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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

DEVELOPMENT OF METHODS AND ALGORITHMS FOR FUZZY TIME SERIES FORECASTING

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Field of science:	Technical sciences
Applicant:	**Parvin Elchin Alizade**

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The work was performed at the department of "Information technologies and programming" of Baku State University,

Scientific supervisor: Doctor of Technical Sciences, professor
Ramin Rza Rzayev

Official opponents: Doctor of Technical Sciences, professor
Fazil Hazin Alekberli

Doctor of Technical Sciences, assoc.prof.
Kamala Rafig Aliyeva

Doctor of Philosophy in techn.sc., assoc.prof
Rahib Aydin Imamguliyev

Disseration council ED 2.48 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at Azerbaijan State Oil and Industry University

Chairman of the Dissertation council: Doctor of Technical Sciences,professor

_______________ **Rafig Aziz Aliyev**

Scientific secretary of the Dissertation council: Doctor of Philosophy in techn.sc. assoc.prof

_______________ **Akif Vali Alizade**

Chairman of the scientific seminar: Doctor of Technical Sciences,professor

_______________ **Mahammad Nurmahammad Nuriyev**

İMZANI TƏSDİQ EDİRƏM
ADNSU-nun Elmi Katibi
dosent N.Əliyeva

ELMİ KATİB

GENERAL CHARACTERISTICS OF WORK

The relevance of the topic and the degree of its development. The main feature of socio-economic systems is their complex behavior, which is characterized by volatile dynamics. Therefore, it is not surprising that time series corresponding to the behavior of socio-economic systems are characterized by a high degree of uncertainty. The main reasons for such uncertainty are the nonstationarity of the processes under study, insufficient duration and accuracy of observations, as well as a weak trend and/or simply unobservable development trend. The presence of the listed properties in the volatility of the behavior of a time series predetermines the reasons why the study of time series is increasingly being considered using various methods of nonlinear dynamics: chaos theory and fractal geometry, fuzzy logic and fuzzy set theory, the theory of neural networks and hybrid modeling[1] systems.

Time series forecasting in the fuzzy paradigm does not claim to have high forecasting accuracy and is mainly used for short-term planning, for example, for forecasting trends in the labor market. Fuzzy time series are rather focused on generating high-quality information about the dynamics of the behavior of the system being studied and provide information support for decision-making as one of the tools for data mining. In other words, fuzzy time series have become an integral part of Data Mining technology, the main directions of which are the analysis and modeling of processes occurring under conditions of uncertainty, including uncertainties of a non-stochastic type; identifying hidden patterns and compiling new knowledge, for example, in the form of predicted time series[2].

Thus, the study of contextual data and the analysis of methods for studying patterns between them have formed a new practical direction

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[1] Минаев Ю.М., Филимонова О.Ю., Бенамеур Лиес. Методы и алгоритмы решения задач идентификации и прогнозирования в условиях неопределенности в нейро-сетевом логическом базисе. М.: Горячая линия-Телеком, 2003. 205 с.

[2] Батыршин И.З. Модели и методы перцептивного дата майнинга временных рядов для систем поддержки принятия решений // Нечеткие системы и мягкие вычисления. Т. 2. – 2007. – №1.

- Data Mining. Over the past two decades, through the efforts of a number of foreign scientists, for example, K. Hirota, H. Tanak, V. Pedrich and J. Kaprzyk, methods of fuzzy regression and analysis of fuzzy time series data have been studied. Among Azerbaijani scientists, first of all, the works of R. Aliyev, K. Imanov, R. Rzayev should be noted, who made a significant contribution to the field of studying the behavior of socio-economic systems under conditions of uncertainty.

The founders of the theory of fuzzy time series are K. Song and B. Chissom, who proposed a methodology for forecasting weakly structured time series using the technique of fuzzy relations and approximate inference[3]. Further, this direction rapidly developed through the efforts of, first of all, S. Chen[4], whose fuzzy models significantly simplified calculations, N. Kumar[5], K. Cheng[6], J. Poulsen[7], and others. The approaches they proposed to forecasting fuzzy time series differ in the rules for fuzzification of historical data and defuzzification outputs of fuzzy predictive models.

Methods of time series analysis in the fuzzy paradigm have already won the right to become the basis for creating libraries of interactive models (simulations) to support decision-making in a variety of areas of management. However, despite the results achieved, many problems in the analysis of fuzzy time series remain unsolved, in particular, the problem of identifying internal fuzzy trends and generating rules for their recognition. Based on the above, it becomes obvious the importance and relevance of the listed features for the

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[3] Song Q., Chissom B.S. Fuzzy time series and its models // Fuzzy Sets and Systems. 1993. №54. pp. 269–277.

[4] Chen S.M. Forecasting enrollments based on high-order fuzzy time series // Cybernetics and Systems: An International Journal. 2002. № 33. pp. 1–16.

[5] Kumar N., Ahuja S., Kumar V., Kumar A. Fuzzy time series forecasting of wheat production // International Journal on Computer Science and Engineering. 2010. Vol. 2, № 3. pp. 635–640.

[6] Cheng C.H., Chang J.R., Yen C.A. Entropy-based and trapezoid fuzzification fuzzy time series approaches for forecasting IT project cost // Technological Forecasting & Social Change. 2006. № 73. pp. 524–542.

[7] Poulsen, J.R.: Fuzzy time series forecasting – developing a new forecasting model based on high order fuzzy time series. – AAUE: CIS 4, 2009. – 67 p.

further development of tools for predicting the behavior of weakly structured systems based on fuzzy time rads.

The main purpose and objectives of the research. The main goal of the dissertation work is to study the predictive capabilities of fuzzy trends in weakly structured time series, develop new rules for fuzzification of historical data and defuzzification of fuzzy forecasts, as well as the construction of a corresponding adequate mathematical model for forecasting volatile time series.

To achieve this goal, using the example of the dynamics of changes in the Dow Jones Industrial Average over a period of more than a year, it is planned to solve the following component tasks:

- conduct a comparative analysis of existing methods for fuzzification of historical time series data, identify their capabilities and disadvantages;
- develop a new rule for fuzzification of weakly structured historical data, both for the case of a fixed arbitrary set of qualitative evaluation criteria, and for the case of a reasonable choice of their quantity;
- build a methodology for identifying fuzzy trends in time series and a methodology for analyzing the time series of fuzzy trends;
- develop an algorithm for restoring a fuzzy time series from the original time series and an inverse algorithm for generating a series in nominal values;
- develop a method for compiling knowledge about internal fuzzy tendencies in the form of implicate rules;
- develop and implement a software shell in the form of a tool for processing historical data for restoring and forecasting a time series;
- develop a neural network model for forecasting time series in nominal values.

Object and subject of research. The object of the study is a volatile fuzzy time series, and the subject of the research is methods for identifying internal fuzzy patterns of time series and methods for modeling time series of fuzzy trends.

Research methodology used. When writing the dissertation, general scientific research methods were used, such as the dialectical method, complex analysis, synthesis, etc., and special scientific methods, implying systematic and formal logical approaches, taking into account the existing developments in fundamental and applied research on the problems of decision-making under conditions of uncertainty, presented in the works of republican and foreign scientists. The intended goal and main objectives of the dissertation predetermined the need to use an interdisciplinary approach that ensures organic calculations using quantitative and qualitative categories, statistical analysis and generalization. At the same time, the analysis of established scientific principles and the use of methodological apparatus - elements of fuzzy logic and modern theory of fuzzy sets, methods of mathematical statistics, theory of neural networks, using specific examples, form the evidence base and ensure the reliability of the conclusions obtained.

The theoretical and information-empirical basis of the dissertation research consists of a number of statistical data from open Internet sources, the work of leading republican and foreign scientists in the subject area, a representative set of which provided reasoned validity for practical recommendations.

Reliability of the results of the dissertation work.

The reliability of scientific propositions, conclusions and recommendations is confirmed by the results of experiments conducted, as well as the results of using the dissertation materials and the developed applied methodology in a specialized organization in accordance with the implementation act.

The main provisions for defense.

- The time series of fuzzy patterns is an effective indicator of the volatile development of the modeled process on the stock exchange, which is described in qualitative categories using terms of linguistic variables
- The key operations for processing fuzzy patterns are algorithms for processing internal fuzzy connections, namely, the technique of

restoring a fuzzy time series from the original time series and the inverse operation of compiling the time series in nominal values, as a reflection of the identified fuzzy trends

- The three-layer neural network of the proposed topology is an effective compiler of rules for predicting fuzzy trends, expressed in the form of defuzzified values of fuzzy interpretations of historical data
- The developed forecasting model based on time series of fuzzy trends makes it possible to forecast volatile time series for a short-term period.
- The developed neural network model of the selected topology makes it possible to predict a volatile time series for a short-term period.

Scientific novelty in the research. The scientific novelty of the dissertation is as follows:

- A definition of a time series in the fuzzy paradigm was proposed and a new method for recognizing internal fuzzy trends was developed
- Algorithms for processing internal fuzzy trends have been developed, including the operation of restoring a fuzzy time series from historical data of the original time series and the inverse operation of generating a series in nominal values
- A method for fuzzification of weakly structured data using a fuzzy inference system is proposed
- A method for compiling knowledge about internal fuzzy patterns in the form of implicative rules and using a neural network model is proposed
- A predictive model has been developed based on time series of fuzzy trends, which allows forecasting volatile time series for the short term
- The effectiveness of predictive models compiled on the basis of time series of fuzzy trends and neural network recognition methods was studied, using average statistical evaluation criteria.

Scientific and practical significance of the study.

The developed methods and algorithms allow potential users to create predictive models of volatile time series with improved quality characteristics and, thereby, significantly reduce the time spent on making operational decisions.

Approbation and implementation of work.

The main provisions and results of the dissertation were reported, discussed and received approval for:

- the 7th International Conference on Control and Optimization with Industrial Applications, COIA – 2020 (2020, Baku, Azerbaijan)
- the 14th International Conference on Applications of Fuzzy Systems, Soft Computing and Artificial Intelligence Tools, ICAFS – 2020 (2020, Budva, Montenegro),
- the 11th International Conference on Theory and Application of Soft Computing, Computing with Words, Perception and Artificial Intelligence ICSCCW-2021, (2021, Antalya, Turkey)
- the 8th World Conference on Soft Computing dedicated to the 100th Birthday anniversary and research heritage of professor Lotfy A. Zadeh (2022, Baku, Azerbaijan)
- the Future of Information and Communication Conference, FICC – 2022 (2022, San Francisco, USA)
- the Intelligent Systems Conference, IntelliSys – 2023, (2023, Amsterdam, The Netherlands),

as well as at extended seminars of the Department of Information Technologies and Programming of the Faculty of Applied Mathematics and Cybernetics of the Baku State University and the Laboratory of Information Decision Support Systems of the Institute of Management Systems of the Ministry of Science and Education of the Republic of Azerbaijan.

Name of the organization where the work was performed. The dissertation work was completed at the Department of Information Technologies and Programming, Faculty of Applied Mathematics and Cybernetics, Baku State University.

Publication of the results of dissertation work. 17 works were published on the topic of the dissertation, including 8 theses-reports and 9 articles, including 6 abroad, of which 5 articles were published in journals with international scientific citation indices from the Web-Science and SCOPUS archives.

Personal contribution. All results that make up the content of the dissertation were obtained by the author independently.

Structure and volume of the dissertation work.

The dissertation consists of an introduction, 3 chapters, a conclusion and a list of references. The work is presented on 123 pages of typewritten text, contains 68 figures and 15 tables, consists of an introduction, three chapters, a conclusion, and a bibliography of 100 titles. The volume of the general and structural sections of the dissertation is distributed approximately as follows:

- total – 235 119 characters,
- table of contents – 1 819 characters,
- introduction – 12 749 characters,
- chapter one – 27 482 characters,
- chapter two – 137 525 characters,
- chapter three – 54 552 characters,
- result – 992 characters.

THE CONTENT OF THE WORK

The introduction of the dissertation outlines the relevance of the research, formulates its main goal and provides a list of tasks and approaches necessary to achieve this goal, describes the structure and content of the work, as well as the results obtained for defense.

The first chapter discusses modern methods for analyzing time series. The existing capabilities and limitations of statistical and intelligent methods for modeling time series are described in a laconic form.

Thus, the conceptual statistical model of a time series is represented in general form as[8]

$$x(t)=\lambda \cdot f(t)+\psi \cdot \varepsilon_t+\xi_t,$$

where the historical data $x(t$ considered as the sum of some systematic component $f(t)$ and random components ε_t; λ and ψ are coefficients that take values from the set {0, 1}; ξ_t – random error with zero mathematical expectation and finite variance. At the same time, despite the diverse tools of statistical analysis, it is not always possible to successfully build predictive models that ensure the reliability of data extrapolation and the correspondence of the model's behavior to the behavior of the original time series.

Neural network modeling comes down to solving the problem of approximating a nonlinear continuous function of many variables using a predetermined set of training examples from the history of a time series. In general, the neural network model of a time series looks like this[9]:

$$\hat{y}_{k+1} = \phi(y_k, y_{k-1}, \dots, y_{k-n+1}) + \varepsilon_{k+1},$$

where $\hat{y}_{k+1}$ is a forecast; $y_k, y_{k-1}, \dots, y_{k-n+1}$ observed values of the time series; $\phi(y_k, y_{k-1}, \dots, y_{k-n+1})$ – nonlinear continuous function $(n$-1) variables, the parametric model of which can be, for example, a three-layer neural network; ε_{k+1} – permissible forecasting error; n – model order.

Particular attention is paid to the use of fuzzy set theory, which is used to control and model weakly structured and complex processes. The main provisions of the theory of fuzzy sets are outlined and models of fuzzy time series are described. To introduce fuzziness, the starting point is the construction of a universe, which in the case of modeling and forecasting a time series is coverage of a range of historical data

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[8] Андерсон, Т. Статистический анализ временных рядов / Т. Андерсон. – М.: Мир, 1976. – 757 с.

[9] Андерсон, Т. Статистический анализ временных рядов / Т. Андерсон. – М.: Мир, 1976. – 757 с.

$$d = [D_{\min} - D_1;\ D_{\max} + D_2],$$

where $D_{\min}$ и $D_{\max}$ – respectively, the minimum and maximum values among time series data; $D_1>0$ и $D_2>0$ are selected from the calculation of dividing the segment d into equal intervals u_j according to the number of selected evaluation criteria C_1, C_2, …, C_n, which in general are specified in the form of the following fuzzy sets:

$$C_1=\mu_{C1}(u_1)/u_1+\mu_{C1}(u_2)/u_2+\ \dots\ +\mu_{C1}(u_n)/u_n,$$
$$C_2=\mu_{C2}(u_1)/u_1+\mu_{C2}(u_2)/u_2+\ \dots\ +\mu_{C2}(u_n)/u_n,$$
$$\dots$$
$$C_n=\mu_{Cn}(u_1)/u_1+\mu_{Cn}(u_2)/u_2+\ \dots\ +\mu_{Cn}(u_n)/u_n,$$

where $\mu_{Ci}(u_j)\in[0, 1]$ $(i, j=1\div n)$ – interval membership function values u_j to the fuzzy set C_i.

The object of research of the dissertation is the time series of the Dow Jones index (DJIA – Dow Jones Industrial Average) $\{x(t)\}$ $(t=1\div T)$ (Pict. 1), where $x(t)$ is considered as a weakly structured data or a fuzzy set A_j $(j=1\div J)$, characterized by cartage [1]:

$$\{x(t)/\mu_{Aj}[x(t)]\},\ \mu_{Aj}[x(t)]: U\rightarrow[0, 1],$$

where U is a discrete universe covering the range of DJIA indicators.

Fig. 1. DJIA time series

The main task is to develop a fuzzification method for DJIA indicators, which would allow a more adequate reconstruction of the DJIA time series $\{x(t)\}$ in terms of fuzzy sets and, thereby, build a more adequate mathematical model of the series for forecasting in nominal units.

The second chapter proposes an approach to fuzzification of historical data, based on the use of the Fuzzy Inference System (FIS), built on trivial statements[10]:

e_1: «If the DJIA index is located closer to the middle of the u_1 segment, then its value is too low »;

e_2: «If the DJIA index is located closer to the middle of the u_2 segment, then its value is very low»;

e_3: «If the DJIA index is located closer to the middle of the u_3 segment, then its value is more than low»;

e_4: «If the DJIA index is located closer to the middle of the u_4 segment, then its value is low»;

e_5: «If the DJIA index is located closer to the middle of the u_5 segment, then its value is high»;

e_6: «If the DJIA index is located closer to the middle of the u_6 segment, then its value is more than high»;

e_7: «If the DJIA index is located closer to the middle of the u_7 segment, then its value is very high»;

e_8: «If the DJIA index is located closer to the middle of the u_8 segment, then its value is too high».

The analysis of these statements, as reflecting cause-and-effect relationships, made it possible to determine the input characteristic in the form of a linguistic variable x = "Localization of the DJIA index", taking values in the form of terms "CLOSER TO THE MIDDLE OF THE SEGMENT u_j» (j=1÷8): u_1=[21771, 22471], u_2=[22471, 23171], …, u_8=[26671, 27371], and the output linguistic variable y = "DJIA index value", the values of which are the terms: "TOO LOW", "VERY LOW", "MORE THAN LOW", "LOW", "HIGH", "MORE THAN HIGH", "VERY HIGH" ", "TOO HIGH". Verbal assessments of DJIA index localization $x(t)$ based on its belonging to a local segment u_j (j=1÷8) displayed as fuzzy subsets of a discrete universe U=[21771, 27371], including index indicators DJIA for 333 days of trading on the

[10] Alizada P.E. Conversion of volatile time series into a fuzzy time series by the example of the Dow Jones index dynamics. Lecture Notes in Networks and Systems, Vol. 362, pp. 662–670, 2022

stock exchange: $U = \{x(t)\}_{t=1}^{333}$. A Gaussian type function is used as a membership function (Fig. 2.)

$$\mu(x) = \exp\left[-(x_i - u_{j0})^2/\sigma^2\right], \tag{1}$$

where $x_t = x(t)$ – index indicator DJIA, established based on the results of trading on the stock exchange on the t-th day; u_{j0} – middle of the interval u_j ($j = 1 \div 8$); σ – standard deviation chosen uniform for all cases in the form of the number 500.

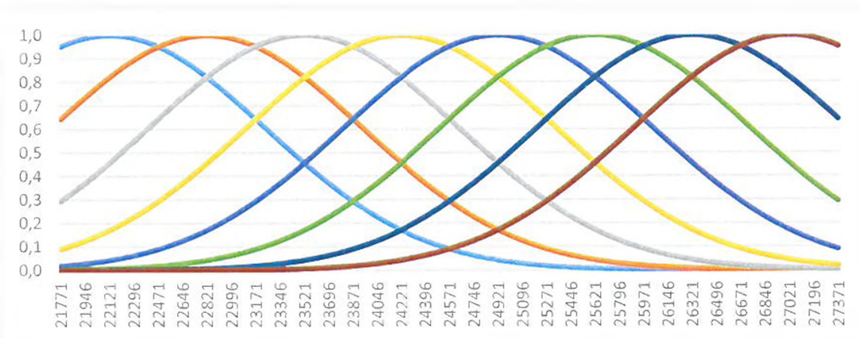

Fig. 2. Membership functions of fuzzy sets reflecting the degree of localization of the DJIA index

Having determined the midpoints of the segments u_j as: u_{10}=22121, u_{20}=22821, ..., u_{80}=27021, in accordance with (1) signs of localization of the DJIA index $x_t = x(t)$ ($t = 1 \div 333$) can be interpreted as:

- «PROXIMITY TO 22121» in the form of a fuzzy set: $X_1 = 0.952181/x_1 + 0.958630/x_2 + \ldots + 0.000793/x_{332} + 0.000472/x_{333}$;
- «PROXIMITY TO 22821» in the form of a fuzzy set: $X_2 = 0.643393/x_1 + 0.656883/x_2 + \ldots + 0.006941/x_{332} + 0.004497/x_{333}$;

....

- «PROXIMITY TO 27021» in the form of a fuzzy set: $X_8 = 0.000016/x_1 + 0.000018/x_2 + \ldots + 0.833393/x_{332} + 0.895873/x_{333}$.

Terms of the output linguistic variable "*The value of the DJIA index" are described in the form of fuzzy subsets of a discrete universe*

$I = \{0, 0.1, 0.2, \ldots; 1\}$[11], that is $\forall i \in I$ as: TL= TOO LOW, $\mu_{TL}(i)=0$, if $i=1$ and $\mu_{TL}(i)=1$, if $i<1$; VL= VERY LOW: $\mu_{VL}(i)=(1-i)^2$; ML= MORE THAN LOW: $\mu_{ML}(i)=(1-i)^{(1/2)}$; L=LOW: $\mu_L(i)=1-i$; H= HIGH: $\mu_H(i)=i$; MH= MORE THAN HIGH: $\mu_{MH}(i)=i^{(1/2)}$; VH=VERY HIGH: $\mu_{VH}(i)=i^2$; TH= TOO HIGH, $\mu_{TH}(i)=1$, if $i=1$ and $\mu_{TH}(i)=0$, if $i<1$.

As a result of applying the fuzzy Lukasiewicz implication

$$\mu_H(u, i)=\min\{1, 1-\mu_X(u)+\mu_Y(i)\}, \qquad (2)$$

fuzzy relations are defined in the form of matrices $R_1, R_2, \ldots, R_8$,in size 333×11, the intersection of which gives a general functional solution $R=R_1 \cap R_2 \cap \ldots \cap R_8$ as the following matrix

$R =$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$x_1 = 25090.5$	0.0114	0.1114	0.2114	0.3114	0.4114	0.5114	0.6114	0.5610	0.4610	0.3610	0.2610
$x_2 = 24987.5$	0.0018	0.1018	0.2018	0.3018	0.4018	0.5018	0.6018	0.5094	0.4094	0.3094	0.2094
$x_3 = 24700.2$	0.0193	0.1193	0.2193	0.3193	0.4193	0.5193	0.4878	0.3878	0.2878	0.1878	0.0878
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$x_{332} = 26346.0$	0.0002	0.0102	0.0402	0.0902	0.1602	0.1666	0.1666	0.1666	0.1666	0.1666	0.8357
$x_{333} = 26496.7$	0.0123	0.0223	0.0523	0.1023	0.1041	0.1041	0.1041	0.1041	0.1041	0.1041	0.8740

,

which reflects the cause-and-effect relationship between the signs of localization of the DJIA index, on the one hand, and its value, on the other. As a result, all historical time series data of the DJIA index are interpreted in the form of corresponding fuzzy sets (FS), which are summarized in Table 1.

[11] Андрейчиков А.В., Андрейчикова О.Н. Анализ, синтез, планирование решений в экономике. – М.: Финансы и статистkиа, 2000. – 368 с.

Table 1. Detailed fuzzy time series of the DJIA index

Data	FS	Membership function values of the fuzzy analogue of the DJIA index						PE of FS
		0	0.1	0.2	…	0.9	1	
15.06.2018	A_1	0.0114	0.1114	0.2114	…	0.3610	0.2610	0.6062
18.06.2018	A_2	0.0018	0.1018	0.2018	…	0.3094	0.2094	0.5939
19.06.2018	A_3	0.0193	0.1193	0.2193	…	0.1878	0.0878	0.5330
20.06.2018	A_4	0.0273	0.1273	0.2273	…	0.1735	0.0735	0.5219
………………………								
07.10.2019	A_{330}	0.0098	0.0198	0.0498	…	0.1113	0.8697	0.9513
08.10.2019	A_{331}	0.0098	0.0198	0.0498	…	0.2546	0.7791	0.8959
09.10.2019	A_{332}	0.0002	0.0102	0.0402	…	0.1666	0.8357	0.9321
10.10.2019	A_{333}	0.0123	0.0223	0.0523	…	0.1041	0.8740	0.9534

As shown in Fig. 3, the dynamic series of the DJIA index in terms of point estimates of fuzzy sets (PE of FS) approximately accurately reproduces the configuration of the DJIA time series on the scale of the interval [0, 1].

Fig. 3. Time series of the DJIA index in PE of FS notation

The number of NMs presented in Table 1 is excessive to form a set of qualitative evaluation criteria. Therefore, according to the following scheme, their optimal number for making value judgments[12] was established

Step 1. Sorting historical indicators x_t (t=1÷333) DJIA index as an ascending sequence $\{x_{p(i)}\}$, where p is a permutation that sorts the DJIA index values in ascending order according to a trivial rule $x_{p(i)} \leq x_{p(i+1)}$.

Step 2. On the set of all pairwise distances $d_i = |x_{p(i)} - x_{p(i+1)}|$ between any two consecutive values $x_{p(i)}$ and $x_{p(i+1)}$ average distance AD calculation according to the formula

$$AD = \sum_{i=1}^{n-1} | x_{p(i)} - x_{p(i+1)} | / (n-1), \quad (3)$$

and corresponding standard deviation σ_{AD} according to the formula

$$\sigma_{AD} = \sqrt{\sum_{i=1}^{n-1} (d_i - AD)^2 / (n-1)} . \quad (4)$$

Step 3. Identification and reset of anomalous values to be eliminated. The values of pairwise distances that do not satisfy the condition are subject to exclusion from consideration:

$$AD - \sigma_{AD} \leq d_i \leq AD + \sigma_{AD}. \quad (5)$$

Step 4. Re-calculate the average AD value on the set of pairwise distances remaining after sorting, taking into account the reset of anomalies.

Step 5. Establishing the optimal number of qualitative criteria for assessing the historical performance of the DJIA industrial index in the form of FS using the formula

$$m = (D_2 - D_1 - AD) / (2 \cdot AD), \quad (6)$$

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[12] Ortiz-Arroyo, D., Poulsen, J.R. A Weighted Fuzzy Time Series Forecasting Model // Indian Journal of Science and Technology, 11(27), pp. 1–11, 2018.

where $D_1=D_{min}-AD$, $D_2=D_{max}+AD$.

As a result of performing these procedures for the DJIA time series with n = 333 the number of criteria was established as m=280. Then, assuming F_1 и F_{333} PE for FS A_1 и A_{333}, accordingly, the segment [F_1, F_{333}] is divided into 280 equal segments a_k (k=1÷280) with lengths ($F_{333}-F_1$)/280. In this. Case. All FS A_t (t = 1÷333) are grouped through the application of an implicative rule[13]:

«If EP of FS A_t from segment a_k, then A_t belongs to the k-th group».

As a result, 144 groups were formed, within which qualitative criteria for assessing the historical performance of the DJIA index were formalized, which are summarized in Table 2.

Table 2. Qualitative criteria for assessing the DJIA index

Evaluation Criteria	Values of the membership function of a fuzzy subset of the universe						
	0	0.1	0.2	0.3	…	0.9	1
C_1	0.9138	0.0412	0.0412	0.0412	…	0.0412	0.0412
C_2	0.8644	0.1200	0.1200	0.1200	…	0.0172	0.0072
C_3	0.8173	0.1960	0.1960	0.1960	…	0.0106	0.0006
C_4	0.8115	0.2051	0.2051	0.2051	…	0.0113	0.0013
………							
C_{141}	0.0117	0.0217	0.0217	0.0217	…	0.0217	0.8730
C_{142}	0.0147	0.0147	0.0147	0.0147	…	0.0147	0.9297
C_{143}	0.0091	0.0091	0.0091	0.0091	…	0.0091	0.9351
C_{144}	0.0010	0.0010	0.0010	0.0010	…	0.0010	0.9423

In terms of the Fuzzy Set C_k (k=1÷144), the DJIA time series, as well as its interpretation in the PE of FS notation, are presented in Table 3.

[13] Rzayev R.R., Alizada P.E., Mehdiyev T.Z. Volatile time series forecasting on the example of the dynamics of the Dow Jones index. Problems of information society, 2023, Vol.14, №1, pp. 14–25.

Table 3. DJIA time series in terms of FS and PE

№	Date	DJIA	FS	PE	CR.	Detalization	PE
1	15.06.2018	25090.5	A_1	0.6062	C_{55}	$A_1 \cap A_{107} \cap A_{164}$	0.6051
2	18.06.2018	24987.5	A_2	0.5939	C_{51}	$A_2 \cap A_{93}$	0.5936
3	19.06.2018	24700.2	A_3	0.5330	C_{41}	$A_3 \cap A_{18} \cap A_{150}$	0.5335
4	20.06.2018	24657.8	A_4	0.5219	C_{39}	A_4	0.5219
………………………………………………………………							
156	29.01.2019	24580.0	A_{156}	0.5022	C_{36}	$A_6 \cap A_{92} \cap A_{156}$	0.5026
157	30.01.2019	25014.9	A_{157}	0.5975	C_{52}	$A_{157} \cap A_{158}$	0.5956
158	31.01.2019	24999.7	A_{158}	0.5956	C_{52}	$A_{157} \cap A_{158}$	0.5956
159	01.02.2019	25063.9	A_{159}	0.6033	C_{54}	$A_{21} \cap A_{24} \cap A_{25} \cap A_{83} \cap A_{159} \cap A_{165}$	0.6021
………………………………………………………………							
330	07.10.2019	26478.0	A_{330}	0.9513	C_{131}	$A_{71} \cap A_{80} \cap A_{285} \cap A_{330} \cap A_{333}$	0.9538
331	08.10.2019	26164.0	A_{331}	0.8959	C_{117}	A_{331}	0.8959
332	09.10.2019	26346.0	A_{332}	0.9321	C_{125}	$A_{204} \cap A_{332}$	0.9320
333	10.10.2019	26496.7	A_{333}	0.9534	C_{131}	$A_{71} \cap A_{80} \cap A_{285} \cap A_{330} \cap A_{333}$	0.9538

Assessing the indicators of the DJIA index using the criteria C_k (k=1÷144), the details of which are presented in Table 2, within the framework of the fuzzy time series of the DJIA index, internal connections of the 1st order were identified in the form of fuzzy relationships, which are divided into 144 groups and summarized in Table 4.

Table 4. Internal connections of the 1st order, divided into groups

Symbol	Group	Symbol	Group
G_1	$C_1 \Rightarrow C_1, C_4$	G_{136}	$C_{136} \Rightarrow C_{130}, C_{141}$
G_2	$C_2 \Rightarrow C_{10}$	G_{137}	$C_{137} \Rightarrow C_{135}, C_{140}$
G_3	$C_3 \Rightarrow C_1$	G_{138}	$C_{138} \Rightarrow C_{139}, C_{141}$
G_4	$C_4 \Rightarrow C_6$	G_{139}	$C_{139} \Rightarrow C_{133}, C_{138}, C_{139}, C_{141}, C_{142}$
G_5	$C_5 \Rightarrow C_8$	G_{140}	$C_{140} \Rightarrow C_{134}, C_{139}$
G_6	$C_6 \Rightarrow C_5$	G_{141}	$C_{141} \Rightarrow C_{142}, C_{143}, C_{144}$
G_7	$C_7 \Rightarrow C_3$	G_{142}	$C_{142} \Rightarrow C_{136}, C_{141}, C_{142}, C_{143}, C_{144}$
G_8	$C_8 \Rightarrow C_9$	G_{143}	$C_{143} \Rightarrow C_{134}, C_{138}, C_{142}, C_{143}, C_{144}$
...	....	G_{144}	$C_{144} \Rightarrow C_{134}, C_{139}, C_{142}, C_{143}, C_{144}$

Internal connections of the 1st order are fuzzy relationships reflecting cause-and-effect relationships between qualitative (fuzzy) estimates of the historical indicators of the DJIA index, as terms of the corresponding linguistic variables, and their fuzzy forecasts. This connection clearly reflects an implicative rule of the form:

«If x_t is C_i, then x_{t+1} is C_j» (t=1÷333; i, j=1÷144), and ambiguously reflects an implicative rule of the form:

«If x_t is C_i, then x_{t+1} is $C_{j(1)}$ or $C_{j(2)}$ or … or $C_{j(p)}$» (t = 1÷333; $i, j(1), j(2),\ldots, j(p)$ = 1÷144).

If in the case of the 1st implication everything is extremely clear, then in the presence of two or more alternative fuzzy conclusions, aggregation of the fuzzy forecast is carried out using the logical operator "OR". In particular, for fuzzy relations: $C_{36} \Rightarrow C_{21}$, C_{51}, C_{52} fuzzy forecast is reflected by the fuzzy set $F = C_{21} \cup C_{51} \cup C_{52}$ c with a membership[14] function

$$\mu_F(u) = \mu_{C_{21} \cup C_{51} \cup C_{52}}(u) = \max\{\mu_{C_{21}}(u),\ \mu_{C_{51}}(u),\ \mu_{C_{52}}(u)\}.$$

Table 5 presents predictions reflecting the consequences in the groups of 1st order internal connections. In the PE of FS notation, the prognostic model of the fuzzy time series DJIA, built on the basis of the analysis of internal connections of the 1st order, induces defuzzified outputs (forecasts) on the scale of the segment [0, 1], which are summarized in Table 6. Geometric interpretation of the prognostic model of the 1st order order in comparison with the DJIA time series in the PE of FS notation is shown in Fig. 4.

Table 5. Fuzzy conclusions as consequences in groups G_k (k=1÷144)

Fuzzy forecast	Membership function values							PE of FS
	0	0.1	0.2	0.3	…	0.9	1	
F_1	0.9138	0.2051	0.2051	0.2051	…	0.0412	0.0412	0.0767
F_2	0.5875	0.4978	0.4978	0.4978	…	0.1492	0.0031	0.2758
F_3	0.9138	0.0412	0.0412	0.0412	…	0.0412	0.0412	0.0226
F_4	0.7193	0.3392	0.3392	0.3392	…	0.0496	0.0396	0.1491
…………								
F_{141}	0.0147	0.0147	0.0147	0.0147	…	0.0147	0.9423	0.9922
F_{142}	0.0369	0.0469	0.0532	0.0532	…	0.0532	0.9423	0.9730
F_{143}	0.0422	0.0422	0.0625	0.0737	…	0.0737	0.9800	0.9662
F_{144}	0.0337	0.0337	0.0625	0.0737	…	0.0737	0.9423	0.9657

[14] Заде Л.А. Основы нового подхода к анализу сложных систем и процессов принятия решений // Математика сегодня. – М.: Знание, 1974. – С. 5–49.

Table 6. DJIA time series forecasting model

Date	Fuzzy analogue of an index		Cr.	Group of 1st order connections	Model output (forecast)		Detailing fuzzy output
	Smb.	PE			Smb.	TO	
15.06.2018	A_1	0.6062	C_{55}	$C_{55} \Rightarrow C_{51}, C_{54}, C_{59}$	–	–	
18.06.2018	A_2	0.5939	C_{51}	$C_{51} \Rightarrow C_{40}, C_{41}$	F_{55}	0.6203	$C_{51} \cup C_{54} \cup C_{59}$
19.06.2018	A_3	0.5330	C_{41}	$C_{41} \Rightarrow C_{28}, C_{39}, C_{49}$	F_{51}	0.5318	$C_{40} \cup C_{41}$
20.06.2018	A_4	0.5219	C_{39}	$C_{39} \Rightarrow C_{32}$	F_{41}	0.4557	$C_{28} \cup C_{39} \cup C_{49}$
......							
07.10.2019	A_{330}	0.9513	C_{131}	$C_{131} \Rightarrow C_{81}, C_{117}, C_{127}, C_{129}$	F_{134}	0.8944	$C_{111} \cup C_{131} \cup C_{132}$
08.10.2019	A_{331}	0.8959	C_{117}	$C_{117} \Rightarrow C_{125}$	F_{131}	0.8127	$C_{81} \cup C_{117} \cup C_{127} \cup C_{129}$
09.10.2019	A_{332}	0.9321	C_{125}	$C_{125} \Rightarrow C_{116}, C_{131}$	F_{117}	0.9320	C_{125}
10.10.2019	A_{333}	0.9534	C_{131}	$C_{131} \Rightarrow C_{81}, C_{117}, C_{127}, C_{129}$	F_{125}	0.9055	$C_{116} \cup C_{131}$
					MSE	0.0020	
					MAPE	4.4879	
					MPE	-0.4391	

Table 6 shows the values of the well-known statistical evaluation criteria: MSE (Mean Squared Error), MAPE (Mean Absolute Percentage Error) and MPE (Mean Percentage Error)[15], which reflect the adequacy of the proposed predictive model. MSE is most often used to select the optimal forecasting model and indicates possible significant errors. In the case under consideration, MSE=0.0020 emphasizes the too low error in the proposed forecasting method. MAPE demonstrates how significant the forecast deviation is relative to the corresponding current value of the time series. MPE is a more informative criterion for assessing the adequacy of a forecasting model. It determines the so-called **"bias"** of the proposed forecast, that is, its constant underestimation or overestimation. In this case, MPE = –0.4391% reflects a slight *bias* in the model, because does not exceed the regulatory threshold on the left of 5%

[15] Lin L., Hedayat A.S., and W. Wu, Statistical Tools for Measuring Agreement, New York: Springer, 2012. – 173 p.

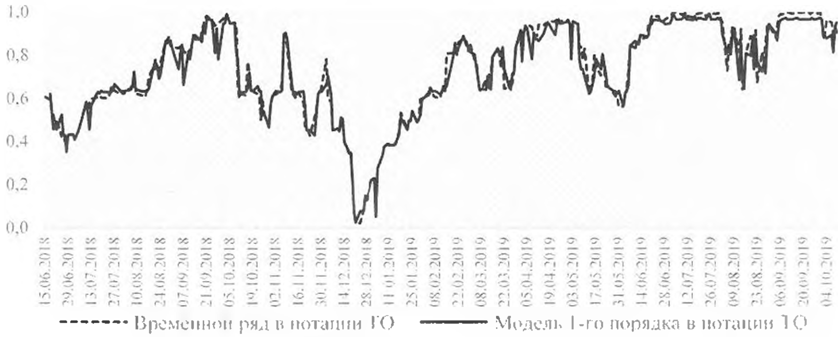

Fig. 4. 1st order prognostic model in PE of FS notation

To finally solve the problem, the actual and predicted outputs of the constructed model must be displayed in nominal values. For this purpose, a three-layer neural network of the proposed topology is used in the form of an effective compiler of rules for predicting fuzzy trends, expressed in the form of defuzzified values of fuzzy interpretations of historical data. To build and adapt an approximation neural network, we will select as a basis a set of training pairs $\{(A_t^{\mathrm{def}}, x_t)\}_{t=1}^{136}$, where x_t – historical DJIA indicator at a point in time t; A_t^{def} – defuzzified value (or point estimate) of a fuzzy set A_t, reflecting indicator x_t. After performing the training, testing and validation processes, the neural network approximates a continuous function $x_t=f(A_t^{\mathrm{def}})$, presented as Table 7.

After approximating the function $x_t=f(A_t^{\mathrm{def}})$ the trained neural network induces at its output nominal forecast values for the DJIA index time series, which correspond to the corresponding defuzzified values of the fuzzy outputs presented in Table 7. The resulting forecasts are summarized in Table 8, and the fuzzy time series model itself is interpreted in Fig. 5 against the background of the original DJIA series.

Table 7. Tabular representation of a function $x_t = f(A_t^{\text{def}})$

t	A_t^{def}	x_t	t	A_t^{def}	x_t	t	A_t^{def}	x_t	t	A_t^{def}	x_t
1	0.6062	25090.5	35	0.6392	25462.6	69	0.9835	26743.5	132	0.0226	22445.4
2	0.5939	24987.5	36	0.6467	25502.2	70	0.9615	26562.1	133	0.0216	21792.2
3	0.5330	24700.2	37	0.6736	25628.9	71	0.9529	26492.2	134	0.0821	22878.5
4	0.5219	24657.8	38	0.6614	25583.8	72	0.9384	26385.3	135	0.1491	23138.8
...	...	...	...	...	...	...	...	...	136	0.1257	23062.4

Table 8. Forecasting DJIA time series

№	Дата	DJIA	Прогноз	№	Дата	DJIA	Прогноз
1	15.06.2018	25090.5		323	26.09.2019	26891.1	26598
2	18.06.2018	24987.5	25299	324	27.09.2019	26820.3	26598
3	19.06.2018	24700.2	24694	325	30.09.2019	26916.8	26661
4	20.06.2018	24657.8	24400	326	01.10.2019	26573.0	26598
5	21.06.2018	24461.7	24463	327	02.10.2019	26078.6	26157
6	22.06.2018	24580.9	24396	328	03.10.2019	26201.0	26129
7	25.06.2018	24252.8	24664	329	04.10.2019	26573.7	26093
8	26.06.2018	24283.1	24285	330	07.10.2019	26478.0	26157
9	27.06.2018	24117.6	24291	331	08.10.2019	26164.0	25895
10	28.06.2018	24216.1	23537	332	09.10.2019	26346.0	26342
...	...	...	...	333	10.10.2019	26496.7	26204
						MSE	72100.5
						MAPE	0.6830
						MPE	-0.2530

Fig. 5. Predictive model in nominal values

Table 8 shows the values of the indicators MSE = 72100.5, MAPE = 0.6830 and MPE = -0.2530, which reflect the adequacy of the proposed forecast model in the nominal values of the DJIA index. In particular, the MSE value indicates a fairly large error in prediction, which we attribute to the insufficiently satisfactory quality of neural network training (ε = 19.4803). At the same time, MAPE demonstrates a fairly acceptable value of the forecast error in comparison with the current values of the DJIA index time series. At the same time, MPE, as a more informative evaluation criterion, reflects a slight "bias" of the prognostic model, not exceeding the normative 5% threshold on the left. Comparing the quality of the two proposed models, it is easy to notice that the predictive model of the time series in nominal values of the DJIA index is significantly inferior to the predictive model in terms of point estimates of the corresponding fuzzy sets. Applying the forecast model in terms of point estimates, we obtain the number A_t^{def} = 0.8127, what is the PE of a fuzzy forecast $F_{131}=C_{81}\cup C_{117}\cup C_{127}\cup C_{129}$. The neural network interprets this prediction as a nominal value as 26204.

The third chapter proposes a method for compiling knowledge about internal fuzzy patterns using trivial (with one linear hidden layer) and multilayer neural networks. The analysis and forecasting of a volatile time series is carried out in the "deep learning" paradigm, i.e. using neural network models[16]. In order to forecast time series, various neural network topologies and their comparative analysis are considered using the example of forecasting the volatile time series of the DJIA index using average statistical evaluation criteria.

In Fig. 6 figuratively shows a neural network consisting of input, hidden and output layers. By activating the *newff* function, the network is formed in MATLAB notation, implying 10 inputs (10 delayed DJIA index values) and one output as the subsequent DJIA[17] index value.

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[16] Круглов В.В. Искусственные нейронные сети. Теория и практика. М.: Горячая линия – Телеком, 2002. 382 с.

[17] Медведев В.С. Нейронные сети. MATLAB 6. Текст. / Медведев В.С., Потемкин В.Г. М.: ДИАЛОГ–МИФИ, 2002.

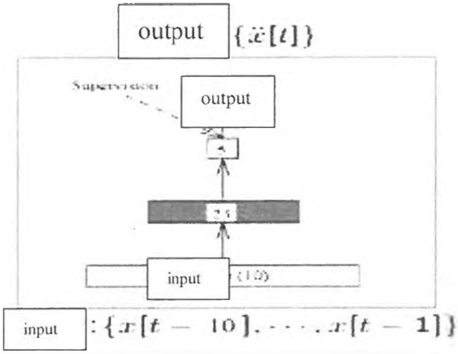

Fig. 6. Figurative representation of a three-layer neural network

Based on the historical indicators of the DJIA index time series, a set of training pairs was constructed in the form:

$$\{x(t\text{-}10), x(t\text{-}9), \ldots, x(t\text{-}1)\} \rightarrow x(t), t = 11 \div 333,$$

which are summarized in Table 9.

Table 9. Set of training pairs for neural network design

t	Inputs of the neural network model							Desired exit
	$x(t\text{-}10)$	$x(t\text{-}9)$	$x(t\text{-}8)$	...	$x(t\text{-}3)$	$x(t\text{-}2)$	$x(t\text{-}1)$	$x(t)$
11	25090.5	24987.5	24700.2	...	24283.1	24117.6	24216.1	24271.4
12	24987.5	24700.2	24657.8	...	24117.6	24216.1	24271.4	24307.2
13	24700.2	24657.8	24461.7	...	24216.1	24271.4	24307.2	24174.8
14	24657.8	24461.7	24580.9	...	24271.4	24307.2	24174.8	24356.7
15	24461.7	24580.9	24252.8	...	24307.2	24174.8	24356.7	24456.5
....								
329	26935.1	26950.0	26807.8	...	26573.0	26078.6	26201.0	26573.7
330	26950.0	26807.8	26970.7	...	26078.6	26201.0	26573.7	26478.0
331	26807.8	26970.7	26891.1	...	26201.0	26573.7	26478.0	26164.0
332	26970.7	26891.1	26820.3	...	26573.7	26478.0	26164.0	26346.0
333	26891.1	26820.3	26916.8	...	26478.0	26164.0	26346.0	26496.7

After training, testing and validation, the initiated neural network FFNN restored the time series of the DJIA index in a fairly acceptable form, presented in Fig. 7, a. Data $x(t)$ The DJIA index time series reconstructed by a neural network are summarized in Table 10, and the dependence of the error when reconstructing a time series by a neural network on the number of iterations is presented in Fig. 7, b.

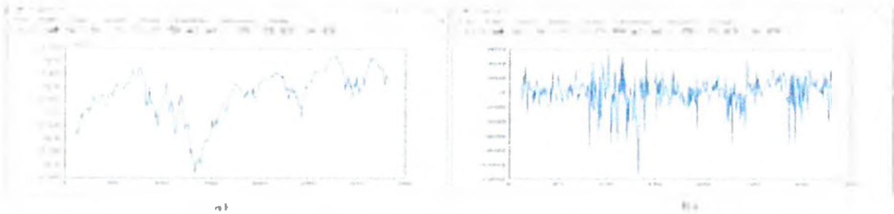

Fig. 7. Neural forecasting of the DJIA index time series: a) restoration of the time series, b) dependence of the error on the number of iterations

Table 10. Neural network modeling of the DJIA index time series

Date	FFNN	Date	FFNN	Date	FFNN	Date	FFNN
15.06.18	×	23.11.18	24328	08.05.19	25979	18.09.19	27146
18.06.18	×	26.11.18	24175	09.05.19	26025	19.09.19	27043
19.06.18	×	27.11.18	24758	10.05.19	25838	20.09.19	26962
20.06.18	×	28.11.18	24851	13.05.19	26012	23.09.19	26844
21.06.18	×	29.11.18	25500	14.05.19	25482	24.09.19	26915
22.06.18	×	30.11.18	25333	15.05.19	25702	25.09.19	26688
25.06.18	×	03.12.18	25611	16.05.19	25681	26.09.19	26819
26.06.18	×	04.12.18	25706	17.05.19	26037	27.09.19	26696
27.06.18	×	06.12.18	25026	20.05.19	25801	30.09.19	26693
28.06.18	×	07.12.18	24805	21.05.19	25864	01.10.19	26770
29.06.18	24153	10.12.18	24339	22.05.19	25784	02.10.19	26485
02.07.18	24222	11.12.18	24394	23.05.19	25892	03.10.19	26042
03.07.18	24313	12.12.18	24489	24.05.19	25541	04.10.19	26238
05.07.18	24109	13.12.18	24648	28.05.19	25650	07.10.19	26508
06.07.18	24352	14.12.18	24514	29.05.19	25345	08.10.19	26410
09.07.18	24414	17.12.18	23957	30.05.19	25207	09.10.19	26250
...	...	...	...	...	...	10.10.19	26365

To assess the adequacy and compare the neural network approaches discussed above for forecasting volatile time series based on the selected DJIA index time series, we will use the statistical evaluation criteria MSE, MAPE and MPE[18][19]. The comparison results using the

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[18] Lewis K.D.: Methods for forecasting economic indicators. Finance and statistics, Moscow, 1986.

[19] Lin L., Hedayat A.S., and W. Wu. Statistical Tools for Measuring Agreement. New York: Springer, 2012. – 173 p.

given evaluation criteria are shown in the following Table 11, and the geometric interpretation of the results is presented in Fig. 8.

Table 11. Results of forecasting the DJIA index time series

Date	DJIA index	Neural networks		Date	DJIA Index	Neural networks	
		LNN	FFNN			LNN	FFNN
15.06.18	25090.5	17389	×	18.09.19	27147.1	26164	27146
18.06.18	24987.5	25226	×	19.09.19	27094.8	26183	27043
19.06.18	24700.2	24955	×	20.09.19	26935.1	26171	26962
20.06.18	24657.8	24910	×	23.09.19	26950.0	26120	26844
21.06.18	24461.7	24680	×	24.09.19	26807.8	26132	26915
22.06.18	24580.9	24784	×	25.09.19	26970.7	26093	26688
25.06.18	24252.8	24465	×	26.09.19	26891.1	26149	26819
26.06.18	24283.1	24554	×	27.09.19	26820.3	26122	26696
27.06.18	24117.6	24367	×	30.09.19	26916.8	26100	26693
28.06.18	24216.1	24512	×	01.10.19	26573.0	26128	26770
29.06.18	24271.4	25277	24153	02.10.19	26078.6	26026	26485
02.07.18	24307.2	25283	24222	03.10.19	26201.0	25868	26042
03.07.18	24174.8	25294	24313	04.10.19	26573.7	25909	26238
05.07.18	24356.7	25243	24109	07.10.19	26478.0	26025	26508
06.07.18	24456.5	25302	24352	08.10.19	26164.0	25990	26410
09.07.18	24776.6	25329	24414	09.10.19	26346.0	25899	26250
11.07.18	24700.5	25460	24987	10.10.19	26496.7	25957	26365
12.07.18	24924.9	25389	24754		MSE	616507	54170
13.07.18	25019.4	25459	24906		MAPE	2.0913	0.6608
...	...	...	...		MPE	0.0970	0.0138

Fig. 8. Geometric interpretation of DJIA time series models

In the example under consideration, the indicators MSE_{LNN} = 616507 and MSE_{FFNN} = 54170 highlight the excessively high errors when using both types of neural networks for time series forecasting. However, the error from using a three-layer neural network is more than an order of magnitude smaller than the standard deviation of the outputs of the DJIA index time series model built using the LNN network.

Indicators $MAPE_{LNN}$ = 2.0913 and $MAPE_{FFNN}$ = 0.6608 demonstrate an insignificant deviation of the forecast relative to the corresponding current value of the time series. Indicators MPE_{LNN} = 0.0970 and MPE_{FFNN} = 0.0138 demonstrate minor “biases” of the proposed models. Both indicators do not exceed the regulatory threshold on the right of 5%. At the same time, the value of this indicator for a model using a three-layer neural network is significantly lower, which also indicates its preference in terms of the accuracy of forecasting the DJIA index time series.

MAIN RESULTS

The main scientific results submitted for defense are formulated in the form of the following statements:

- The time series of fuzzy patterns is an effective indicator of the volatile development of the modeled process on the stock exchange, which is described in qualitative categories using terms of linguistic variables.
- The key operations for processing fuzzy patterns are algorithms for processing internal fuzzy connections, namely, the technique of restoring a fuzzy time series from the original time series and the inverse operation of compiling the time series in nominal values, as a reflection of the identified fuzzy trends.
- The three-layer neural network of the proposed topology is an effective compiler of rules for predicting fuzzy trends, expressed in the form of defuzzified values of fuzzy interpretations of historical data.
- The developed predictive model based on time series of fuzzy trends makes it possible to predict a volatile time series for a short-term period.

- The developed neural network model of the selected topology makes it possible to predict a volatile time series for a short-term period.

The main results of the dissertation work were published in the following scientific articles:

1. Mardanov M. Dzh., Rzayev R.R., Alizade P.E. Ob odnom podkhode k fazzifikatsii dannykh na primere vremennogo ryada indeksa Dou-Dzhonsa. Matematicheskiye Mashiny i Sistemy, Institut Problem Matematicheskikh Mashin i Sistem, Kiyev, 2020, №2, str. 3–13 (Higher Attestation Commission of Ukraine).
2. Mardanov M.J., Rzayev R.R., Alizada P.E. About one approach to modeling and forecasting the fuzzy time series. Proceeding of the 7th International Conference on Control and Optimization with Industrial Applications, COIA – 2020, Vol. 1, pp. 269-271, Baku, 26–28 August 2020.
3. Alizada P.E., Mehdiyev T.Z. One approach to volatile time series forecasting. Advances in Intelligent Systems and Computing, Vol. 1306, pp. 433–441, 2021 (indexed by WoS and Scopus).
4. Alizada P.E. Conversion of volatile time series into a fuzzy time series by the example of the Dow Jones index dynamics. Lecture Notes in Networks and Systems, Vol. 362, pp.662–670, 2022 (indexed by WoS and Scopus).
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Personal contribution of the applicant in works published in co-authorship:

[1] Defining the universe as the coverage of historical data of the Dow Jones time series, the formation of a set of qualitative evaluation criteria and the establishment of internal 1st order relationships, reflecting cause-effect relationships between historical data.
[2] Carrying out calculations related to defuzzification of fuzzy outputs of a predictive model of the Dow Jones index time series.
[3] Fuzzification of historical data, formation of an optimal set of qualitative evaluation criteria and construction of a 1st order predictive model for the time series of the Dow Jones index.
[4] Analysis of historical data for volatility, fuzzification of historical data using the Gaussian membership function, construction of a predictive model and its assessment of adequacy.
[5] Carrying out simulations in the Neural Networks Toolbox software shell of the MATLAB application package for neural approximation of dependencies within the volatile time series of the Dow Jones index.
[6] Adaptation of a fuzzy inference system for fuzzification of historical data using Gaussian-type membership functions.
[7] Forecasting the Dow Jones index time series in terms of point estimates of the fuzzy outputs of the predictive model, as well as in nominal values of the Dow Jones index through neural

approximation of a continuous function of the defuzzified values of the model's fuzzy outputs.

[8] Development of rules for fuzzification of historical data and defuzzification of fuzzy time series forecasts.

[9] Construction of a predictive model for forecasting the time series of the Dow Jones index based on identifying internal connections of the 1st order.

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