{J_{x x}} w_{x}(t) w_{z}(t)=\frac{k l}{J_{y y}}\left(\omega_{1}^{2}-\omega_{3}^{2}\right) \\
\frac{d^{2}}{d t^{2}} \psi(t)+\cos \vartheta(t) \frac{d^{2}}{d t^{2}} \gamma(t)+\frac{J_{y y}-J_{x x}}{J_{z z}} w_{x}(t) w_{y}(t)= \\
=\frac{b}{J_{z z}}\left(\omega_{1}^{2}-\omega_{2}^{2}+\omega_{3}^{2}-\omega_{4}^{2}\right)
\end{array}\right.
$$

Here,
$w_{x}(t), w_{y}(t), w_{z}(t)$ are the angular rotational velocities of the unmanned aerial vehicle,
$v_{x}(t)=\frac{d}{d t} x(t), \quad v_{y}(t)=\frac{d}{d t} y(t), \quad v_{z}(t)=\frac{d}{d t} z(t) \quad$ are $\quad$ the components of the velocity vector, $\psi(t), \vartheta(t), \gamma(t)$ are the yaw, pitch and roll angles,
$\omega_{1}, \omega_{2}, \omega_{3}, \omega_{4}$ the angular rotational velocities of the blades,
$m$ is the total mass of the unmanned aerial vehicle,
$l$ is the length of the arm of the quadcopter-type unmanned aerial vehicle,
$k$ is the coefficient of proportionality between the rotation velocity of the blades and the force generated,
$C_{a}$ is the coefficient of air resistance,
$b$ is the coefficient of proportionality between the rotation velocity of the blades and the torque created around the vertical axis of the quadcopter,
$J_{x x}, J_{y y}, J_{z z}$ are elements of a diagonal rotational inertial matrix,
$g$ is the gravitational acceleration.
Thus, the mathematical model of the problem expressing the relationship between the input data $\omega_{1}, \omega_{2}, \omega_{3}, \omega_{4}$ and the output data $\psi(t), \vartheta(t), \gamma(t), x(t), y(t), z(t)$ is constructed. All quantities included in the model are calculated according to the inertial system relative to earth.

The modeling-related studies were published in [13], pp. 141147.

Section 2 of Chapter 4 is devoted to the problem of automatic calculation of some characteristic parameters that are subject to change during the operation of a quadcopter-type unmanned aerial vehicle. Thus, depending on the tasks to be performed, the devices attached to the unmanned aerial vehicle (photo/video camera, radiorepeater, various packages, cargo, etc.) to some extent change its aerodynamic and technical characteristics. Changes in aerodynamic properties cause the blades to rotate in the calculated modes in order for the aerial vehicle to move along the intended trajectory, and it becomes necessary to change these modes. Thus, the problem arises of maximum correction of some parameters included in the mathematical model of the aerial vehicle in order to calculate the adequate operating modes of the blades.

During the research, it was proposed that a number of important parameters included in the control model (mathematical model) of a quadcopter-type unmanned aerial vehicle should be automatically calculated during the flight of the quadcopter and the model should be corrected. It is considered that in order to solve this problem, it is possible to use the data obtained in the first seconds of the flight from the control parameters (rotation speed of the blades) and navigation devices installed on a quadcopter-type unmanned aerial vehicle.

The following points were taken into account during the investigation of the problem:

- the physical and technical parameters in the mathematical model of the aerial vehicle include the parameters $l, m, J_{x x}, J_{y y}, J_{z z}, C_{a}, k, b ;$
- the symmetry of the body of a quadcopter-type unmanned aerial vehicle with respect to its arms gives grounds to consider that $J_{x x}=J_{y y}$;
- the system of equations expressing the balance of torques is invariant with respect to the ratios $\frac{J_{x x}}{l}=\frac{J_{y y}}{l}$ and $\frac{b}{l}$;
- it is sufficient to know the quantities $m, \frac{J_{x x}}{l}=\frac{J_{y y}}{l}, \frac{J_{z z}}{l}, C_{a}, k, \frac{b}{l}$ to control the quadcopter.

In view of the above, the system of equations expressing the balance of torques can be written as follows:

$$
\left\{\begin{array}{c}
\left(\begin{array}{r}
\left.+\sin \psi(t) \frac{d^{2}}{d t^{2}} \vartheta(t)+\cos \psi(t) \sin \vartheta(t) \frac{d^{2}}{d t^{2}} \gamma(t)+w_{y}(t) w_{z}(t)\right) \frac{J_{x x}}{l}+ \\
+w_{y}(t) w_{z}(t) \frac{J_{z z}}{l}=k\left(\omega_{2}^{2}-\omega_{4}^{2}\right), \\
\left(-\cos \psi(t) \frac{d^{2}}{d t^{2}} \vartheta(t)+\sin \psi(t) \sin \vartheta(t) \frac{d^{2}}{d t^{2}} \gamma(t)+w_{x}(t) w_{z}(t)\right) \frac{J_{x x}}{l}- \\
-w_{x}(t) w_{z}(t) \frac{J_{z z}}{l}=k\left(\omega_{1}^{2}-\omega_{3}^{2}\right),
\end{array}\right. \\
\left(\frac{d^{2}}{d t^{2}} \psi(t)+\cos \vartheta(t) \frac{d^{2}}{d t^{2}} \gamma(t)\right) \frac{J_{z z}}{l}-\left(\omega_{1}^{2}-\omega_{2}^{2}+\omega_{3}^{2}-\omega_{4}^{2}\right) \frac{b}{l} .
\end{array}\right.
$$

This notation of the moment balance system allows calculating the quantities $\frac{J_{x x}}{l}=\frac{J_{y y}}{l}, \frac{J_{z z}}{l}, \frac{b}{l}$ on the basis of the data related to the first moments of the flight, and then determining the remaining
quantities $m, C_{a}, k$ from the force balance system. Thus, the proposed solution algorithm for the problem allows for the automatic determination of a sufficient number of parameters for the application of the mathematical model expressing the equations of control of the motion in the initial period of flight.

The studies described in this section were published in [13], pp. 147-150.

Any task that an unmanned aerial vehicle has to perform, of course, involves it flying along a certain trajectory plotted by the operator or generated by an automated planning system and described relative to earth.

A simplified model of the motion of a quadcopter-type unmanned aerial vehicle causes the aircraft to deviate from the planned trajectory due to the factors considered (wind, errors in the balancing of the quadcopter, etc.). In this case, we have a problem of minimizing the deviations of the unmanned aerial vehicle along the entire flight path. Similar problems were considered, for example, in the studies of I.M. Gostev ${ }^{8}$, V. Klein ${ }^{9}$, K.P. Lievens ${ }^{10}$ and others. In these studies, the orientation of the drone is given by Euler angles, and these angles are very suitable for computer simulation. However, navigation devices currently used in unmanned aerial vehicles measure Krylov angles, or derivatives (variations) of these angles. Based on the above, Section 3 of Chapter 4 is devoted to the study of a mathematical model correcting the motion of a quadcopter-type unmanned aerial vehicle based on the processing of data obtained from its sensors (navigation devices).

[^5]To solve the problem, it is assumed that the coordinates of the current location and velocity of the unmanned aerial vehicle were calculated on the basis of gyroscope and accelerometer (or GPS) data. It is also assumed that the orientation of the unmanned aerial vehicle and the rotation frequency of the blades of its propulsion systems (motors) are known.

As a rule, the intended trajectory of the aircraft is described as a broken line with given nodal points. The control problem can be expressed as follows:

The planned trajectory $L_{p}=\left\{x_{p}(t), y_{p}(t), z_{p}(t)\right\}$ of the quadcopter, the mass $M$, the coordinates $\left\{x\left(t_{0}\right), y\left(t_{0}\right), z\left(t_{0}\right)\right\}$ at the current instant $t_{0}$, and the flight velocity $\left\{\frac{d}{d t} x\left(t_{0}\right), \frac{d}{d t} y\left(t_{0}\right), \frac{d}{d t} z\left(t_{0}\right)\right\}$ are known. It is necessary to find a distributed control force $\Delta \mathbf{F}$ such that the deviation of the unmanned aerial vehicle from the planned flight trajectory is eliminated in a short period of time $\Delta t$.

If we denote the difference between the coordinates of the point on the planned trajectory where the unmanned aerial vehicle should be at the considered instant and the coordinates of its current location by $\Delta x=x(t)-x_{p}(t), \Delta y=y(t)-y_{p}(t), \Delta z=z(t)-z_{p}(t)$, respectively, then the system of equations for the additional force compensating for the deviation of the trajectory can be written as follows:

$$
\left\{\begin{array}{l}
M \frac{d^{2}}{d t^{2}} \Delta x(t)=\Delta F_{x} \\
M \frac{d^{2}}{d t^{2}} \Delta y(t)=\Delta F_{y} \\
M \frac{d^{2}}{d t^{2}} \Delta z(t)=\Delta F_{z}
\end{array}\right.
$$

By adding this difference to the force generated by the motors, we can find the force $\hat{\mathbf{F}}$ required to eliminate the deviation. It is assumed that
the force generated by the motors during the elimination of the deviation should not create additional torques. If we denote the known current torques of the quadcopter with respect to the local coordinate system by $M_{O x}, M_{O y}, M_{O z}$, then the new values of the rotation frequencies of the blades to eliminate the deviation can be calculated from the following system of equations:

$$
\left\{\begin{array}{c}
k \sum_{i=1}^{4} \omega_{i}^{2}=|\hat{\mathbf{F}}|, \\
k l\left(\omega_{2}^{2}-\omega_{4}^{2}\right)=M_{O x}, \\
k l\left(\omega_{1}^{2}-\omega_{3}^{2}\right)=M_{O y}, \\
b\left(\omega_{1}^{2}-\omega_{2}^{2}+\omega_{3}^{2}-\omega_{4}^{2}\right)=M_{O z} .
\end{array}\right.
$$

The studies described in this section were published in [12].
In Section 4 of Chapter 4, the problem of determining the location of an unmanned aerial vehicle based on video camera footage is investigated.

As mentioned above, the location of an unmanned aerial vehicle can be determined by processing data from navigation devices. However, due to the integral nature of the processing results, the accuracy of the calculation of the current location of the aircraft is reduced because of measurement errors. In order to reduce the errors accumulated during flight control, it is considered appropriate to make regular adjustments to the current coordinates of the unmanned aerial vehicle using alternative data sources. Images taken by video cameras installed on the vehicle can be one of such alternative data sources.

An aircraft can determine its location by image processing on the basis of different principles ${ }^{11,12,13}$. Existing studies offer different approaches, depending on the nature of the problem being solved, the characteristics of video and photo cameras that take pictures, as well as the number of pictures taken, the accuracy of object identification and other criteria.

The approach presented in this section addresses the problem of calculating the location of an unmanned aerial vehicle based on video footage and photo images, taking the following principles as a basis for its solution:

- Since the areas shown in the images taken by the unmanned aerial vehicles have a fairly small scale, they can be considered part of a plane.
- It is possible to construct a bijective mapping of a regular nature between the coordinates calculated relative to the image for three real objects recognized in the images and their geographic coordinates.
- The constructed bijective mapping can be used to determine the location of the unmanned aerial vehicle relative to the observed objects in accordance with the orientation determined from navigation data.

Since the problem of identification (recognition) of objects in video footage and photo images has been explored in various studies ${ }^{14}$, we assume that the problem of identification of objects in video footage and photo images has been solved and their geographical coordinates are known.

[^6]To solve the problem, a $O \xi \eta \zeta$ rectangular coordinate system relative to the video camera monitor was introduced (Fig. 3).


Fig. 3. $O \xi \eta \zeta$ local coordinate system relative to the monitor
Suppose that an unmanned aerial vehicle observes through its video camera three points $A_{1}, A_{2}, A_{3}$ on the earth's surface, and the geographical coordinates of these points (latitude and longitude) are known. Denote the geographical coordinates of the point $A_{j}$ accordingly by $\left(v_{j}, u_{j}\right),(j=1,2,3)$. Points $A_{1}, A_{2}, A_{3}$ are mapped onto some points $B_{1}, B_{2}, B_{3}$ on the video camera monitor. Let the coordinates of these points relative to the $O \xi \eta \zeta$ system be $\left(\xi_{1}, \eta_{1}, 0\right)$ , $\left(\xi_{2}, \eta_{2}, 0\right)$, and $\left(\xi_{3}, \eta_{3}, 0\right)$, respectively.

It is shown that the bijective mapping $\Psi:(v, u) \rightarrow(\xi, \eta)$ can be constructed as $\Psi=\Psi_{A}^{-1} * \Psi_{M}$. Here $\Psi_{M}:(v, u) \rightarrow(x, y)$ is the mapping of the geographical coordinates in the Mercator projection ${ }^{15}$ :

$$
\left\{\begin{array}{c}
x=R \cdot v \\
y=R \cdot \ln \left(\operatorname{tg}\left(\frac{\pi}{4}+\frac{u}{2}\right)\left(\frac{1-\varepsilon \cdot \sin u}{1+\varepsilon \cdot \sin u}\right)^{\frac{\varepsilon}{2}}\right)
\end{array}\right.
$$

[^7]The affine transformation $\Psi_{A}:(\xi, \eta) \rightarrow(x, y)$ can be written as:

$$
\left\{\begin{array}{l}
a_{11} \xi+a_{12} \eta+b_{1}=x \\
a_{21} \xi+a_{22} \eta+b_{2}=y
\end{array}\right.
$$



Fig. 4. Schematic representation of the points on the earth's surface in the monitor plane

Based on the data of the problem, the coefficients of the mapping $\Psi_{A}$ are found. Then, based on the orientation angles of the unmanned aerial vehicle, we calculate the geographical coordinates of the projection $\mathbf{A}_{0}$ of the focal point $\mathbf{F}$ of the camera, identified with the vehicle's location, on the earth's surface (Fig. 4).

The studies described in this section were published in [20].
The problem of determining the trajectory for moving along the intended route bypassing obstacles in the area is studied in Section 5 of Chapter 4.

As mentioned above, the intended flight route of the unmanned aerial vehicle is given by including nodal points along this route. These points are usually entered by the operator in the background of the map
on the tablet. However, the unmanned aerial vehicle's route through the nodal points is complicated by obstacles and terrain. Therefore, the problem arises of determining the optimal trajectory for automatic control of the unmanned aerial vehicle's flight along the route.

Information on the irregularities of the terrain is usually given by terrain maps compiled in different formats ${ }^{16,17,18}$. However, obtaining and processing the required information from such maps requires additional effort. Therefore, a different method of providing terrain information was proposed in the article by A.B. Pashayev in $2019^{19}$. The distinguishing feature of this map, called a geometric map, is that the obstacles and terrain in the area are given as simple geometric shapes characterized by a small number of parameters, so the calculations for determining the trajectory are quite simple. In this section, a mathematical model of the problem of finding the safe optimal trajectory for crossing an area with obstacles given as inequalities $\quad\left(x-x_{i}\right)^{2}+\left(y-y_{i}\right)^{2}+\left(z-z_{i}\right)^{2} \leq r_{i}^{2}, \quad i=1,2, \ldots \quad$ is constructed.

It is assumed that the unmanned aerial vehicle flies over a certain plane above the earth's surface, i.e., $h_{0} \leq z$, where $h_{0}$ is a prescribed number.

Suppose that the flight path $M_{k},(k=1,2, \ldots)$ is given in the form of a sequence of nodal points, the finite number of elements of terrain obstacles is given as the inequalities $\left(x-x_{B_{i}}\right)^{2}+\left(y-y_{B_{i}}\right)^{2}+\left(z-z_{B_{i}}\right)^{2} \leq r_{B_{i}}^{2}, \quad(i=1,2,3, \ldots)$. Here,

[^8]$x_{B_{i}}, y_{B_{i}}, z_{B_{i}}, r_{B_{i}}$ are prescribed numbers. Let us apply the following transformations successively to each of the nodal points $M_{k-1} M_{k}$ and all $B_{i}$ points of the route:

- $A_{\mu}$ is a parallel shift of the coordinates up to the vector $\mu$;
- $A_{\alpha}$ is a $\alpha$ times compression (extension) of the length of the segments;
- $A_{\varphi}=\left(\begin{array}{ccc}\cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1\end{array}\right)$ is the rotation around the $O z$ axis until the angle $\varphi$; here

$$
\begin{gathered}
\mu=\left(-x_{M_{k-1}},-y_{M_{k-1}},-z_{M_{k-1}}\right), \\
\alpha=\sqrt{\left(x_{M_{k}}-x_{M_{k-1}}\right)^{2}+\left(y_{M_{k}}-y_{M_{k-1}}\right)^{2}+\left(z_{M_{k}}-z_{M_{k-1}}\right)^{2}}, \\
\varphi=\arccos \left[\alpha^{-1}\left(x_{M_{k}}-x_{M_{k-1}}\right)\right] .
\end{gathered}
$$

Then the segment $M_{k-1} M_{k}$ is mapped onto the interval $[0,1]$ of the Ox coordinate axis. Suppose that during this transformation the nodal point $M_{k}$ is mapped onto the point $\breve{M}_{k}=\left(0,0, \tilde{z}_{M}\right)$, and the points $B_{i}$ are mapped onto the points $\widetilde{B}_{i}\left(\tilde{x}_{i}, \tilde{y}_{i}, \tilde{z}_{i}\right)$. By replacing the radius $r_{B_{i}}$ with $r_{i} \equiv \alpha r_{B_{i}}$ accordingly, a mathematical model for calculating the optimal trajectory along the interval $[0,1]$ can be constructed.

If we denote the sought trajectory by $y(x), z(x)$, based on the condition of minimality of the functional $\int_{0}^{1} \sqrt{1+\left(\frac{d}{d x} y(x)\right)^{2}+\left(\frac{d}{d x} z(x)\right)^{2}} d x$, the constraint conditions $r_{i}^{2}-\left(x-\tilde{x}_{i}\right)^{2}-\left(y(x)-\tilde{y}_{i}\right)^{2}-\left(z(x)-\tilde{z}_{i}\right)^{2} \geq 0$ and $h_{0} \leq z(x)$, it is possible to formulate an optimization problem for determining the
required trajectory. By entering Lagrange multipliers (coefficients) and then applying the Kuhn-Tucker theorem, the problem

$$
\left\{\begin{array}{l}
\frac{d^{2} y}{d x^{2}}(x) \frac{1+\left(\frac{d z}{d x}(x)\right)^{2}}{2\left(1+\left(\frac{d y}{d x}(x)\right)^{2}+\left(\frac{d z}{d x}(x)\right)^{2}\right)^{\frac{3}{2}}}-\frac{d y}{d x}(x)-y(x) \sum_{j=1}^{n} \lambda_{j}=\sum_{j=1}^{n} \lambda_{j} \tilde{y}_{j} \\
\frac{d^{2} z}{d x^{2}}(x) \frac{1+\left(\frac{d y}{d x}(x)\right)^{2}}{2\left(1+\left(\frac{d y}{d x}(x)\right)^{2}+\left(\frac{d z}{d x}(x)\right)^{2}\right)^{\frac{3}{2}}}-\frac{d z}{d x}(x)-z(x) \sum_{j=1}^{n} \lambda_{j}=\sum_{j=1}^{n} \lambda_{j} \tilde{z}_{j}
\end{array}\right.
$$

is reduced to the boundary value problem for a system of differential equations. The appropriate boundary conditions are written as follows:

$$
\left\{\begin{array}{lc}
y(0)=0, & z(0)=0 \\
y(1)=0, & z(1)=\tilde{z}_{M}
\end{array}\right.
$$

As we can see, the resulting problem is nonlinear, and its numerical solution requires the use of more complex calculation methods ${ }^{20}$. Therefore, in practice, the preference is often given to simplified algorithms for determining the trajectory. Such trajectories are not "optimal", but are considered more efficient in terms of use. For this reason, an algorithm for building an efficient trajectory is

[^9]developed in this dissertation. The algorithm is essentially a 1dimensional analogue of a 2-dimensional surface drawn on a contour.

The studies described in this section were published in [14].
Chapter 5 is devoted to the development of a mathematical model and algorithm for the joint flight control of a group of unmanned aerial vehicles.

It is known that it is very difficult to prevent a swarm attack by a large number of military equipment. It is also a good tactic to sacrifice a certain number of relatively cheap unmanned aerial vehicles to destroy enemy's targets of special significance in a swarm attack. In this regard, the need for joint control of a group of unmanned aerial vehicles arises. The main aspect of this problem is that each unmanned aerial vehicle participating in a group flight should not be operated individually, all unmanned aerial vehicles in the group should be operated efficiently on the basis of instructions or commands from one operator.

It is clear that a decision-making system designed to achieve joint flight of unmanned aerial vehicles must be universal, the output data must be unambiguous regardless of the amount of input data, and the results of the program executed on different unmanned aerial vehicles must not contradict each other. Based on this, it was proposed to design a group of unmanned aerial vehicles as a System of Systems ${ }^{21}$. The main conjectures and principles that will be used to create the system are given below.

- All unmanned aerial vehicles have the same flight characteristics - their maximum speed, cruise speed, permissible acceleration (acceleration capacity) are assumed to be identical.
- The system clocks of all unmanned aerial vehicles participating in the joint flight are synchronized with the clock of the operator's station, and the time difference between different vehicles due to technical issues is negligibly small.

[^10]- Data received (transmitted) by unmanned aerial vehicles has a single structure: "vehicle number", "moment of data transmission (reception)", "flight speed", "current coordinates". In the process of data exchange, each unmanned aerial vehicle receives a full set of single-format information about the unmanned aerial vehicles in its immediate vicinity.
- The flight direction of an unmanned aerial vehicle is of a set of primary and randomly oriented "auxiliary" directions along a predetermined trajectory. The auxiliary direction is formed by differences in the thrust forces of the motors during the rotation of the blades, delays in the decision-making system, as well as in the transmission of commands to the actuating units of the vehicle, the impact of the wind and other similar random factors.
- The movement of unmanned aerial vehicles is regulated by changing the value and direction of its velocity in its current position.

To implement these principles, the velocity of each unmanned aerial vehicle has been taken as its control parameter. The goal here is to select the velocity of movement after each exchange of information between the vehicles so that they can safely participate in the joint flight without colliding with each other.

An algorithm has been developed for the decision-making system of each unmanned aerial vehicle to calculate its adjustable velocity depending on the segment of the joint flight path in which the vehicle under consideration is located, as well as the coordinates and velocity of other vehicles flying in its immediate vicinity.

In order to simplify the calculations, all the data in the problem are first subjected to coordinate transformation around the center of gravity of the aerial vehicle under consideration in such a way that the direction of motion is directed along the $O z$ axis in the Oxyz coordinate system fixed to the vehicle. The boundaries of the safe zone of motion in that direction are described as a truncated cone, and the velocity vector is calculated to ensure that the aerial vehicle does not go outside that cone. Then, using the inverse coordinate
transformation, the velocity of the unmanned aerial vehicle in the coordinate system relative to the earth is found.

The studies described in this section were published in [18].

In conclusion, I would like to express my deepest gratitude to my scientific consultant Professor T.A. Aliev, academician of ANAS, Doctor of Technical Sciences, for his constant attention.

## MAIN RESULTS OF THE WORK

The following results have been obtained in the dissertation.

1. The problem of eliminating load distortions in the flight data of a military aircraft equipped with a TECTEP-У3-type device has been studied, and a formula for calculating the loads relative to the normal coordinate system has been proposed.
2. An algorithm has been given for calculating the data fusion coefficients based on the loads, orientation angles and speed data recorded in the black box for the initial flight period of a military aircraft with a platform-type navigation system has been given.
3. A neural network method has been developed for recovering flight data lost during recording in the black box; a numerical computation algorithm has been given for calculating the coefficients of the network.
4. Algorithms have been proposed for filtering the yaw angle of the aircraft, correcting the flight velocity according to the data of the Doppler velocity sensor and adjusting the pitch angle.
5. The idea has been put forward of fusing the data of the equation of the flight path of a military aircraft as a linear combination. An algorithm based on the Newton-Raphson iteration method has been proposed for finding the coefficients of the equations.
6. The mathematical model of the problem of the fusion of aircraft flight data has been constructed as a problem of minimization of a functional. The minimization problem has been reduced to the
boundary value problem for fourth-order ordinary differential equations with constant coefficients using the variation method. An analogue of the classical sweep method has been developed for solving the boundary value problem.
7. The possibility of applying the approximation criterion in the $L_{2}$ metric instead of the regular approximation criterion when adjusting the data received from the transmitters on board unmanned aerial vehicles with the help of additional flight information has been substantiated.
8. An algorithm has been developed for calculating and refining a number of characteristic physical and technical parameters of a quadcopter that have changed due to the addition of various devices and loads during operation on the basis of data related to the initial flight period based on mathematical model that expresses the relationship between the trajectory and spatial orientation of a quadcopter-type unmanned aerial vehicle and the factors and control parameters.
9. The problem of the control of a quadcopter-type unmanned aerial vehicle by eliminating the deviations that occur during its movement along the intended trajectory has been investigated, an algorithm has been developed for calculating the corresponding control.
10. Constructing a bijective mapping between the geographical coordinates and the images on the camera monitor has been proposed to solve the problem of determining the location of the unmanned aerial vehicle from the video and photo images taken by it; an iterative computation algorithm has been developed for calculating the geographical coordinates of the current flight location.
11. A mathematical model of the problem of determining the optimal trajectory of movement along the intended route has been constructed on the basis of a geometric map of obstacles and irregularities of the terrain in the area given as spherical elements. The concept of efficient trajectory has been given; a heuristic
numerical computation algorithm has been proposed for finding one of the possible efficient trajectories.
12. A group of unmanned aerial vehicles has been described as a system of systems to create an effective control mechanism for the joint flight of unmanned aerial vehicles by one operator; basic universal principles of the control system have been proposed.
13. A universal decision-making algorithm has been developed for controlling the movement of each unmanned aerial vehicle participating in a joint flight along the intended trajectory by regulating its speed. Formulas have been given for calculating the speed mode that allows an unmanned aerial vehicle to participate in a safe joint flight without colliding with other UAVs in different parts of the trajectory under consideration. The principles for determining the debugging parameters for the model of joint flight control of unmanned aerial vehicles have been developed.

## The main results of the dissertation appeared in the following publications:

1. Алгулиев, Р.М., Оруджев, Г.Г., Сабзиев, Э.Н. Об одном методе восстановления потерянной полетной информации // Труды III международной конференции "Идентификация систем и задачи управления" (SICPRO'04), - Москва, 28-30 янв., - 2004. с. 348-352.
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## The author's personal contribution to coauthored works:

In $[1,2,3,4,9,13]$ the author contributed to the discussion of the formulation of the problem statement, proposed a solution method, and developed a research mechanism.

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