

REPUBLIC OF AZERBAIJAN

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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

**FUZZY IDENTIFICATION OF KEY FACTORS OF
CONSUMER DECISIONS IN DIGITAL MARKETING**

Specialty: 3338.01 – Systems analysis, control systems and
information processing (control and decision making)

Field of science: Technical sciences

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Baku - 2025

The dissertation work was performed at the scientific-research laboratory "Intelligent Control and Decision-Making Systems in Industry and Economy" of Azerbaijan State Oil and Industry University.

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GENERAL CHARACTERISTICS OF THE RESEARCH

Relevance of the topic and degree of elaboration. It is still a relevant issue today, as the main factors of consumer decisions in digital marketing are characterized by uncertainty. The study of this problem depends on the main factors affecting the digital marketing process, the main criteria of their characteristics and the generation of alternatives. With the advancement of digital technology, marketing strategies and processes have experienced considerable transformation. Technological innovations enable companies to better understand consumer needs, thereby enhancing customer relationships and gaining a competitive edge.

Marketing plans such as products manufactured, prices offered to customers, online stores, customer contact and communication tools, personalized promotions, and target market selection have been radically transformed by digital marketing technologies. Consequently, the demand for digital marketing technologies has also increased significantly. This study focuses on the selection of near-optimal technology under uncertainty among digital marketing technologies such as artificial intelligence (AI), Internet of Things (IoT) and virtual/augmented reality (VR/AR) and is based on relevant research in this area to solve the problem. The incompleteness and perception-based nature of the information used requires taking into account the degree of inaccuracy and confidence in its description. To address this, the study employs the selection of the main criteria, determination of the weight vector of the criteria and the use of fuzzy and Z-theory to solve the problem of selecting the best digital marketing technology. The complexity and high investment required to select a digital marketing technology under conditions of uncertainty, as well as the inevitability of the problem of fuzzy identification of the key factors of consumer decisions, prove the relevance of the issue of selecting a digital marketing technology that is near-optimal.

Object and subject of the study. The object of this research is the decision-making processes within digital marketing. The subject

is the development of models and methods for choosing the most suitable digital marketing technology under uncertainty.

Goals and objectives of the study. The purpose of the study is to develop a fuzzy logic-based identification for key consumer decision factors in digital marketing under uncertainty and a decision-making model for selecting the optimal digital marketing technology. To accomplish this goal, the following specific tasks have been defined and solved accordingly.

- 1) Study of types of uncertainty in digital marketing and formation of decision preferences;
- 2) Development of a model for solving the problem of selecting digital marketing technologies;
- 3) Determination of consistent decision preferences in the evaluation and selection of the most effective and efficient digital marketing technologies for companies with Z-numbers and solution of the decision-making problem;
- 4) Development of a sensitivity analysis to determine the stability of the decision-making model.

Research methods. To solve the problem, fuzzy logic theory, Z-numbers theory, multi-attribute decision-making methods were used. The stability of the model was assessed based on the Z-numbers approach by applying the sensitivity analysis. The “Z-Lab” application software package was used for computer simulation.

The main provisions for defense. In the dissertation, the following clauses are submitted for defense:

- 1) Proposal of a decision-making model based on Z-numbers for solving the problem of selecting the optimal digital marketing technology;
- 2) Proposal of approaches for determining and analyzing the main criteria in the formation of consumer decisions using Z-numbers to solve the problem of choosing the optimal digital marketing technology;
- 3) Proposal of an approach for expressing and evaluating alternative digital marketing technologies using Z-numbers by pairwise comparisons;

4) Proposal of an approach for determining the robustness of the proposed Z-number-based decision-making model by applying the sensitivity analysis.

Scientific novelty of the research.

1) Formulation of the problem of fuzzy identification of the main criteria of consumer or company decisions in the selection of digital marketing technology under conditions of uncertainty and the issue of optimal technology selection;

2) Synthesis of consistent decision preference (in decision matrix) based on the consistency factor for solving the problem of digital marketing technology selection;

3) Ranking of alternatives in the Z-information environment based on decision preference and finding a solution close to the optimal;

4) Study of the stability of the proposed Z-number-based decision-making model for digital marketing technology selection.

Theoretical and practical significance of the research. The theoretical significance of the study is the identification of the main factors of consumer decisions under uncertain information conditions in digital marketing technology selection based on Z-numbers and the construction and development of models related to the issue of optimal technology selection, the selection of decisions close to optimal, and the verification of the accuracy and sensitivity of the results obtained. The practical significance is that it can be used in decision-making of consumers or companies in digital marketing technology selection, in solving problems of optimal digital marketing technology selection, in determining the most effective and efficient technology for consumer companies or individual consumers, and in decision-making processes under uncertainty in other areas.

Approbation of dissertation. The results obtained were applied to the solution of the problem of digital marketing technology selection.

The main clauses and results of the dissertation were discussed at the seminar of the scientific research laboratory "Intelligent Control and Decision-Making Systems in Industry and Economy" and presented at the following international conferences:

1. 9th International Conference on Theory and Application of Soft Computing, Computing with Words and Perception, ICSCCW-2017, Budapest, Hungary.
2. 13th International Conference on Theory and Application of Fuzzy Systems and Soft Computing—ICAFS-2018, Warsaw, Poland.
3. 14th International Conference on Theory and Application of Fuzzy Systems and Soft Computing – ICAFS-2020, Budva, Montenegro.
4. 11th World Conference “Intelligent System for Industrial Automation” (WCIS-2020). WCIS 2020, Tashkent, Uzbekistan.
5. Innovative Tendencies in Ensuring Regional Development: Materials of the Republican Scientific Conference on Realities and Modern Challenges, 2020, Mingachevir, Azerbaijan.
6. Dedicated to the 98th Anniversary of the National Leader of Azerbaijan, Heydar Aliyev, V International Scientific Conference of Young Researchers, 2021, Baku, Azerbaijan.
7. 11th International Conference on Theory and Application of Soft Computing, Computing with Words and Perceptions and Artificial Intelligence-ICSCCW-2021, Antalya, Turkey.
8. 15th International Conference on Applications of Fuzzy Systems, Soft Computing and Artificial Intelligence Tools – ICAFS-2022, Budva, Montenegro.
9. 16th International Conference on Applications of Fuzzy Systems, Soft Computing and Artificial Intelligence Tools – ICAFS-2023, Antalya, Turkey.
10. International Conference on Smart Environment and Green Technologies-ICSEGT2024, Baku, Azerbaijan.
11. 12th International Conference on Theory and Application of Soft Computing, Computing with Words, Perceptions and Artificial Intelligence - ICSCCW-2024, Budva, Montenegro.

The name of the organization where the dissertation work was performed. Azerbaijan State Oil and Industry University, “Intelligent Control and Decision-Making Systems in Industry and Economy” scientific research laboratory.

Published scientific works. 17 scientific works have been published on the topic of the dissertation, including 7 local publications, 5 of which are without co-authorship, and 10 international publications, 2 of which are without co-authorship. 8 of these publications were included in the SCOPUS and 2 in the Web of Science databases.

The volume of the dissertation's structural sections separately and the general volume. The dissertation consists of an introduction (7869 characters), 5 chapters (chapter I – 24356 characters, chapter II – 58550 characters, chapter III – 24723 characters, chapter IV – 36702, chapter V – 58165 characters), results (3009 characters) and a list of 136 references. In total, the volume of the dissertation is 213374 characters, and it consists of 24 tables and 9 figures.

MAIN CONTENT OF THE WORK

The introduction presents the relevance of the topic in the field of research, the objectives set, research methods, theoretical and practical significance of the study.

The first chapter presents an analysis of existing scientific works and the state of the problem. The application of digital marketing technologies, in addition to requiring less financial resources and optimizing marketing costs compared to existing traditional technologies, offers important advantages such as real-time monitoring of performance indicators, establishing more accurate targeting strategies, and ensuring interactive communication with customers.

Factors such as changes in consumer behavior and the actualization of two-way communication between brands and customers encourage marketers to apply innovative digital marketing technologies¹. Organizations that take advantage of the opportunities offered by the technology strengthen their relationships with

¹ Tiago, M. T. P. M. B., Verissimo, J. M. C. Digital marketing and social media: Why bother?// Business horizons, – 2014, 57, – p.703-708.

customers through digital platforms, and are able to manage order fulfillment and sales more effectively by increasing customer value². Among existing digital marketing tools, advanced technologies like artificial intelligence (AI), virtual and augmented reality (VR/AR), and the Internet of Things (IoT) play a particularly important role³. The application of these technologies requires significant financial and technical investment. Therefore, it is strategically important for enterprises to select the most efficient and expedient technology. However, one of the main problems facing enterprises is to correctly evaluate existing technologies and determine the optimal alternative. In addition to the limited scope of existing research in this area, most of them do not sufficiently take into account uncertainty and reliability factors in the decision-making process. In the scientific literature, multi-criteria decision-making methods (MCDM) have been mainly used to evaluate digital marketing technologies⁴. Most existing studies are limited to covering only the theoretical aspects of the selected technologies and do not provide sufficient evidence of their practical application. Thus, there is a need to apply new methodological approaches to determine the optimal digital marketing technology. In this regard, the application of fuzzy logic-based evaluation methods can act as an effective decision-making tool in selection of the optimal digital marketing technology.

The second chapter examines the main factors considered in digital marketing, the main advantages of digital marketing over traditional marketing, and considers the issue of identifying alternatives.

Digital marketing can be characterized as a modern marketing approach based on the use of digital communication channels to

² Mort, G. S., Drennan, J. Mobile digital technology: Emerging issue for marketing// *Journal of Database Marketing & Customer Strategy Management*, – 2002, 10, – p. 9-23

³ Imanova, G. E., Imanova G. Digital marketing technologies selection under Z-environment // *Advances in Intelligent Systems and Computing*, Springer, – 2020, – p. 378-387

⁴ Imanova, G. E., Imanova, G. Some Aspects of Fuzzy Decision Making in Digital Marketing Analysis // *Lecture Notes in Networks and Systems*, Springer, – 2022, – p. 465-473

promote products and services on a global scale⁵. As the intensity of competition increases in the current market conditions, businesses are more actively applying digital marketing technologies to gain a competitive advantage. By using digital marketing tools such as social media platforms and search engines, businesses can reach customers more effectively at a more convenient time⁶. Businesses can significantly increase their profits, sales revenue and market share, by combining digital marketing strategies and methods such as SMM, SEM, PPC, email marketing, content marketing with digital marketing technologies⁷.

Proper identification of the target market, selection of effective marketing channels, technological knowledge and skills, company budget, professional competence of team members, big data, content marketing and effective use of social media marketing are considered as the main factors affecting digital marketing. The following criteria are put forward to solve the problem of optimal digital marketing technology selection.

C_1 : Budget/financial capabilities;

C_2 : Efficiency of technology;

C_3 : Ease of use of technology;

C_4 : Ability to communicate with customers;

C_5 : Targeting ability.

Company budget/financial capabilities (C_1): One of the main factors in the process of selecting digital marketing technologies is the financial capabilities of the enterprise. Although digital approaches

⁵ Lawrence, S., Deshmukh, S., Navajivan, E. A comparative study of digital marketing vs. traditional marketing // IIBM'S Journal of Management Research, – 2018, 3(1-2), – p. 112-121

⁶ Durmaz, Y., Efendioglu, I. H. Travel from traditional marketing to digital marketing// Global journal of management and business research, – 2016, 16(2), – p. 34-40.

⁷ Imanova, G. E., Difference Between Digital Marketing and Traditional Marketing Models // Lecture Notes in Networks and Systems, Springer, – 2023, – p. 522-529

seem more profitable than traditional marketing methods, a certain amount of investment is required for their effective implementation. In this regard, it is necessary for companies to conduct specific budget planning before implementing digital marketing technologies. Also, different digital marketing channels, technologies and related applications require different investments. Technologies such as Artificial Intelligence (AI), Internet of Things (IoT), Virtual Reality (VR) and Augmented Reality (AR) require larger investments due to high-tech infrastructure and special software.

Efficiency of technology (C_2): The efficiency of digital technology depends on the extent to which the positive results it brings to the company outweigh the costs incurred. More precisely, the efficiency of technology is measured by its ability to generate maximum revenue with minimal resources. According to relevant studies, the implementation of digital marketing strategies is accompanied by higher financial results, in addition to increasing the effectiveness of marketing communications⁸. This approach allows for a higher return on investment (ROI). In this sense, companies should compare existing systems and comprehensively evaluate both costs and expected benefits when choosing a technology.

Ease of use of technology (C_3): In addition to the presence of technology itself, the ability of the staff to adopt and use it efficiently is also essential for its successful implementation. A smooth and intuitive interaction between users and the technology directly impacts the effectiveness of its implementation. The integration of technologies such as Artificial Intelligence, Internet of Things, Virtual Reality, Augmented Reality requires a high level of technical knowledge and special training, which necessitates additional preparation for their successful implementation.

Ability to communicate with customers (C_4): One of the main advantages of digital marketing is that it expands the opportunities of

⁸ Kapustina, L., Gaiterova, O., Izakova, N., Lazukov, M. Digital marketing communications: selection criteria // KnE Social Sciences, –2021, – p. 181-190

interaction with customers. Through the use of various online platforms, social media, websites and mobile applications, companies can create a more transparent and interactive environment for consumers. This, in turn, strengthens customer trust and brand loyalty⁹. Artificial intelligence-based chatbots, IoT technologies and VR/AR applications make customer communication channels more interactive and personalized.

Targeting ability (C_5): The basis of effective marketing campaigns is a correctly selected target audience. If advertising and content are directed at a random audience, this can lead to additional financial losses. Digital marketing technologies, such as AI, IoT, VR and AR, provide companies with powerful analytical capabilities in terms of analyzing customer behavior and delivering messages tailored to specific customer segments. Through these technologies, more precise targeting strategies can be implemented, which in turn ensures efficient use of financial resources and increases marketing performance.

The alternative digital marketing technologies considered in accordance with the purpose of the study are listed below.

Artificial Intelligence (A_1): it is a program or computer that can learn and replicate decision-making processes quickly and effectively and produce meaningful results¹⁰. Chatbots are artificial intelligence (AI) systems that communicate with users through natural language user interfaces on internet platforms or social media sites such as Instagram and Facebook. Therefore, chatbots create the opportunity to build better relationships with customers through interactive communication and operational response capabilities that improve

⁹ Imanova, G. E. Identification and ranking of key factors for pattern of consumer buying decisions in digital marketing // 11th World Conference "Intelligent System for Industrial Automation" (WCIS-2020), Tashkent, Uzbekistan, Springer International Publishing, –2021, – p. 237-245

¹⁰ Imanov, E., Aiyeyika, L.I., Imanova, G. E. Development and Assessment of Energy-Efficient Approaches for AI-Based Green Computing // International Conference on Smart Environment and Green Technologies – ICSEGT 2024, Baku, Azerbaijan, Springer Nature Switzerland, – 2024, – p. 179-187.

customer satisfaction¹¹. Artificial intelligence technologies offer numerous advantages in the field of digital marketing¹². Thus, presenting products and services in the market at optimal time and at competitive prices, allowing to design e-mail campaigns according to individual user behavior, personalized product offers and content prepared through analytical approaches based on customer behavior can more effectively influence buyer decisions and sales.

Internet of Things (IoT) (A_2): Includes a set of smart devices, software, and systems that are connected to the Internet and can interact. This technology enables deeper customer interactions by keeping the connection between brands and consumers closer and more dynamic, and ultimately serves to increase brand loyalty. IoT solutions are widely used in the operation of smart TVs, computers, watches, vehicles and living spaces, security systems, medical technologies such as MRI equipment, and other "smart" devices¹³.

Businesses are using this technology to improve the quality of their products and services¹⁴. In addition, personalized advertising patterns based on online user data, more effective customer segmentation, and the formation of advertising strategies for target audiences are examples of the integration of IoT technologies into digital marketing strategies.

Virtual/Augmented Reality (VR/AR) (A_3): A simulation created by special devices and computers that combine and stimulate the user's senses and perception and provide realistic interaction with the environment is called virtual reality (VR). Various devices such as desktop computers, head-mounted displays, large screens, and

¹¹ Brandtzaeg, P. B., Følstad, A.: Why people use chatbots // In International Conference on Internet Science, Springer, Cham, – 2017, – p. 377-392 .

¹² Brandtzaeg, P. B., Følstad, A.: Why people use chatbots // In International Conference on Internet Science, Springer, Cham, – 2017, – p. 377-392 .

¹³ Nguyen, B., Simkin, L. The Internet of Things (IoT) and marketing: The state of play, future trends, and the implications for marketing // Journal of Marketing Management. – 2017, 33, – p. 1-6.

¹⁴ Imanova, G. E. Assessing the technological progress of Internet of Things (IoT) in post-pandemic period // Transactions of Azerbaijan Institutes of Technology, – 2023, №25, – p.27-33.

smartphones can be used for VR experiences. Augmented reality (AR) is a subset of virtual reality (VR), where additional digital information such as sound, 3D objects, and images are integrated by digital devices in real time to enrich the user's view of the physical world. Virtual and augmented reality (VR/AR) technologies are widely used in the retail sector to optimize online product search, eliminate potential obstacles during the purchasing process, enhance user experience, and improve navigation in physical stores, particularly in areas such as the fashion and apparel industry, car test drives, and 3D product presentations¹⁵.

In the third chapter, basic concepts related to the problem of selecting digital marketing technologies are given, decision preferences under uncertainty and existing decision-making methods are reviewed, decision-making methods in a fuzzy environment and sensitivity testing are investigated. Unlike classical logic systems, fuzzy logic is a system that operates with imprecise and uncertain information, aiming to model human reasoning and approximate judgment¹⁶. Fuzzy logic provides more appropriate modeling in conditions with a high degree of uncertainty and imprecision. This approach makes information processing possible with the presence of various intermediate values between 0 and 1.

Linguistic variables are often used in fuzzy logic. These variables are defined by expressions such as “very high”, “high”, “medium” and are characterized by their closeness to natural language. Such variables are transformed into fuzzy sets, which allow the subjective judgments of the decision-maker to be taken into account^{17 18}.

¹⁵ Wedel, M., Bigné, E., Zhang, J. Virtual and augmented reality: Advancing research in consumer marketing // *International Journal of Research in Marketing*, – 2020, 37(3), – p. 443-465.

¹⁶ Zadeh, L. A. Fuzzy sets // *Information and control*, – 1965, 8(3), – p. 338-353.

¹⁷ Eyupoglu, S. Z., Imanova, G. E. Self-confidence Preference Based Decision Making in Personnel Selection // *Advances in Intelligent Systems and Computing*, Springer, – 2019, – p. 288-292

¹⁸ Eyupoglu S. Z., Imanova G.E., Dovlatova Kh. J. Application of self-confidence preference to personnel selection problem // *Transactions of Azerbaijan Institutes of Technology*, – 2019, №6, – p.115-119.

In order for information to be useful and functional, its reliability is one of the main conditions. In this context, the concept of Z-numbers aims to model not only the value of information, but also its reliability level. Such evaluations are made possible through fuzzy numbers, and Z-numbers represent a formal vector-structured approach that simultaneously expresses two aspects – the degree of membership and the reliability on that degree. The Z number has two parts, $Z(A, B)$ ¹⁹.

A is the first component and imposes a fuzzy constraint on the values that an uncertain or random variable X can take. B is the second component and reflects the degree of certainty of A . Both components, A and B , can be modeled as triangular or trapezoidal fuzzy sets and are expressed in natural language (NL). Z-number theory provides a comprehensive uncertainty modeling framework that takes into account both the degree of membership and its degree of reliability together. The main advantage of this approach is that it allows modeling not only type-1 and type-2 uncertainties, but also the degree of reliability on these uncertainties. Z-number theory has wide applications, especially in areas such as economic modeling, risk assessment, decision-making processes, data synthesis and forecasting.

Sensitivity analysis serves to assess the stability and reliability of decisions made on the basis of a fuzzy pairwise comparison matrix. Here, the weights assigned to the criteria determine how the alternatives are ranked.

In the fourth chapter, the issue of selecting digital marketing technologies under uncertainty is posed as a scientific problem and a method for constructing a Z-number model is proposed for its solution.

Initially, five important criteria for the problem of selecting digital marketing technologies are identified and evaluated by pairwise comparisons. Next, we evaluate the digital marketing technologies under consideration to select the alternative that best satisfies each of the five criteria. Using the methodology proposed in the literature, the

¹⁹ Zadeh, L. A. A note on Z-numbers // Information sciences, – 2011, 181(14), – p. 2923-2932.

most suitable consistent pairwise comparison matrix for the constructed matrix is found²⁰. The consistent matrix is obtained based on the following equation.

$$J = \sum_{i=1}^n \sum_{j=1}^n D(Z_{ij}, Z'_{ij}) \rightarrow \min \quad (1)$$

In the pairwise comparison matrix, the variables Z'_{ij} are defined as Z-valued variables. Here, the smallest possible distance between the elements of Z_{ij} representing the original inconsistent matrix and the newly formed Z'_{ij} is found.

The constraints are as follows:

Multiplicative reciprocity constraints²⁰: In the equation $Z(1) = (A, B)$, A and B are fuzzy singletons $A = 1, B = 1$, whereas the i -th criterion is considered as more important than the j -th criterion for the Z-number valued Z'_{ij} , the j -th criterion is considered less important than the i -th criterion.

$$\begin{aligned} Z'_{ij} \cdot Z'_{ji} &= Z(1) \\ (Z'_{ij} \cdot Z'_{ji} \approx Z(1) \quad i = \overline{1, n}, j = \overline{1, n}) \end{aligned} \quad (2)$$

Multiplicative transitivity constraints²⁰: These constraints are based on the principle of consistency. These constraints state that if the i -th criterion is Z'_{ij} times more important than the j -th criterion, and the j -th is Z'_{jk} times more important than the k -th, then the i -th criterion is superior to the k -th (in the ideal case) by $Z'_{ik} = Z'_{ij}Z'_{jk}$.

$$Z'_{ij} \cdot Z'_{jk} = Z'_{ik} \quad \text{for all } i, j, k - \quad (3)$$

Non-negativity constraints²⁰: All preference degrees must be non-negative, i.e.:

²⁰ R.A. Aliev, B. G. Guirimov, O. H. Huseynov, R. R. Aliyev // A consistency-driven approach to construction of Z-number-valued pairwise comparison matrices, – 2021, 18(4).

$$Z'_{ij} \geq Z(0) \quad i, j=1, \dots, \quad (4)$$

According to the constraints given in formula (4), the preference degrees are non-negative, where $Z(0) = (A, B)$ is a function and A, B are fuzzy singletons ($A = 0, B = 1$). The non-linear optimization problem given in formulas (1)–(4) is characterized by probabilistic and fuzzy uncertainties. A method based on the DE optimization method is applied to solve this problem²¹.

In the second phase, the eigenvector and (highest) eigenvalue are calculated to obtain the priority weights of the criteria. The eigenvector $(Z_{x_1} = (A_{x_1}, B_{x_1}), Z_{x_2} = (A_{x_2}, B_{x_2}), Z_{x_3} = (A_{x_3}, B_{x_3}), Z_{x_4} = (A_{x_4}, B_{x_4}), Z_{x_5} = (A_{x_5}, B_{x_5}))^T$ and the eigenvalue Z_λ meets the following equations:

$$(Z'_{ij})(Z_{x_j}) = Z_\lambda(Z_{x_j}) \quad (5)$$

$Z_\lambda = (A_\lambda, B_\lambda)$ and $Z_{x_j} = (A_{x_j}, B_{x_j}), j = 1, \dots, 5$ are the main problems to be solved. The solution has two parts. First, by solving the fuzzy problem related to formula (5), the parts A_λ and $A_{x_j}, j = 1, \dots, 5$ are determined in the form of triangular fuzzy numbers.

After obtaining A_λ and A_{x_j} , the second components B_λ and B_{x_j} are determined in the form of triangular fuzzy numbers by solving in the form of an optimization problem²¹: $B_\lambda = (b_{\lambda l}, b_{\lambda m}, b_{\lambda r})$ and $B_{x_j} = (b_{x_j l}, b_{x_j m}, b_{x_j r})$

$$\sum_{i=1}^5 d \left(\sum_{j=1}^5 Z_{ij} Z_{x_j}, Z_\lambda Z_{x_i} \right) \rightarrow \min \quad (6)$$

$$b_{\lambda m} \geq 0.5, \quad (7)$$

²¹ R. Storn, K. Price, Differential Evolution – A simple and efficient adaptive scheme for global optimization over continuous spaces // J. Global Optim., – 1997, 11 – p. 341-359

$$b_{x_i m} \geq 0.5, i = 1, \dots, 5. \quad (8)$$

In the above mathematical expressions, $i=1, \dots, 5$, d is equal to the distance between the left and right sides of the i th equation. The sum of these distances regarding all equations satisfies the above objective function. The vector of criterion weight is defined by $(Z_{w_1} = (A_{w_1}, B_{w_1}), Z_{w_2} = (A_{w_2}, B_{w_2}), Z_{w_3} = (A_{w_3}, B_{w_3}), Z_{w_4} = (A_{w_4}, B_{w_4}), Z_{w_5} = (A_{w_5}, B_{w_5}))^T$, given the eigenvector $(Z_x) = (Z_{x_1} = (A_{x_1}, B_{x_1}), \dots, Z_{x_5} = (A_{x_5}, B_{x_5}))^T$. The weights are normalized.

The components of A A_{w_j} are determined by the method proposed in the study²². The solution of the linear programming problem $A_{w_j} = (a_{w_j,1}, a_{w_j,2}, a_{w_j,3}), j = 1, \dots, 5$ uses triangular fuzzy numbers of $A_{x_j} = (a_{x_j,1}, a_{x_j,2}, a_{x_j,3})$ to find triangular fuzzy numbers.

In the third stage, the weight of the criterion with the highest Z -value leads to the recognition of the corresponding alternative as the optimal selection.

The distance between each alternative $a_i = (Z_{C_{i1}}, \dots, Z_{C_{i5}})$ and the ideal alternative, $a^* = (Z_1^*, \dots, Z_5^*)$, is calculated:

$$D(a_i, a^*) = \sqrt{\sum_{j=1}^5 D^2(Z_{ij}, Z_j^*)} \quad (9)$$

In the case of selecting digital marketing technologies under uncertainty, a 5×5 Z -number pairwise comparison matrix is first created to solve the problem of selecting the optimal digital marketing technology based on five criteria $(C_1, C_2, C_3, C_4, C_5)$. Based on the analysis, the following criteria are used for the optimal solution of this problem: C_1 -budget/financial capabilities, C_2 -efficiency of technology, C_3 - ease of use of technology, C_4 - ability to communicate with customers, C_5 - targeting ability. The alternatives are as follows:

²² Y.-M. Wang, K.-S. Chin, An eigenvector method for generating normalized interval and fuzzy weights // Appl. Math. Comput., - 2006, 181(2), - p. 1257-1275

A_1 - Artificial Intelligence (AI), A_2 - Internet of Things (IoT), A_3 - Virtual Reality/Augmented Reality (VR/AR).

The pairwise comparison matrix, decision matrix and consistent Z-matrix are given as follows (Tables 1, Table 2 and Table 3):

Pairwise comparison matrix
Table 1

	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 3), (0.6, 0.7, 0.8)	(2, 3, 4), (0.5, 0.6, 0.7)	(2, 3, 4), (0.4, 0.5, 0.6)	(3, 4, 5), (0.4, 0.5, 0.6)
C ₂	(0.33, 0.5, 0.5), (0.6, 0.7, 0.8)	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 3), (0.5, 0.6, 0.7)	(1, 2, 3), (0.4, 0.5, 0.6)	(3, 4, 5), (0.4, 0.5, 0.6)
C ₃	(0.25, 0.33, 0.5), (0.5, 0.6, 0.7)	(0.33, 0.5, 1), (0.5, 0.6, 0.7)	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 3), (0.4, 0.5, 0.6)	(2, 3, 4), (0.4, 0.5, 0.6)
C ₄	(0.25, 0.33, 0.5), (0.4, 0.5, 0.6)	(0.33, 0.5, 1), (0.4, 0.5, 0.6)	(0.33, 0.5, 1), (0.4, 0.5, 0.6)	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 2, 3), (0.4, 0.5, 0.6)
C ₅	(0.2, 0.25, 0.33), (0.4, 0.5, 0.6)	(0.2, 0.25, 0.33), (0.4, 0.5, 0.6)	(0.25, 0.33, 0.5), (0.4, 0.5, 0.6)	(0.33, 0.5, 1), (0.4, 0.5, 0.6)	(1, 1, 1), (0.8, 0.9, 1.0)

Decision matrix
Table 2

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	(2.0, 2.5, 3.0), (0.6, 0.7, 0.7, 0.8)	(3.0, 3.5, 4.0), (0.6, 0.7, 0.7, 0.8)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)
A ₂	(1.0, 1.5, 2.0), (0.6, 0.7, 0.7, 0.8)	(2.0, 2.5, 3.0), (0.6, 0.7, 0.7, 0.8)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)
A ₃	(3.0, 3.5, 4.0), (0.6, 0.7, 0.7, 0.8)	(2.0, 2.5, 3.0), (0.6, 0.7, 0.7, 0.8)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)

The weights of the criteria are given by Z-numbers and are as follows:

$$Z_{w_1} = (A_{w_1}, B_{w_1}) =$$

$$[0.1856 \ 1.0 \ 1.0 \ 1.0] [0.4823 \ 0.4843 \ 0.4843 \ 0.4924]$$

$$Z_{w_2} = (A_{w_2}, B_{w_2}) =$$

$$[0.0928 \ 0.5 \ 0.5 \ 0.5002] [0.4536 \ 0.5261 \ 0.5261 \ 0.5462]$$

$$Z_{w_3} = (A_{w_3}, B_{w_3}) =$$

$$[0.0619 \ 0.3333 \ 0.3333 \ 0.3337] [0.3924 \ 0.4397 \ 0.4397 \ 0.44]$$

$$Z_{w_4} = (A_{w_4}, B_{w_4}) =$$

$$[0.0619 \ 0.3333 \ 0.3333 \ 0.3339] [0.4651 \ 0.4806 \ 0.4806 \ 0.4811]$$

$$Z_{w_5} = (A_{w_5}, B_{w_5}) =$$

$$[0.0465 \ 0.2501 \ 0.2501 \ 0.2505] [0.4871 \ 0.572 \ 0.572 \ 0.5741]$$

Consistency analysis determines the consistency of a matrix by evaluating the consistency of the preference values of the given criteria based on a pairwise comparison in the decision matrix. Answers in the range of 0-0.5 are considered as consistent. The consistent decision matrix, expressed in Z-numbers (consistent Z-matrix) has been determined and is given below. In this case, the inconsistency index is set as 0.2891517523781516.

Consistent Z-matrix
Table 3

	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.9, 2.0, 2.1), (0.95, 0.97, 0.99)	(2.85, 3.0, 3.15), (0.95, 0.97, 0.99)	(2.85, 3.0, 3.15), (0.95, 0.97, 0.99)	(3.8, 4.0, 4.2), (0.95, 0.97, 0.99)
C ₂	(0.476, 0.5, 0.526), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.425, 1.5, 1.575), (0.95, 0.97, 0.99)	(1.425, 1.5, 1.575), (0.95, 0.97, 0.99)	(1.9, 2.0, 2.1), (0.95, 0.97, 0.99)

Table 3 continues

C ₃	(0.317, 0.333, 0.351), (0.95, 0.97, 0.99)	(0.635, 0.667, 0.702), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.266, 1.333, 1.4), (0.95, 0.97, 0.99)
C ₄	(0.317, 0.333, 0.351), (0.95, 0.97, 0.97, 0.99)	(0.635, 0.667, 0.702), (0.95, 0.97, 0.99)	(0.952, 1.0, 1.053), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.266, 1.333, 1.4), (0.95, 0.97, 0.97, 0.99)
C ₅	(0.238, 0.25, 0.263), (0.95, 0.97, 0.99)	(0.476, 0.5, 0.526), (0.95, 0.97, 0.99)	(0.714, 0.75, 0.79), (0.95, 0.97, 0.99)	(0.714, 0.75, 0.79), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)

Normalized values of the criteria weights, alternatives, and their ranking are represented. The normalized vector of criteria is expressed as follows:

[0.4133 0.4138 0.4138 0.414] [0.4823 0.4843 0.4843 0.4924],

[0.2068 0.2069 0.2069 0.2071] [0.4536 0.5261 0.5261 0.5462],

[0.1378 0.1379 0.1379 0.1382] [0.3924 0.4397 0.4397 0.44],

[0.1379 0.1379 0.1379 0.1382] [0.4651 0.4806 0.4806 0.4811],

[0.1035 0.1035 0.1035 0.1037] [0.4871 0.572 0.572 0.5741]

Normalized decision matrix:

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	(0.3802, 0.4874, 0.7016, 0.8088), (0.6, 0.7, 0.7, 0.8)	(0.1615, 0.2686, 0.4829, 0.5901), (0.6, 0.7, 0.7, 0.8)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)
A ₂	(0.41, 0.75, 1.0, 1.0), (0.44, 0.57, 0.57, 0.66)	(0.0248, 0.132, 0.3463, 0.4534), (0.6, 0.7, 0.7, 0.8)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)	(0.0, 0.0, 0.1302, 0.2374), (0.7, 0.8, 0.8, 0.9)	(0.0, 0.0, 0.1302, 0.2374), (0.7, 0.8, 0.8, 0.9)
A ₃	(0.0725, 0.1797, 0.394, 0.5011), (0.6, 0.7, 0.7, 0.8)	(0.0248, 0.132, 0.3463, 0.4534), (0.6, 0.7, 0.7, 0.8)	(0.0, 0.0, 0.0, 0.0), (0.62, 0.76, 0.76, 0.87)	(0.0, 0.0, 0.0, 0.0), (0.62, 0.76, 0.76, 0.87)	(0.0, 0.0, 0.0, 0.0), (0.62, 0.76, 0.76, 0.87)

After the relevant calculations, the results obtained for the alternatives are formulated as follows:

$$A_1 = [0.22 \ 0.32 \ 0.53 \ 0.64] [0.5333 \ 0.6933 \ 0.6933 \ 0.8],$$

$$A_2 = [0.19 \ 0.36 \ 0.56 \ 0.62] [0.1067 \ 0.16 \ 0.16 \ 0.2133],$$

$$A_3 = [0.04 \ 0.1 \ 0.23 \ 0.3] [0.0533 \ 0.1067 \ 0.1067 \ 0.16]$$

Result for A_1 : 1.7149958975312196

Result for A_2 : 2.2249728914353883

Result for A_3 : 2.808191898708996

The ranking of alternatives is given as follows: $A_1 > A_2 > A_3$.

As a result, artificial intelligence (AI) is selected as the optimal technology based on the given criteria. The second best (optimal) digital marketing technology is selected as the Internet of Things, and the third is virtual/augmented reality.

In the fifth chapter, the issue of sensitivity analysis of the given model was considered.

Sensitivity analysis tests the robustness and reliability of decisions based on a fuzzy pairwise comparison matrix. It examines how changes in fuzzy set weights affect the final ranking of alternatives. The results show how variations in pairwise inputs impact criterion weights and their relative importance. Sensitivity analysis ensures consistent outcomes.

Sensitivity analysis tests the robustness and reliability of decisions based on a fuzzy pairwise comparison matrix. It examines how changes in fuzzy set weights affect the final ranking of alternatives. The results show how variations in pairwise inputs impact criterion weights and their relative importance. It should be noted that since criterion weights are often based on subjective considerations, sensitivity analysis provides important information to ensure the reliability of the results.

In this study, sensitivity analysis was performed using the Z-Lab program and model robustness was tested by altering criterion weights through several iterations. Z-Lab supports processing of Z-numbers with A and B components defined as triangular or trapezoidal fuzzy numbers.

The analysis involved adjusting weights in the fuzzy comparison matrix. A three-parameter approach was used, modifying parameters (a, b, c) – a-lower, b-central, c-upper values separately. Initially, 0.5 units were added to each fuzzy number in part A of the Z-matrix, assuming the first criterion C_1 is "slightly superior" to the second C_2 .

In the next step, preference values increased by another 0.5 units (total 1.0), and the resulting changes in alternative rankings were evaluated.

The initial sensitivity analysis results are shown below. The relative preference of the first criterion C_1 over the second C_2 was increased to the maximum degree (5.5) using the Z-Lab program. Tables and rankings for each iteration are provided in the dissertation.

The corresponding tables (Table 4, Table 5 and Table 6) and rankings for the first iteration are shown below.

Pairwise comparison matrix for the first iteration
Table 4

	C_1	C_2	C_3	C_4	C_5
C_1	(1, 1, 1), (0.8, 0.9, 1.0)	(1.5, 2.5, 3.5), (0.6, 0.7, 0.8)	(2, 3, 4), (0.5, 0.6, 0.7)	(2, 3, 4), (0.4, 0.5, 0.6)	(3, 4, 5), (0.4, 0.5, 0.6)
C_2	(0.29, 0.4, 0.4, 0.67), (0.6, 0.7, 0.8)	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 3), (0.5, 0.6, 0.7)	(1, 2, 3), (0.4, 0.5, 0.6)	(3, 4, 5), (0.4, 0.5, 0.6)
C_3	(0.25, 0.33, 0.5), (0.5, 0.6, 0.7)	(0.33, 0.5, 1), (0.5, 0.6, 0.7)	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 3), (0.4, 0.5, 0.6)	(2, 3, 4), (0.4, 0.5, 0.6)
C_4	(0.25, 0.33, 0.5), (0.4, 0.5, 0.6)	(0.33, 0.5, 1), (0.4, 0.5, 0.6)	(0.33, 0.5, 1), (0.4, 0.5, 0.6)	(1, 1, 1), (0.8, 0.9, 1.0)	(1, 2, 3), (0.4, 0.5, 0.6)

Decision matrix
Table 5

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	(2.0, 2.5, 3.0), (0.6, 0.7, 0.7, 0.8)	(3.0, 3.5, 4.0), (0.6, 0.7, 0.7, 0.8)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)
A ₂	(1.0, 1.5, 2.0), (0.6, 0.7, 0.7, 0.8)	(2.0, 2.5, 3.0), (0.6, 0.7, 0.7, 0.8)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)
A ₃	(3.0, 3.5, 4.0), (0.6, 0.7, 0.7, 0.8)	(2.0, 2.5, 3.0), (0.6, 0.7, 0.7, 0.8)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)

Consistent Z-matrix
Table 6

	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	(0.95, 1.0, 1.0, 1.05), (0.95, 0.97, 0.97, 0.99)	(2.375, 2.5, 2.5, 2.625), (0.95, 0.97, 0.97, 0.99)	(2.85, 3.0, 3.0, 3.15), (0.95, 0.97, 0.97, 0.99)	(2.85, 3.0, 3.0, 3.15), (0.95, 0.97, 0.97, 0.99)	(3.8, 4.0, 4.0, 4.2), (0.95, 0.97, 0.97, 0.99)
C ₂	(0.381, 0.4, 0.4, 0.421), (0.95, 0.97, 0.97, 0.99)	(0.95, 1.0, 1.0, 1.05), (0.95, 0.97, 0.97, 0.99)	(1.14, 1.2, 1.2, 1.26), (0.95, 0.97, 0.97, 0.99)	(1.14, 1.2, 1.2, 1.26), (0.95, 0.97, 0.97, 0.99)	(1.52, 1.6, 1.6, 1.68), (0.95, 0.97, 0.97, 0.99)
C ₃	(0.317, 0.333, 0.333, 0.351), (0.95, 0.97, 0.97, 0.99)	(0.794, 0.833, 0.833, 0.877), (0.95, 0.97, 0.97, 0.99)	(0.95, 1.0, 1.0, 1.05), (0.95, 0.97, 0.97, 0.99)	(0.95, 1.0, 1.0, 1.05), (0.95, 0.97, 0.97, 0.99)	(1.266, 1.333, 1.333, 1.4), (0.95, 0.97, 0.97, 0.99)
C ₄	(0.317, 0.333, 0.333, 0.351), (0.95, 0.97, 0.97, 0.99)	(0.794, 0.833, 0.833, 0.877), (0.95, 0.97, 0.97, 0.99)	(0.952, 1.0, 1.0, 1.053), (0.95, 0.97, 0.97, 0.99)	(0.95, 1.0, 1.0, 1.05), (0.95, 0.97, 0.97, 0.99)	(1.266, 1.333, 1.333, 1.4), (0.95, 0.97, 0.97, 0.99)
C ₅	(0.238, 0.25, 0.25, 0.263), (0.95, 0.97, 0.97, 0.99)	(0.595, 0.625, 0.625, 0.658), (0.95, 0.97, 0.97, 0.99)	(0.714, 0.75, 0.75, 0.79), (0.95, 0.97, 0.97, 0.99)	(0.714, 0.75, 0.75, 0.79), (0.95, 0.97, 0.97, 0.99)	(0.95, 1.0, 1.0, 1.05), (0.95, 0.97, 0.97, 0.99)

The inconsistency index is set as: 0.2891517523781516.

The vector of criteria is given below:

[0.1617 0.4269 0.4269 0.4269] [0.3752 0.5442 0.5442 0.5921],
 [0.0647 0.1708 0.1708 0.1708] [0.5358 0.6076 0.6076 0.7065],
 [0.054 0.1423 0.1423 0.1424] [0.5623 0.5921 0.5921 0.6508],
 [0.054 0.1423 0.1423 0.1425] [0.556 0.5818 0.5818 0.6576],
 [0.0405 0.1067 0.1067 0.107] [0.544 0.6131 0.6131 0.7083]

The normalized vector of criteria is given below:

[0.431 0.4317 0.4317 0.432] [0.3752 0.5442 0.5442 0.5921],
 [0.1725 0.1727 0.1727 0.1729] [0.5358 0.6076 0.6076 0.7065],
 [0.1438 0.1438 0.1438 0.1441] [0.5623 0.5921 0.5921 0.6508],
 [0.1438 0.1438 0.1438 0.1442] [0.556 0.5818 0.5818 0.6576],
 [0.1079 0.1079 0.1079 0.1082] [0.544 0.6131 0.6131 0.7083]

Normalized decision matrix:

	C ₁	C ₂	C ₃	C ₄	C ₅
A ₁	(0.3802, 0.4874, 0.7016, 0.8088), (0.6, 0.7, 0.7, 0.8)	(0.1615, 0.2686, 0.4829, 0.5901), (0.6, 0.7, 0.7, 0.8)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)
A ₂	(0.41, 0.75, 1.0, 1.0), (0.44, 0.57, 0.57, 0.66)	(0.0248, 0.132, 0.3463, 0.4534), (0.6, 0.7, 0.7, 0.8)	(0.0725, 0.1797, 0.394, 0.5011), (0.7, 0.8, 0.8, 0.9)	(0.0, 0.0, 0.1302, 0.2374), (0.7, 0.8, 0.8, 0.9)	(0.0, 0.0, 0.1302, 0.2374), (0.7, 0.8, 0.8, 0.9)

A_3	(0.0725, 0.1797, 0.394, 0.5011), (0.6, 0.7, 0.7, 0.8)	(0.0248, 0.132, 0.3463, 0.4534), (0.6, 0.7, 0.7, 0.8)	(0.0, 0.0, 0.0, 0.0), (0.62, 0.76, 0.76, 0.87)	(0.0, 0.0, 0.0, 0.0), (0.62, 0.76, 0.76, 0.87)	(0.0, 0.0, 0.0, 0.0), (0.62, 0.76, 0.76, 0.87)
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The vector of alternatives is as follows:

[0.22 0.34 0.54 0.64] [0.352 0.512 0.512 0.8],
 [0.19 0.37 0.58 0.64] [0.256 0.352 0.352 0.64],
 [0.03 0.1 0.23 0.3] [0.032 0.064 0.064 0.256]

Result for A_1 : 1.8359738988510375

Result for A_2 : 1.9599778999085016

Result for A_3 : 2.8158659041322514

Based on the answer obtained above, the ranking of alternatives is given as follows: $A_1 > A_2 > A_3$.

No change in the ranking of alternatives was observed. AI remained the optimal technology, followed by IoT and then VR/AR. In the second and next three iterations, the relative preference of the first criterion C_1 over the second C_2 was increased by 0.5 units. Still, the rankings stayed the same.

In the sixth iteration, the B part of the Z-number (i.e., the reliability of the A part) for the first criterion C_1 was raised to its maximum values (0.8, 0.9, 1.0) compared to the second C_2 . Even then, no change in the ranking was detected. Results are presented:

Result for A_1 : 1.801377895729411

Result for A_2 : 1.9494868974248272

Result for A_3 : 2.6157369156799994

Based on the results, the ranking remains unchanged:

$A_1 > A_2 > A_3$. Using the Z-Lab program, even when the first criterion's relative preference C_1 over the second C_2 is increased to the maximum (5.5), no change in ranking occurs. This confirms that altering the weight range from (1, 2, 3) to (3.5, 4.5, 5.5) for the first criterion's preference C_1 over the second criterion C_2 does not impact the rankings. The model remains stable.

According to the sensitivity analysis, although the B component of the Z-number, which expresses the superiority of the first criterion C_1 over the second criterion C_2 in the pairwise comparison matrix, in other words, the degree of reliability of the A component, as well as the B component of the Z-number corresponding to the first criterion of the first alternative in the decision matrix are increased to the maximum reliability values (0.8, 0.9 and 1.0), no significant change was recorded in the final ranking of the alternatives.

Based on the answer obtained, the ranking of the alternatives is given as follows: $A_1 > A_2 > A_3$. Based on this result, no change is detected in the ranking of the alternatives. Artificial intelligence (AI) technology is selected as optimal technology again. Internet of Things (IoT) technology remains in the second place, and virtual/augmented reality (VR/AR) technology remains in the third place.

In the next step, although the pairwise comparison matrix remains the same, the decision matrix is modified with several iterations. In general, no change in the ranking of alternatives is observed until the ninth iteration. This result indicates the stability of the decision-making model against relevant parameter changes and the persistence of the existing priority order. The results of the eighth iteration are given below.

The vector of alternatives:

[[0.16 0.27 0.48 0.59] [0.512 0.672 0.672 0.8]],

[[0.2 0.39 0.61 0.67] [0.096 0.224 0.224 0.352]],

[[0.03 0.09 0.22 0.28] [0.0 0.128 0.128 0.192]]

Result for alternative A_1 : 1.8359499016503866

Result for alternative A_2 : 2.0909868921875514

Result for alternative A_3 : 2.8278619020950932

Based on the above answer, the ranking of alternatives is given as follows: $A_1 > A_2 > A_3$. Even with multiple iterations, the ranking of alternatives remained unchanged. Through this approach, the stability and robustness of the model have been assessed and proven.

In the next step, we modify the model and conduct a sensitivity test on 4×4 Z-numbered pairwise comparison matrix. Thus, we will analyze the effect of additional criteria on the solution of the problem.

The criteria are as follows:

C_1 : Budget/financial capabilities;

C_2 : Efficiency of technology;

C_3 : Ease of use of technology;

C_4 : Customer communication skills.

The alternatives remained the same:

A_1 : Artificial Intelligence (AI);

A_2 : Internet of Things (IoT);

A_3 : Virtual Reality/Augmented Reality (VR/AR).

Using the notations mentioned, the pairwise comparison matrix and decision matrix are given as follows (Table 7 and Table 8).

Pairwise comparison matrix

Table 7

	C ₁	C ₂	C ₃	C ₄
C ₁	(1, 1, 1), (0.6, 0.7, 0.8)	(1, 2, 3), (0.6, 0.7, 0.8)	(2, 3, 4), (0.6, 0.7, 0.8)	(2, 3, 4), (0.6, 0.7, 0.8)
C ₂	(0.33, 0.5, 1), (0.6, 0.7, 0.8)	(1, 1, 1), (0.6, 0.7, 0.8)	(1, 2, 3), (0.6, 0.7, 0.8)	(1, 2, 3), (0.6, 0.7, 0.8)
C ₃	(0.25, 0.33, 0.5), (0.6, 0.7, 0.8)	(0.33, 0.5, 1), (0.6, 0.7, 0.8)	(1, 1, 1), (0.6, 0.7, 0.8)	(1, 2, 3), (0.6, 0.7, 0.8)
C ₄	(0.25, 0.33, 0.5), (0.6, 0.7, 0.8)	(0.33, 0.5, 1), (0.6, 0.7, 0.8)	(0.33, 0.5, 1), (0.6, 0.7, 0.8)	(1, 1, 1), (0.6, 0.7, 0.8)

Decision matrix

Table 8

	C ₁	C ₂	C ₃	C ₄
A ₁	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)
A ₂	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)
A ₃	(3.0, 3.5, 4.0), (0.7, 0.8, 0.9)	(2.0, 2.5, 3.0), (0.7, 0.8, 0.9)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)	(1.0, 1.5, 2.0), (0.7, 0.8, 0.9)

Consistent Z-matrix:

	C ₁	C ₂	C ₃	C ₄
C ₁	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.645, 1.732, 1.819), (0.95, 0.97, 0.99)	(2.582, 2.718, 2.854), (0.95, 0.97, 0.99)	(3.651, 3.843, 4.035), (0.95, 0.97, 0.99)
C ₂	(0.55, 0.577, 0.608), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.491, 1.569, 1.647), (0.95, 0.97, 0.99)	(2.108, 2.219, 2.33), (0.95, 0.97, 0.99)
C ₃	(0.35, 0.368, 0.387), (0.95, 0.97, 0.99)	(0.607, 0.637, 0.671), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)	(1.344, 1.414, 1.485), (0.95, 0.97, 0.99)

C4	(0.248, 0.26, 0.274), (0.95, 0.97, 0.99)	(0.429, 0.451, 0.474), (0.95, 0.97, 0.99)	(0.673, 0.707, 0.744), (0.95, 0.97, 0.99)	(0.95, 1.0, 1.05), (0.95, 0.97, 0.99)
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Inconsistency index=0.2891517523781516

Vector of alternatives:

[[0.23 0.34 0.57 0.66] [0.448 0.608 0.608 0.8]],

[[0.21 0.4 0.63 0.68] [0.256 0.544 0.544 0.736]],

[[0.04 0.11 0.27 0.35] [0.064 0.288 0.288 0.576]]

Result for alternative A_1 : 1.733979892966485

Result for alternative A_2 : 1.7699918961922436

Result for alternative A_3 : 2.5608769122962496

Thus, based on the obtained answer, the ranking of the alternatives is given as follows: $A_1 > A_2 > A_3$.

Then, sensitivity analysis is applied by changing the weights of the criteria and the stability of the model is tested. To perform the sensitivity analysis, we change the weights of the criteria in the pairwise comparison matrix in five iterations and analyze the results. By adding 0.5 to each of the fuzzy values given in section A of the Z-matrix, the relative preference of the first criterion (C_1) compared to the second criterion (C_2) are adjusted with a "slightly stronger preference" in each iteration. The results of the sixth iteration are given below:

Vector of alternatives:

[[0.24 0.35 0.57 0.68] [0.4622 0.64 0.64 0.8]],

[[0.22 0.44 0.66 0.71] [0.3556 0.5156 0.5156 0.6756]],

[[0.04 0.11 0.26 0.32] [0.1778 0.3911 0.3911 0.5689]]

Result for A_1 : 1.6944338945839634

Result for A_2 : 1.7194028972107482

Result for A_3 : 2.5026479129033925

Based on the results of our research, in the sensitivity test of the 4×4 pairwise comparison matrix (without changing the decision matrix), we observed that the ranking of the alternatives did not change until the seventh iteration. Thus, in this model, AI was also identified as the optimal digital marketing technology, which is followed by IoT and AR/VR respectively. It should be noted that when the partial reliability or the B component in the Z-matrix is increased from “sure” to “very sure” (0.8, 0.9, 1.0), the ranking of the alternatives remains unchanged. Thus, the robustness of the model has been assessed and proven using the sensitivity test.

MAIN SCIENTIFIC RESULTS OF THE WORK

The results obtained in the dissertation are as follows:

1. Types of uncertainty in digital marketing have been studied and a new approach to be applied to the solution of the problem has been determined. A decision-making model based on Z-numbers has been more adequately evaluated for solving the problem of selecting the optimal digital marketing technology.
2. A model based on Z-numbers has been created for the optimal selection of advanced digital marketing technologies such as Artificial Intelligence, Internet of Things and Virtual and Augmented Reality under uncertainty.
3. A sensitivity analysis was applied to the created model and the stability of the model was checked by changing the criteria weights.
4. An approach based on the evaluation and selection of alternative digital marketing technologies with Z-numbers has been developed.
5. The problem of decision-making based on Z-numbers under uncertainty of the most effective and efficient technology for consumer companies or individual consumers has been solved.

6. The main advantage of the proposed scientific approach over existing decision-making approaches to digital marketing technology selection is that the reliability of the decisions made is analyzed in this approach and the stability and robustness of the decision analysis under consideration are ensured.

The results obtained can be applied in various fields under uncertainty conditions.

The main content of the dissertation is published in the following works:

1. Imanov, E., Ozkiloglu, O., Imanova, G. E. Flight Information system by using Fuzzy Expert Inference // *Procedia Computer Science*, – 2017, – p. 304-310
2. Eyupoglu, S. Z., Imanova, G. E. Self-confidence Preference Based Decision Making in Personnel Selection // *Advances in Intelligent Systems and Computing*, – 2019, – p. 288-292
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4. Imanova G. E. Qeyri-səlis üstünlük münasibətləri ilə nisbi etibarlılıq səviyyələrinə əsaslanan mütəxəssis seçimi // *Regional İnkişafın Təmin Olunmasında İnnovativ Tendensiyalar: Reallıqlar Və Müasir Çağırışlar Respublika Elmi Konfransının Materialları, Mingəçevir, Azərbaycan – 2020, – s. 190-192*
5. Imanova, G. E. Identification and ranking of key factors for pattern of consumer buying decisions in digital marketing // 11th World Conference “Intelligent System for Industrial Automation” (WCIS-2020), Tashkent, Uzbekistan, Springer International Publishing, –2021, – p. 237-245
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Personal contribution of the applicant in the works published in co-authorship.

- [1] – Computer simulation and analysis of results
- [2] – Problem formulation, solution method and analysis of results
- [3] – The author of the idea, problem formulation and solution method
- [6]– The author of the idea, problem formulation and computer simulation
- [9] – The author of the idea, problem formulation, solution method and calculations
- [11] –The author of the idea, problem formulation and solution method
- [14] – Problem formulation, solution method and analysis of results
- [15] – Problem formulation and analysis of results
- [16] – Problem formulation, solution method and analysis of results
- [17] –The author of the idea, problem formulation and computer simulation

The defense of the dissertation will be held on September 30, 2025 at 2 PM at the meeting of the Dissertation Council ED 2.48 operating under the Azerbaijan State Oil and Industry University.

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The dissertation is available in the library of the Azerbaijan State Oil and Industry University.

Electronic versions of the dissertation and abstract are posted on the official website of the Azerbaijan State Oil and Industry University.

The abstract was sent to the necessary addresses on August 20, 2025.

A handwritten signature in blue ink, appearing to be the initials 'SAD' or similar, is centered on the page.

Signed for publication: 13.08.2025

Paper format: A5

Volume: 38091

Circulation: 30