REPUBLIC OF AZERBAIJAN

On the rights of the manuscript

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

LAUNCH OF SATELLITES FOR EARTH RADIATION MONITORING USING COMBINED THRUST

Specialty: 3324.04 - Operation of Ground Complexes, Launch Equipment, Aircraft and their Systems

Field of science: Technical sciences

Applicant: **Tarlan Hamlet Mammadzada**

Baku – 2025

The dissertation was completed at the Department of "Aircrafts and Aircraft Engines" of the National Aviation Academy.

Scientific supervisor: Doctor of physical and mathematical sciences, academician
Roald Zinnurovich Sagdeyev

Official opponents: Doctor of physical and mathematical sciences, professor
Latif Khalil Talibli

Doctor of technical sciences, professor
Tofig Ibrahim Suleymanov

Doctor of philosophy in technical sciences, associate professor
Farhad Alif Mirzoyev

Dissertation council ED 2.01 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at National Aviation Academy

Chairman of the Dissertation council: Full member of ANAS, doctor of physical and mathematical sciences, academician
Arif Mir Jalal Pashayev

Scientific secretary of the Dissertation council: Doctor of geographical sciences, associate professor
Surkhay Hasan Safarov

Chairman of the scientific seminar: Doctor of technical sciences, professor
Mustafa Rahim Mustafayev

GENERAL CHARACTERISTICS OF THE WORK

Relevance of the study: The Earth is surrounded by powerful radiation belts – highly charged energetic particles (mainly protons and electrons) trapped in the geomagnetic field and constantly influenced by solar activity. Their radiation has a wide range of effects, posing a significant threat to the operational reliability of spacecraft (SC) onboard equipment and the health of space crews. According to some estimates, more than half of onboard SC system failures occur due to the adverse effects of various space factors, particularly penetrating and accumulating radiation. For example, solar panels operate less efficiently and eventually fail after absorbing significant doses of radiation, while microchips can experience sudden malfunctions and burn out. A notable example is the "Elektron-1" and "Elektron-2" satellites launched in 1964 – their silicon solar panels degraded within 40 days and 5 months, respectively. The second pair, "Elektron-3" and "Elektron-4," launched six months later, were equipped with solar panels coated with a thin film that absorbed most of the radiation, allowing them to last longer.

Earth's inner radiation belt begins at an altitude of several thousand kilometers and consists mainly of positively charged protons with high kinetic energy, while the outer belt is located at an altitude of about 10,000 km and also contains electrons. The toroidal radiation belts of Earth encompass nearly all artificial satellite orbits. To take timely measures to maintain a satellite's operational capability, it is essential to have accurate data on the radiation environment in its orbit.

Currently, to record real-time values of particle fluxes and theoretically study radiation dynamics, instruments for measuring fluxes of energetic charged particles are installed on artificial Earth satellites. The fluxes of energetic particles in near-Earth space can exhibit significant variations over time intervals ranging from several days to several years, which cannot be described by static models of Earth's radiation belts. Moreover, most existing satellites measure radiation only in a limited region of near-Earth space. Therefore, regularly obtaining up-to-date satellite data on current energetic particle fluxes is a necessary condition for studying the flux dynamics.

This study examines the problem of launching satellites into orbits for Earth's radiation monitoring using combined propulsion engines. The considered orbits intersect a wide range of charged particle trajectories at different altitudes, enabling the construction of a three-dimensional dynamic model that reflects the current spatial distribution of fluxes. This allows real-time observation of radiation dynamics across a significant portion of near-Earth space.

It is known that electric propulsion systems (EPS) can significantly reduce fuel consumption, thereby increasing the maximum possible payload mass due to their substantially higher specific impulse compared to chemical rocket engines (CRE) and, consequently, extending interorbital transfer duration. Theoretical studies show that a satellite-carrying propulsion module equipped with both chemical and electric propulsion systems in sequence can deliver a larger payload mass compared to CRE in less time than using EPS alone. Thus, developing satellite launch schemes using combined propulsion is crucial for improving the efficiency of satellite deployment in near-Earth space.

Subject elaboration degree: Designing a satellite launch system requires preliminary trajectory modeling and ballistic analysis as a fundamental basis for engineering justification of thrust parameters, flight control strategies, and compliance with target orbit and mission requirements.

The promising use of electric propulsion systems (EPS) has driven the development of low-thrust spaceflight mechanics, addressing the selection of optimal design parameters for spacecraft and the determination of optimal thrust control. Foundational works in optimizing thrust control for low-thrust transfers include the monographs by G.L. Grodzovsky, Y.N. Ivanov, V.V. Tokarev[1], and V.N. Lebedev[2]. The first work examines the characteristics of low-thrust propulsion systems, equations of spacecraft motion in near-Erath space, and the formulation and solution of optimization problems for transfers using high-thrust engines, low-thrust engines, and their combinations. The second work focuses on optimizing transfers between circular non-coplanar orbits, including the maximization of the payload mass of a spacecraft.

Methods for optimizing inter-orbital transfers are also explored in the works of V.V. Salmin[3], S.A. Ishkov and V.A. Romanenko[4], M.S. Konstantinov[5], V.V. Vasilyev[6]. The principle of maximum by L.S. Pontryagin is often applied to find optimal control, reducing the optimization problem to a boundary-value problem for a system of ordinary differential equations. The complexity lies in finding initial approximations for conjugate variables.

Significant advancements in research on transfers with combined propulsion systems have been made by V.G. Petukhov[7] through the application and development of the parameter continuation method. The optimization of interplanetary transfers with combined control using high- and low-thrust engines has been studied by I.S. Grigoriev[8] and G.G. Fedotov[9]. In most cases, the operation of high-thrust propulsion systems is approximated by impulsive actions. R.Z. Akhmetshin, S.S. Beloglazov, and N.S. Belousov[10] analyzed spacecraft flights equipped with high- and low- thrust engines, maximizing the payload delivered to an asteroid or comet. In their approach, the high-thrust engine operates on the first segment of the trajectory within Earth's sphere of influence, while the low-thrust engine operates on the subsequent segment. The calculation is sequential: first for the high-thrust segment and then for the low-thrust segment.

---

[1] Grodzovsky, G.P. Mechanics of Space Flight with Low Thrust / Grodzovsky, G.P., Ivanov, Yu.N., Tokarev, V.V. – Moscow: Nauka, 1975. – p. 679.

[2] Lebedev, V.N. Calculation of the Motion of a Spacecraft with Low Thrust / Moscow: Computing Center of the USSR Academy of Sciences, 1968. – p. 106.

[3] Salmin, V.V. Methods for Solving Variational Problems in the Mechanics of Space Flight with Low Thrust / Samara: Samara Scientific Center of the Russian Academy of Sciences, 2006. – 164 pages.

[4] Ishkov, S.A., Romanenko, V.A. Formation and Correction of a Highly Elliptical Orbit of an Earth Satellite Using a Low-Thrust Engine // – Moscow: Kosmicheskie Issledovaniya – Cosmic Research, 1997, vol. 35, no. 3, – pp. 278–296.

[5] Konstantinov, M.S. Methods of Mathematical Programming in the Design of Aerospace Vehicles / Moscow: Mashinostroenie, 1975. – p. 168.

V.V. Salmin also demonstrated that when using a combined propulsion system with a limited exhaust velocity engine, the optimal set of maneuvers involves utilizing the high-thrust engine during the initial phase, followed by low-thrust propulsion for the subsequent transfer phase. This scheme is considered a compromise, combining the advantages of impulsive maneuvers with short transfer times and low-thrust maneuvers with relatively lower propellant consumption.

Methods for solving spacecraft trajectory optimization problems are classified into direct, indirect, and hybrid approaches. Direct methods involve dividing the trajectory into small segments, each with its own control program, reducing the trajectory optimization problem to a conditional minimization problem in a high-dimensional space with numerous constraints, effectively a nonlinear programming problem. Indirect methods optimize the transfer trajectory without segmentation, using necessary and sufficient optimality conditions in classical variational form, Pontryagin's maximum principle, or Krotov optimality conditions. Hybrid methods combine aspects of both direct and indirect approaches, leveraging their advantages to serve as a "compromise" methodology for optimizing transfer trajectories.

---

[6] Vasiliev, V.V. Optimal Control of the Elliptical Orbit of an Earth Satellite with Low Thrust // – Moscow: Kosmicheskie Issledovaniya – Cosmic Research, 1980, vol. 18, no. 5, – pp. 707–714.

[7] Petukhov, V.G. Continuation Method for Optimizing Interplanetary Trajectories with Low Thrust // Moscow: Kosmicheskie Issledovaniya – Cosmic Research, 2012, vol. 50, no. 3, – pp. 258–270.

[8] Grigoryev, I.S. Optimization of a Mission to Phobos Using a Spacecraft with Combined Thrust and Return to Earth // Engineering Journal: Science and Innovations, – Moscow: 2017, no. 7, – pp. 1–24.

[9] Fedotov, G. Possibilities of Combining High- and Low-Thrust Engines in Flights to Mars // – Moscow: Kosmicheskie Issledovaniya – Cosmic Research, 2001, vol. 39, no. 6, – pp. 613–621.

[10] Akhmetshin, R.Z. Optimization of Spacecraft Transfers to Asteroids and Comets Using a Combination of High and Low Thrust / Akhmetshin, R.Z., Beloglazov, S.S., Belousova, N.S. – Moscow: Keldysh Institute of Applied Mathematics, Preprint No. 144, 1985. – pp. 38–47.

Object of the research: The investigation focuses on the integration of chemical rocket engines and electric propulsion systems within an orbital transfer vehicle to enhance satellite injection efficiency across diverse orbital configurations.

Subject of the research: The research examines the operational principles and flight trajectories of a transfer stage equipped with combined propulsion system, specifically for delivering satellites to Earth radiation monitoring orbits.

Purpose and objectives of the research: The study aims to validate the efficiency of hybrid propulsion systems (combining chemical and electric thrusters) in satellite injection stages, demonstrating their potential to enhance propellant efficiency and reduce transfer time across diverse orbital configurations. The analysis accounts for constraints typical of rideshare launches for small satellites targeting Earth radiation monitoring orbits.

To achieve this goal, the following tasks were undertaken:

1. Feasibility Analysis: Conducted a comprehensive assessment of the rationality and technical viability of using hybrid propulsion systems in orbital transfer vehicles for deploying Earth radiation monitoring satellites.
2. Propulsion System Characterization: Performed detailed analysis of chemical and electric propulsion system characteristics, along with optimization methods for interorbital transfers, examining both standalone and combined operational modes.
3. Mission-Specific Trajectory Design: Developed specialized satellite injection schemes utilizing hybrid propulsion, specifically tailored for Earth radiation monitoring mission requirements.
4. Numerical Validation: Executed computational simulations of trajectory parameters and orbital maneuvers, providing quantitative verification of the proposed injection schemes' technical feasibility.
5. Comparative Performance Evaluation: Conducted systematic comparison between hybrid propulsion and conventional (chemical-only or electric-only) injection methods,

demonstrating the hybrid approach's advantages in both propellant consumption (up to 30% reduction) and mission duration (50% shorter than all-electric systems).

Research method: The research methodology is computational-theoretical and is based on the application of classical spaceflight mechanics methods. The task of transferring satellites from a parking orbit to target orbits using a chemical rocket engine is considered in an impulsive formulation and approximated by two apsidal maneuvers without changing the orbital plane or orientation. The problem of inter-orbital transfer with low thrust is solved using Pontryagin's Maximum Principle, which reduces the optimization problem to a boundary value problem for a system of ordinary differential equations.

To solve the boundary value problem with the required accuracy, a modified Newton method is applied. To improve convergence in the construction of multi-revolution trajectories, an averaging method over one orbital revolution is used, which significantly reduces the computational load while maintaining the necessary accuracy. An initial approximation of the adjoint variables is generated through the preliminary solution of an energy-minimization trajectory problem.

The mathematical model of motion is constructed in equinoctial orbital elements, which helps avoid singularities at low eccentricities and inclinations. The second zonal harmonic J_2 of Earth's gravitational field, which has the greatest influence on near-Earth orbital motion, is taken into account in the calculations. Gravitational perturbations from third bodies, solar radiation pressure, and atmospheric drag are not considered due to their limited influence in the context of comparative analysis of deployment schemes.

The combined deployment scheme assumes the use of a chemical propulsion stage in the initial phase to form an intermediate elliptical orbit, followed by insertion into the target orbit using an electric propulsion system with a minimized transfer time. To reflect practical mission conditions, the duration of the low-thrust phase is fixed to be twice as short as that of a transfer using only electric propulsion. This approach enables an accurate comparison of alternative schemes in terms of fuel efficiency and time expenditure.

The sources of the research include the works of both domestic and foreign scientists.

Key points for defense:

1. The engineering concept of using a hybrid propulsion orbital transfer vehicle as part of a satellite launch system has been substantiated - operational schemes have been proposed for the sequential delivery of satellites to target orbits for Earth radiation monitoring.
2. An analysis of satellite injection parameters using chemical rocket engines in impulsive maneuvers has been conducted – the efficiency of the traditional approach based on two-impulse maneuvers has been evaluated for radiation monitoring applications.
3. A model for optimal control of interorbital transfers using electric propulsion has been investigated and adapted, focusing on reducing transfer duration – numerical integration methods have been applied, including averaging over an orbit and a modified Newton's method.
4. Numerical simulation of the transfer vehicle's trajectory has been performed for different types of propulsion systems, with comparative analysis of operational characteristics based on propellant consumption and injection duration criteria.
5. The practical efficiency of hybrid propulsion in launch systems has been demonstrated – achieving up to 73% reduction in propellant costs compared to chemical-only systems and halving the injection time compared to electric propulsion-only systems.

The scientific novelty of the research lies in the development and justification of technically feasible satellite deployment schemes for Earth's radiation monitoring using combined thrust, adapted to the conditions of the current project by the National Aviation Academy in collaboration with Azercosmos and Lomonosov MSU.

Unlike previous studies that primarily focused on hybrid propulsion for interplanetary or single interorbital transfers, this work specifically addresses its application for deploying constellations of small satellites from a reference orbit to Earth radiation monitoring orbits. The study incorporates critical constraints of rideshare launches while meeting mission requirements for reduced propellant consumption and shorter transfer durations, necessitating holistic optimization of the orbital transfer vehicle's performance.

Within the framework of the dissertation, an applied approach has been developed that allows:

- Reducing the mission duration by almost half compared to the use of electric propulsion only.
- Decreasing fuel consumption to 63–73% compared to chemical propulsion.
- Adapting the scheme to the parameters of small satellites deployed alongside larger spacecraft.
- Developing a numerically stable and resource-efficient method for constructing transfer trajectories based on optimal control methods and specially developed software.

The trajectory decomposition method, previously known from fundamental works (for example, by V.V. Salmin), has been extended in this research by introducing a practice-oriented methodology for selecting an intermediate orbit, specifically adapted to the task of radiation monitoring under the conditions of a real-world project.

An important result is the development of a software module implementing a numerically stable hybrid propulsion injection model. The program calculates orbital maneuver parameters based on time, propellant, and thrust constraints. The module's interface allows for specifying satellite mass and target orbit selection, and can be adapted to meet various mission requirements.

Practical significance of this research lies in developing a methodology to improve transfer stage efficiency through hybrid propulsion for launching small Earth radiation monitoring satellites, accounting for mass, time and energy constraints typical of rideshare launches.

The results can be used in designing and operating launch systems, developing transfer stage control software, and planning cluster launches. One application direction is the methodology's proposed implementation in a National Aviation Academy project conducted jointly with Azercosmos and Lomonosov Moscow State University, aimed at establishing a small satellite constellation for Earth radiation monitoring.

Validation of the work. The results presented in the dissertation are considered as a methodological and computational foundation for subsequent application within the framework of the project implemented by the National Aviation Academy in collaboration with Azercosmos and Lomonosov Moscow State University, aimed at creating a constellation of small satellites to study the dynamics of Earth's radiation belts. The developed deployment schemes and numerical trajectory calculations using combined thrust are intended for use during the preliminary design phase and for the optimization of orbital configurations of satellites for Earth's radiation monitoring.

The main results of the dissertation research are presented in 10 scientific articles by the author in journals such as the "Engineering Journal: Science and Innovations" of Bauman Moscow State Technical University, the "Herald of the National Engineering Academy of the Republic of Kazakhstan", the "Herald of Samara University. Aerospace Technology, Technologies and Engineering", the "Herald of the Azerbaijan Engineering Academy", "Scientific Notes of the National Aviation Academy of Azerbaijan" and conference proceedings from the "Fifth IAA Conference on Dynamics and Control of Space Systems", "11th European CubeSat Symposium, University of Luxembourg", "IV International Conference, National Aviation Academy of Azerbaijan", and the "III International Conference, National Aviation Academy of Azerbaijan". The scientific supervisor of some of the works was Professor M.I. Panasyuk, the head of the Russian Radiation Monitoring Program for Near-Earth Space and Director of the National Research Institute of Nuclear Physics at Moscow State University.

Institution where the dissertation was completed. The dissertation was carried out at the National Aviation Academy of Azerbaijan under the "Azerbaijan Airlines" (AZAL).

Volume of the dissertation by structural sections with character count. The dissertation has been prepared in accordance with the requirements of the Higher Attestation Commission under the

President of the Republic of Azerbaijan. The work consists of an introduction (23,000 characters), three chapters (Chapter I – 5 paragraphs, 44,600 characters; Chapter II – 4 paragraphs, 46,200 characters; Chapter III – 6 paragraphs, 51,100 characters), a conclusion (4,300 characters), a list of 85 references, 7 appendices, 21 figures, and 21 tables. The total volume of the dissertation text (excluding spaces, figures, tables, appendices, and the reference list) is 169,100 characters and is presented on 173 printed pages.

SUMMARY OF THE WORK

The first chapter presents a comprehensive review of the problem of studying Earth's radiation belts using artificial satellites, along with an analysis of chemical and electric propulsion technologies and their combinations for solving satellite injection tasks. The chapter examines the history of discovery and research of radiation belts beginning with the launches of early satellites. It emphasizes the necessity for regular and continuous monitoring of the radiation environment, driven by the high variability of energetic particle fluxes, their dependence on solar activity, and the presence of short-term bursts that can affect spacecraft operations. The text notes that existing models of radiation background suffer from insufficient current measurements, particularly in highly elliptical orbits, leading to reduced forecast accuracy and increased mission risks.

The chapter thoroughly investigates the specifics of transferring small satellites from a reference orbit to various target orbits, including elliptical orbits with apogee altitudes of 800 km, 2000 km, and 8000 km, as well as Molniya-type orbits with a semi-major axis of approximately 26,555 km (Figure 1). The selection of such orbits is justified from the perspective of covering different regions of radiation belts and enabling observation of spatiotemporal dynamics of charged particles across various altitude ranges.

Special attention is given to the characteristics of electric propulsion systems (EPS), their advantages over chemical rocket engines (CRE) due to higher specific impulse, as well as limitations related to low thrust levels and the need for extended time intervals to reach target orbits. The analysis includes modern types of EPS (electrostatic, electrothermal, electromagnetic) used in various missions, along with examples of their successful applications, including missions with extended transfer periods and orbit corrections.

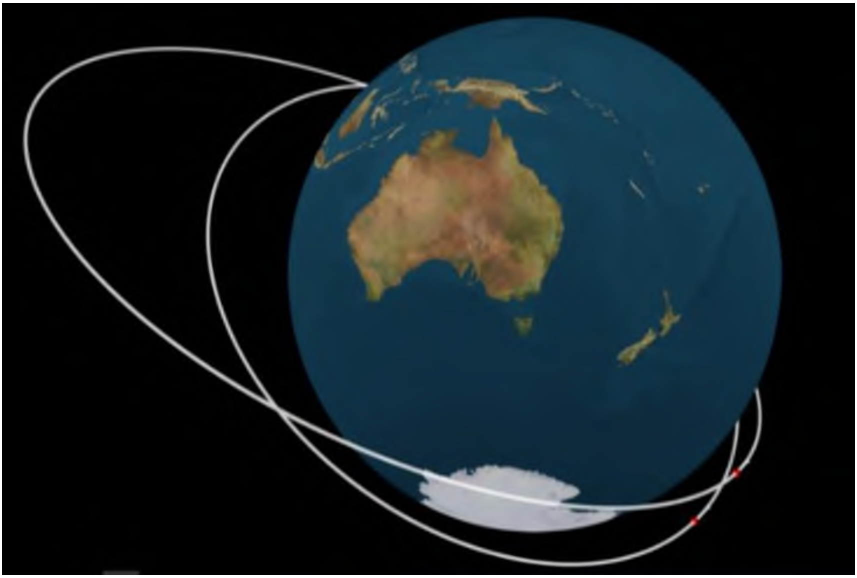

Fig. 1. Orbits of satellites considered for studying the dynamics of Earth's radiation belts.

A dedicated section of the chapter explores the potential of combining chemical and electric propulsion systems to enhance satellite injection efficiency. It examines global examples of hybrid propulsion applications and substantiates the advantages of combined approaches in terms of increased payload capacity and reduced transfer time. The text highlights how hybrid propulsion enables flexible mission profile adaptation to existing constraints, including rideshare launch limitations, restricted power resources of small satellites, and mission duration requirements. The analysis demonstrates that hybrid propulsion combines the benefits of chemical engines (high initial thrust) and electric propulsion (fuel efficiency in final transfer phases), while also improving system reliability through load distribution between the two propulsion types.

Based on the review of existing research, the chapter concludes with the promising potential of hybrid propulsion for Earth radiation monitoring tasks and formulates the research problem addressed in the dissertation, which includes: developing optimal injection schemes, performing numerical trajectory simulations, and justifying engineering parameters for a hybrid propulsion transfer stage.

In the second chapter, mathematical models of spacecraft motion, methods for trajectory construction in impulsive formulation, and optimization of low-thrust flight trajectories are considered, as well as a methodology for launching a spacecraft using combined propulsion. The problem of minimizing the inter-orbital flight time with low thrust is solved using the maximum principle of L.S. Pontryagin and is reduced to solving a boundary value problem for an extended system of ordinary differential equations. The boundary value problem is transformed into a Cauchy problem using a modified Newton method, and its solution is obtained by numerically integrating the system of ordinary differential equations. To improve convergence, the system of differential equations is replaced by an averaged one over an orbit cycle. To select an initial guess for the unknown variables in the time minimization problem, the energy cost minimization problem is solved first.

For the high-thrust engine operation phase, two-impulse apsidal maneuvers are considered without changing the orbital plane and orientation. The transition to a coplanar orbit using CRE is justified by the fact that such a flight results in the smallest chemical fuel consumption. Changing the orbital plane and orientation is assumed to be carried out with low-thrust engines. Thus, the problem of joint optimization of high and low-thrust phases is reduced to determining the parameters of an intermediate orbit, from which the continuation of the flight will proceed using low thrust.

To analyze undisturbed multi-revolution trajectories of spacecraft with electric rocket engines, equinoctial elements are used [11]:

$$a,\ e_x = e\cos(\omega + \Omega),\ e_y = e\sin(\omega + \Omega),\ h_x = \tan\left(\frac{i}{2}\right)\cos(\Omega),$$
$$h_y = \tan\left(\frac{i}{2}\right)\sin(\Omega),\ L = \omega + \Omega + \upsilon,$$

where a is the semi-major axis, e – eccentricity, ω – argument of periapsis, i – inclination, Ω – longitude of the ascending node, υ – true anomaly, and L is the true longitude (Fig. 2).

The change in mass is determined as follows:

$$\frac{dm}{dt} = -\frac{|u|}{g_e I_{sp}}$$

where |u| is the magnitude of the thrust vector, g_e is the gravitational constant, I_{sp} is the specific impulse.

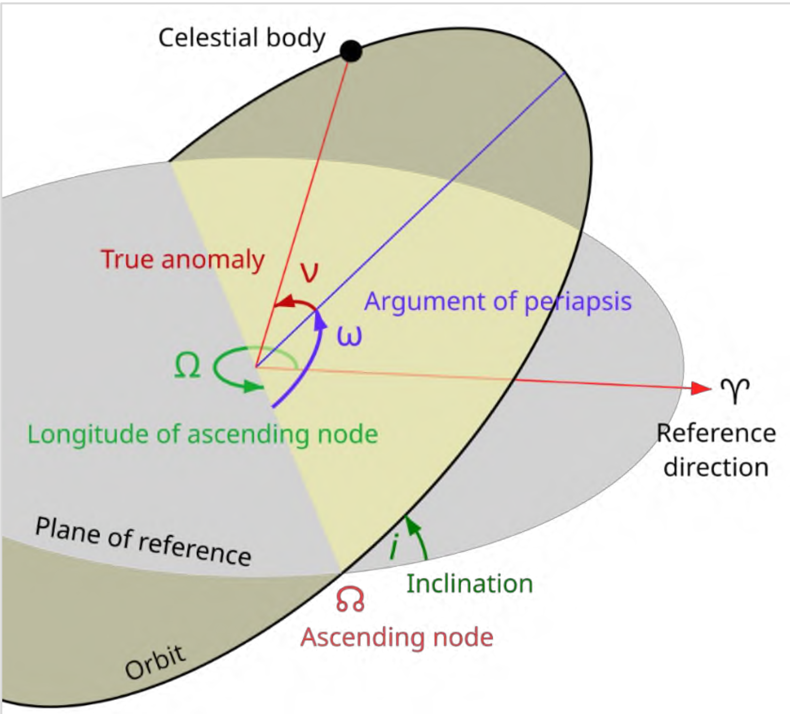

Fig. 2. Orbital elements.

[11] Fourcade, J. An averaging optimal control tool for low-thrust minimum-time transfers. / Fourcade, J., Geffroy, S., Epenoy, R. – Paris: Centre National d'Etudes Spatiales, – 1991. – p. 17.

The change in the orbital position of the spacecraft is described by Gauss's equations:

$$\begin{cases} \dfrac{dx}{dt} = f(x, L)\dfrac{u}{m} \\ \dfrac{dL}{dt} = g_0(x, L) + g_1(x, L)\dfrac{u}{m} \end{cases}$$

where $x = [a, e_x, e_y, h_x, h_y]$, $g_0(x, L)$, $g_1(x, L)$ and $f(x, L)$ are described by the following equations (μ – gravitational parameter):

$$g_0(x, L) = \sqrt{\frac{\mu}{a^3}}\frac{D^2}{A^3}$$

$$g_1(x, L) = \begin{bmatrix} 0 & 0 & \sqrt{\dfrac{a}{\mu}}\dfrac{A}{D}(h_x sinL - h_y cosL) \end{bmatrix}$$

$$f(x, L) = \sqrt{\frac{a}{\mu}}\frac{A}{D}\begin{bmatrix} \dfrac{2aBD}{A^2} & 0 & 0 \\ \dfrac{2(e_x + cosL)D}{B} & \dfrac{-C}{B} & -e_y(h_x sinL - h_y cosL) \\ \dfrac{2(e_y + sinL)D}{B} & \dfrac{E}{B} & e_x(h_x sinL - h_y cosL) \\ 0 & 0 & \dfrac{1}{2}(1 + {h_x}^2 + {h_y}^2)cosL \\ 0 & 0 & \dfrac{1}{2}(1 + {h_x}^2 + {h_y}^2)sinL \end{bmatrix}$$

A, B, C, D, E are described as follows:

$$A = \sqrt{1 - e_x^2 - e_y^2}$$

$$B = \sqrt{1 + 2e_x cosL + 2e_y sinL + e_x^2 + e_y^2}$$

$$C = 2e_x e_y cosL - sinL(e_x^2 - e_y^2) + 2e_y + sinL$$

$$D = 1 + e_x cosL + e_y sinL$$

$$E = 2e_x e_y sinL + cosL(e_x^2 - e_y^2) + 2e_x + cosL$$

The optimal control problem (flight for minimum time with a free final true anomaly) is formulated as follows:

$$\begin{cases} \min t_1 \\ \dfrac{dx}{dt} = f(x, L)\dfrac{u}{mg} \\ \dfrac{dL}{dt} = g_0(x, L) + g_1(x, L)\dfrac{u}{m} \\ \dfrac{dm}{dt} = -\dfrac{|u|}{g_e I_{sp}} \\ x(t_0) = x_0 \quad x(t_1) = x_1 \\ L(t_0) = 0 \quad L(t_1)\ free. \\ m(t_0) = m_0 \quad m(t_1)\ free. \\ t_1\ free. \end{cases}$$

The movement of the spacecraft on the passive sections of the impulsive trajectory occurs along a Keplerian arc [12]:

$$\begin{cases} \dot{x}(t) = v_x(t) \\ \dot{y}(t) = v_y(t) \\ \dot{z}(t) = v_z(t) \\ \dot{v_x}(t) = -\mu\dfrac{x(t)}{r^3(t)}\{1 + \dfrac{3}{2}J_2\left(\dfrac{r_{eq}}{r}\right)^2(1 - 5\dfrac{z^2}{r^2})\} \\ \dot{v_y}(t) = -\mu\dfrac{y(t)}{r^3(t)}\{1 + \dfrac{3}{2}J_2\left(\dfrac{r_{eq}}{r}\right)^2(1 - 5\dfrac{z^2}{r^2})\} \\ \dot{v_z}(t) = -\mu\dfrac{z(t)}{r^3(t)}\{1 + \dfrac{3}{2}J_2\left(\dfrac{r_{eq}}{r}\right)^2(3 - 5\dfrac{z^2}{r^2})\} \end{cases}$$

with initial conditions

$x_0 = x(t_0), y_0 = y(t_0), z_0 = z(t_0), \dot{x}_0 = \dot{x}(t_0), \dot{y}_0 = \dot{y}(t_0),\ \dot{z}_0 = \dot{z}(t_0)$

where $x(t), y(t), z(t)$ – coordinates of the center of mass of SC at time t, $r = \sqrt{x^2(t) + y^2(t) + z^2(t)}$ – distance from the SC to the center of Earth at time t, v_x, v_y, v_z – components of velocity of SC at time t, μ – gravitational parameter, r_{eq} – equatorial radius of the Earth, J_2 – second zonal harmonics.

The inertial Cartesian coordinate system used is the International Celestial Reference System (ICRS), defined by the 2003 convention of the International Earth Rotation and Reference Systems Service (IERS).

For simplification of the optimization dynamics of satellite launches, the spacecraft (including the booster with satellites) was treated as a point mass, with the spacecraft's design parameters not considered.

In the calculations, the second zonal harmonic J_2, which has the greatest influence on orbital motion in near-Earth space, is taken into account. Gravitational perturbations from third bodies, atmospheric drag, and solar radiation pressure were not considered in the modeling. A full accounting of these perturbations would require significant computational resources, whereas the objective of this work is a comparative analysis of satellite deployment schemes with different types of thrust for preliminary mission design.
The limitation of the model to considering only the J_2 harmonic is justified and provides sufficient accuracy with a reasonable computational load.

In the third chapter of the dissertation, a preliminary analysis of the parameters of target orbits is conducted, and calculations of the characteristics of satellite launch trajectories for radiation monitoring of the Earth are presented, using chemical rocket engines, electric propulsion systems, and a combination of electric and chemical rocket engines. The trajectories with chemical rocket engines were considered in impulse formulation, with fuel consumption and required velocity increments calculated. The trajectories with electric propulsion systems were optimized using Pontryagin's maximum principle with the objective of minimizing flight duration.

Combined launch schemes for the spacecraft were constructed using the decomposition method: launching satellites to a certain intermediate orbit using chemical rocket engines and then boosting to target orbits using low-thrust engines.

---

[12] Izzo, D. Problem description for the 9th Global Trajectory Optimisation Competition. / Izzo D. – Noordwijk: European Space Agency, – 2017, – p. 9.

The study examines five satellite injection tasks to specified orbits, differing in target orbit configurations, payload mass, and rideshare launch conditions. The selection of these tasks is driven by the need to provide representative coverage of the most characteristic scenarios encountered by modern missions focused on Earth radiation monitoring:

1. A 100 kg satellite is launched to an orbit with parameters [Ra = 8000 km, Rp = 600 km, i = 64.3º, ω = 300º, Ω – free] alongside the launch of a large 1000 kg satellite to a "Molniya" orbit with a semi-major axis of 26,555 km — apogee height of 39,754 km and perigee height of 600 km.
2. A 100 kg satellite is launched to a circular orbit with parameters [Ra = 800 km, Rb = 800 km, i = 64.3º, ω = 300º, Ω – free] alongside the launch of a large 1000 kg satellite to an orbit with an apogee height of 8000 km and a perigee height of 600 km.
3. A 1000 kg satellite is launched to a circular orbit with parameters [Ra = 800 km, Rp = 800 km, i = 80º, ω = 300º, Ω – free] alongside the launch of a 100 kg satellite to an orbit with an apogee height of 2000 km.
4. A 100 kg satellite is launched to an orbit with parameters [Ra = 2000 km, Rp = 800 km, i = 80º, ω = 300º, Ω – free] alongside the launch of a 1000 kg satellite to an orbit with an apogee height of 8000 km and a perigee height of 600 km.
5. A 1000 kg satellite is launched to an orbit with parameters [Ra = 2000 km, Rp = 800 km, i = 64.3º, ω = 300º, Ω – free] alongside the launch of a 100 kg satellite to an orbit with an apogee height of 39,754 km and a perigee height of 600 km.

A comparative analysis of the results of launching satellites using chemical and electric rocket engines individually, as well as their combination for launching a constellation of satellites into the considered near-Earth orbits for radiation monitoring of Earth, is presented. Thus, the advantages of using a combination of chemical rocket engines and electric propulsion systems for launching satellites into orbits for radiation monitoring of Earth are demonstrated.

In the first task, the launch of a 100 kg satellite to an orbit with an apogee height of 8000 km and a perigee height of 600 km was considered, along with the launch of a 1000 kg satellite to an orbit with a semi-major axis of 26,555 km (Molniya orbit). The fuel consumption using chemical rocket engines (CRE) was 2534.3 kg, with small-thrust engines it was 1090 kg, and with combined propulsion, it was 1872.3 kg. In this case, the satellite launch using electric rocket engines (ERE) required 559 days, while using combined propulsion reduced the launch time to 276.5 days (Fig. 3).

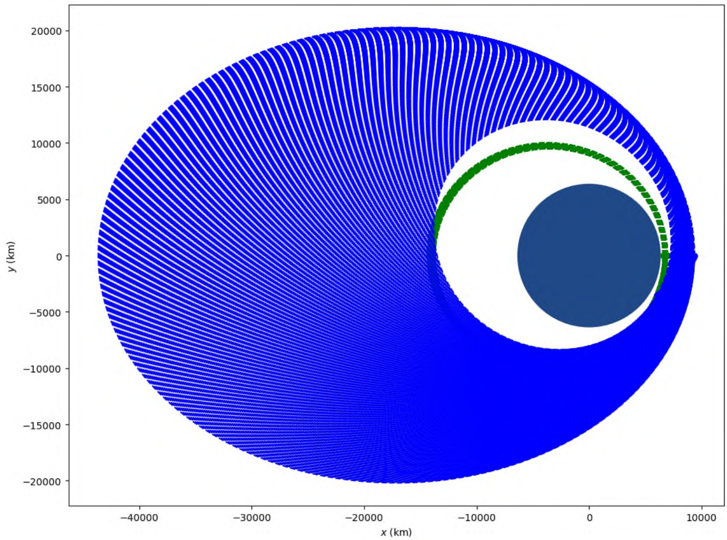

Fig. 3. Illustration for the first task of launch with combined propulsion.

In the second task, the launch of a 100 kg satellite to a circular orbit with an altitude of 800 km was considered, along with the launch of a 1000 kg satellite to an orbit with an apogee of 8000 km and a perigee of 600 km. The fuel consumption in the impulsive formulation was 1257.4 kg, with low thrust it was 439 kg, and with combined propulsion it was 1089.6 kg. Using EPS for the launch of the satellites took 225 days, and using combined propulsion took 118 days.

In the third task, the launch of a 1000 kg satellite to a circular orbit with an altitude of 800 km was considered, along with the launch of a 100 kg satellite to an orbit with an apogee of 2000 km. The fuel consumption in the impulsive formulation was 404.8 kg, with low thrust it was 115 kg, and with combined propulsion it was 308 kg. Using EPS for the launch of the satellites took 59 days, and using combined propulsion took 29.3 days.

In the fourth task, the launch of a 100 kg satellite to an orbit with an apogee of 2000 km and a perigee of 800 km was considered, along with the launch of a 1000 kg satellite to an orbit with an apogee of 8000 km and a perigee of 600 km. The fuel consumption in the impulsive formulation was 1264 kg, with low thrust it was 422 kg, and with combined propulsion it was 932.5 kg. Using EPS for the launch of the satellites took 216 days, and using combined propulsion took 108.8 days.

In the fifth task, the launch of a 1000 kg satellite to an orbit with an apogee of 2000 km and a perigee of 800 km was considered, along with the launch of a 100 kg satellite to an orbit with an apogee of 39754 km and a perigee of 600 km. The fuel consumption in the impulsive formulation was 1654.3 kg, with low thrust it was 581 kg, and with combined propulsion it was 1303.2 kg. Using EPS for the launch of the satellites took 298 days, and using combined propulsion took 149.4 days.

The table (Table 1) presents the results for five satellite launch tasks: Task 1, Task 2, Task 3, Task 4, and Task 5. In each task, the launch of satellites was performed using three different propulsion technologies: impulsive formulation, low thrust, and combined propulsion. For each case, the total velocity increment, fuel consumption, and the duration of the satellite's flight were calculated.

Table 1. Main Results of Satellite Launching Tasks to Designated Orbits (ΔV – Velocity Increment (m/s), M – Fuel Consumption (kg), T – Mission Duration (in days)).

		Impulsive	Low thrust	Combined thrust
Task 1	ΔV	2658.4	6255	4855
	M	2534.3	1090	1872.3
	T	0.62	559	276.5

Task 2	ΔV	1634.2	2972	2613
	M	1257.4	439	1089.6
	T	0.26	225	118
Task 3	ΔV	799.1	1198	1153
	M	404.8	115	308
	T	0.14	59	29.3
Task 4	ΔV	1632	2858	2461.8
	M	1264	422	932.5
	T	0.27	216	108.8
Task 5	ΔV	2748.2	5700	4947.4
	M	1654.3	581	1303.2
	T	0.82	298	149.4

A comparative analysis of the values for velocity increment, fuel mass, and flight duration using the considered satellite launch technologies was conducted:

1. Velocity increment is one of the most important parameters, as it directly influences the energy required for launching a satellite into orbit. The results show that impulsive formulation requires a smaller velocity increment to launch a satellite into orbit compared to low thrust and combined propulsion. In Task 1, the impulsive formulation requires a velocity increment of 2658.4 m/s, while low thrust and combined propulsion require 6255 m/s and 4855 m/s, respectively. A similar trend is observed in the following tasks: in Task 2 – 1634.2 m/s, 2972 m/s, 2613 m/s, respectively; in Task 3 – 799.1 m/s, 1198 m/s, 1153 m/s, respectively; in Task 4 – 1632 m/s, 2858 m/s, 2461.8 m/s, respectively; and in Task 5 – 2748.2 m/s, 5700 m/s, 4947.4 m/s, respectively, using CPE, EPS, and their combination. Thus, if the velocity increment is a critical factor, the impulsive

formulation appears to be a preferred technology for launching satellites into orbit.

2. Fuel mass is related to the velocity increment through the Tsiolkovsky equation. The results show that impulsive formulation requires a larger mass of fuel to achieve the desired velocity increment compared to low thrust and combined propulsion. In Task 1, the impulsive formulation requires 2534.3 kg of fuel, while low thrust and combined propulsion require 1090 kg and 1872.3 kg, respectively. The same trend is observed for the other tasks: in Task 2 – 1257.4 kg, 439 kg, 1089.6 kg; in Task 3 – 404.8 kg, 115 kg, 308 kg; in Task 4 – 1264 kg, 422 kg, 932.5 kg; and in Task 5 – 1654.3 kg, 581 kg, 1303.2 kg, respectively with CPE, EPS, and their combination. Thus, if fuel consumption and, consequently, the initial mass of the spacecraft is a critical factor, low thrust and combined propulsion may be more preferred technologies for launching satellites into orbit.
3. Flight duration determines how much time is required to reach the target orbit. The results show that impulsive formulation requires less flight time compared to low thrust and combined propulsion. In Task 1, the impulsive formulation requires 0.62 days, while low thrust and combined propulsion require 559 days and 276.5 days, respectively. A similar pattern is observed in the other tasks: in Task 2 – 0.26 days, 225 days, 118 days; in Task 3 – 0.14 days, 59 days, 29.3 days; in Task 4 – 0.27 days, 216 days, 108.8 days; and in Task 5 – 0.82 days, 298 days, 149.4 days, respectively with CPE, EPS, and their combination. Thus, if time is a critical factor, impulsive formulation may be a preferred technology for launching satellites into orbit.
4. One of the most interesting results is the effect of combining technologies. As shown in the table, combined propulsion has advantages in launching satellites into orbit. In the considered tasks, combined propulsion requires a smaller velocity increment than EPS and consumes less fuel than CPE.

Based on the results, it can be concluded that each propulsion technology has its advantages and disadvantages. Impulsive formulation allows for a large velocity increment, which can be important for certain missions. However, it is also characterized by the highest fuel consumption. Low thrust allows for reduced fuel consumption but requires more time to launch the satellite into orbit and has a limited payload capacity. Combined propulsion can combine the advantages of different propulsion technologies – it launches satellites in less time than with EPS, but with lower fuel consumption than with CPE.

Thus, the efficiency of using a particular propulsion technology depends on the specific goals of the mission. If the critical factor is velocity increment, then impulsive thrust may be more preferable. If the critical factor is mass, then low thrust and combined thrust technologies appear more favorable. If the critical factor is flight time, then impulsive thrust is the most optimal.

Overall, the results of the study highlight the importance of choosing the optimal propulsion technology for a given mission and the possibility of combining different technologies to achieve the mission's objectives. They also demonstrate the need to consider various factors, such as velocity increment, mass, time, and costs, when selecting the most suitable propulsion technology.

RESULTS

During the research conducted within the framework of the dissertation, the stated scientific objectives were achieved, and the following main results were obtained:

1. Based on the conducted analysis, the feasibility and technical viability of using a combined propulsion system as part of a booster stage for deploying satellites into orbits intended for Earth's radiation monitoring were identified and confirmed.
2. A combined scheme for launching satellites into orbits for radiation monitoring of the Earth was developed. The scheme includes the formation of an intermediate elliptical orbit using two-impulse apsidal maneuvers (without changing the orbital plane and orientation) and subsequent transfer to the target orbit using electric propulsion. The model is theoretically

substantiated and implemented based on the equations of motion using Pontryagin's maximum principle.

3. Numerical trajectory modeling was performed using the developed software, taking into account the effect of the second zonal harmonic J2. The simulation confirmed the applicability of the approach to multi-revolution trajectories and ensured the required accuracy at acceptable computational costs. A comparative analysis was conducted based on fuel consumption and transfer duration criteria for various propulsion options.
4. Quantitative estimates of time and fuel resource expenditures were obtained for different schemes. The calculation results showed that the combined scheme allows for a twofold reduction in transfer duration compared to electric propulsion and up to a 73% reduction in fuel consumption compared to chemical propulsion. These figures were obtained under conditions close to the parameters of current small satellite projects.
5. Based on the analysis of propulsion system characteristics, a conclusion was made regarding the feasibility of using combined propulsion. The justification is based on a comparison of specific impulse, payload mass, and time constraints for Earth observation and radiation monitoring missions. It was established that the combined approach offers a compromise between fuel efficiency and transfer time.

Thus, the dissertation represents a comprehensive study encompassing both theoretical and applied aspects of using a combined propulsion system as part of a booster stage for deploying satellites into orbits intended for Earth's radiation monitoring.

Tasks for the future research

The results of the dissertation lay the foundation for expanding research in the field of satellite deployment into orbits using combined propulsion, aimed at addressing Earth's radiation monitoring tasks. The analysis carried out confirms the potential of this approach in optimizing both time and resource expenditures during the launch of artificial Earth satellites.

In the context of future research, the following directions are considered advisable:

1. This study examined five satellite deployment tasks for radiation monitoring of the Earth. In the future, it is advisable to extend the research to a broader range of orbital parameters and configurations in order to enhance the spatial and temporal coverage of the radiation belts and to optimize the placement of the satellite constellation. This approach will improve monitoring efficiency and provide a more representative data sample across various zones of near-Earth space.
2. In the current trajectory modeling, perturbations caused by Earth's oblateness—specifically the second zonal harmonic J_2 – are taken into account. For greater accuracy and closer alignment of the model with practical conditions, it is necessary to also consider the gravitational influence of the Sun, the Moon, and nearby planets.
3. This study focused on chemical and electric propulsion systems as the most commonly used types of propulsion units. However, the combined satellite deployment scheme allows for the possible use of other engines, whose capabilities and limitations have yet to be investigated, including their comparative analysis. This will make it possible to identify optimal engine combinations for different tasks and select the most suitable propulsion system for the specific mission.
4. Another promising direction is the integration of artificial intelligence and machine learning elements into the trajectory optimization process. Neural network-based algorithms can be used to predict optimal thrust parameters, which would significantly reduce computation time and increase the model's adaptability to various mission conditions.

These research directions will significantly improve the accuracy and efficiency of satellite deployment methods and contribute to the further development of optimization strategies tailored to diverse space missions and modern technological challenges.

Publications on the dissertation topic

1. Iskanderov, M.G., Mammadzada, T.G. Satellite launch for Earth radiation monitoring using combined propulsion. // Moscow: Engineering Journal: Science and Innovations, 2023. Vol. 135, No. 3, pp. 1-14.
2. Mammadzada, T.G. Comparative analysis of satellite launch for radiation monitoring using high-thrust and low-thrust engines. // Astana: Bulletin of the National Engineering Academy of the Republic of Kazakhstan, 2023. Vol. 87, No. 1, pp. 143-148.
3. Mammadzada, T.G. Satellite launch for Earth radiation monitoring using low-thrust propulsion. // Baku: Bulletin of the Azerbaijan Engineering Academy, 2023. Vol. 15, No. 1, pp. 16-25.
4. Samedov, A.C., Mammadzada, T.G., Podzolko, M. Launch of small satellites to orbits for studying Earth's radiation belts. // Baku: Scientific Notes of the National Aviation Academy of Azerbaijan, 2020. No. 1-2, pp. 1-10.
5. Osedlo, V. Russian-Azerbaijan small satellite project for radiation monitoring and upper atmosphere. / Osedlo, V., Panasyuk, M., Mammadzada, T., et al. // Advances in Astronautical Sciences, San Diego: 2020. Vol. 173, No. AAS 20-250, pp. 519-527.
6. Samedov, A.S. Russian-Azerbaijani small satellite project for scientific and technological experiments. / Samedov, A.S., Panasyuk, M.I., Mammadzada, T.G., et al. // Bulletin of Samara University. Aerospace Engineering, Technologies, and Machine Engineering, Samara: 2019. Vol. 18, No. 1, pp. 128-139.
7. Svertilov, S. Small Satellite Constellation for TGF and TLE Observations. / Svertilov S., Panasyuk M., Mammadzada T., et al. // 11th European CubeSat Symposium. Luxembourg, 2019, p. 74.
8. Mammadzada, T.G. About one problem of satellite launch into low-Earth orbit. // IV International Scientific Conference, Baku, 2019, pp. 18-20.

9. Mammadzada, T.G. GTOC 9: Problem and Solution. // III International Conference, Baku, 2018, pp. 17-19.
10. Badirkhanov, D., Mammadzada, T. Interference mitigation methodology improvement for non-geostationary satellites. // Baku: Scientific Reports (National Aviation Academy), 2018. No. 3, pp. 139-145.

Personal contribution of the applicant to scientific works

[1, 4, 5, 6] – mathematical models were formulated, deployment schemes were developed, software for computations was developed, results were analyzed and summarized.

[7] – deployment schemes were developed, calculations were performed, and a presentation was delivered at a scientific conference.

[10] – a mathematical model was formulated, software for calculations was developed, and results were summarized.

[2, 3] – the works with the specified numbers were written independently by the applicant.

[8, 9] – the works with the specified numbers were written independently by the applicant and presented at a conference.

The defense will be held on 9 September 2025 at 11:00 at the meeting of the Dissertation council ED 2.01 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at National Aviation Academy.

Address: AZ1045, Baku, Mardakan Avenue 30, Meeting Hall of the 3rd Academic Building, National Aviation Academy.

Dissertation is accessible at the National Aviation Academy Library

Electronic versions of dissertation and its abstract are available on the official website of the NAA (naa.edu.az).

Abstract was sent to the required addresses on 20 June 2025.

Signed for print: 03.06.2025

Paper format: A5

Volume: 37726

Number of hard copies: 20