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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

DETERMINATION OF THE MAIN CHARACTERISTICS OF WELLS CONSIDERING THE HYDROMECHANICS OF TWO-PHASE SYSTEMS

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GENERAL CHARACTERISTICS OF THE WORK

Relevance of the Topic and Degree of Development of the Problem.

The flow of two-phase gas-liquid systems represents a complex hydrodynamic process characterized by the interconnected transfer of mass, momentum, and energy between the phases. The study of such processes is one of the fundamental problems of continuum mechanics, hydrodynamics, and multiphase flow theory and is widely applied in the analysis of natural and engineering systems.

The theoretical foundations for the study of gas-liquid systems were established in classical works on hydrodynamics and turbulence, which consider the laws governing the motion of continuous and multiphase media, the characteristics of turbulent flows, and the mechanisms of interfacial interaction.

In the oil and gas industry, the flow of gas-liquid mixtures plays a key role in well operation and the transportation of production through pipelines. The flow structure, phase distribution, and flow regime determine hydraulic losses and the efficiency of hydrocarbon production.

During field development, changes in reservoir pressure, temperature, and the filtration properties of the porous medium lead to the occurrence of complex hydrodynamic processes in the "reservoir–well" system. Particular importance is attached to the condition of the near-wellbore zone, since its contamination and changes in filtration characteristics significantly reduce well productivity.

The object of the study is the "reservoir–well" system containing a gas-liquid mixture, the flow of which is determined by the complex interaction between the phases, the rheological properties of the liquid, and the geometry of the flow channel.

The rheological properties of oil and oil emulsions differ significantly from those of Newtonian fluids, which requires the use of special models to describe the flow of viscoplastic and anomalous media.

Two-fluid models, drift-flux models, and averaged equations of multiphase media developed in the works of leading researchers are widely used to describe gas-liquid flows.

Despite the considerable number of studies conducted in this field, a number of unresolved problems remain, particularly those related to the accurate determination of two-phase flow parameters, the consideration of the rheological properties of the liquid, and the influence of the skin factor on filtration processes.

Engineering practice has established that the efficiency of oil and gas field development is largely determined by well productivity, which depends on the condition of the near-wellbore zone and the conditions of fluid filtration.

Therefore, methods for enhancing oil recovery and intensifying fluid inflow aimed at improving filtration characteristics and reducing hydrodynamic resistance are of particular relevance.

Thus, the development of methods that account for the combined effects of multiphase flow, rheological properties, and the skin factor represents an important scientific and practical problem.

Research Objective

The objective of the study is to develop an improved methodology for determining gas-liquid flow parameters and bottom-hole pressure, taking into account hydrodynamic and rheological factors.

Research Tasks

To achieve the stated objective, the following tasks are addressed:

— to develop a mathematical model of two-phase flow in the "reservoir–well" system;

— to establish the regularities of phase distribution and flow parameters;

— to develop a method for determining the velocity profile;

— to determine the flow-rate characteristics of the phases;

— to develop a method for calculating hydraulic losses;

— to develop a methodology for determining bottom-hole pressure based on wellhead data;

— to investigate the influence of the skin factor on flow parameters and natural-flow operation.

Research Methods

The study employs methods of theoretical analysis, mathematical modeling, numerical calculations, as well as experimental and field investigations.

Object and Subject of the Study

The object of the study is the flow of two-phase gas-liquid systems in the wellbores of oil and gas wells. The subject of the study is the regularities governing the formation of hydrodynamic parameters that determine bottomhole pressure, the inflow profile, and the characteristics of interfacial interaction under nonequilibrium conditions.

Scientific Novelty

A new hydrodynamic model of two-phase gas-liquid flow has been developed, taking into account the rheological properties of the liquid.

For the first time, the regularities governing the influence of the skin factor on flow parameters and natural-flow operation have been established.

An original method for determining flow parameters based on a comprehensive analysis of hydrodynamic characteristics has been proposed.

An improved methodology for calculating bottom-hole pressure based on wellhead information, taking into account the multiphase nature of the flow, has been developed.

Main Provisions Submitted for Defense

The following main provisions are submitted for defense:

— an engineering-mathematical approach to determining the main hydrodynamic characteristics of oil and gas wells, taking into account the hydromechanics of two-phase gas-liquid systems;

— mathematical models for determining bottom-hole pressure and the inflow profile, taking into account the effects of interfacial interaction, flow structure, surface forces, and nonequilibrium processes;

— a methodology for determining the hydrodynamic characteristics of wells using wellhead measurement data without the use of downhole measuring instruments;

— computational algorithms for predicting the operating characteristics of oil and gas wells, providing improved accuracy of engineering calculations.

Theoretical Significance of the Study

The theoretical significance of the dissertation lies in the further development of methods for mathematical modeling of the hydromechanics of two-phase gas-liquid systems. The analytical relationships obtained expand the understanding of the mechanisms governing the formation of bottom-hole pressure, interfacial interaction, and the influence of flow structural heterogeneity on the hydrodynamic characteristics of oil and gas wells.

Practical Significance

The developed methodology makes it possible to improve the accuracy of determining flow parameters and bottom-hole pressure, thereby contributing to the optimization of well operation and increased oil recovery.

Approbation and Publications

The main results of the dissertation were presented and discussed at international and national scientific conferences, scientific seminars, and forums devoted to the development of oil and gas fields, the hydromechanics of two-phase systems, and the mathematical modeling of filtration processes, including the 9th International Conference on Energy Science and Applied Technology (ESAT 2024), the 4th International Conference on Advances in Materials, Machinery, Electrical Engineering (AMMEE 2025), and the Republican Scientific Conference of PhD Students and Young Researchers (2025). The reported results were used to refine the mathematical models presented in the dissertation.

Institution Where the Dissertation Was Conducted

The dissertation research was conducted at the Department of Oil and Gas Engineering of Azerbaijan State Oil and Industry University (ASOIU).

Publications

Eleven scientific works have been published on the dissertation topic, including seven journal articles and four papers in proceedings

of international and national scientific conferences. Two of the published articles are indexed in the international Scopus database.

Volume and Structure of the Dissertation

The dissertation consists of an introduction, three chapters, conclusions, and a list of references.

The dissertation has a total length of 206092 characters. The introduction contains 8480 characters; Chapter 1, 40635 characters; Chapter 2, 96157 characters; Chapter 3, 59102 characters; and the conclusions, 1718 characters. The reference list includes 160 sources.

BRIEF CONTENT OF THE WORK

The introduction substantiates the relevance of the research topic, which is associated with the need to improve methods for determining the main characteristics of oil and gas wells, taking into account the hydromechanics of two-phase gas-liquid systems. The flow of gas-liquid mixtures in the "reservoir–well" system is a complex multifactorial process in which the phase distribution, pressure, flow velocity, density, and effective viscosity of the mixture change simultaneously. During the operation of oil and gas fields, changes in reservoir pressure, temperature, composition of the produced fluids, and filtration properties of the reservoir are accompanied by significant changes in fluid-flow conditions.

Of particular importance is the fact that, as pressure decreases in the wellbore, gas dissolved in the oil is released from the liquid phase, resulting in the formation of a two-phase flow with variable gas content. Changes in the volumetric gas content lead to changes in the effective density and hydraulic resistance of the mixture, as well as transitions between different flow regimes. Depending on operating conditions, bubbly, slug, annular, and transitional flow regimes may occur. Each of these regimes is characterized by a different phase distribution and a different contribution of gravitational, inertial, and interfacial forces to the overall pressure gradient. In the oil and gas industry, accurate determination of the hydrodynamic characteristics of wells is of direct importance for evaluating well productivity, calculating bottom-hole pressure, determining the inflow profile, and selecting an appropriate operating regime. Particularly complex conditions arise when the liquid phase exhibits non-Newtonian or anomalous rheological properties. Oils containing asphaltenes, resins, and other high-molecular-weight components may exhibit viscoplastic, pseudoplastic, and thixotropic behavior. Under such conditions, the use of models based exclusively on the assumption of Newtonian fluid behavior may result in systematic errors.[1]

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[1]Chhabra R. P., Richardson J. F. Non-Newtonian Flow and Applied Rheology: Engineering Applications. — 2nd ed. — Oxford: Butterworth-Heinemann, 2008. — 536 p.

An additional factor is the condition of the near-wellbore zone. Formation damage, changes in permeability, additional hydrodynamic resistances, and the formation of a skin zone directly affect the inflow of liquid and gas into the well. Under two-phase filtration conditions, the influence of the skin factor becomes even more complex, since a change in the resistance of the near-wellbore zone affects not only the flow rate but also the pressure distribution and the parameters of the flowing gas-liquid mixture.

Despite the considerable amount of research conducted, existing models do not always provide the required accuracy when applied to real oil and gas wells. Classical correlations, including Lockhart–Martinelli, Hagedorn–Brown, and Beggs–Brill, have been widely used because of their relative simplicity and applicability to engineering calculations. At the same time, their applicability is determined by the ranges of experimental conditions under which the original correlations were developed. When fluid properties, flow regime, well geometry, or near-wellbore conditions change significantly, calculation accuracy may decrease. The dissertation notes that the application of existing approaches under complex field conditions may result in significant discrepancies between calculated and actual values of bottom-hole pressure, flow rate, and inflow profile.[2]

Accordingly, the objective of the study is to develop an improved methodology for determining gas-liquid flow parameters and bottom-hole pressure, taking into account hydrodynamic and rheological factors. To achieve this objective, a set of interrelated tasks is addressed, including development of a mathematical model of two-phase flow in the "reservoir–well" system, investigation of phase distribution and flow parameters, determination of velocity profiles and phase flow-rate characteristics, development of a method for calculating hydraulic losses, determination of bottom-hole pressure from wellhead data, and investigation of the influence of the skin factor on flow parameters and natural-flow operation.

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[2]Govier G. W., Aziz K. The Flow of Complex Mixtures in Pipes. — New York: Van Nostrand Reinhold, 1972. — 800 p.

The study employs methods of theoretical analysis, mathematical modeling, numerical calculation, as well as experimental and field investigations.

The object of the study is the flow of two-phase gas-liquid systems in the wellbores of oil and gas wells, while the subject of the study comprises the regularities governing the formation of hydrodynamic parameters that determine bottom-hole pressure, the inflow profile, and the characteristics of interfacial interaction under nonequilibrium conditions.

The scientific novelty of the study consists in developing a new hydrodynamic model of two-phase gas-liquid flow that accounts for the rheological properties of the liquid; establishing the regularities governing the influence of the skin factor on flow parameters and natural-flow operation; proposing an original method for determining flow parameters from a comprehensive analysis of hydrodynamic characteristics; and developing an improved methodology for calculating bottom-hole pressure from wellhead information while accounting for the multiphase nature of the flow.

The theoretical significance of the study is associated with the further development of methods for mathematical modeling of the hydromechanics of two-phase gas-liquid systems. The obtained relationships expand the understanding of the mechanisms governing bottom-hole pressure formation, interfacial interaction, and the influence of flow structural heterogeneity on the hydrodynamic characteristics of oil and gas wells. The practical significance lies in the possibility of improving the accuracy of determining flow parameters and bottom-hole pressure, thereby creating prerequisites for optimizing well operation and increasing the efficiency of field development.

Chapter 1 presents a systematic analysis of the current state of research in the field of hydromechanics of two-phase gas-liquid systems. The main theoretical concepts of multiphase flow, modern mathematical and empirical models, historical stages in the development of two-phase flow research, existing problems and unresolved issues, as well as the main directions for further

improvement of methods for determining the characteristics of oil and gas wells are considered.

The flow of a two-phase gas-liquid system is considered a complex hydrodynamic process involving the simultaneous transfer of mass, momentum, and energy between the phases. Unlike single-phase flow, the characteristics of a two-phase flow are determined not only by the physical properties of individual components but also by the characteristics of their mutual movement. The most significant factors include the relative velocity of the phases, gas content, flow structure, interfacial friction, gravitational forces, surface tension, rheological characteristics of the liquid phase, and geometric parameters of the channel.

Under oil and gas well conditions, two-phase flow develops directly during the upward movement of production through the wellbore. As pressure decreases, dissolved gas is released, causing the gas phase to gradually increase while the mixture parameters change with depth. Consequently, the mixture density, effective viscosity, and hydraulic resistance are not constant. This results in a variable distribution of pressure and velocity along the wellbore.

The theoretical description of these processes is based on the conservation equations of mass, momentum, and energy for individual phases. Depending on the required accuracy and level of detail, homogeneous models, separated models, two-fluid models, drift-flux models, and other multiphase approaches are applied. Homogeneous models assume identical phase velocities and a common pressure field, which considerably simplifies the calculation. However, this assumption limits their ability to describe the actual relative motion of gas and liquid.

More complex separated and two-fluid models account for differences in phase velocities, while their interaction is described using appropriate interphase exchange coefficients. This approach makes it possible to analyze phase distribution and transitions between different flow regimes in greater detail. However, the increased physical detail of the model is accompanied by a larger number of input parameters and greater computational complexity.

One of the main parameters of a two-phase flow is gas content, which characterizes the volumetric fraction of the gas phase in the mixture. As gas content increases, the effective mixture density, phase velocities, and hydraulic losses change significantly. Under well conditions, these changes directly affect pressure distribution and may have a substantial influence on bottom-hole pressure.

Particular attention in Chapter 1 is given to existing engineering correlations. The most widely used approaches considered include the Lockhart–Martinelli, Hagedorn–Brown, and Beggs–Brill models. These models have been extensively applied in petroleum engineering because they allow pressure gradients and other two-phase flow parameters to be determined relatively quickly.

The Lockhart–Martinelli model is one of the early engineering correlations for calculating the hydraulic resistance of two-phase flow. Its advantages include relative simplicity and a small number of input parameters. However, the model was developed using a limited set of experimental data and does not account for many characteristics of modern oil and gas wells, including complex transitional flow regimes, changes in the rheological properties of the liquid phase, and the influence of near-wellbore conditions.

The Hagedorn–Brown correlation is widely used for calculating pressure gradients in vertical wells. Compared with earlier models, it provides a more detailed description of phase distribution and gas-liquid mixture flow. However, its accuracy also depends on the correspondence between the conditions under consideration and the range of the original experimental data. The influence of the rheological properties of the liquid and the skin factor is not directly considered in the model.

The Beggs–Brill model has become widely used because it can calculate two-phase flow in pipes with different orientations. It accounts for changes in the flow regime depending on the flow conditions and is therefore a convenient engineering tool. However, the use of empirical coefficients and correlations limits its applicability under complex geological and technical conditions and when dealing with non-standard reservoir-fluid properties. A comparison of these models in the dissertation shows that none of

them fully accounts for the combined influence of the rheological properties of the liquid, near-wellbore conditions, and interfacial interaction.

Chapter 1 also examines the historical development of two-phase flow research. Initial studies were primarily associated with problems in thermal and nuclear power engineering, where gas-liquid flows were important for the analysis of heat transfer and equipment stability. Subsequently, the theoretical and experimental approaches developed in these fields were widely applied in the petroleum industry.

Significant contributions to the development of two-phase flow theory were made by the studies of Zuber, Wallis, and Griffith concerning bubble formation and coalescence mechanisms, interfacial interaction, and interface dynamics. In the petroleum industry, methods for calculating gas-liquid flow in wells, tubing, and production transportation systems were subsequently developed.[3]

An important stage was the emergence of empirical and semi-empirical correlations, which made it possible to move from predominantly experimental studies of flows to engineering calculations. Subsequently, advances in computing technology facilitated the widespread implementation of computational fluid dynamics (CFD). CFD approaches made it possible to model individual phase motion, interfacial interaction, turbulence, capillary and gravitational effects, as well as more complex channel geometries. However, the high computational complexity of CFD models limits their application in operational field calculations.[4]

A separate section of Chapter 1 is devoted to the problems and unresolved issues of existing research. One of the main problems identified is the absence of a universal mathematical model capable of adequately describing different two-phase flow regimes over a wide range of operating conditions.

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[3]Zuber N., Findlay J. A. Average Volumetric Concentration in Two-Phase Flow Systems // Journal of Heat Transfer. — 1965. — Vol. 87, No. 4. — P. 453–468.

[4]Versteeg H. K., Malalasekera W. An Introduction to Computational Fluid Dynamics: The Finite Volume Method. — 2nd ed. — Harlow: Pearson Education, 2007. — 520 p.

Most classical correlations are based on simplifying assumptions concerning phase equilibrium, constant physical properties, and stable flow regimes. Such conditions are not always satisfied in real wells. Gas-liquid mixtures are characterized by continuous phase redistribution, changes in velocity and pressure, local fluctuations, and transitions between flow regimes.

Another significant limitation is the transfer of laboratory research results to field conditions. The geometric dimensions of laboratory facilities differ substantially from those of operating wells, resulting in changes in similarity criteria, flow regimes, and the relative influence of surface and capillary forces. Therefore, the direct use of laboratory correlations without additional adaptation may lead to systematic errors in determining the hydrodynamic parameters of wells.

Another important problem is the insufficient consideration of transient, thermodynamic, and capillary effects. During the movement of a gas-liquid mixture, changes in pressure and temperature are accompanied by changes in the physical properties of the phases and the conditions of interfacial interaction. The influence of capillary forces may become particularly significant in low-permeability reservoirs and at small characteristic pore sizes.

The rheological properties of the liquid also have a significant influence on the structure of two-phase flow. In the presence of anomalous viscosity, changes in the content of high-molecular-weight components of oil may result in changes in effective viscosity, hydraulic resistance, and phase velocity distribution. Consequently, the use of models assuming Newtonian behavior of the liquid phase does not always allow real filtration and production-flow processes to be adequately described.

Particular attention is given to the problem of flow-regime identification. In a real well, the flow regime may change along the wellbore due to pressure reduction, changes in gas content, and phase interaction. The transition from bubbly to slug and then to annular flow is accompanied by changes in hydraulic resistance and pressure distribution. Therefore, correct identification of the flow regime is an

important prerequisite for improving the accuracy of subsequent calculations.

Chapter 1 also demonstrates that near-wellbore conditions should be considered together with the characteristics of two-phase flow. The skin factor reflects additional hydrodynamic resistance in the near-wellbore zone and directly affects fluid inflow. Under two-phase filtration conditions, changes in the skin factor may be accompanied by changes in the inflow profile, pressure distribution, and natural-flowing conditions. Therefore, excluding this factor from the mathematical model may distort the results of well-productivity calculations.

Based on the analysis performed, the main requirements for improving mathematical models were formulated. A prospective model should account for the relative motion of phases, interfacial friction, changes in flow structure, rheological properties of the liquid, nonequilibrium processes, and additional resistance in the near-wellbore zone. At the same time, the model should maintain acceptable computational complexity so that it can be used not only for fundamental analysis but also for engineering calculations.

A modern direction of development involves integrating classical hydrodynamic models with computational and intelligent methods. CFD methods allow a more detailed investigation of local flow structure, whereas machine-learning methods and neural networks can be used to identify nonlinear relationships between measured parameters and calculated well characteristics. The dissertation notes that such approaches represent a promising direction for improving the accuracy of predicting bottom-hole pressure and other operating parameters.

At the same time, the exclusive use of intelligent methods does not solve all the problems. Neural-network models require representative training data and an appropriate selection of input parameters, whereas physicomathematical models make it possible to directly account for the physical laws governing phase motion. Therefore, the combination of physically based mathematical models with modern computational and intelligent methods represents the most promising approach.

As a result of the analysis conducted in Chapter 1, the scientific problem of the present study was formulated: the need to develop an engineering-mathematical approach that makes it possible to determine the main hydrodynamic characteristics of oil and gas wells while simultaneously accounting for two-phase flow, the rheological properties of the liquid, the characteristics of interfacial interaction, and the influence of the near-wellbore zone.

Thus, the literature analysis demonstrates that further development of the hydromechanics of two-phase systems should be aimed not only at increasing the physical detail of models but also at improving their practical applicability. The methods developed should provide the possibility of determining bottom-hole pressure, inflow profile, hydraulic losses, and other characteristics based on available field information.

The review provides the basis for moving to the mathematical modeling and investigation of two-phase systems presented in Chapter 2. Chapter 2 is devoted to the mechanics of gas-liquid systems, modeling of the hydrodynamic characteristics of the flow, mathematical description of filtration and inflow processes, and the development of a methodology for determining bottom-hole pressure based on information obtained at the wellhead.

In Chapter 2, based on the A. Armand model, an in-depth investigation of the motion of a two-phase gas-liquid mixture in pipes and in a vertical wellbore is carried out. Particular attention is paid to the derivation of equilibrium equations, the distribution of phase velocities, determination of the volumetric flow rate of the mixture, the hydraulic-resistance law, and analysis of the effective viscosity of the two-phase system, taking into account the influence of volumetric gas content and interfacial interaction.

As the initial physical model, an elementary cylindrical volume of a gas-liquid mixture undergoing steady uniform flow in a pipeline is considered. According to the fundamental principles of continuum mechanics, the sum of all forces acting along the flow axis must be equal to zero. The equilibrium condition is written in the following form:

$$F_1-F_2-T=0 \qquad (1)$$

or in expanded form:

$$p_1\pi r^2-p_2\pi r^2+\tau_{CM}2\pi rL=0 \quad (2)$$

where:

F_1 and F_2 — pressure forces acting at the inlet and outlet of the selected section;

T —friction force;

τ_{mix} — shear stress of the mixture;

r — pipe radius;

L — length of the elementary section.

This equation reflects the physical balance between the pressure drop and the resistance to motion caused by viscous friction at the pipe wall. This relationship serves as the fundamental basis for the further development of the mathematical model of two-phase flow.

For a more accurate description of the flow structure, the shear stresses for the liquid and gas phases are determined separately:

$$\tau_l=\mu_l\frac{dV_l}{dr},$$
$$\tau_g=\mu_g\frac{dV_g}{dr} \quad (3)$$

where:

μ_l, μ_g — dynamic viscosities of the liquid and gas phases;

V_l, V_g — velocities of the corresponding phases.

Since a two-phase system involves interphase interaction, the concept of volumetric gas content φ, which characterizes the fraction of the gas phase in the total mixture volume, plays an important role. As φ increases, the structure of the flow changes, interphase forces become stronger, and the radial distribution of velocities in the pipe is altered.

Based on the equilibrium equation and after integrating the equation of motion, the velocity distribution of the liquid phase in steady laminar flow is obtained:

$$V_l=\frac{\Delta PR^2}{\mu_l L}\left(1-\frac{g^2}{R^2}\right) \quad (4)$$

This equation expresses the classical parabolic velocity profile characteristic of laminar flow. It shows that the maximum velocity occurs at the pipe centerline, while the velocity gradually decreases toward the wall, reaching zero at the boundary.

The boundary conditions are assumed in the following form:

$$\left.\frac{dV}{dr}\right| r=0 \; u(R)=0, \tag{5}$$

The first condition reflects the no-slip effect of the liquid on the pipe wall, while the second condition represents the symmetry of the flow relative to the central axis of the pipe.

Thus, the velocity profile has a strictly parabolic form, which is fully consistent with classical hydrodynamic concepts of laminar flow.

If $\varphi = 0$, i.e., when the gas phase is absent, the classical Stokes equation for single-phase laminar flow is obtained:

$$V_{max\,l}=\frac{\Delta p}{4\mu_l L}R^2 \tag{6}$$

The volumetric flow rate of the mixture is derived from the velocity distribution. For the circular area element:

$$dF=2\pi r dr \tag{7}$$

The elementary flow rate is determined as follows:

$$dQ=V_l 2\pi r dr \tag{8}$$

Integration over the entire pipe cross-section gives the total volumetric flow rate:

$$Q_l=\frac{\pi\Delta P}{8\mu_l}\frac{R^4}{L}(1-\varphi)^{1.53} \tag{9}$$

The obtained equation shows that an increase in the gas volume fraction φ leads to a decrease in the liquid-phase flow rate because part of the pipe cross-section is occupied by the gas phase and interfacial energy losses increase.

When $\varphi = 0$, this expression reduces to the classical Poiseuille law, confirming the physical validity of the developed model.

The hydraulic resistance law is used to estimate pressure losses in a pipe network. By incorporating the Reynolds number and the hydraulic resistance coefficient, the following expression is derived for the laminar regime:

$$\lambda_l = \frac{64}{Re_l} \quad (10)$$

where Re_l— Reynolds number of the liquid phase.

It has been established that when Re <2300, the flow maintains a steady laminar character, and hydraulic resistance is primarily determined by viscous forces. As the Reynolds number increases, a transition occurs, followed by the onset of turbulent flow, accompanied by a significant increase in pressure losses.

Figure 1 shows the dependence of the pressure drop on volumetric flow rate for different gas-volume fractions φ.

Special attention in Chapter 2 is given to determining the effective viscosity of the two-phase mixture. Under real conditions, the flow cannot be treated as a simple sum of two independent phases because complex interfacial interaction between the liquid and gas changes the shear stresses and redistributes energy.

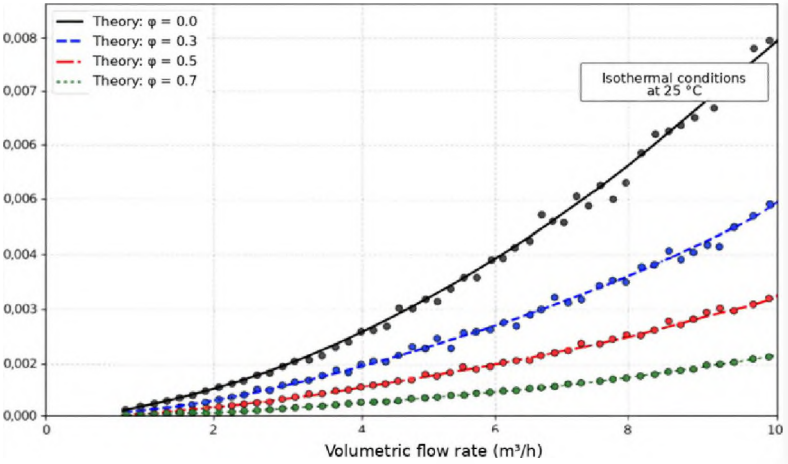


Figure 1. Dependence of pressure drop on volumetric flow rate at different values of gas volume fraction φ

To describe this effect, the following expression is used:

$$\tau_{mix} = \tau_l (1-\varphi)^n \quad (11)$$

where n is an empirical coefficient accounting for the flow structure. It has been established that an increase in gas content leads to an increase in effective shear stress and hydrodynamic resistance.

This statement indicates that the effective shear stress is determined not only by the properties of the liquid phase but also by the spatial distribution of the gas component. As the gas content increases, the flow structure changes, turbulence at the interphase boundary intensifies, resistance to motion rises, and hydrodynamic losses increase.

The analysis conducted has shown that applying the A. Armand model allows for a more accurate consideration of the impact on gas-phase flow parameters compared to classical single-phase models. The derived dependencies are in good agreement with experimental data and can serve as a basis for engineering calculations when analyzing the operation of oil and gas wells.

Thus, in the second chapter, a physical-mathematical foundation has been established for further research on the hydromechanics of two-phase systems, enabling a transition from simplified empirical assumptions to a more accurate description of the actual dynamics of gas–liquid mixtures within wells and pipeline networks.

This flow-rate equation was first obtained experimentally by the French physician Jean Léonard Marie Poiseuille in 1840 while studying the flow of water through capillary tubes. In the present work, this relationship is extended to a two-phase gas-liquid mixture by accounting for the gas volume fraction φ and interfacial interaction.

The following expression is used to determine pressure losses during the laminar flow of a mixture in circular pipes:

$$h_w = \frac{\Delta P}{Y_1} = \frac{32\mu_1 \cdot \nu_1 L}{Y_1 D^2 (1-\varphi)^{1.53}} \tag{12}$$

This equation represents the law of hydraulic resistance. When $\varphi = 0$, it reduces to the expression for a homogeneous liquid. Multiplying both sides of this equation by $(2\nu_1)$, we obtain the pressure losses:

$$h_w = \frac{32\mu_1 \cdot \nu_1 L}{Y_1 D^2 (1-\varphi)^{1.53}} \quad \frac{2\nu_1}{2\nu_1} = \frac{64\mu_1}{Y_1 \cdot \nu_1 D^2} \quad \frac{L}{D} \frac{\nu_1^2}{2} \frac{1}{(1-\varphi)^{1.53}} \tag{13}$$

The specific weight of the liquid and its dynamic viscosity are related through the corresponding kinematic flow parameter.

$$h_w=\frac{64\upsilon_l}{r_l D}\ \frac{L}{D}\frac{v_l^2}{2g}\frac{1}{(1-\varphi)^{1.53}} \tag{14}$$

This equation incorporates Reynolds number:

$$h_w=\frac{64}{Re_l}\ \frac{L}{D}\frac{1}{(1-\varphi)^{1.53}} \tag{15}$$

Multiplying both sides of this expression by ($\nu_l g$), we obtain the frictional pressure losses:

$$\Delta P_l=\frac{64}{Re_l}\ \frac{L}{D}\frac{v_l^2}{2}\rho_l\frac{1}{(1-\varphi)^{1.53}} \tag{16}$$

Direct determination of bottom-hole pressure makes it possible to establish the filtration behavior. Bottom-hole pressure data are used in the design of downhole equipment for artificial-lift operation and in evaluating the efficiency of natural-flow lifting systems.

The equation for the vertical pipe can be written as follows:

$$\frac{dP}{dx}=\rho_l g(1-\varphi)+\rho_g g\varphi+\frac{i_l}{(1-\varphi)^{1.53}} \tag{17}$$

Here:

$\frac{dP}{dx}$— pressure gradient;

i_l— hydraulic gradient;

ρ_g— gas density under operating conditions.

$$\rho_g=\rho_0\frac{PT_0}{P_0TZ} \tag{18}$$

Here:

ρ_0— gas density under atmospheric conditions;

T_0— reference temperature;

T — specified temperature;

P_0 — atmospheric pressure;

Z — compressibility factor;

P — specified pressure.

The following concept is introduced:

$$a=\rho_0\frac{T_0}{P_0TZ}$$

$$b=\rho_l g(1-\varphi)+\frac{i_l}{(1-\varphi)^{1.53}}\rho_l g \quad (19)$$

Then we obtain:

Based on the methods of the integral theory of filtration, we proceed as follows:

$$\int_{P_{wh}}^{P_{bh}}\frac{dP}{aP+b}=\int_0^L dx \quad (20)$$

Here:

P_{bh} — bottomhole pressure;

P_{wh} — wellhead pressure;

L — length of the production tubing.

Integrating, we obtain:

$$\ln\frac{aP_{bh}+b}{aP_{wh}+b}=La \quad (21)$$

To determine the actual gas content, the experimental correlation of A. Armand is used:

$$\varphi=(1+0.073\ln X) \quad (22)$$

X — mass gas fraction.

$$X=\frac{M_g}{M_g+M_l}$$

Here:

M_g— gas mass flow rate;

M_l— liquid mass flow rate.

$$\frac{aP_{bh}+b}{aP_{wh}+b}=e^{La} \quad (23)$$

Expanding the power term in a series:

$$e^{La}=1+La \quad (24)$$

Then we obtain:

$$\frac{aP_{bh}+b}{aP_{wh}+b}=1+\frac{L\rho_0 gT_0}{P_0 TZ} \quad (25)$$

In the first approximation, the compressibility factor is determined under wellhead conditions.

Introducing the notation:

$$P_{bh}=P_{wh}+\frac{L\rho_0 gT_0}{P_0 TZ_y}\left[P_{wh}+\frac{\rho_l g(1-\varphi)+i_l\rho_l g(1-\varphi)^{-1.53}}{\frac{\rho_0 gT_0}{P_0 TZ_y}\varphi}\right] \quad (26)$$

When $\varphi = 0$, the equation reduces to the equation of motion for a homogeneous liquid.

$$P_3=P_{wh}+\rho_l gL+i_l\rho_l gL=P_{wh}+\rho_l gL(1+i_l) \quad (27)$$

When $\varphi = 1$, the equation reduces to the equation of motion for a homogeneous gas.

$$P_{bh}=P_{wh}(1+\frac{L\rho_0 gT_0}{P_0 TZ_y}) \quad (28)$$

Table 1.

Comparison of calculated and experimental well data

Well No.	ρ oil, kg/m³	L tubing, m	$\frac{P_{wh}}{P_0}$ (wellhead)	Q_l m³/s	Q_g m³/s	X (mass fraction)	φ (actual gas content)	ρ mixture, kg/m³	$\frac{P_{bh}}{P_0}$ (calc.)	$\frac{P_{bh}}{P_0}$ (exp.)
392	855	1708	17	0.00095	0.0624	0.528	0.297	601.5	137.5	134.9
----		----	14.5	0.00127	0.0838	0.506	0.305	615.9	134.1	133.6
382		1744	13.0	0.00122	0.0803	0.507	0.303	595.9	135.0	132.2
----		----	15.5	0.00099	0.0659	0.505	0.284	633.1	139.4	140.6
99		1663	11.0	0.00146	0.0962	0.509	0.319	605.1	133.2	123.1
904		1606	7.5	0.00143	0.0945	0.511	0.290	604.5	108.9	112.1
37		1405	14.0	0.00107	0.0704	0.506	0.252	627.5	112.0	112.6
39		1780	15.0	0.00077	0.0472	0.505	0.315	658.5	139.1	143.6
580		1770	11.9	0.00139	0.0912	0.508	0.289	608.5	133.5	134.0
581		1482	14.5	0.00102	0.0685	0.506	0.288	629.6	115.8	116.3

Dimensionless pressure:

$$\overline{P_y}=\frac{P_{wh}}{P_0},$$

where P_y— wellhead pressure, P_0— atmospheric pressure.

The table presents a comparison of calculated and experimental data for the wells.

As it appears, the calculated and measured values are in good agreement.

The discrepancies are small and remain within acceptable engineering limits.

This confirms the model's validity and applicability.

Chapter 3 investigates the effects of the skin factor, anomalous viscosity, and structural features of two-phase flow on filtration processes in the near-wellbore zone, pressure distribution, and well flow rate. Particular attention is paid to the motion of an elastoviscoplastic fluid in the presence of a skin zone, which substantially alters the hydrodynamic characteristics of the system and affects the operating efficiency of oil and gas wells.

It is well known that damage to the near-wellbore zone, contamination of the pore space, deposition of asphaltene-resin-paraffin components, salt deposition, and mechanical plugging lead to the formation of an additional filtration-resistance region — the skin zone. The presence of this zone increases the pressure drop, reduces well productivity, and worsens reservoir-fluid inflow conditions.

To describe filtration under these conditions, an elastoviscoplastic-fluid flow model accounting for structural resistance is used. At the next stage, an expression for the elementary pressure drop in radial flow is obtained:

$$\frac{Q\eta}{2\pi h k_1} r\cdot dr+\frac{\alpha\tau_0}{\sqrt{k_g}} dr=dP \qquad (29)$$

This equation reflects the combined effects of viscous resistance and the additional yield stress arising in an elastoviscoplastic medium. Unlike the classical Darcy law, it accounts for nonlinear resistance associated with the limiting shear stress and changes in the flow structure.

In addition, because a skin coefficient is formed in the skin-effect zone under certain boundary conditions, special filtration boundary conditions must be considered. The skin zone is assumed to have one or more boundaries perpendicular to the horizontal plane and parallel to the reservoir surface.

For a horizontally oriented skin zone, the following conditions apply:

$$r=R_K;P=P_K$$
$$r=R_C;P=P_C$$
$$r=R_S;P=P_S \quad (30)$$

Substituting these boundary conditions into the governing equation and integrating with respect to the filtration radius gives:

$$\frac{Q\eta}{2\pi hk}\left[\frac{k}{k_s}\ln\frac{R_S}{R_C}+\ln\frac{R_K}{R_S}\right]+\frac{\alpha\tau_0}{\sqrt{k_g}}(R_K-R_C)=P_K-P_C \quad (31)$$

After successive algebraic transformations, the expression is reduced to a more convenient engineering form:

$$\frac{Q\eta}{2\pi hk}\left[\left(\frac{k}{k_s}-1\right)\ln\frac{R_S}{R_C}+\ln\frac{R_K}{R_S}\right]+\frac{\alpha\tau_0}{\sqrt{k_g}}(R_K-R_C)=P_K-P_C \quad (32)$$

Here:

R_C— well radius;

R_K — external drainage-boundary radius;

R_S — near-wellbore (skin-zone) radius;

P_K — pressure at the external drainage boundary;

P_C — bottomhole pressure.

The resulting equation shows that the pressure drop is determined not only by the flow rate and geometric parameters of the system, but also by the degree of near-wellbore damage expressed through the permeability ratio.

If the permeability of the skin zone decreases, the pressure-drop term increases, leading to an increase in the skin factor and a reduction in well productivity.

Under these conditions, the volumetric oil flow rate is determined by the following formula:

$$Q=\frac{kF}{\eta}\left[\frac{P'_K-P_C}{L-L_K}-\frac{\alpha\tau_0}{\sqrt{k_g}}\right] \quad (33)$$

Assume that $F=2\delta hL_K=R_K$ is the distance between wells. Then the following expression can be obtained:

$$Q=\frac{2\delta kh}{\eta}\left[\frac{P'_K-P_C}{L-R_K}-\frac{\alpha\tau_0}{\sqrt{k_g}}\right] \quad (34)$$

$$P'_K-P_c=\frac{Q(L-R_K)}{2\delta kh}+\frac{\alpha\tau_0}{\sqrt{k_g}}(L-R_K) \quad (35)$$

Using expressions (34) and (35), the total pressure drop corresponding to the considered flow region is calculated:

$$\Delta P=\Delta P_B+\Delta P_H \quad (36)$$

Thus, the final expression takes the following form:

$$\Delta P=(P_K-P_C)+(P'_K-P_K)=P'_K-P_C \quad (37)$$

Joint application of equations (36)–(37) gives:

$$\Delta P=\frac{Q\eta}{2kh}\left\{\left(S+\ln\frac{\rho}{2R_S}\right)+\frac{L}{\sigma}\right\}\frac{\alpha\tau_0}{\sqrt{k_g}}(L-R_C) \quad (38)$$

Assuming the radius of the well drainage zone to be equal to (n), the total system flow rate and the corresponding number of wells can be determined. In this case, the total production volume is written as:

$$\sum Q=Q\cdot n, Q=\frac{\sum Q}{n} \quad (39)$$

Substituting into equation (39), where $B=2\sigma\cdot n\sigma=\frac{B}{2n}$ the total well flow rate is determined as follows :

$$\sum Q=\frac{\frac{2\pi kh}{\eta}nL\left[\frac{P_K-P_C}{L}-\frac{\alpha\tau_0}{\sqrt{k_g}}\left(1-\frac{R_C}{L}\right)\right]}{\left(S+\ln\frac{B}{4nR_C}\right)+\frac{2nL}{B}} \quad (40)$$

The results show that increasing the well radius and decreasing the skin factor reduce the pressure drop and increase the flow rate, whereas increasing the distance to the reservoir boundary and deterioration of the filtration properties of the near-wellbore zone have the opposite effect.

The figure shows the dependence of the pressure drop in the filtration region on the well radius and the distance to the reservoir boundary.

The analysis confirms that accounting for the skin factor and anomalous viscosity is essential when calculating well productivity and selecting optimal operating regimes. The developed model significantly improves the accuracy of engineering calculations and provides a basis for practical forecasting of oil and gas well performance under complex two-phase filtration conditions.

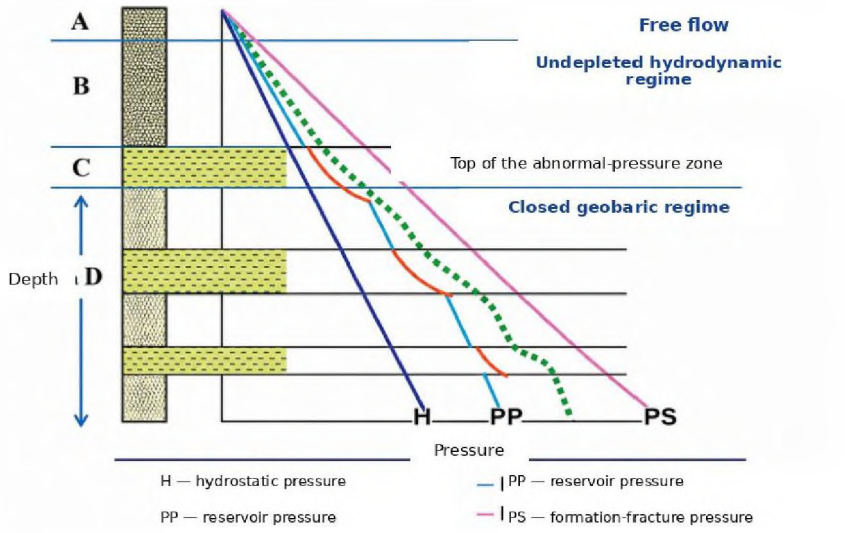


Figure 2. Dependence of pressure drop on well radius and distance to the reservoir boundary in the filtration region

CONCLUSIONS

1. A comprehensive study of the hydromechanics of two-phase gas-liquid systems occurring in oil and gas wells was carried out. The principal regularities of multiphase flow in pipes and reservoir systems, as well as their influence on the technological parameters of well operation, were considered [3, 4, 7, 11].

2. Based on the analysis of existing theoretical and experimental studies, the specific features of gas-liquid mixture flow under complex hydrodynamic regimes were clarified. It was shown that accounting for the rheological properties of the liquid and the flow regime substantially improves the accuracy of hydrodynamic-parameter calculations [1, 2, 4, 5].

3. A mathematical model of two-phase-system flow based on the conservation equations of mass, momentum, and energy was developed. The proposed model accounts for gravitational forces, friction, and the characteristics of phase distribution in the flow [3, 4, 7].

4. Computational relationships were obtained for determining pressure distribution and phase velocities in the well. On the basis of the proposed relationships, a method for determining bottomhole pressure from wellhead measurements was developed [10, 11].

5. A numerical analysis of two-phase flow was performed using mathematical-modeling methods. The results obtained show good agreement with known experimental and calculated data [3, 4, 10].

6. It was established that the developed model makes it possible to account more accurately for the influence of the flow regime, phase distribution, and the rheological characteristics of the liquid on the hydrodynamic parameters of the system [1, 2, 4, 5, 7].

7. The proposed methods and computational relationships can be used in the design and optimization of oil and gas production processes, as well as in the interpretation of field data [6, 8, 10, 11].

8. The practical significance of the research results lies in the possibility of improving the efficiency of oil and gas well operation through more accurate determination of bottomhole pressure and analysis of two-phase-flow processes [8, 10, 11].

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1. Dadash-Zade, M.A. Characteristic features of the movement of elastic-viscoplastic liquid in pipes and layers / M.A. Dadash-Zade, R. Cao // Probe: Environmental Science and Technology. – 2023. – Vol. 5, No. 4. – P. 49–53.

2. Dadash-Zade, M.A. The impact of abnormal oil viscosity and oil well skin coefficient on production efficiency: a study based on inter-well interference / M.A. Dadash-Zade, R. Cao // Petrochemical. – 2024. – No. 3. – P. 40–42.

3. Dadash-Zade, M.A. Rheological properties of elastoviscoplastic liquid in tubes / M.A. Dadash-Zade, R. Cao // Advances in Resources and Environmental Science. – 2024. – Vol. 3, No. 2. – P. 21–24.

4. Dadash-Zade, M.A. Study of the motion of two-phase systems based on fluid mechanics and physical modelling of fluid / M.A. Dadash-Zade, R. Cao // Proceedings of the 9th International Conference on Energy Science and Applied Technology (ESAT 2024). – 2024. – P. 1–3.

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Personal contribution of the applicant

In all published works [1–11], the candidate was directly involved in formulating the research problems, performing calculations and mathematical modelling, processing and analysing the results, providing a scientific interpretation of the findings, and preparing the materials for publication.

In works [1–8, 10, 11], the candidate was directly involved in conducting research on the dissertation topic and synthesising the findings.

Cao Ru

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